

Ductile crack growth based on damage criterion: Experimental and numerical studies

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Abstract

The continuum mechanical simulation of the microstructural damage process is important in the study of ductile fracture mechanics. In this paper, the continuum damage mechanics framework for ductile materials developed by Lemaitre has been validated experimentally and numerically for A533-B1 alloy steel under triaxial stress conditions. An experimental procedure to identify the damage parameters was established and the experimental calibrated damage parameters were then used in a finite element model. A fully coupled constitutive elastic–plastic–damage model has been developed and implemented inside the ABAQUS implicit FEA code. The model is based on a simplified Lemaitre ductile damage model whose return mapping stage requires the solution of only one scalar non-linear equation. A local crack growth criterion based on the critical damage parameter was proposed; the validity of this criterion was examined by comparing the simulation with the experimental results on standard three point bending (3PB) test. The critical load at crack growth initiation and the fracture toughness, J_{IC} , has also been predicted from the simulation. These numerically predicted values compared favourably with those obtained from experiments.

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1. Introduction

Fracture of engineering components is often preceded by considerable changes in the microstructure of its material. It is now well known that the applicability of the J -based single parameter fracture mechanics is restricted to high constraint crack geometries and materials of low ductility. For highly ductile materials, the fracture process zone

is large and the crack tip fields are no longer adequately characterized by the J -integral alone. Micro-mechanically based damage models, which simulate the physical process of void nucleation, growth and coalescence using continuum mechanics equations, are among the most promising methods to investigate fracture behaviour in ductile materials (Lemaitre and Chaboche, 1990). The advantage of a micromechanical damage model, compared with conventional fracture mechanics, is that, in general, the model parameters are only material dependent, and not geometry dependent. The damage model

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allows damage assessments at every point of a structure for any geometry or loading, as long as the damage mechanisms and stress/strain fields are known (Lemaitre, 1992).

The development of microstructural damage in engineering materials can be effectively modelled using continuum damage mechanics (CDM) (Lemaitre, 1992). CDM introduces a field variable to represent the damage in a continuum sense. In this paper, this concept has later been used to model the initiation and growth of cracks.

Accurate failure predictions can only be obtained if microstructural damage is taken into account in the fracture modelling. This requirement has led to the development of the so-called local or continuum approaches to fracture, in which fracture is regarded as the ultimate consequence of the material degradation process (Lemaitre and Chaboche, 1990; Lemaitre, 1985). In these methods, the degradation is often modelled using continuum damage mechanics (Lemaitre, 1985; Rice and Tracey, 1969; Needleman and Tvergaard, 1984). Continuum damage theory introduces a set of field variables (damage variables) which explicitly describe the local loss of material integrity. A crack at macro scale size is represented by that part of the material domain in which the damage has become critical, i.e., where the material cannot sustain stress anymore. Redistribution of stresses results in the concentration of deformation and damage growth in a relatively small region in front of the crack tip. It is the growth of damage in this process zone that determines in which direction and at what rate the crack will propagate. Crack initiation and growth thus follow naturally from the standard continuum mechanics theory, instead of from separate fracture criteria.

In the CDM model, the effect of void growth on material behaviour is incorporated by introducing an (internal) damage variable in the constitutive relation. The effect of void nucleation can also be included by modifying the damage growth law appropriately. But as far as void coalescence is concerned, it has to be incorporated as an additional condition (in terms of continuum parameters) based on a suitable micro model. Dhar et al. (1996) combined Lemaitre's (1985) CDM model and Thomason's (1990) void coalescence condition in a large deformation finite element analysis of different case studies to show that the critical value of damage variable is a geometry independent material parameter and can be used for predicting micro-crack initiation. The literature contains quite a few local

criteria for crack growth initiation. Ritchie et al. (1973) proposed a critical stress criterion for cleavage fracture. For mild steel at low temperature, they found that predictions from this criterion agreed well with experimental results. For ductile fracture, Rice and Johnson (1970) expressed the critical crack tip opening displacement in terms of an inter-particle distance. For structural steels, the predictions from the RJ model match well with experimental results. Ritchie et al. (1979) used a strain based criterion to predict the fracture toughness of A533-B and A302-B alloy steels.

In this paper, an experimental procedure to identify the damage parameters has been established for A533-B1 alloy steel under triaxial stress conditions. The CDM model has been used to simulate the ductile damage behaviour of flat rectangular notched bar specimen tests. The model is validated by modelling the three point bending fracture specimen (3PB). The comparisons indicate good agreement between the simulated and experimental results. The critical load for crack growth initiation and the fracture toughness are predicted by using the proposed criterion for crack growth initiation. Comparison of numerical and experimental results shows that the proposed criterion reasonably predicts the crack growth initiation in Mode-I ductile fracture.

The present paper has been organized as follows. In Section 2, the CDM model formulation is briefly reviewed and a local criterion for crack growth initiation and propagation is proposed. In Section 3, the experimental procedure used to calibrate the damage parameters in A533-B1 steel is explained. The CDM model has been used to simulate the ductile damage behaviour of flat rectangular notched bar specimens. Comparisons between the simulated and experimental results are also presented in this section. In Section 4, to validate our present criterion, experiments have been conducted on standard 3PB specimens. By using the damage parameters identified in Section 2, the CDM model with damage propagation criterion are applied to crack growth in 3PB tests. The FEA simulation compared with the experimental results. In Section 5, the main highlights of the work are summarized.

2. Ductile fracture model

2.1. Constitutive law

In the Lemaitre damage model (Lemaitre, 1985), the damage variable is defined as the net area of a

unit surface cut by a given plane corrected for the presence of existing cracks and cavities. By assuming homogeneous distribution of microvoids and the hypothesis of strain equivalence, which states that the strain behaviour of a damaged material is represented by constitutive equations of the virgin material (without damage) in the potential of which the stress is simply replaced by the effective stress (Lemaitre, 1992), the effective stress tensor, $\tilde{\sigma}$, can be represented as

$$\tilde{\sigma} = \frac{\sigma}{1-D} \quad (1)$$

where σ is the stress tensor for the undamaged material. The corresponding effective stress deviator, $\tilde{s}$, is related to the stress deviator, s , by an analogous expression:

$$\tilde{s} = \frac{s}{1-D} \quad (2)$$

Therefore, the evolution equation for internal variables can be derived by assuming the existence of a potential of dissipation, Ψ , as a scalar convex function of the state variables, which is decomposed into plastic, Ψ^p , and damage, Ψ^d , components as

$$\Psi = \Psi^p + \Psi^d = \Phi + \frac{r}{(1-D)(s+1)} \left(\frac{-Y}{r} \right)^{s+1} \quad (3)$$

for a process accounting for isotropic hardening and isotropic damage, in which r and s are material and temperature-dependent properties and Φ and Y are, respectively, the yield function and the damage strain energy release rate, given by

$$\Phi(\sigma, \varepsilon_{eq}^p, D) = \frac{\sigma_{eq}}{(1-D)} - [\sigma_Y^0 + R(\varepsilon_{eq}^p)] \quad (4)$$

and

$$-Y = \frac{\sigma_{eq}^2}{2E(1-D)^2} \left[\frac{2}{3}(1+\nu) + 3(1-2\nu) \left(\frac{p}{\sigma_{eq}} \right)^2 \right] \quad (5)$$

where σ_Y^0 is the initial yield stress, R represents the radial growth of the yield surface, ε_{eq}^p is the equivalent plastic strain, $\varepsilon_{eq}^p = \sqrt{2/3} \|\varepsilon^p\|$, ε^p is the plastic strain tensor, σ_{eq} is the equivalent stress, $\sigma_{eq} = \sqrt{3/2} \|s\|$, p is the hydrostatic stress, $p = (1/3)tr(\sigma)$, E is the Young's modulus, and ν is Poisson's ratio.

By the hypothesis of generalized normality, the plastic flow equation is

$$\dot{\varepsilon}^p = \dot{\gamma} \sqrt{\frac{3}{2}} \frac{s}{\|\varepsilon^p\|} = \dot{\gamma} \frac{3}{2} \frac{s}{\sigma_{eq}} \quad (6)$$

and the evolution law of the internal variables are

$$\begin{aligned} \dot{\varepsilon}_{eq}^p &= -\dot{\gamma} \frac{\partial \Psi}{\partial R} = \dot{\gamma} \\ \dot{D} &= -\dot{\gamma} \frac{\partial \Psi}{\partial Y} = \dot{\gamma} \frac{1}{1-D} \left(\frac{-Y}{r} \right)^s \end{aligned} \quad (7)$$

where $\dot{\gamma}$ is the plastic consistency parameter, which is subject to the so-called Kuhn-Tucker conditions (Simo and Hughes, 1998) for loading and unloading as

$$\dot{\gamma} \geq 0, \quad \Phi \leq 0, \quad \dot{\gamma} \Phi = 0 \quad (8)$$

The evolution problem is highly non-linear thereby requiring an efficient integration algorithm, as discussed in Section 2.2.

2.2. Numerical integration for damaged solids

An algorithm for the numerical integration of the elastic–plastic–damage constitutive equations will be presented in this section. Algorithms based on the operator split concept, resulting in the standard elastic predictor/plastic corrector format, are widely used in computational plasticity (Simo and Hughes, 1998). Let us consider what happens to a typical Gauss point of the finite element mesh within a (pseudo-) time interval $[t_n, t_{n+1}]$.

Given the incremental strain:

$$\Delta \varepsilon = \varepsilon_{n+1} - \varepsilon_n \quad (9)$$

and the values σ_n , ε_n^p , $\varepsilon_{eq,n}^p$ and D_n at t_n , the numerical integration algorithm should obtain the updated values at the end of the interval, σ_{n+1} , ε_{n+1}^p , $\varepsilon_{eq,n+1}^p$ and D_{n+1} such that they become consistent with the constitutive equations of the model.

The elastic predictor/return mapping algorithm for the elastic–plastic–damage model can be summarised as following:

Step 1. Elastic predictor: Given the incremental strain, $\Delta \varepsilon$, and the state variables at t_n , compute elastic trial stresses:

$$\begin{aligned} \varepsilon_{n+1}^{trial} &= \varepsilon_n^e + \Delta \varepsilon, \quad \varepsilon_{eq,n+1}^{trial} = \varepsilon_{eq,n}^p, \\ s^{trial} &= s_n + 2G\Delta \varepsilon, \\ p^{trial} &= p_n + K\Delta v \quad D_{n+1} = D_n. \end{aligned} \quad (10)$$

In the above equations G and K are, respectively, the shear and bulk moduli, e and v denote, respectively, the strain deviator and the volumetric strain.

Step 2. Plastic consistency check: First compute

$$\Phi^{\text{trial}} = \sqrt{\frac{3}{2}} \frac{\|s^{\text{trial}}\|}{(1 - D_n)} - [\sigma_Y^0 + R(\varepsilon_{\text{eq},n}^p)] \quad (11)$$

and then check:

IF $\Phi^{\text{trial}} \leq \varepsilon_{\text{tol}}$ **THEN** (elastic state)

Update $(\cdot)_{n+1} = (\cdot)^{\text{trial}}$ **RETURN**

ELSE (plastic state)

Step 3. Return mapping: Find $\Delta\gamma$ such that

$$D(\Delta\gamma) - D_n - \frac{\Delta\gamma}{1 - D(\Delta\gamma)} \left(\frac{-Y(\Delta\gamma)}{r} \right)^s = 0 \quad (12)$$

Step 4. Update the variable:

$$\begin{aligned} s_{n+1} &= \left(1 - \sqrt{\frac{3}{2}} \frac{2G\Delta\gamma}{\|s^{\text{trial}}\|} \right) s^{\text{trial}}, \quad p_{n+1} = p^{\text{trial}}, \\ \sigma_{n+1} &= s_{n+1} + p_{n+1} \mathbf{I}, \quad \varepsilon_{\text{eq},n+1}^p = \varepsilon_{\text{eq},n}^p + \Delta\gamma, \\ \varepsilon_{n+1}^e &= \frac{1}{2G} s_{n+1} + \frac{1}{3K} p_{n+1} \mathbf{I}, \quad D_{n+1} = D(\Delta\gamma) \end{aligned} \quad (13)$$

ENDIF

RETURN

The full details of the above algorithm can be found elsewhere Mashayekhi et al. (2005).

2.3. Damage growth model

The fracture of ductile materials is mainly due to growth and coalescence of microscopic voids existing within the material. In the numerical simulation, when the fracture threshold within an element is reached, that element fractures and a crack occurs. The direction of crack propagation and the crack tip position are then determined by the damage parameter value at each element of the model.

Crack propagation in the structure is often simulated with finite element (FE) analysis based on continuum mechanics. There are four common methods to simulate crack propagation in a finite element model. These are element splitting, nodes separation, decreasing elements stiffness and deleting damaged elements. In this paper, the last technique has been used in order to simulate crack propagation. The displacement control method was adopted for loading the structure. After each increment in FE

analysis, the damage parameter for all the elements in the model was calculated. The initiation of a crack in the structure is assumed to occur at any point when the damage parameter reaches its critical value, D_c , a value at which ductile failure will occur. In our numerical analysis this value at any Gauss point was chosen 0.9. At this stage the damaged elements are removed and the crack will advance.

3. Measurement of damage parameters and numerical validation

3.1. Material specification

Calibration and validation of the continuum damage model was conducted on sample of A533-B1 steel, extracted from a 110 mm width rolled steel block. The chemical composition of the material is given in Table 1. Engineering stress–strain properties of the steel block were obtained using standard smooth round-bar tensile specimens with 25 mm gauge length and 6.2 mm diameter, tested according to ASTM Standard E8 in a servo-hydraulic 100 kN capacity testing machine. The measured yield and ultimate stresses of the material are 430 MPa and 600 MPa, respectively. Other mechanical properties of the material extracted from tests are tabulated in Table 2.

3.2. Identification of the damage parameters

Selection of the damage parameter is one of the most important and most contentious aspects of damage mechanics. Many experimental techniques exist to measure damage or damage related events (Lemaitre, 1992). It is often difficult to perform a quantitative measurement with the majority of these techniques. The simplest method to do this is by the use of one-dimensional damage measurement through tension tests.

The first natural way is that damage is measured from the microstructure, such as measuring of the fraction area of voids at each different plastic strain level (Lemaitre, 1992):

Table 1
Chemical composition of A533-B1 steel

%C	%Mn	%P	%S	%Si	%Cr	%Ni	%Mo
0.19	1.41	0.011	0.011	0.20	0.13	0.69	0.48

The balance is Fe.

Table 2
Identified mechanical and damage parameters for A533-B1 steel

Yield stress σ_Y^0 (MPa)	Ultimate stress σ_{TS} (MPa)	Ultimate strain e_u (%)	Young's Modulus E (GPa)	Poisson's ratio, ν	Hardening coefficient ^a k (MPa)	Hardening power ^a n	Damage parameters	
							r (MPa)	s
430	600	9.3	200	0.3	998	0.18	2.8	1

^a Power-law work hardening is: $\sigma_Y = k\epsilon_p^n$.

$$D = 1 - \frac{A_{\text{eff}}}{A} \quad (14)$$

where A_{eff} is the effective resisting area and A is the overall cross-sectional area.

The second method is that damage is measured through physical parameters, such as electrical potential difference measured at each different plastic strain level:

$$D = 1 - \frac{\bar{V}}{V} \quad (15)$$

In which $\bar{V}$ and V are electrical potential difference for the virgin and the damaged material, respectively.

A straightforward manner to quantify the isotropic elastic damage variable is to measure the decrease of the stiffness of the material. Successive loading and unloading of the material permits to measure different stages of damage which is then computed through:

$$D = 1 - \frac{E_D}{E_0} \quad (16)$$

where E_D is the effective elastic modulus of the damaged material, derived from measurements, and E_0 is the Young's modulus of the virgin material.

Lemaitre (1985) was the first to measure D through the degradation of the elastic modulus. A specimen similar to the one depicted in Fig. 1, called the damage specimen, was loaded in tension. The plastic strain was recorded by local strain gauges fixed at the minimum cross-section. The specimen

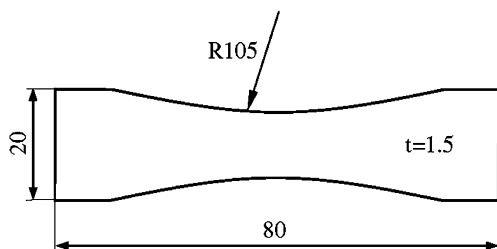


Fig. 1. The usual specimen geometry for measuring the damage parameter, D , [Lemaitre, 1992]. All dimensions in mm.

is then unloaded and the elastic modulus is measured from the slope of the unloading stress–strain curve. After some level of deformation, typically producing strains less than 0.10, the strain gauges fail and they are replaced by a new set. The specimen is further loaded to increase plastic strain and then unloaded to obtain the current elastic modulus. This process is repeated until a visual crack is detected. For ductile metals with failure strains of around unity, at least ten pairs of strain gauges were used.

In this work, in order to evaluate the efficiency of the CDM model in describing damage evolution under stress triaxiality, flat rectangular notched bar specimens were tested. The experimental testing procedure is subdivided into the following steps:

- (i) Two flat rectangular notched bar specimens were prepared. The block material is machined into rectangular specimens of length 78 mm and minimum cross-sectional area of 8×8 mm. The opposite faces of the specimen were polished parallel to each other. The specimen geometry and dimensions are shown in Fig. 2a.

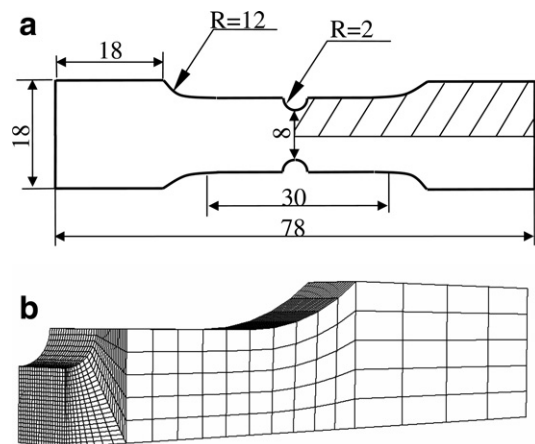


Fig. 2. Flat rectangular notched bar specimen. (a) Geometry and dimension (all in mm), (b) FE model of 1/8 of the specimen using symmetry condition.

- (ii) Strain gauges with size of 10×8 mm were attached to the minimum cross-section of the specimen using epoxy glue; the strain gauge resistance was 120Ω and its nominal deformation limit is $\pm 20.0\%$. A sample flat rectangular notched bar specimen with the attached strain gauge is shown in Fig. 3.
- (iii) A monotonic load was applied to the specimen under displacement control. The rate of displacement has been fixed at 0.5 mm/min on a servo-hydraulic Instron testing machine, with a load capacity of 100 kN.
- (iv) The tests were carried out with a series of partial unloading–reloading to measure the change in the elastic slope while the strain increased.
- (v) The tests were continued until the deformation limit for the strain gauge is reached. Then the specimen was unloaded and removed from the testing machine and a new strain gauge was attached, as required.

The test results are presented in Table 3. The coefficients s and r are calibrated from these results. According to experimental results reported by Lemaitre (1992), the s parameter for this material was assumed to be 1. For the one-dimensional case, Eq. (5) changes to:

$$-Y = \frac{\sigma^2 \times 1}{2E(1-D)^2} \quad (17)$$

Then, we can substitute Y into Eq. (7) and determine $dD/d\varepsilon^p$:

Table 3

Measured effective Young's modulus, plastic strain and damage parameter in flat rectangular notched bar specimen during deformation

E_D (GPa)	ε^p (mm)	D
187.4	0.0445	0.1309
168.4	0.0723	0.1720
158.3	0.0956	0.2042
153.6	0.1095	0.2225
149.4	0.1244	0.2413
145.6	0.1396	0.2596
145.0	0.1425	0.2630
140.9	0.1623	0.2853
136.3	0.1885	0.3126
131.3	0.2231	0.3447
122.4	0.3064	0.4038
118.1	0.3613	0.4286

$$\frac{dD}{d\varepsilon^p} = \frac{\sigma^2}{2Er(1-D)^2} \quad (18)$$

$dD/d\varepsilon^p$ can be calculated from the slope of the line fitted to D_i and ε_i^p data. The parameter r , can then be obtained from Eq. (18):

$$r = \frac{\sigma^2}{2E(1-D)^2 \frac{dD}{d\varepsilon^p}} \quad (19)$$

Up to 12 data points were considered in order to calculate an accurate r .

A sample load versus displacement diagram measured on a flat rectangular notched bar specimen is shown in Fig. 4. For this specimen, three sets of strain gauges were used to measure the deformation of the specimen until the sudden failure of the last

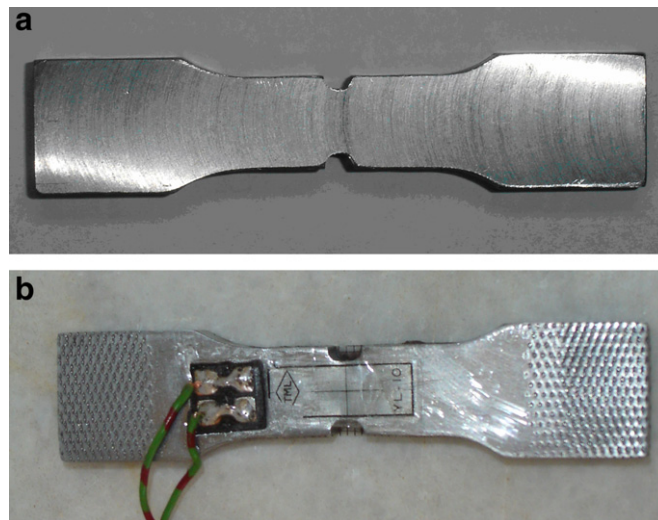


Fig. 3. Specimen used for the calibration of damage parameters (a) specimen before loading (b) specimen with attached strain gauge.

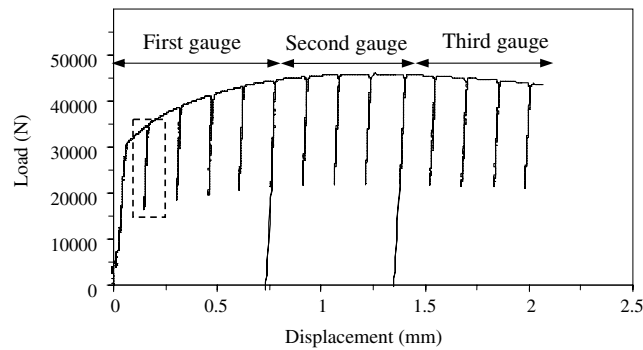


Fig. 4. Experimental load–displacement at the strain gauge position in flat rectangular notched bar specimen.

strain gauge. The loading–unloading stages used for damage measurement are clearly visible. Part of an elastic unloading–reloading ramp is enlarged in Fig. 5 showing a limited hysteresis loop. According to the suggestions made by Lemaitre (1992), the Young's modulus was measured during the unloading ramp. However, the uncertainty in the estimation of the Young's modulus during either the unloading or the elastic reloading is less than 2%. The calibrated damage parameters extracted from the tests results are summarised in Table 2.

3.3. Finite element model of flat rectangular notched bar

In reality, the state of stress in a flat rectangular notched bar specimen geometry does not completely follow fully plane stress or plane strain conditions. An accurate study of the evolution of stress triaxiality with plastic strain across the minimum section requires a three-dimensional finite element simulation. Simulations have been carried out with ABAQUS Standard code and a user's subroutine

UMAT. Due to symmetry, only one fourth of the specimen has been modelled by using eight-noded, isoparametric, hexahedral elements (Fig. 2b). In order to have a detailed map of stress and damage, the minimum section was meshed with evenly shaped bricks with a side length of 0.2 mm. The predicted load–displacement results from FEA are compared with the experimental values in Fig. 6. The FEA results follow very closely the experimental values. The maximum difference was found to be less than $\pm 10\%$.

In the next step, the damage evolution across the minimum section was analysed in order to determine the location of first ductile failure. The contour plots of damage evolution are shown in Figs. 7a–c. These results shows that ductile failure does not initiate at the centre of the minimum section as it would be expected. Here, the competition of stress triaxiality and plastic strain accumulation, controlled by the notch effect, determines the failure initiation site to become near the notch root. From there, ductile fracture spreads across the minimum section. The failure of the elements along the minimum section

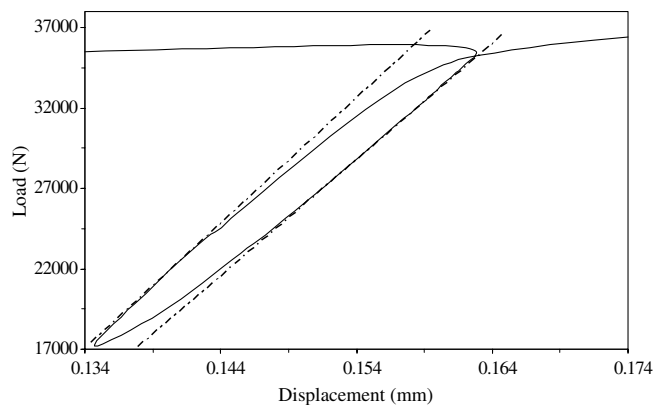


Fig. 5. A stage of elastic unloading–reloading for measurement of elastic moduli.

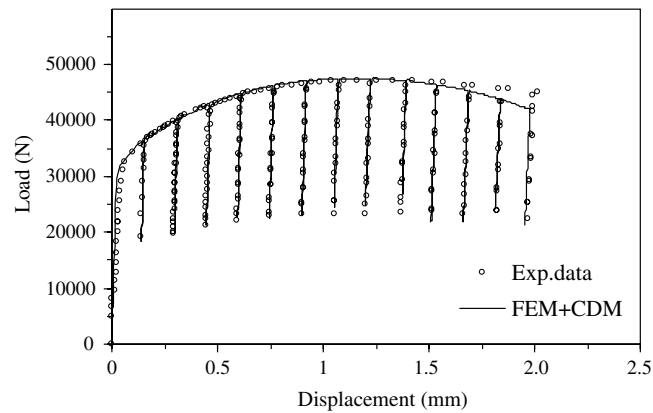


Fig. 6. Comparison of experimental and FE simulation of load-displacement at the strain gauge position in flat rectangular notched bar specimen.

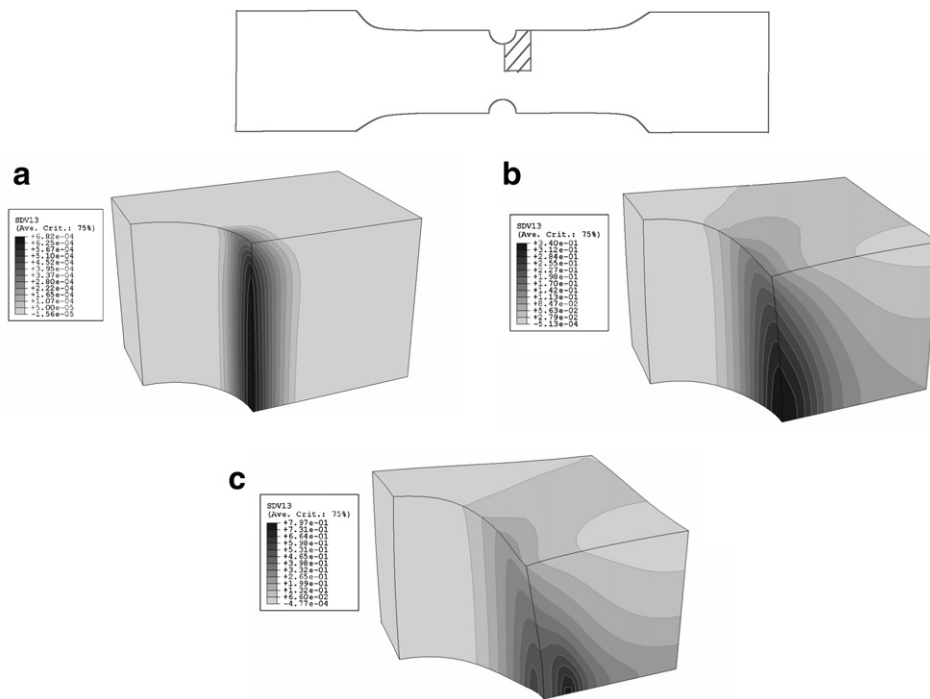


Fig. 7. Damage evolution across the minimum section (notch). (a) At applied displacement of 0.04 mm, (b) at applied displacement of 1 mm, (c) at applied displacement of 1.2 mm.

occurs in a few load increments. Once a ductile crack initiated by the failure of few elements, catastrophic failure immediately follows it.

4. Continuum approach to fracture

After a certain amount of loading which results in some damage growth, three regions can generally be distinguished in the material domain S_0 (Fig. 8).

In region S_0 , the damage variable has its initial value ($D = 0$) and the material properties are those of the virgin material. In the second region, S_d , some development of damage has occurred, but the damage is not yet critical, i.e., $0 < D < 1$. The limiting value $D = 1$ has been reached in the third region, i.e., S_c . The mechanical integrity and strength of the material have been completely lost in this region. The completely damaged region, S_c ,

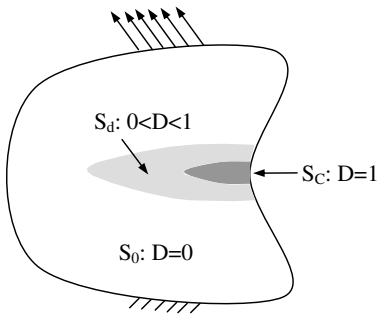


Fig. 8. Damage distribution in a solid.

is the continuum damage representation of a crack. It is important to realize that the complete loss of strength in S_c , implies that in this region stresses are identically zero for any arbitrary deformation fields.

During the initiation stage, the damage variable satisfies $D < 1$ everywhere in the domain S and no cracks are therefore present. Cracks initiate at positions when the damage parameter, D , approaches its critical value, i.e., $D = 1$. As soon as the crack has been initiated, the deformation field contains a discontinuity. This means that the most critical point in front of the crack tip will fail instantaneously and the crack starts to grow.

In this section, the prediction of crack initiation, crack propagation and resistance curve of $J - \Delta a$ for a three point bending (3PB) specimen of the A533-B1 steel has been carried out using the damage growth model suggested in Section 2.3.

4.1. Criterion for setting the element size in FEA

The FE analysis will be valid in the continuum mechanics sense if an appropriate element size was chosen in the model. The upper and lower bounds size of the elements in the model can be set according to the following rules.

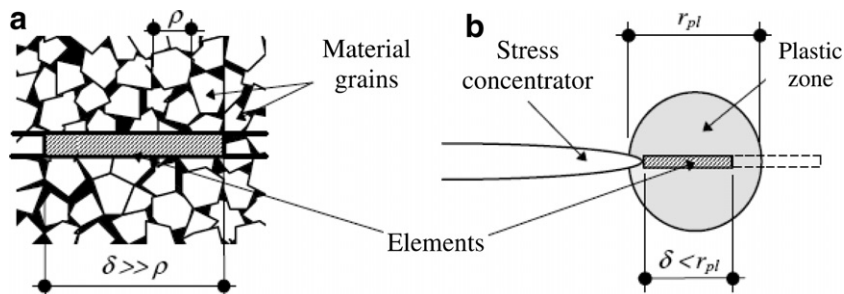


Fig. 9. Element size limits: (a) lower bound, (b) upper bound.

In order to satisfy the requirements of a continuum media, the element size should be many times greater than the size of the material grain size

$$\delta \gg \rho \quad (20)$$

where ρ is the mean grain size and δ is the element size around the crack tip (see Fig. 9).

On the other hand, since local damage processes occur essentially within the plastic zone, the size of the elements must be smaller than the size of the smallest plastic zone possible around the stress concentrator:

$$\delta \ll r_{pl} \quad (21)$$

The mean material grain size of most *steels* used in engineering application is about $\rho \leq 0.01$ mm. Assuming that the requirements of continuum mechanics are satisfied if the element size exceeds five times the grain size, according to Eq. (20) the minimum size for the element will be

$$\delta_{min} = 0.05 \text{ mm} \quad (22)$$

An estimate for the size of the plastic zone at a crack tip can be obtained using Dugdale-Barenblatt cohesive zone model. Based on their analysis, the plastic zone size for plane strain condition is (Anderson, 1995):

$$r_p = \frac{1}{3\pi} \frac{K_c^2}{\sigma_Y^2} \quad (23)$$

where K_c is the crack tip stress intensity factor. The ratio of K_c^2/σ_Y^2 in Eq. (23) can be varied between 1 mm and 200 mm for commonly used steels. Thus the smallest plastic zone size can be estimated as:

$$\delta_{max} < \frac{1}{3\pi} \times 1^2 = 0.106 \text{ mm} \quad (24)$$

From the above discussion, one can conclude that for commonly used steels, the element size should vary between $0.05 \text{ mm} \leq \delta < 0.106 \text{ mm}$.

In this study, the element size around the crack tip was set at 0.1 mm and remained the same for all the simulations to minimize the effect of mesh dependency.

4.2. Experiment on three point bending specimen

Fracture toughness testing was performed according to the [ASTM E1820-99 standard](#) 3PB specimen. The geometry and dimension of the specimen is shown in [Fig. 10](#). The notch was “sharpened” by “precracking” using cyclic fatigue loading. The final precrack length (notch plus pre-

crack) was 19 mm, with a presumed atomically sharp crack tip. The specimen was then loaded to failure under displacement control with a Universal Testing Machine Model 1195 at ambient temperature with a cross-head displacement rate of 0.5 mm/min. The magnitude of the loads and the corresponding displacements were simultaneously recorded during the test. [Fig. 11](#) presents the load versus load-line-displacement for the 3PB specimen. In order to calculate J_1 value at a certain load level, the following formulae were used ([ASTM, 1999](#)):

$$J_1 = \frac{2A}{Bb} \quad (25)$$

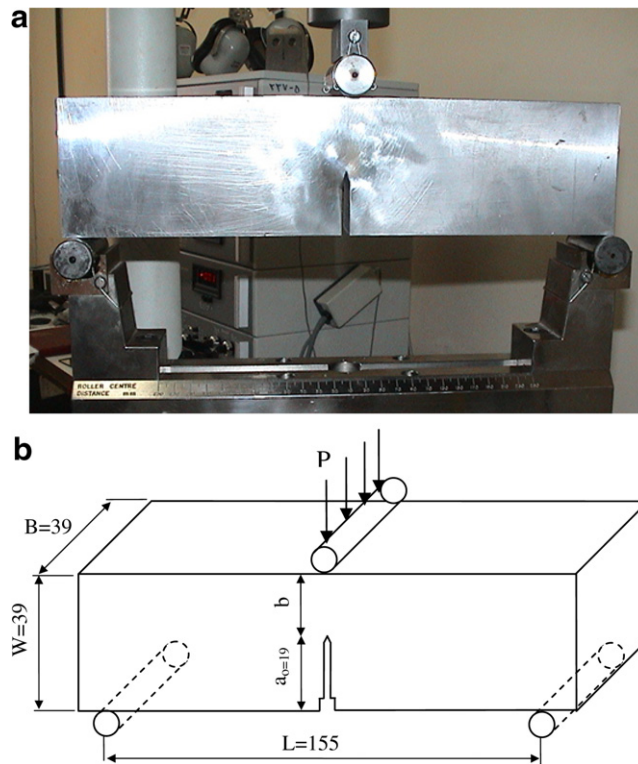


Fig. 10. Experimental setup for three point bending test (a) experimental setup (b) specimen geometry. All dimensions in mm.

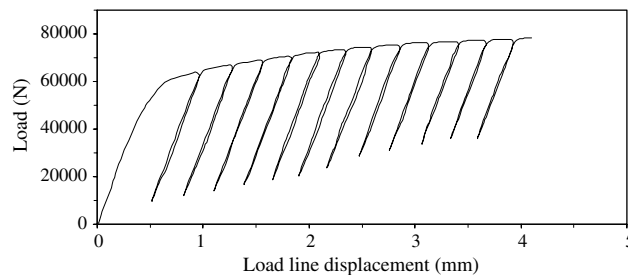


Fig. 11. Load versus load-line-displacement in three point bending specimen.

where A is the energy absorbed during loading, measured by the area under the tensile loading curve, B is the thickness of the specimen and b is the ligament length.

A $J - \Delta a$ resistance curve is produced according to ASTM E1820, from which the critical fracture toughness J_{Ic} , corresponding to ductile crack initiation, is calculated. The resistance curve ($J - \Delta a$) illustrated in Fig. 12. Exclusion lines are drawn at crack extension (Δa) values of 0.15 mm and 1.5 mm. The slope of the exclusion lines corresponds approximately to the component of crack extension that is due to crack blunting, as opposed to ductile tearing. All data that fall within the exclusion limits are fit to a power-law expression:

$$J = C_1(\Delta a)^{C_2} \quad (26)$$

where C_1 and C_2 are constants and for our accepted data $C_1 = 350$ and $C_2 = 0.4632$.

The J_Q is defined as the intersection between power-law expression and 0.2 mm offset line. If all

other validity criteria are met, $J_{Ic} = J_Q$ as long as the following size requirements are satisfied:

$$B, b \geq \frac{25J_Q}{\sigma_Y} \quad (27)$$

The value of J_{Ic} is found to be 240 kN/m.

4.3. Simulation of crack growth in 3PB test

A finite element model was constructed to evaluate how well the CDM predicts the measured fracture initiation from the test. Also, the transferability of damage parameters to fracture mechanics test specimens ought to be validated. The A533-B1 steel was characterised by fracture test of three point bending specimen in previous section. Since for the given tests $B, b \geq 25J_Q/\sigma_Y$, a plane strain state could be expected, allowing a two-dimensional FE analysis. The analysis was performed by using ABAQUS/standard FEA software. A representative finite element mesh for one half of the 3PB model with

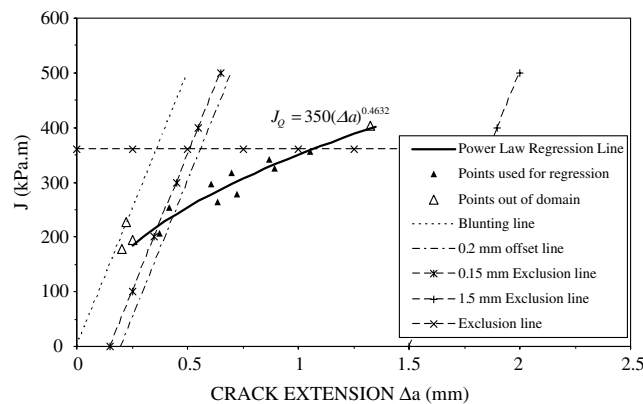


Fig. 12. The resistance curves for A533-B1 steel.

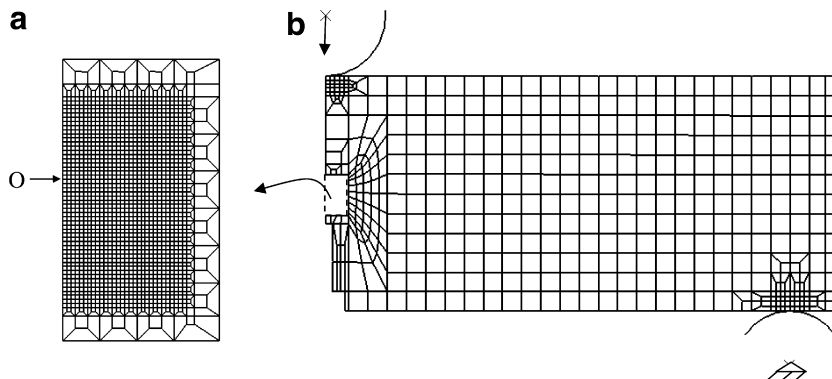


Fig. 13. Finite element model of the three point bending test. (a) Global mesh, (b) local mesh near the crack tip. The initial crack tip is at the position O.

$a/W = 0.48$ is shown in Fig. 13. The size of the elements surrounding the crack tip is about 0.1 mm. The stable ductile crack growth is simulated auto-

matically by the damage model and leads to a chain of failed elements along the ligament. The details in Fig. 14 show how the crack propagates to form a

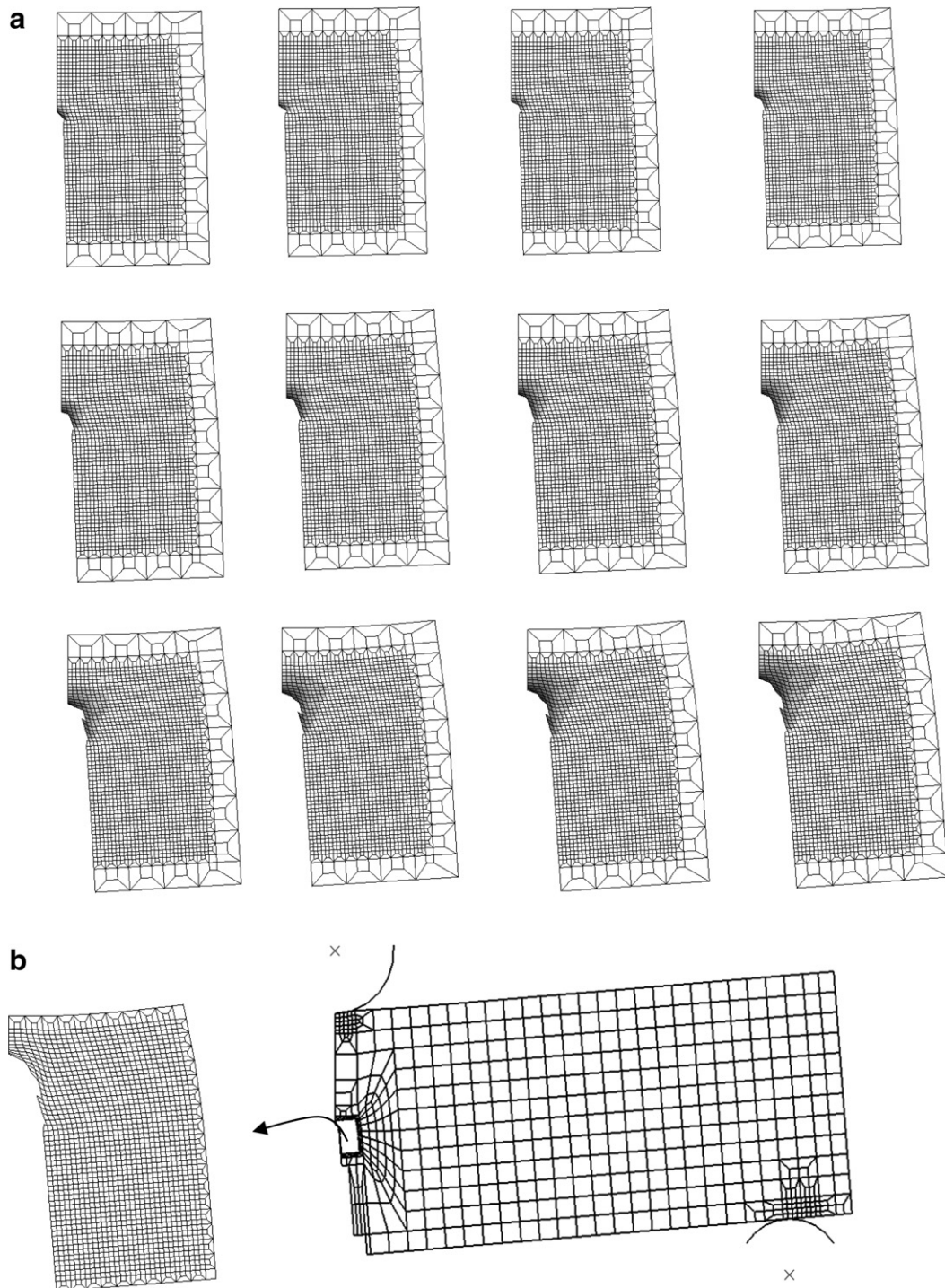


Fig. 14. Simulation of crack growth in three point bending test. (a) Details of local mesh near the crack tip, (b) details of global mesh at the end of loading.

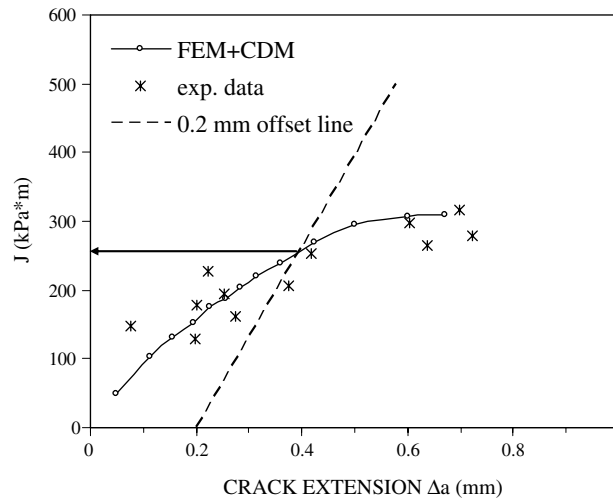


Fig. 15. Experimental and FE simulation of crack resistance curves for A533-B1 steel.

blunted crack tip. In Fig. 14a the evolution of damage growth around the crack tip and advance of the crack front has been consequently shown. By increasing the loading, the damage around the crack tip will grow until eventually the first element will reach to the critical damage parameter, D_c . At this stage this element will be removed from the model and the crack will advance as much as the deleted element length. The loading will continue to increase and the above process will be checked at the end of each loading increment. This process continues until final failure of the specimen. In Fig. 14b the final step of the crack growth in 3PB test has been shown. A few cases of FE simulation with crack length less than 0.1 mm were also performed and no significant difference in the crack path was observed.

The far field J -integral was also computed at all increments. The crack growth resistance curve $J - \Delta a$ was obtained from these analyses. These results were compared with the experimental $J - \Delta a$ in Fig. 15 with very good agreement. This confirms that the experimental damage parameters calibration is very accurate and the measured $J - \Delta a$ curve is very well predicted by the simulations. The results of J_{Ic} value from FE analysis of 3PB specimen is 260 kN/m, while the experimental value of J_{Ic} for 3PB specimen is 240 kN/m. The simulated value is 8.3% higher than the experimental value. This discrepancy partly attributed to the different conditions in two-dimensional plane strain FE model and the condition in the real three-dimensional test.

5. Conclusions

In this paper, the effect of stress triaxiality on ductile damage evolution in metals was investigated from both experimental and theoretical points of view. Ductile fracture process is influenced by both the plastic strain and triaxiality. Stress triaxiality plays a major role on the damage evolution, which is demonstrated by the progressive reduction of material ductility under increasing triaxial states of stress. These effects have been studied by examining damage evolution in notched flat rectangular bar specimen. The CDM model predictions are in very good agreement with the experimental damage measurements. The validity and transferability of damage parameters were checked by applying them to 3PB specimen commonly used for materials testing in damage mechanics.

The elastic–plastic–damage analysis was successfully applied to study ductile fracture. Furthermore, by numerical simulation of ductile crack growth, fracture toughness values were successfully deduced. Comparison of the computer simulated results with the experimental data showed that the overall agreement is satisfactory. The proposed crack growth initiation criterion is reasonably good in explaining Mode-I fracture.

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