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Continuum Damage Mechanics: Part I—General Concepts

Continuum Damage Mechanics (C.D.M.) has developed continuously since the early works of Kachanov and Rabotnov. It constitutes a practical tool to take into account the various damaging processes in materials and structures at a macroscopic continuum level. The main basic features of C.D.M. are considered in the first part together with its present capabilities, including damage definitions and measures, and its incorporation into a thermodynamic general framework. Practical damage growth equations will be reviewed in the second part of the paper.

1 Introduction

In the classical sense, Continuum Mechanics allows us to describe the heterogeneous microprocesses involved during the straining of materials and structures at the macroscale; elastic and plastic strains and the corresponding hardening effects are generaly accepted to be represented by global continuum variables, even if microdefects, such as grains, subgrains, dislocations, and cells, are not really homogeneous. The damaging processes correspond to localizations and accumulations of the strains and are considerably more irreversible. Due to the larger scale to be considered, continuum concepts are more difficult to introduce. Since the early works of Kachanov (1958) and Rabotnov (1969), who considered the creep of metals, the concept of macroscopic damage variables has developed markedly. The distributed defects in materials and structures not only lead to crack initiation and final fracture, but also induce progressive material deterioration (material damage) which can be measured through the decrease of strength, stiffness, toughness, stability, and residual life.

The new concepts initiated the development of Continuum Damage Mechanics (C.D.M.) (Hult, 1979; Chaboche, 1981; Krajcinovic, 1984). They are supported by the general framework of thermodynamics of irreversible processes and offer complementary possibilities to Fracture Mechanics.

The aim of the present paper is to review some general features of C.D.M. and to summarize its main possibilities, considering successively the following aspects:

(*a*) Definitions and measures of damage, including the possibility of describing the microstructural damaged state in terms of appropriate mechanical variables

(*b*) Description of the mechanical behavior of the damaged material. This can be studied in the framework of thermodynamics and can include the influence of damage anisotropy

(*c*) Formulation of equations governing the evolution of these damage variables (see Part II).

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2 Damage Measures and Definitions

The first step in developing a damage theory concerns the definition of the damage variable. Obviously damage is not directly measurable as strain or plastic strain. In the present section, we consider different ways of defining the damage internal variable through indirect measurement procedures. In fact, such measurements are not always practicable but furnish conceptual definitions. Let us note that interpretation of each measure in terms of a damage variable requires a corresponding model.

2.1 The Problem of Crack Initiation. Before any damage theory can be developed, it is necessary to know precisely what we mean by the ultimate state of the damage processes; under the present development of C.D.M., this final state corresponds generally to macroscopic crack initiation, that is the "breaking up" of the continuum volume element. A large degree of arbitrariness is present in the definition of crack initiation, especially in fatigue where the behavior of newly nucleated cracks and short cracks shows various complex interactions with the microstructure (see the schematic view of Fig. 1(*a*) and Jeal, 1985).

In fact the consideration of a macroscopic crack in the framework of Fracture Mechanics supposes a defect sufficiently large as compared to the microscopic heterogeneities (grains, subgrains, other defects and microcracks. . .). The main macroscopic crack is assumed to be developed through several grains, in order to show a sufficient macroscopic homogeneity, in size, geometry and direction, leading to a possible treatment through the Fracture Mechanics concepts (see schematic illustration by Fig. 1).

2.2 Damage Measures Through the Remaining Life. From an engineer's point of view, the main objective of a damage theory is to allow predictions of the lifetime of a structure. Then the remaining life concept is a natural way to define damage. The most conventional definition for such a damage parameter is the life ratio, N/N_F in fatigue, where N and N_F

represent, respectively (for a given loading condition), the present number of cycles already applied and the total number of cycles to crack initiation (or failure). In this case the damage theory corresponds to the linear Miner's rule. For instance, the remaining life at the second level, N_2, after damaging to N_1/N_{F_1} at a first level, is:

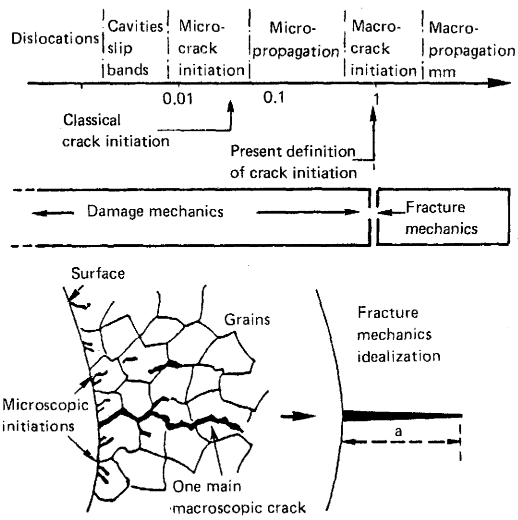

Fig. 1 (*a*) Schematics of the fatigue crack growth; (*b*) illustration of a macroscopic crack initiation concept

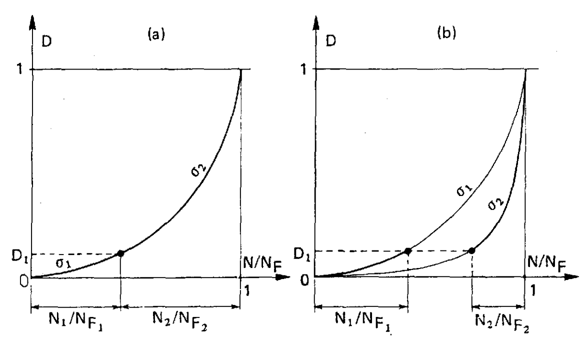

Fig. 2 Schematic of damage evolution curves as deduced from remaining life measures

$$\frac{N_2}{N_{F_2}} = 1 - D = 1 - \frac{N_1}{N_{F_1}} \tag{1}$$

More generally, the remaining life concept does not necessarily lead to the linear rule; after a certain damaging process, the present damage is measured by performing a "measure test," with a fixed loading under which the nominal life (for an initially undamaged specimen) is N_{F_2}. If the measured remaining life is N_2, the damage after the initial damaging process is:

$$D = 1 - \frac{N_2}{N_{F_2}} \tag{2}$$

The remaining life measurements provide evidence of interesting properties of damage. For instance, in fatigue there is not a unique damage evolution curve as a function of the life ratio N/N_F, but a dependency on the applied loading (Fig. 2) (Manson, 1979; Chaboche, 1974). This leads to the conclusion that damage growth equations have to show unseparability, of the damage and loading variables (Krempl, 1977). However, such measurements are not sufficient to completely fix the values of the damage. As shown by Chaboche (1980), a one-to-one mapping changes the damage value without changing any remaining life result. Let us note that the remaining life measures are also practicable for creep damage (Woodford, 1973).

2.3 Damage Measures from the Microstructure. A second natural way to define a damage variable is to observe and quantify the irreversible defects: intergranular cavities in creep, surface microcracks in fatigue, dimensions of cavities in ductile fracture. Such measurements have already been used in many situations (see Dyson and McLean, 1977; Levaillant and Pineau, 1982). Some difficulties arise when interpreting these results:

(*a*) Such measures are destructive, which limit their use to observe the development of damage.

(*b*) Defects are difficult to observe during the first phase of the damaging processes. Moreover, the initial state is not easily characterized.

(*c*) The quantification has to be done in terms of macroscopic variables which are usable in structure computation. Then, in each a case, convenient hypotheses have to be considered and a particular model developed.

Interpretation in terms of mechanical parameters can be

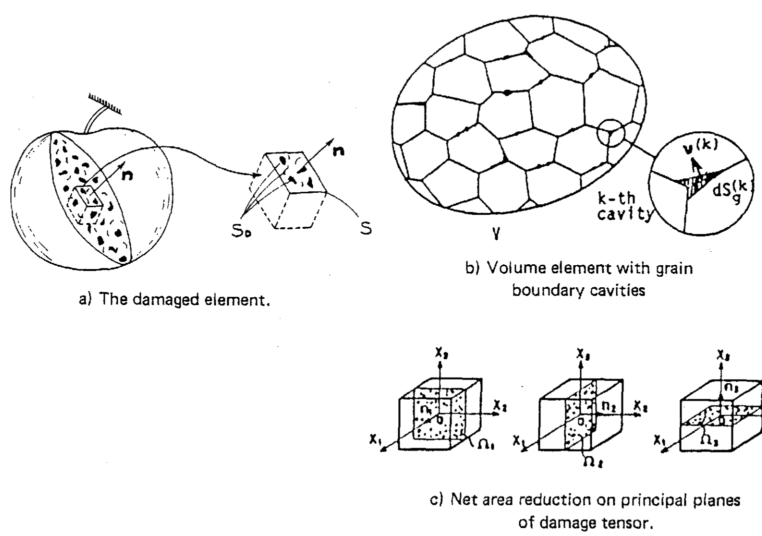

Fig. 3 Net area reduction: (*a*) The damaged element; (*b*) volume element with grain boundary cavities; (*c*) net area reduction on principal planes of damage tensor

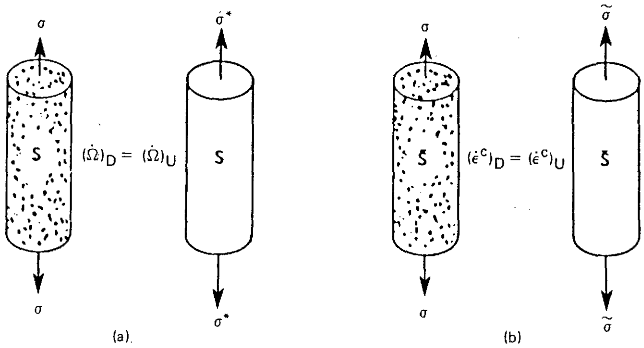

Fig. 4 (*a*) Net stress tensor for damage growth; (*b*) effective stress tensor

made by means of a net area reduction (Fig. 3(*a*)). By definition, the damage variable D_n associated with direction n is:

$$D_n = \frac{S_D}{S} = \frac{S - S^*}{S} \tag{3}$$

$D_n = 0$ corresponds to the undamaged state, $D_n = D_c$, a critical value, corresponds to the rupture of the element ($0.2 < D_c < 0.8$ for metals).

The corresponding net stress concept has been developed by Murakami and Ohno (1980) in the case of creep damage of polycrystalline metals. The microstructural changes can be characterized mainly by nucleation and growth of various cavities on grain boundaries. Assuming that the principal effect consists of a net area reduction due to the distribution of cavities, the state of material damage may be described by a second rank symmetric tensor (Murakami, 1983) (Fig. 3(*b*)), generalizing equation (3) as:

$$\Omega = \frac{3}{S_g(V)} \sum_{k=1}^{N} \int_V \vec{\nu}^{(k)} \otimes \vec{\nu}^{(k)} dS_g^{(k)} \tag{4}$$

where $dS_g^{(k)}$ and $\vec{\nu}^{(k)}$ are, respectively, the area of a given boundary occupied by the kth cavity in volume V and the unit normal vector to it. Figure 4(*a*) indicates schematically that some equivalence is supposed between the present damaged material and the undamaged one (in terms of damage rate) to define the net stress σ^*.

The above definition takes into account the directional nature of damage in a natural way. Other methods can be used to introduce the anisotropy of damage. An attractive one, developed in a framework similar to the slip theory of plasticity, is to use a family of vectors (Krajcinovic, 1983) each of them being associated with to a typical direction of microcracks.

2.4 Damage Measures Through Physical Parameters and the Effective Stress Concept. The influence of damage on physical quantities can be measured and used to define properly the damage variable:

- Density change (Jonas and Baudelet, 1977), which can be interpreted as a damage variable in ductile failure;
- resistivity change (Cailletaud et al., 1980) which, through a convenient model, leads to very similar damage measures to that for mechanical parameters (see below);
- acoustic emission, change in sound velocity. . . ;
- change in the fatigue limit (Bui-Quoc et al., 1971) which can also be interpreted in terms of the remaining life;
- change in the mechanical behavior of the material, interpreted through the effective stress concept:

"A damaged volume of material under the applied stress σ shows the same strain response as the undamaged one submitted to the effective stress $\tilde{\sigma}$." If the damage D represents the

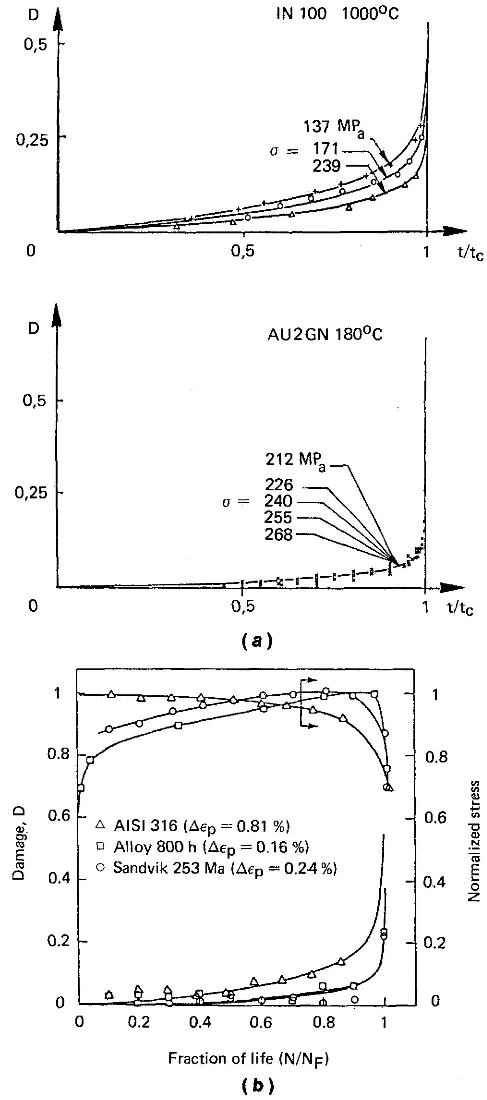

Fig. 5 (*a*) Creep damage measures—alloys IN 100 at 1000°C and AU2GN at 180°C; (*b*) damage accumulation curves as measured from the change in elastic response with corresponding cyclic stress response

loss of effective area taking into account decohesions and local stress concentrations, one can write:

$$\tilde{\sigma} = \sigma \frac{S}{\tilde{S}} = \frac{\sigma}{1-D} \tag{5}$$

This definition through the gross behavior of the material is supported by the results of homogenization techniques (Duvaut, 1976), considering periodic arrays of defects.

Damage measures through the effective stress concept have been performed in several situations. Let us mention:

- The case of ductile rupture (Lemaître, 1985), which measures the change of the elastic modulus. From elasticity equations for both damaged and undamaged material

$$\sigma = \tilde{E}\,\epsilon_e \quad \tilde{\sigma} = E\,\epsilon_e$$

one obtains:

$$\tilde{\sigma} = \frac{E}{\tilde{E}}\,\sigma = \frac{\sigma}{1-D} \rightarrow D = 1 - \frac{\tilde{E}}{E} \tag{6}$$

- Brittle creep damage (Lemaître and Chaboche, 1978), using the power law to describe secondary creep, which can be considered as an undamaged state:

$$\dot{\epsilon}_s = \left(\frac{\sigma}{\lambda}\right)^N \tag{7}$$

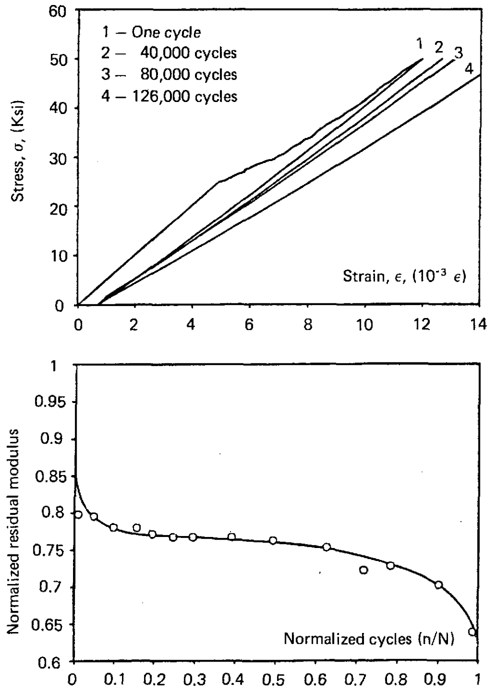

Fig. 6 Fatigue of $[0/90_4]_s$ graphite/epoxy specimens, σ_{max} = 345 MPa (50 ksi), R = 0.1, from Charewicz and Daniel (1985). (*a*) stress-strain curves at various stages of the fatigue life; (*b*) normalized residual elastic modulus.

Several tests give the exponent N. Damage follows easily from the measurement of strain rate during tertiary creep and the effective stress concept (5):

$$D = 1 - \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_s}\right)^{1/N} \tag{8}$$

Figure 5(*a*) shows the case of the superalloy IN 100 (Lemaître and Chaboche, 1978).

- In the case of fatigue damage, the measures are more difficult to interpret, due to its particular localization (often near the surface) and to the superposition of cyclic hardening or softening processes (which cannot be directly considered as damage processes!). In fact, the effective stress concept (Chaboche, 1974) allows some useful correlations, both through elastic and inelastic behaviors, as shown in Fig. 5(*b*) taken from Plumtree and Nilsson (1986). In the three materials, showing different cyclic responses, the damage measured through elastic modulus changes (6) agrees fairly well with the decreasing part of the peak stress evolution. Such measurements also correlates well with quantitative microcrack evaluations (Cailletaud and Levaillant, 1984; Hua and Socie, 1984).
- Damage in composites develops continuously and grows by various mechanisms at the microscale (as in fiber debonding, matrix microcracking, delamination). The case of fatigue is illustrated in Fig. 6 for graphite/epoxy laminates (from Charewicz and Daniel, 1985). One observes simultaneously the change in the elastic modulus on the first cycle (due to partial debonding), its progressive decrease during fatigue (Fig. 6(*b*)), accompanying microcracking of the matrix with some stabilization, and the final delamination giving rise to rupture. Let us note some similarities with the case of metals, with much more pronounced effects. Charewicz and Daniel (1985) show clearly the correspondence between the decrease of elastic modulus, the decrease of residual strength, and the remaining life in fatigue.

This last example shows the adequacy of Damage Mechanics to describe completely different kinds of materials.

Table 1

Observable variable	Internal variables	Associated variables
Elastic strain tensor ϵ_e		Stress tensor σ
Temperature T		Entropy S
	Isotropic hardening r	Size increase of yield surface R
	Damage D	Damage strain energy release rate Y

Talreja (1985) gives further theoretical developments on the case of composites.

Let us note that damage can be active or passive (Krajcinovic and Fonseka, 1981). When the defects, especially microcracks, are closed, they do not affect the macroscopic behavior. The damage state is not eliminated but has to be considered as passive; this can be the case under compression, for instance. The theory developed by Krajcinovic and Fonseka (1981) allows a natural treatment of this effect, which is particularly important for materials like concrete. Ladevèze and Lemaître (1984) gave also some developments around this point.

Another important feature of damage is its probabilistic nature (at the microscale). When considering the residual strength of the material, that leads to statistical definitions of the damage variables (Kracjcinovic and Silva, 1982; Chrzanowski, 1976).

3 Thermodynamic Aspects

The present developments are based on a thermodynamic theory of irreversible processes with internal state variables (Sidoroff, 1975; Germain et al., 1983). The presentation is limited here to the simple case of isotropic hardening within the small strain hypothesis and to an isotropic damage. The extension to kinematic hardening is well known (Halphen and Nguyen, 1975), and theories with anisotropic damage evolution have been developed by Chaboche (1978) and Lemaître and Chaboche (1985). Generalization to finite strain can be found in Rousselier (1980).

The chosen state variable are given in Table 1 for the present case. The plastic strain tensor is defined from the total strain tensor by $\epsilon_p = \epsilon - \epsilon_e$ and the accumulated plastic strain by $\dot{p} = (2/3\ \dot{\epsilon}_p : \dot{\epsilon}_p)^{1/2}$. In the case of a nondamaging material and neglecting the time recovery effects, it can be demonstrated (Lemaître and Chaboche, 1985) that r reduces to the accumulated plastic strain p (in the case of a generalized associative flow rule).

3.1 Thermodynamic Potential. The specific free energy Ψ, taken as the thermodynamic potential in which elasticity and plasticity are uncoupled, gives the law of thermoelasticity coupled with damage:

$$\Psi = \Psi_e(\epsilon_e, T, D) + \Psi_p(T, r) \tag{9}$$

As proposed by Chaboche (1977), the damaged elastic behavior is described through a strain equivalence and, referring to the effective stress concept:

$$\Psi_e = \frac{1}{2\rho}(1-D)\Lambda : \epsilon_e : \epsilon_e \tag{10}$$

The stress is defined as:

$$\sigma = \rho \frac{\partial \Psi}{\partial \epsilon_e} = (1-D)\Lambda : \epsilon_e \quad \text{or} \quad \sigma = \tilde{\Lambda} : \epsilon_e \tag{11}$$

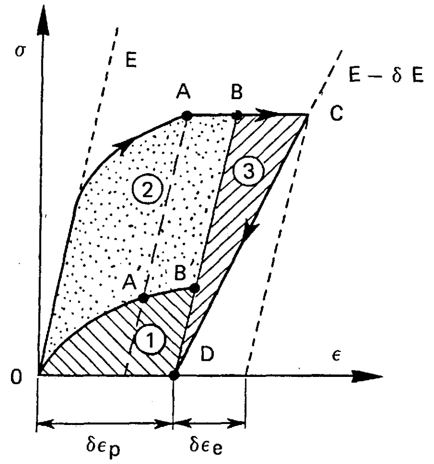

Fig. 7 Schematic of the dissipation during plastic flow and damage growth

The thermodynamic forces Y associated with D and R associated with r are defined as:

$$Y=\rho\frac{\partial\Psi}{\partial D}=-\frac{1}{2}\Lambda:\epsilon_e:\epsilon_e \qquad R=\rho\frac{\partial\Psi}{\partial r} \tag{12}$$

Let us note that D includes all the damaging effects, the density ρ being considered as constant. In the finite strain case (Rousselier, 1980), the change of ρ (due to the growth of cavities) can be used as a damage variable. Moreover, in the present case, complete separability of the hardening and damage processes is assumed, Ψ_p does not depend on D.

W_e being the density of elastic strain energy defined by $dW_e = \sigma: d\epsilon_e$, the expression for Y shows that (Chaboche, 1977):

$$-Y=\frac{W_e}{1-D}=\frac{1}{2}\frac{dW_e}{dD} \text{ at constant } \sigma \text{ and Temperature} \tag{13}$$

Then, the variable $-Y$ can be considered as the elastic strain energy release rate associated with a unit damage growth. The analogy with Fracture Mechanics concepts is clear. $-Y$ may be calculated as a function of the hydrostatic stress $\sigma_H = 1/3\, Tr(\sigma)$ and the Von Mises equivalent stress $\sigma_{eq} = (3/2\ \sigma':\sigma')^{1/2}$ where σ' is the stress deviator (Lemaître, 1984):

$$-Y=\frac{\sigma_{eq}^2}{2E(1-D)^2}\left[\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] \tag{14}$$

The relation (13) defining $-Y$ as the elastic strain energy release rate and the above dissipation aspects can be illustrated in Fig. 7, showing the different parts of dissipation during the plasticity and rupture processes (Lemaître and Chaboche, 1985). The curve $OA'B'$ represents the evolution of hardening during plastic flow OAB. Parts AB and BC correspond, respectively, to the plastic flow and the elastic strain increase during the damaging process (schematized at constant stress). The total dissipated energy separates into: ① the energy stored in the system (hardening); ② the heat dissipated energy; ③ the energy released by the system during the damaging process $-Y\,\delta D$, eventually converted into heat.

The fact that energy stored in the material is the work done above the initial yield limit is a consequence of some simplifying assumptions on the isotropic hardening. In fact, additional energies are converted into heat, which can be described by using the superposition of a nonlinear kinematic hardening as in Lemaître and Chaboche (1985) and specific choices for the part Ψ_p of the free energy.

3.2 Dissipation. The second principle of thermodynamics imposes that the intrinsic dissipation has to be positive:

$$\sigma:\dot{\epsilon}_p-R\,\dot{r}-Y\,\dot{D}>0 \tag{15}$$

Hardening and damage being uncoupled, it is sufficient to assume:

$$-Y\,\dot{D}>0 \tag{16}$$

As $-Y$ is a quadratic function, this leads to $\dot{D} > 0$.

The rupture criterion "$-Y = |Y| = Y_c \rightarrow$ crack initiation" corresponds to an elastic energy criterion. It may be written in terms of D through the one-dimensional rupture stress σ_R:

$$Y_c=\frac{\sigma_R^2}{2E(1-D_c)^2} \rightarrow D_c=1-\frac{\sigma_R}{(2EY_c)^{1/2}} \tag{17}$$

Many experiments have shown that: $0.2 < D_c < 0.8$, which allows $(1 - D_c)^x$ to be neglected with regard to 1 when x is much greater than 1.

The potential of dissipation is a scalar convex function of flux variables ($\dot{\epsilon}_p$, $\dot{r}$, $\dot{D}$ and the heat flux q) or their dual variables (by means of the Legendre-Fenchel transform), the state variables acting as parameters (Germain et al., 1983):

$$\phi(\sigma, R, Y; \epsilon_e, T, p, D)$$

It gives the constitutive equations for the evolution of dissipative variables, written here as:

$$\dot{\epsilon}_p=\frac{\partial\phi^*}{\partial\sigma} \qquad \dot{r}=-\frac{\partial\phi^*}{\partial R} \qquad \dot{D}=\frac{\partial\phi^*}{\partial Y} \tag{18}$$

If ϕ^* is a convex function of $-Y$, the damage dissipation (16) is automatically positive (Chaboche, 1977).

In the case of time independent plasticity and isotropic hardening, the plastic flow can be particularized with the Von Mises plastic potential (Lemaître and Chaboche, 1985):

$$f(\sigma, R, D)=\tilde{\sigma}_{eq}-R-k<0 \tag{19}$$

where $\tilde{\sigma}_{eq}$ is the equivalent effective stress, here $\sigma_{eq}/(1-D)$.

It follows then from the normality rule (σ' is the stress deviator):

$$\dot{\epsilon}_p=\lambda\frac{\partial f}{\partial\sigma}=\frac{3}{2}\frac{\lambda}{1-D}\frac{\sigma'}{\sigma_{eq}} \tag{20}$$

$$\dot{r}=\lambda\frac{\partial f}{\partial R}=\lambda=\left(\frac{2}{3}\dot{\epsilon}_p:\dot{\epsilon}_p\right)^{1/2}=(1-D)\dot{p}$$

The plastic multiplier is determined from the consistency condition $\dot{f} = 0$. When damage is zero, the state variable reduces to the accumulated plastic strain p.

In fact the above normality rule is a sufficient but not a necessary condition (Onat, 1985). In some cases it appears as too restrictive, especially for materials or rupture conditions where the energetic rupture criterion (17) does not allow a correct description, as is the case in creep. The only condition to be verified is $\dot{D} > 0$ (because $-Y$ is always positive).

The present theory can be generalized to the case of large strains, including some modifications. In the theory developed by Rousselier (1980), two damage parameters are used, the first one corresponds to the density change and acts on the elastic behavior, the second one follows from an explicit dependency between the plastic potential and the hydrostatic stress and generalizes in some way the Rice and Tracey (1969) formula for the growth of spherical voids under high triaxiality.

4 Conclusion

The general concepts of Damage Mechanics have been reviewed, considering the measures and definitions of damage variables and their incorporation into a general thermodynamic framework. One of the main features of C.D.M. is to take into account the coupling effects between damaging processes and stress-strain behavior.

Different damage growth equations can be specified and particularized, including the description of creep and fatigue processes, ductile damage, and brittle damage. This is the subject of the second part of this review (Chaboche, 1988), which will also consider the possibilities of C.D.M. in the lifetime and crack prediction techniques.

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