Engineering Fracture Mechanics Vol. 28, No. 5/6, pp. 643–661, 1987
Printed in Great Britain

0013-7944/87 $3.00 + .00

DAMAGE MEASUREMENTS

JEAN LEMAITRE and JACQUES DUFAILLY
Laboratoire de Mécanique et Technologie, E.N.S. de CACHAN, C.N.R.S., Université Paris 6,
61, av. du Président Wilson, 94230 Cachan, France

Abstract—Eight different methods are described to measure damage defined as the effective surfacic density of micro-cracks and cavities in any plane of a representative volume element: (i) direct measurements such as the observation of micrographic pictures and the measurement of the variation of the density; (ii) non direct measurements which are destructive such as the measurement of the variations of the elasticity modulus, of the ultrasonic waves propagation and of the cyclic plasticity or creep responses, or non destructive such as the measurement of the variation of the micro-hardness and of the electrical potential.

1. INTRODUCTION

DAMAGE of materials is taken here in the sense of continuum mechanics that is a property which diminishes the strength until failure. It mainly consists of the creation and development of discontinuities in the solid media:

micro-cracks or micro-cavities in metals,
breakage of bounds and cracks in polymers,
decohesions of cells and fibres in woods,
decohesions of interfaces between cement, sand and aggregates in concrete.

All those phenomena correspond to different mechanisms and, even in metals, different kinds of damage can be distinguished:

brittle damage which corresponds to defects created without measurable macroscopic plastic strain,
ductile damage which corresponds to cavities induced by large plastic strains,
creep damage which corresponds to intergranular decohesions at high temperature,
fatigue damage which mostly corresponds to a transgranular cracking phenomenon due to the repetition of loadings.

Historically, the need for damage evaluation has probably been most important in the field of fatigue. Indeed, it is possible to have a rough estimation of the rupture conditions due to other kinds of damage by means of a rupture stress criterion (this criterion, function of the time to rupture for creep, may be the von Mises' equivalent stress or better, the damage equivalent stress [see Section 2]). But for fatigue, it is necessary to have a law to accumulate damages due to a succession of different loading levels. The success of the Miner's rule is due to its simplicity:

$$\sum \frac{N_i}{N_{Ri}} = 1.$$

where N_i denotes the number of cycles of an amplitude of stress σ_i and, N_{Ri} the number of cycles to failure corresponding to a periodic loading of constant amplitude σ_i. Implicitly, N/N_R is a measure of damage which is linear with N.

Many other evaluations of damage have been used in the past to estimate the value of damage before failure[1] but, most often without any precise definition of a damage variable.

The need to measure damage covers at least three large domains of applications:

(i) The necessity of formulating the evolution laws for damage and to identify them for each engineering material. Those constitutive equations are then used to predict by calculation the damage evolution and the failure of structures in service.

(ii) The necessity of determining "*in situ*" the damage of a component for control and security purposes.

(iii) The necessity of evaluating damage, at least qualitatively within broken components in order to find out the causes of accidents.

2. DEFINITION OF A DAMAGE VARIABLE

The definition used herein is in accordance with continuum mechanics in order to solve problems of solid mechanics involving stresses, strains and failure[2]. Those variables must be defined at each mathematical point, a concept which needs careful examination because of the scale of the phenomena.

The goal is to represent a discontinuous state (micro-cracks, micro-cavities) by a continuous variable. If the geometrical nature of the defects is simple and known, micro-mechanics[3] and homogeneization techniques[4] are the tools to do it. But, most often, the mechanisms are not well known and a simple mean value of a defect characteristic is sufficient. A mean value on which domain? This is a crucial point.

A "representative" volume element in mechanics, is the smallest volume on which a density may represent a field of discontinuous properties. For damage, because of the scale of the defects, the linear size of the representative volume element is of the order of:

0.05 to 0.5 mm for metals,
0.1 to 1.0 mm for polymers,
1.0 to 10.0 mm for woods,
10.0 to 100.0 mm for concrete.

With these numbers it is now possible to state the following definition:

> *The damage variable* D *is the effective surfacic density of micro-defects on a representative volume element.*

If $\mathbf{n}$ is the normal which defines a surface of intersection δS of a volume element δV and if δS_0 is the "effective" surface of intersection of micro-cracks and micro-cavities with the plane of δS

$$D_{(\mathbf{n})} = \frac{\delta S_0}{\delta S}$$

$D_{(\mathbf{n})} = 0 \rightarrow$ no damage in the direction $\mathbf{n}$
$D_{(\mathbf{n})} \rightarrow 1 \rightarrow$ the element breaks up into two parts along the plane $\mathbf{n}$.
The meaning of "effective" will be given later.

It is possible to consider this variable as an internal variable in the sense of thermodynamics of irreversible processes[2] introducing its associated variable Y so that $-Y\dot{D}$ is the power dissipated in the damage processes of creation of a new discontinuity surface $\mathrm{d}(\partial S_0) = \dot{D}\,\mathrm{d}t\partial S$.

This variable Y plays a very important role in damage mechanics. Its significance is given by the expression of the thermodynamic potential based on the concept of the effective stress[5] associated to the principle of strain equivalence[6]. If we restrict ourselves to isotropic damage, D does not depend on $\mathbf{n}$ and the effective stress related to the area which effectively resists is defined by

$$\tilde{\boldsymbol{\sigma}} = \frac{\boldsymbol{\sigma}}{1-D},$$

$\boldsymbol{\sigma}$ being the Cauchy stress tensor.

The principle of strain equivalence states that any strain constitutive equation of a damaged material is written exactly as for a virgin material except that the stress is replaced by the effective stress.

When applied to linear elasticity, this principle allows to write the elastic thermodynamic potential ψ as

$$\rho\psi = \tfrac{1}{2}\mathbf{a}:\boldsymbol{\epsilon}^{e}:\boldsymbol{\epsilon}^{e}(1-D) = \tfrac{1}{2}a_{ijkl}\epsilon^{e}_{ij}\epsilon^{e}_{kl}(1-D)$$

where ρ denotes the density and from which the law of elasticity coupled with damage is

$$\boldsymbol{\sigma} = \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}^{e}} = \mathbf{a}:\boldsymbol{\epsilon}^{e}(1-D).$$

For isotropic elasticity (E being the Young's modulus and ν the Poisson's ratio).

$$\boldsymbol{\epsilon}^{e} = \frac{1+\nu}{E}\frac{\boldsymbol{\sigma}}{1-D} - \frac{\nu}{E}\frac{\mathrm{tr}(\boldsymbol{\sigma})}{1-D}\mathbf{1}$$

and the associated variable Y is then defined by

$$Y = \rho\frac{\partial\psi}{\partial D} = -\frac{1}{2}\mathbf{a}:\boldsymbol{\epsilon}^{e}:\boldsymbol{\epsilon}^{e}.$$

This allows one to define a damage equivalent stress σ^{*} derived from this expression of Y for a three-dimensional case and for a one-dimensional equivalent case[8]. Together with linear elasticity the result is

$$\sigma^{*} = \sigma_{eq}R_{\nu}^{1/2}$$

with

$$R_{\nu} = \frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_{H}}{\sigma_{eq}}\right)^{2}$$

$$\sigma_{eq} = (\tfrac{2}{3}\boldsymbol{\sigma}^{D}:\boldsymbol{\sigma}^{D})^{1/2}$$

$$\boldsymbol{\sigma}^{D} = \boldsymbol{\sigma} - \sigma_{H}\mathbf{1}$$

$$\sigma_{H} = \tfrac{1}{3}\mathrm{tr}\,(\boldsymbol{\sigma}).$$

For the one-dimensional case

$$\sigma^{*} = \sigma$$

and

$$Y = \frac{-\sigma^{2}}{2E(1-D)^{2}}.$$

3. DIRECT MEASUREMENTS

Direct measurements consist of the evaluation of ∂S_{D} within a surface ∂S

$$D = \frac{\partial S_{D}}{\partial S}.$$

3.1 *Observation of micrographic pictures*

First of all, ∂S has to be chosen of an order of magnitude coherent with that of the representative volume element. This means that for metals a maximum magnification of $\simeq 1000\times$ is sufficient to observe a picture of 100×100 mm size. For concrete, a small magnification of only $10\times$ is needed.

Due to the need to prepare the samples, this method is of course a fully destructive method except in the case of observations of surfaces by means of the replica technique[7]. The authors being not competent to give advice about metallographical work, let us assume a "good" micrography which can be analysed either automatically or manually.

In order to evaluate the damage, in accordance with its definition, a micromechanics model of the volume element is needed (Fig. 1).

If the damage mainly consists of cavities, the macroscopic model is made of cells within each of which it exists a spherical cavity. In this case, the effective surface ∂S_D coincides with the total area of intersection of cavities with the plane of the picture. An example taken from ref.[8] is given in Fig. 2, showing ductile micro-cracks and dimple formation in a high alloy steel. (Evidence for slip can be seen on the surface (600 : 1).) In the plane of the picture at scale 1, $\partial S_D = 1163$ mm^2, $\partial S = 11{,}317$ mm^2. Damage relative to a plane normal to the picture, $D \simeq 0.1$.

If the damage mainly consists of micro-cracks, the macroscopic model is made of cells within each of which exists a planar flat circular or square crack randomly oriented (Fig. 1). Therefore, there is almost no chance to obtain even only one crack in any plane of intersection. Some equivalence must be pointed out. The best way to do it is to write that the energy involved in the growth of a crack by means of classical fracture mechanics is equal to the energy involved in the equivalent damage of the cell by means of damage mechanics.

First of all, consider a single cell of size d (volume d^3) with a crack of area a^2 created by brittle fracture. If G_C is the toughness of the material, the amount of energy dissipated in that process calculated by means of classical fracture mechanics is:

$$W = G_C a^2.$$

Consider now the same cell uniformly damaged by isotropic brittle damage in such a way that the same energy W has been dissipated. Variables Y and D are defined so that $-Y\dot{D}$ is the power per unit volume dissipated in the damage process. For brittle damage, $-Y$ is constant and equal to its critical value Y_C, then, the amount of energy dissipated to damage a volume d^3 is:

$$W = Y_C D d^3.$$

Writing equality of both energies gives a relation between the damage variable and the crack area

$$G_C a^2 = Y_C D d^3$$

$$D = a^2 \frac{G_C}{Y_C d^3}.$$

We see that the damage in the sense of the definition used herein is equal to the relative area of

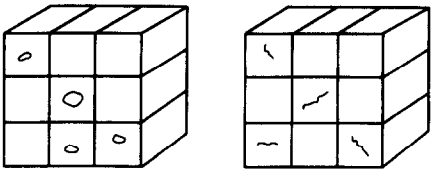

Fig. 1. Micro-mechanics model of micro-cavities and micro-cracks.

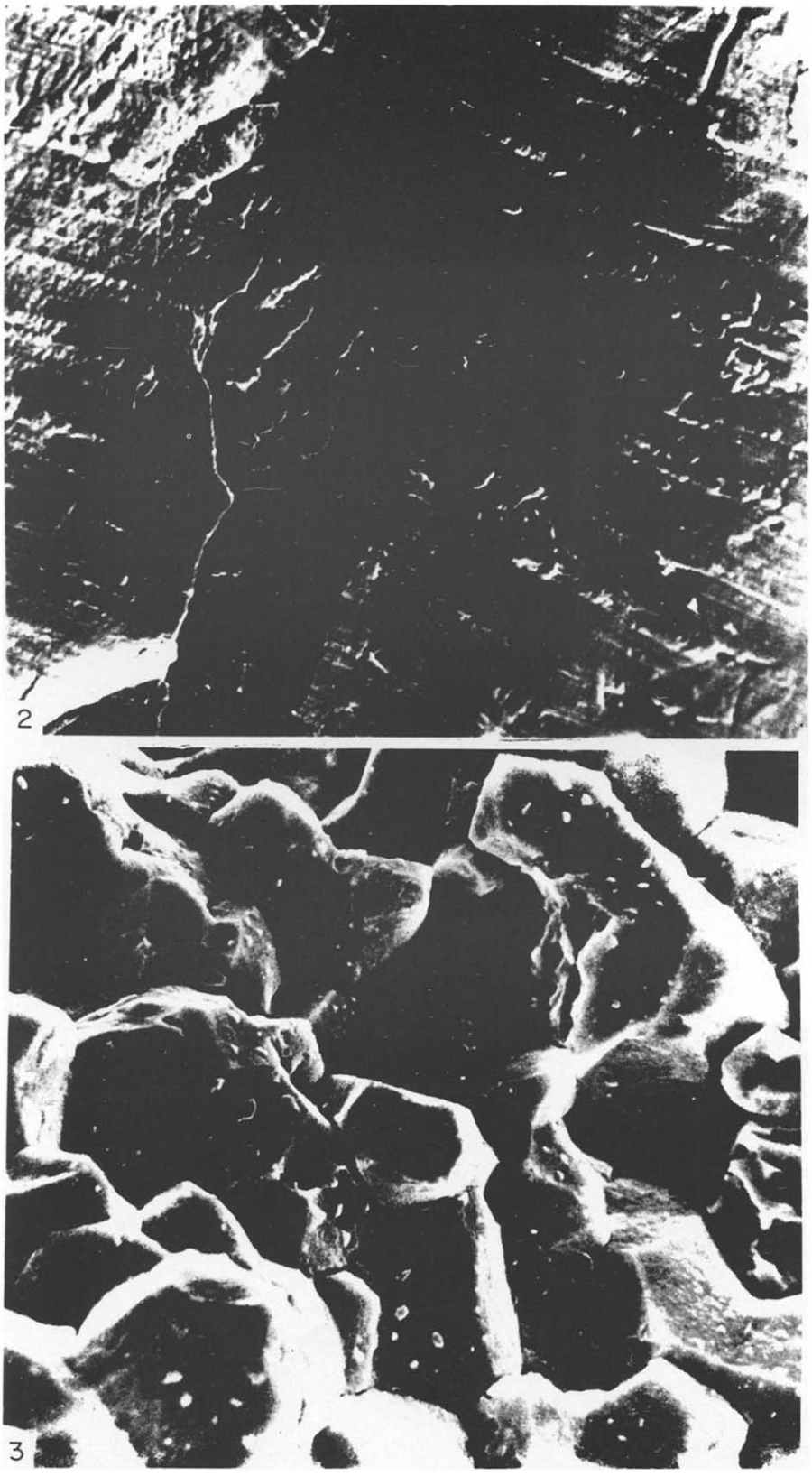

Fig. 2. Evaluation of ductile damage on high alloy steel. (After C. Lienard.)

Fig. 3. Evaluation of intercrystalline failure damage in a martensitic Ni Co Mo steel. (After C. Lienard.)

crack a^2/d^2 multiplied by a correction factor $G_C/Y_C d$ and the effective area S_D (identical with δS_D of the Section 2) of a crack surface a^2 is:

$$S_D = a^2 \frac{G_C}{Y_C d}.$$

The evaluation of the correction factor requires the knowledge of:

(i) The critical value of the strain energy release rate which may be deduced from the toughness K_{IC} by

$$G_C = \frac{K_{IC}^2}{E}(1-\nu^2),$$

plane strain conditions being assumed. For engineering materials K_{IC} is found in all good books of fracture mechanics.

(ii) The critical value of the strain energy density release rate which may be calculated from the rupture condition in tension[2]

$$Y_C = \frac{\sigma_u^2}{2E(1-D_C)^2}.$$

σ_u is the ultimate rupture stress, D_C is the critical value of damage at failure in tension. Few books give this characteristic! Take $D_C = 0.5$ if you do not know!

(iii) The size of the cells or of the grains for metals.

Consider now a microscopic volume element of size $l \times l \times d$ with cells of size $d \times d \times d$ (Fig. 1) the number of cells is l^2/d^2.

If damage is known as isotropic, the damage of the volume element is taken as the mean value of the partial damages of the cells

$$D = \frac{\sum \frac{a_i^2 G_C}{Y_C d^3}}{\text{number of cells}}.$$

Practically, if the size d of the cells (or of the grains for metals) do not differ too much on the analysed picture and as G_C and Y_C do not depend upon the cells:

$$D = \frac{d^2}{l^2}\frac{G_C}{Y_C d^3}\sum a_i^2$$

$$D = \frac{G_C}{Y_C l^2 d}\sum a_i^2.$$

It is possible to develop an approximate expression easiest to use. Considering that the volume element is fully damaged ($D = 1$) or broken when all the cells are broken by cracks of size d^2:

$$\sum a_i^2 = d^2 \cdot \frac{l^2}{d^2} = l^2$$

$$1 = \frac{G_C}{Y_C l^2 d} l^2 \longrightarrow \frac{G_C}{Y_C} = d$$

and

$$\boxed{D = \frac{\sum a_i^2}{l^2}}$$

which is simply the square of the one-dimensional density of cracks. An example taken from ref. [8] is given in Fig. 3 showing grain boundary precipitates of titanium carbide caused intercrystalline failure in a martensitic NiCoMo steel component which has been subjected to triaxial stresses *(2000 : 1). Measured on the picture at scale 1, $\sum a_i^2 = 9000\,\text{mm}^2$, $l^2 = 10{,}200\,\text{mm}^2$. Equivalent isotropic damage, $D \simeq 0.88$.

3.2 *Variations of density*

In the case of pure ductile damage, the defects are cavities which can be assumed as roughly spherical; this means that the volume increases with damage. The corresponding decrease of the density is measurable with apparatuses based on the Archimedean principle[9].

If $(\tilde{\rho} - \rho)/\rho$ is the relative variation of density between the damaged state $\tilde{\rho}$ and the initial non-damaged state ρ, by means of micromechanics, considering a spherical cavity of radius r in a spherical volume element of initial radius R and mass m, it is easy to derive the following relations between the surfacic damage D and the variation of density or porosity[10] assuming no residual micro-stress

$$\rho = \frac{m}{\frac{4}{3}\pi R^3}$$

$$\tilde{\rho} = \frac{m}{\frac{4}{3}\pi(R^3 + r^3)}$$

$$\frac{\tilde{\rho} - \rho}{\rho} = \frac{R^3}{R^3 + r^3} - 1 = \frac{-r^3}{R^3 + r^3}$$

$$D = \frac{\pi r^2}{\pi(R^3 + r^3)^{2/3}} = \left(\frac{r^3}{R^3 + r^3}\right)^{2/3}$$

$$\boxed{D = \left(1 - \frac{\tilde{\rho}}{\rho}\right)^{2/3}}.$$

An example taken from ref. [9] is given in Fig. 4.

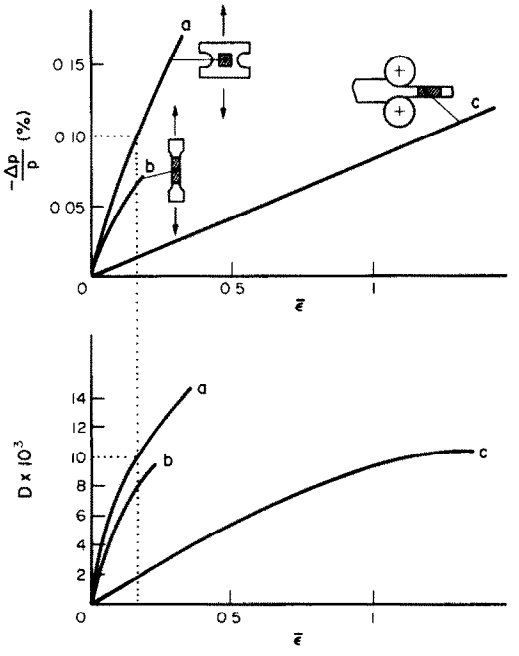

Fig. 4. Evaluation of ductile damage on steel for different strain hardenings.

4. NON-DIRECT MEASUREMENTS

The concept of the effective stress associated to the principle of strain equivalence gives rise to a full set of methods of measurement when applied to elasticity, plasticity or visco-plasticity and damage constitutive equations. Damage is not measured directly but through its effects on strain properties.

4.1 *Destructive methods*

These methods require the manufacture of specimens in order to run mechanical tests.

Variation of the elasticity modulus. The law of elasticity coupled with damage as written in Section 2 through the thermodynamic potential allows to evaluate the damage by means of its influence on the elasticity modulus[2]. For the one dimensional case

$$\epsilon_e = \frac{\sigma}{E(1-D)} = \frac{\sigma}{\tilde{E}}.$$

We see that the effective elasticity modulus $\tilde{E} = E(1-D)$ decreases as D increases. If Young's modulus E is known from a test on the non-damaged material and if $\tilde{E}$ is measured in the damaged state, the corresponding value of damage is:

$$\boxed{D = 1 - \frac{\tilde{E}}{E}.}$$

Since this method was first developed in 1977[11–12], it has been very extensively used in many laboratories. It needs very accurate strain measurements by strain gages and it has been found that the best accuracy is obtained during unloadings. An example of measurement of ductile damage during a classical tension test is given on Fig. 5.

This technique may be used for any kind of damage as long as damage is uniformly distributed in the volume on which strain is measured. This is the main limitation of that method.

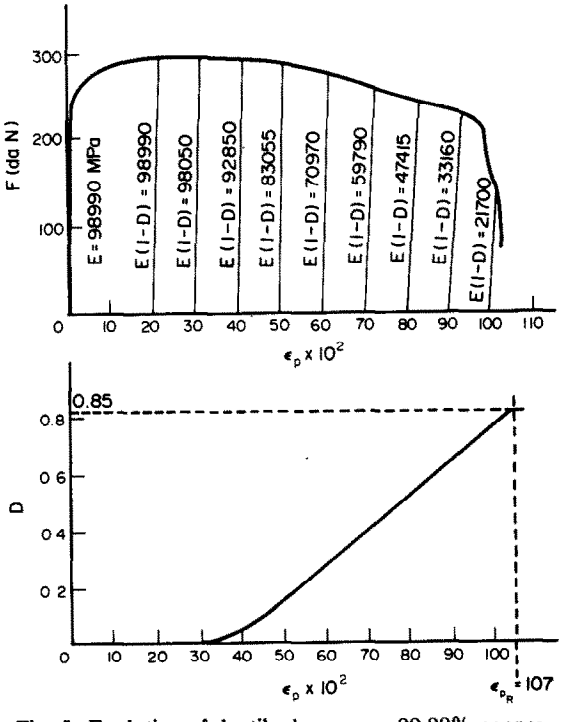

Fig. 5. Evolution of ductile damage on 99.99% copper.

If damage is too much localized, as for high cycle fatigue of metals for example, another method must be used.

Some other precautions related to non-linearities have to be taken.

At the beginning and at the end of the unloading paths in the plane (σ, ϵ) there are small non-linearities due to viscous or hardening effects and also due to experimental devices. Then, the best is to ignore them and to identify $\tilde{E}$ in the range

$$0.15\frac{F_{MAX}}{S} < \frac{F}{S} < 0.85\frac{F_{MAX}}{S}.$$

The most important is to keep the same procedure to evaluate E and the evolution of $\tilde{E}$.

For ductile or low cycle fatigue damages in metals, the procedure could be somewhat disturbed by an early decrease of $\tilde{E}$ at small strain levels or during the first cycles. This is due to microplasticity related to reversible movements of dislocations and to texture development but not to damage. As this phenomenon is rapidly saturated, it is easy to consider:

$$D = 0 \quad \text{for} \quad \epsilon < \epsilon_0 \qquad \text{(a damage strain threshold)}$$

or

$$D = 0 \quad \text{for} \quad N < N^* \qquad \text{(number of cycles to stabilization)}.$$

For polymers or composites, to avoid the viscous effect of visco-elasticity the strain rate during unloadings must be the same for the measurement of E and $\tilde{E}$. An example of a damage measurement on a carbon–carbon composite is given on Fig. 6.

On concrete it is important to check the "uniform" distribution of cracks as the method is no more valid if a single big crack develops. An example of a good result is given on Fig. 7.

Ultrasonic waves propagation. Another technique based on the variation of the elasticity modulus consists in measuring the speed of ultrasonic waves[15].

For frequencies higher than 20 kHz in a linear isotropic elastic cylindrical medium the longitudinal wave speed v_L and the transversal wave speed v_T are:

$$v_L^2 = \frac{E}{\rho}\frac{1-\nu}{(1+\nu)(1-2\nu)} \qquad v_T^2 = \frac{E}{\rho}\frac{1}{2(1+\nu)}.$$

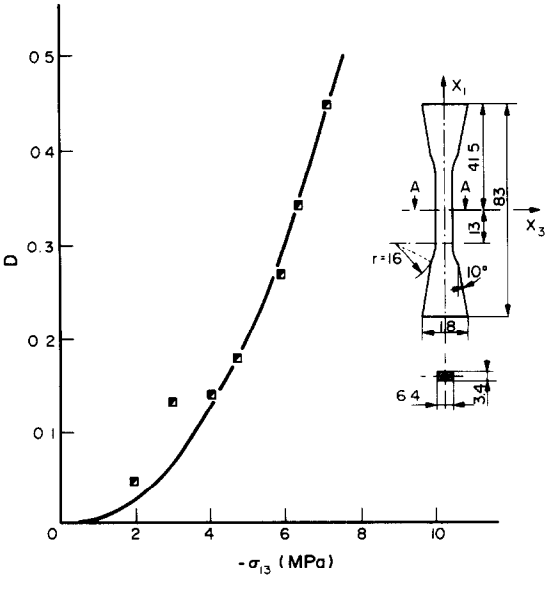

Fig. 6. Evolution of brittle damage in a shear direction of a three-dimensional carbon–carbon composite—from [13].

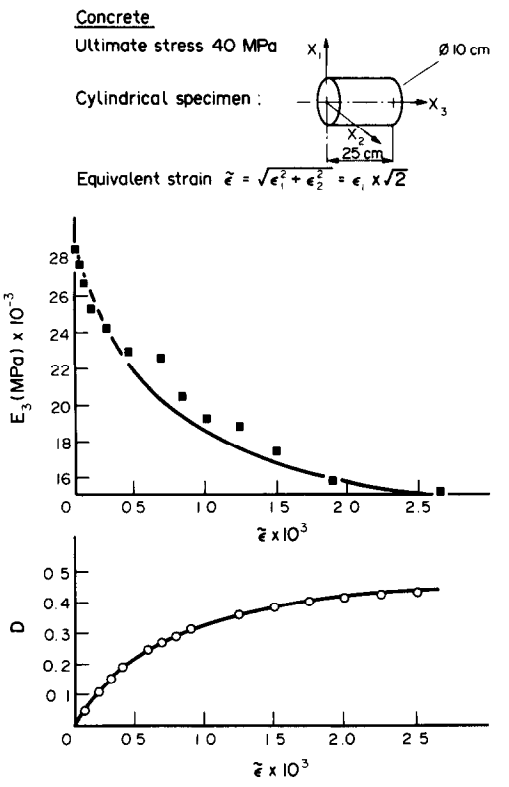

Fig. 7. Evolution of brittle damage in compression for a concrete ($\sigma_u = 40$ MPa)—from [14].

A measure of the longitudinal wave speed of a damaged material gives

$$\tilde{v}_L^2 = \frac{\tilde{E}}{\tilde{\rho}} \frac{1-\nu}{(1+\nu)(1-2\nu)}$$

where $\tilde{E}$ and $\tilde{\rho}$ are the actual damaged elasticity modulus and density. The Poisson's ratio does not vary with damage if elasticity and damage are isotropic.

Damage is calculated by:

$$D = 1 - \frac{\tilde{E}}{E} = 1 - \frac{\tilde{\rho}}{\rho} \frac{\tilde{v}_L^2}{v_L^2} \quad \text{with} \quad \frac{\tilde{\rho}}{\rho} = 1 - D^{3/2}.$$

For small values of D, $(\rho/\tilde{\rho}) \simeq 1$, and one can assume

$$\boxed{D \simeq 1 - \frac{\tilde{v}_L^2}{v_L^2}.}$$

This method is considered here as destructive: indeed, to measure the speed V_L or the time a wave takes to propagate through a certain thickness, one needs to materialize that thickness by two surfaces. If the space distribution of the damage is not uniform, this thickness must be of an order of magnitude coherent with that of the representative volume element and therefore that requires to cut the body to be analysed into parts.

The present limitations of this method are related to this size which, for metals, is too small in comparison with ultrasonic transducers and accuracy of time measurements.

The range of frequency to be used are[15]

1.0 to 50 MHz for metals,

0.1 to 5 MHz for polymers,

0.1 to 5 MHz for woods,

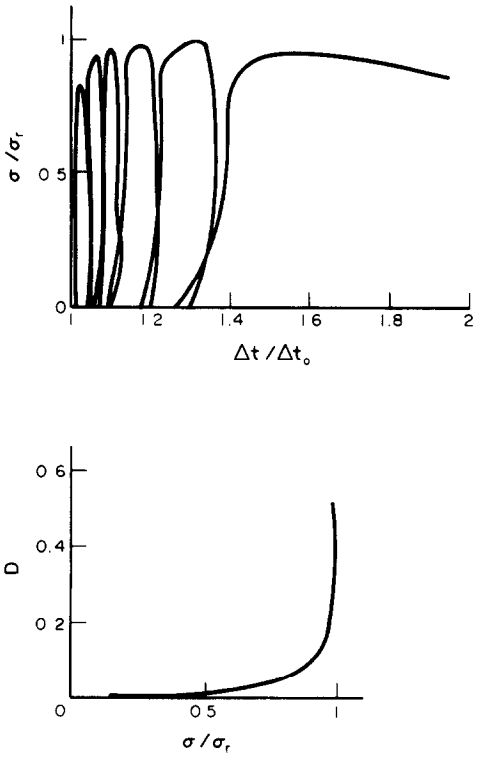

Fig. 8. Evolution of brittle damage from the propagation time Δt of ultrasonic waves in concrete.

0.1 to 1 MHz for concrete.

An example is given in Fig. 8.

A way to improve this method in order to use it "*in situ*" on structures without any destruction consists in working with surfacic waves and also with attenuation of the ultrasonic signal[16].

Variation of the cyclic plasticity response. The influence of damage on plasticity may be used to measure the low cycle fatigue damage.

The cyclic law of plasticity at stabilization may be written as

$$\Delta\epsilon_p = \left(\frac{\Delta\sigma}{K_C}\right)^{M_C}$$

for a non-damaged material, and

$$\Delta\epsilon_p = \left[\frac{\Delta\sigma}{K_C(1-D)}\right]^{M_C}$$

for a damaged material in application of the principle of strain equivalence[2].

Considering a test at constant plastic strain amplitude, if $\Delta\sigma^*$ is the stress amplitude at stabilization at the end of the cyclic softening or hardening period and before the beginning of the damage process:

$$\Delta\epsilon_p = \left(\frac{\Delta\sigma^*}{K_C}\right)^{M_C} = \left[\frac{\Delta\sigma}{K_C(1-D)}\right]^{M_C}$$

from which

$$\boxed{D = 1 - \frac{\Delta\sigma}{\Delta\sigma^*}.}$$

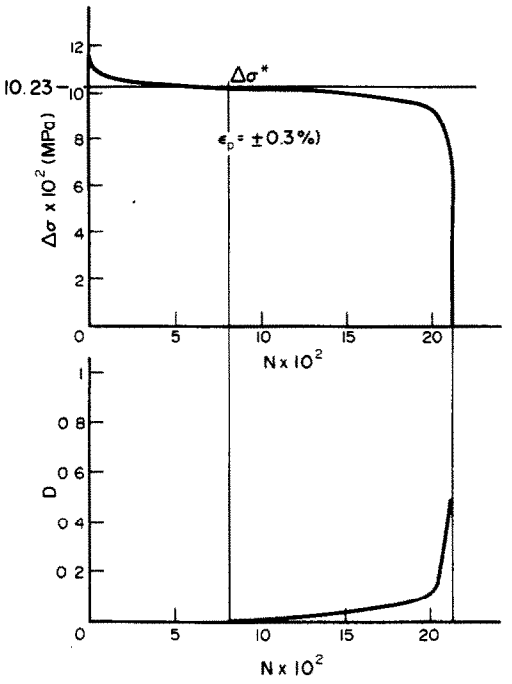

Fig. 9. Evolution of low cycle fatigue damage on AISI 10 deep drawn steel.

This method gives good results to identify the evolution of damage during low cycle fatigue of metal except when the stabilization of the cyclic softening or hardening does not exist. An example is given in Fig. 9[17].

Tertiary creep response. Creep damage occurs in metals loaded at temperatures above approximately one third of the melting temperature. To identify creep damage during creep tests (constant σ) it is convenient to use once more the principle of strain equivalence applied here to the Norton's law of secondary creep.

$$\dot{\epsilon}_p^* = \left(\frac{\sigma}{K_n}\right)^N$$

K_n and N being temperature dependent material coefficients.

Assuming that the damage process begins at the end of secondary creep, during tertiary creep one may write[2]:

$$\dot{\epsilon}_p = \left[\frac{\sigma}{K_N(1-D)}\right]^N$$

from which, one derives

$$\boxed{D = 1 - \left(\frac{\dot{\epsilon}_p^*}{\dot{\epsilon}_p}\right)^{1/N}}$$

where $\dot{\epsilon}_p^*$ is the minimum creep rate.

This method gives good results which are in accordance with those given by the measure of the variation of the elasticity modulus. An example is given in Fig. 10.

4.2 *Non-destructive methods*

The non-destructive methods reported here are those for which, in principle, no specimen or sample is needed. They can be applied "*in situ*" but, of course, a better accuracy is obtained if

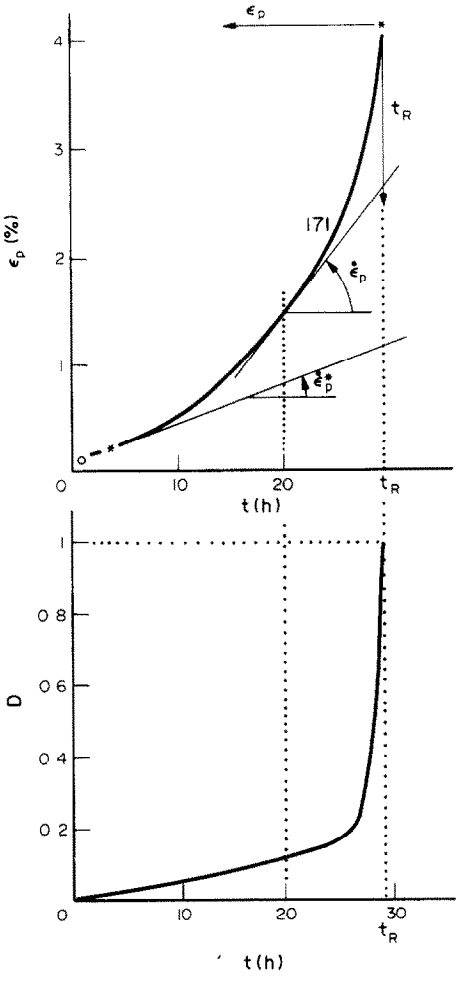

Fig. 10. Evaluation of creep damage on IN 100 superalloy at 1000°C—from [2].

a good preparation of the region tested is made. Generally speaking, they are not so accurate as the destructive methods if applied "*in situ*" but, if applied on specific samples, the accuracy is of the same order of magnitude: ±10% in relative value for D.

Variations of the micro-hardness. The most promising non-destructive method is probably to deduce damage from micro-hardness measurements. This method has been developed very recently but has already given good results[18].

The micro-hardness test consists in indenting a diamond indenter in the material, the hardness H being defined by the mean stress in space:

$$\sigma = H = \frac{F}{S}.$$

The load F on the indenter is chosen to obtain a projected indented area S of the same order of magnitude as that of the representative volume element.

Theoretical analyses and many experimental results prove a linear relationship between H and the plasticity threshold σ_s,

$$H = k\sigma_s.$$

The plasticity criterion can be coupled with damage once more through the concept of the effective stress together with the principle of strain equivalence[2].

$$\frac{\sigma_s}{1-D} - R - \sigma_y = 0$$

where R is the strain hardening variable and σ_y is the yield stress. Consequently, it can be derived that

$$H = k(R + \sigma_y)(1 - D).$$

In fact, the micro-hardness test itself increases the strain hardening by an amount which corresponds to an accumulated plastic strain p_H ($\dot{p} = (\frac{2}{3}\dot{\epsilon}^p_{ij} : \dot{\epsilon}^p_{ij})^{1/2}$) of the order of 5 to 8% [19]. H is then always related to $(p + p_H)$, p being the current accumulated plastic strain.

If $H^* = k(R + \sigma_y)(1 - D)$ is the micro-hardness of the material which would exist without any damage for $(p + p_H)$ and $H = k(R + \sigma_y)(1 - D)$ the actual micro-hardness for $(p + p_H)$, then

$$\boxed{D = 1 - \frac{H}{H^*}.}$$

H is measured and H^* has to be evaluated as proposed below.

(i) For high cycle fatigue, damage occurs while the stress remains below the yield stress so that $p = 0$ and

$$H^*_{(p_H)} = k\sigma_{s(p_H)}.$$

H^* may be measured on a non-damaged part of the material.

(ii) For low cycle fatigue, we may consider that the strain hardening is saturated so that $R = R_\infty$ and:

$$H^*_{(p+p_H)} = k(R_\infty + \sigma_y) = cte.$$

H^* is obtained by a measure on the material fully strain hardened but non-damaged.

(iii) For ductile damage, damage and strain hardening occur simultaneously and H^* has to be obtained by some extrapolation procedure.

Figure 11 shows two examples of measurement of the evolution of ductile damage compared with the results obtained by the measurement of the elasticity modulus decrease.

Apart from the quasi non-destructive character of this method, it enables the evaluation of damage fields in a nice way if the micro-hardness test is automatized.

Variations of the electrical potential. The surfacic definition of damage allows to define an effective intensity of the electrical current in the same way as the effective stress is defined[20].

$$\tilde{i} = \frac{i}{1 - D},$$

$\tilde{i}$ being the intensity which effectively exists in the cohesive parts of a damage volume element.

Given the potential difference V, Ohm's law for a non-damaged element of length l, area s and resistivity r is written as

$$V = r\frac{l}{s}i$$

whereas, for a damaged element of the same size

$$\tilde{V} = \tilde{r}\frac{l}{s}\tilde{i}$$

where $\tilde{r}$ is the resistivity affected by the damage only by means of the change of volume $(\tilde{\rho} - \rho)/\rho$

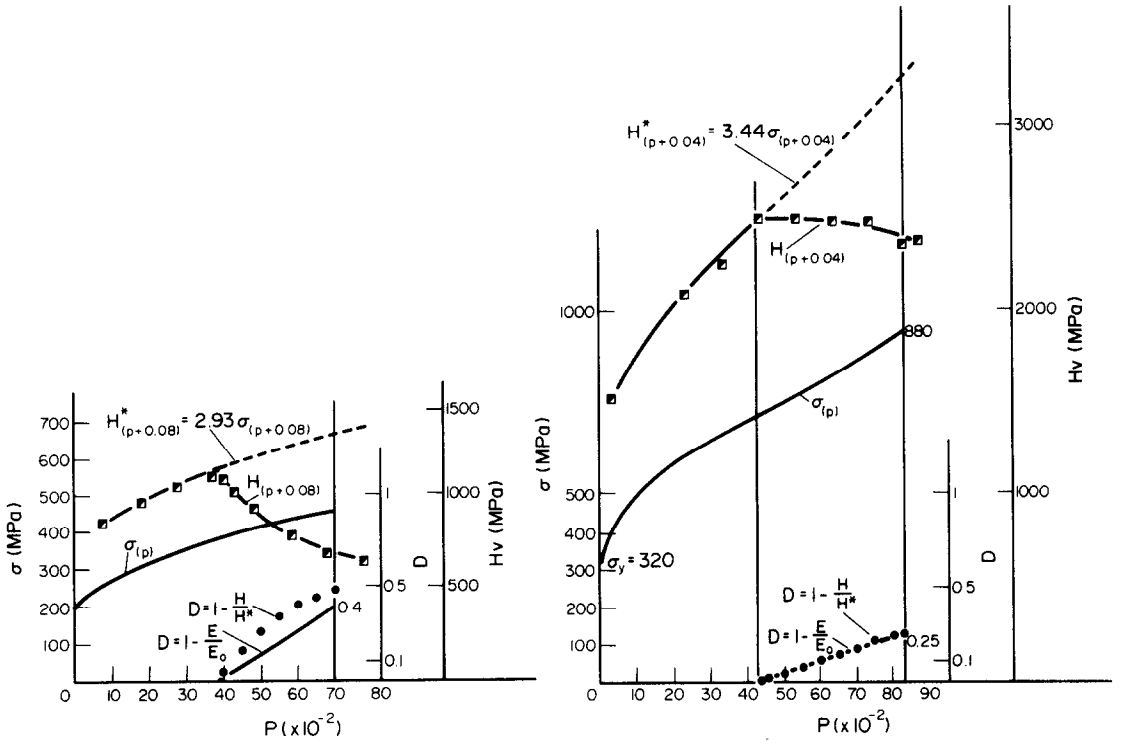

Fig. 11. Evolution of ductile damage on copper and on AISI 1010 annealed steel.

(ρ being the density). Bridgman's law is

$$\tilde{r} = r\left(1 + K\frac{\rho - \tilde{\rho}}{\rho}\right) = 2(1 + KD^{3/2})$$

K is a coefficient and $K \simeq 2$ for metals.

If the same intensity i is considered for the non-damaged and the damaged elements, the damage D may be derived from the two expressions

$$\frac{V}{\tilde{V}} = \frac{r}{\tilde{r}}\frac{\frac{l}{s}i}{\frac{l}{s}\frac{i}{1-D}}$$

$$D = 1 - \frac{V}{\tilde{V}}\frac{\tilde{r}}{r} \quad \text{with} \quad \frac{\tilde{r}}{r} = 1 + KD^{3/2}.$$

For small values of D, the correction term $(\tilde{r}/r - 1)$ due to the volume change is negligible (for instance $D \simeq 0, 1 \rightarrow \tilde{r}/r = 1.064$), then

$$\boxed{D = 1 - \frac{V}{\tilde{V}}.}$$

This method is well known as the "potential drop method"[21].

Some comparisons between measurements of damage obtained with this method and from elasticity modulus variations $D = 1 - \tilde{E}/E$ are given on Fig. 12.

On these examples, as the damage measurements are made during the tension tests, a correction is necessary to take account of the length change $l(1 + \epsilon)$, of the area change $s(1 - 2\nu\epsilon_e - \epsilon_p)$ and of the volume change due to elastic strain: $(1 - 2\nu)\epsilon_p$. The agreement between the results obtained by the two methods is sufficient for the practical accuracy involved.

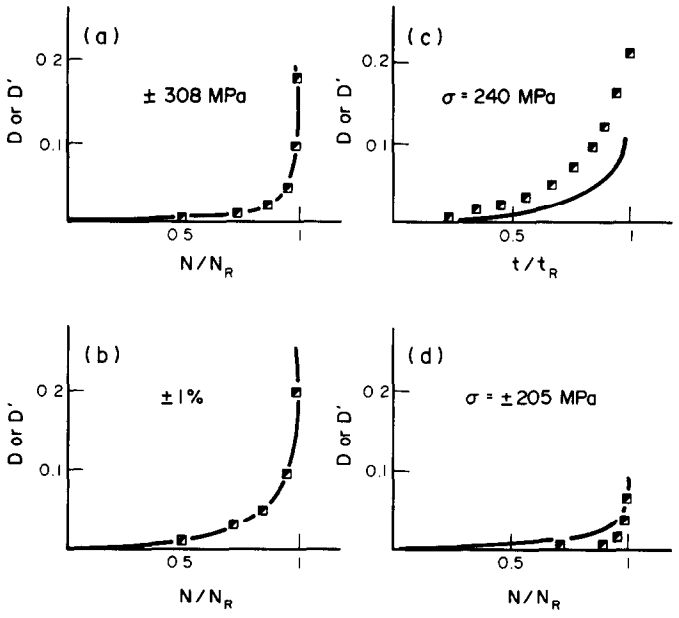

Fig. 12. Comparison between damage measurements by variation of the electrical potential and by (a) the elasticity modulus change on stress controlled fatigue of A 316 stainless steel, (b) the elasticity modulus change on strain controlled fatigue of A 316 stainless steel, (c) the tertiary creep method on creep damage of IN 100 superalloy at 1000°C, (d) the cyclic plasticity response on stress controlled fatigue of IN 100 superalloy at 1000°C.

4.3 *Other methods*

Since damage still remains difficult to define, many other methods have been investigated. They often correspond to different definitions and are difficult to use and to compare with each other: Acoustic emission[22]; Hydrogen absorbtion[23]; X-Ray diffusion and particularly small angle X-Ray diffraction[24] for physical measurements.

Damage may also be deduced from its constitutive equations, for example[25]:

$$\dot{D} = f(Y, D, \dot{p}).$$

If such a law of evolution is identified for a given material, as long as the local history of loading $\sigma(M, t)$ is known, in general, a simple calculation gives the histories of the variables $Y(t)$ (strain energy release rate density) and $\dot{p}(t)$ (accumulated plastic strain rate) and an integration of the constitutive equation yields $D(t)$. This calculation represents more a predictive method than a damage measurement.

5. CONCLUSION

If about ten methods may be applied to measure damage, it means that no one is perfect with excellent results! This is mainly due to the fact that they depend on the definition of damage and on the scale of the phenomenon.

Due to the different mechanisms involved in the damage process in different materials, it is difficult to ensure one basic definition of damage. The surface density of defects used here is probably the most useful but it leads to some difficulties when the damage is not uniformly distributed.

The scale at which measures have to be done may be more crucial for measurement. The "size" of the representative volume element is still a subjective question which needs further studies, but how?

Nevertheless, it is now possible to give some advice about the measure of damage for practical purposes. The chart of Fig. 13 gives an idea (the idea of the authors!) on which method to select in function of the quality required and the difficulty tolerated for each specific type of

Damage		Brittle	Ductile	Creep	Low cycle fatigue	High cycle fatigue
Micrography	$D=\frac{\partial S_D}{\partial S}$	*	**	**	*	*
Density	$D=1-\frac{\tilde{\rho}}{\rho}^{2/3}$		**	*	*	
Elasticity modulus	$D=1-\frac{\tilde{E}}{E}$	**	***	***	***	
Ultrasonic waves	$D=1-\frac{\tilde{V}_L^2}{V_L^2}$	***	**	**	*	*
Cyclic stress amplitude	$D=1-\frac{\Delta\sigma}{\Delta\sigma^*}$		*	*	**	*
Tertiary creep	$D=1-\left(\frac{\dot{\epsilon}_p^*}{\dot{\epsilon}_p}\right)^{1/N}$		*	***	*	
Micro-hardness	$D=1-\frac{H}{H^*}$	**	***	**	***	*
Electrical resistance	$D=1-\frac{V}{\tilde{V}}$	*	**	**	*	*

Fig. 13. Quality chart of methods of damage measurement.

damage, like for a wine selection!:

three stars	***	means "very good"	like "Bourgogne Chambertin 1976"
two stars	**	means "good"	like "Bourgogne Pommard 1979"
one star	*	means "try to see"	like "Beaujolais 1983"
no star		means "do not try"	like ???

A votre santé!

REFERENCES

[1] T. Yokobori, *An Interdisciplinary Approach to Fracture and Strength of Solids*. Wolters, Noordhoff, Groningen (1968).
[2] J. Lemaitre and J. L. Chaboche, *Mécanique des Matériaux Solides*—Dunod (1985).
[3] D. Krajcinovic and G. U. Fonseka, Continuous damage theory of brittle materials. *J. appl. Mech.* **48**, 809 (1981).
[4] P. Suquet, Elements of homogenisation for inelastic materials. CISM Course lecture notes in Physics (1987).
[5] L. M. Kachanov, Time of the rupture process under creep conditions. *Tvz. Akad. Nauk SSR. Otd. Tech. Nauk.* **8**, 26–31 (1958).
[6] J. Lemaitre, Evaluation of dissipation and damage in metals subjected to dynamic loading. *Proc. ICM 1*. Kyoto, Japan (1971).
[7] C. T. Hua and D. F. Socie, Fatigue damage in 1045 steel under constant amplitude biaxial loading. *Fatigue Engng Mater. Struct.* 7(3), 000–000 (1984).
[8] L. Engel and H. Klingele, *Atlas of Metal Damage*. Wolfe Science Books, C.H. Verlag, Munich (1981).
[9] F. Moussy, Microstructure, endommagement et rupture ductile. *Proc. Ecole d'été franco—canadienne* (1986).
[10] D. P. Bompard, Effets endommageants de la porosité sur la propagation des fissures dans le Nickel fritté. Thèse UTC (1986).
[11] J. Lemaitre, J. P. Cordebois and J. Dufailly, Sur le couplage endommagement-elasticité. *C. R. Acad. Sci. Paris*, p. 288 (1979).
[12] J. Dufailly, Modélisation mécanique et identification de l'endommagement plastique des matériaux. Thèse 3ème cycle Université Paris 6 (1980).
[13] M. Poss, Endommagement et rupture des matériaux composites carbon-carbon. Thèse 3ème cycle Université Paris 6 (1982).
[14] J. Mazars, Application de la mécanique de l'endommagement au comportement non linéaire et à la rupture du béton de structure. Thèse 3ème cycle Université Paris 6 (1984).
[15] Y. Berthaud, Mesure de l'endommagement mécanique du béton par méthode ultrasonore. *1er Congrès Int. RILEM*, Paris (Sept. 1987).
[16] C. Maridet, C. Joder and A. Vincent, Mesure de la Vitesse des ondes ultrasonores appliquée à la caractérisation des matériaux. *Matér. Techniques* (Sept., Oct. 1986).

[17] A Benallal, R. Billardon, I. Doghri and L. Moret-Bailly, Crack initiation and propagation analysis taking into account initial strain hardening and damage fields. *Fourth Int. Conf. Numerical Methods in Fracture Mechanics*. San Antonio, Texas, U.S.A., *Proc.* (Edited by A. R. Luxmore). Pineridge Press, Swansea, U.K. (1987).
[18] J. Lemaitre, J. Dufailly and R. Billardon, Evaluation de l'endommagement par mesures de micro-dureté. *C.R. Acad. Sci Paris* T **304**, Sér. II, no. 12 (1987).
[19] F. A. MacClintock and A. S. Argon, *Mechanical Behaviour of Materials*, Chap. 13. Addison Wesley, Reading, MA (1966).
[20] G. Cailletaud, M. Policella and G. Baudin, Influence de l'endommagement sur la résistance électrique des métaux. *C.R. Acad. Sci. Paris* T **292**, *Sér.* 2, 1103 (1981).
[21] M. Policella, G. Baudin and G. Cailletaud, Mesures de longueur de fissures, déformations et endommagement par une technique de potentiel électrique. 60th meeting of S.M. Panel AGARD, San Antonio, U.S.A. (1985).
[22] D. Rouby and P. Fleischmann, *Physique State Solid* (a), 48–439 (1978).
[23] S. Blouet, D. Rault and M. Entringer, Journée sur l'hydrogène dans les aciers. *Soc. Française de Métallurgie*. Metz (1981).
[24] T. Lepisto *et al. Physique State Solid* (a) 67, 93 (1981).
[25] J. Lemaitre, Formulation and Identification of damage kinetic constitutive equations. CISM Course on Continuum damage mechanics. UDINE Italy (Sept. 1986).