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Continuum Damage Mechanics: Part II—Damage Growth, Crack Initiation, and Crack Growth

Continuum Damage Mechanics (CDM) allows the description of the influence of damage on the stress-strain behavior of materials. In the present part, some practical damage growth equations are reviewed for creep, fatigue, creep-fatigue interaction, ductile damage, and brittle damage. The capabilities of CDM to improve both the crack initiation and crack propagation predictive tools are then discussed. Particular attention is given to the new developments of the "local approaches to fracture."

1 Introduction

The general purpose of Continuum Damage Mechanics is to introduce the possibility of describing the coupling effects between damage processes and the stress-strain behavior of materials (Hult, 1979; Chaboche, 1981; Krajcinovic, 1984). Part I of the present paper (Chaboche, 1988) reviewed the main features of CDM and its incorporation in a general thermodynamic framework. The damage growth equations were written in a general form, by means of a dissipative potential.

The present part develops some useful and practical damage rate equations. Different damage variables are associated with different damage processes such as creep, fatigue, ductile and brittle damage. Various growth equations are considered and discussed on the basis of some examples on various kinds of materials. They can be incorporated in the general framework presented in Part I.

Applications of CDM and, more precisely, of the damage growth equations are possible in two domains:

(*a*) To improve the life prediction techniques, with the purpose of calculating damage growth and crack initiation in structures. Several domains can be considered: high cycle and low cycle fatigue, creep, and creep-fatigue.

(*b*) To improve the macroscopic crack growth calculation techniques, usually based on the Fracture Mechanics concepts. Including the damage processes explicitly allows additional possibilities of improving the "local approaches to fracture," which are now extensively developed. Sections 3.2 and 3.3 consider the respective capabilities of the two approaches.

2 Damage Growth Equations

2.1 Creep Damage. Continuum Damage Mechanics was developed first for the case of creep damage (Kachanov, 1958; Rabotnov, 1969). The Rabotnov-Kachanov equation can be considered as very classical. With D varying between 0 for the undamaged material and 1 for the rupture, under pure tensile stresses, the expressions reduce to:

$$dD=\left(\frac{\sigma}{A}\right)^{r}(1-D)^{-k}dt \quad (1)$$

where r, k, A are material and temperature dependent coefficients. Their determination is made from constant stress creep data, for which the integration from 0 to 1 gives the rupture time:

$$t_c=\frac{1}{k+1}\left(\frac{\sigma}{A}\right)^{-r} \quad (2)$$

while evolution of damage is given by:

$$D=1-\left(\frac{t}{t_c}\right)^{1/k+1} \quad (3)$$

The concept of effective stress, introduced in the secondary creep law (Norton's equation), allows the calculation of tertiary creep curves as well as the prediction of the change in creep ductility (Chaboche, 1978, 1984). Figure 1 gives an example:

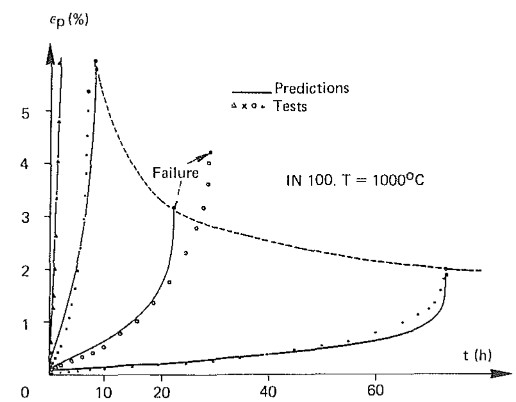

Fig. 1 Calculated and measured creep curves on superalloy IN 100. Prediction of creep ductility.

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$$\dot{\epsilon}_p = \left[\frac{\sigma}{K(1-D)}\right]^n \tag{4}$$

Under multiaxial stress conditions the damage equations can be generalized by describing isodamage surfaces (or isochronous surfaces) defined with proper stress invariants: (*a*) the octahedral shear stress $J_2(\sigma)$ related to the effects of shear; (*b*) the hydrostatic stress $J_1(\sigma) = Tr(\sigma) = \sigma\text{: } \mathbf{1}$, which greatly affects the growth of the cavities; and (*c*) the maximum principal stress $J_0(\sigma) = \sigma_1$, which opens the microcracks and causes them to grow.

Following the method of Hayhurst (1972), the equivalent stress can be defined through a linear combination:

$$\chi(\sigma) = \alpha\, J_o(\sigma) + \beta\, J_1(\sigma) + (1-\alpha-\beta) J_2(\sigma), \tag{5}$$

where α, β are coefficients dependent on the material and temperature. The time to failure under a fixed multiaxial stress is expressed as:

$$t_c = \frac{1}{k+1}\left\langle\frac{\chi(\sigma)}{A}\right\rangle^{-r} \tag{6}$$

The creep damage equations can be considered as taking into account the evolution of microstructural defects in an indirect manner. Works by the material scientists have shown that the increase of damage results from a combination of two mechanisms, the nucleation and growth of cavities (Greenwood, 1973; Dyson, 1979). It is possible to make some connections between the equations obtained from material science and the more macroscopic ones developed in the framework of CDM (Hayhurst, 1983).

For the multiaxial stress criterion, the physical interpretations often lead to a product form:

$$\chi(\sigma) = [J_o(\sigma)]^{\alpha}[J_2(\sigma)]^{1-\alpha} \tag{7}$$

which can describe adequately the isochronous surfaces in a large domain (Hayhurst, 1983; Delobelle, 1985). However, the form (7) automatically gives no damage under pure compression, which is not perfectly true in polycrystalline metals, as shown by two level creep tests (Policella et al., 1982). Let us mention a possible different form for the damage growth equation, using the creep strain instead of time (Constesti, 1986):

$$dD = A[J_o(\sigma)]^{\alpha}\epsilon_{eq}^{\beta} d\epsilon_{eq} \tag{8}$$

such a form was deduced from cavity measurements on notched specimens. It is possible to show that equations (1) and (4) are equivalent to equation (8) through the following one-to-one mapping (Contesti, 1986):

$$D \rightarrow [1-(1-D)^{k-n+1}]^{\beta+1} \tag{9}$$

2.2 Fatigue Damage. In the case of fatigue several aspects have to be considered:

(*a*) the existence of microinitiation and micropropagation stages;

(*b*) the nonlinear-cumulative effects for two-level tests or block-program loading conditions;

(*c*) the existence of a fatigue limit, but its marked decrease after prior damage;

(*d*) the effect of mean-stress either for the fatigue limit or for the S-N curves.

Fatigue damage accumulation models have been considered by Marco and Starkey (1954), Manson (1979), and Chaboche (1974). A common general form is obtained with:

$$dD = D^{\alpha(\sigma_M, \bar{\sigma})}\left[\frac{\sigma_M - \bar{\sigma}}{M(\bar{\sigma})}\right]^{\beta} dN \tag{10}$$

where σ_M and $\bar{\sigma}$ are, respectively, the maximum and mean stresses. Several choices for α (Chaudonneret and Chaboche, 1986) lead to the rules considered by Manson (1979), Subramanyan (1976), Hashin and Laird (1980). The key of the nonlinear effect is the dependency of α on σ_M and $\bar{\sigma}$ which, after integrating for a two-level test, gives:

$$\frac{N_2}{N_{F_2}} = 1 - \left(\frac{N_1}{N_{F_1}}\right)^{\frac{1-\alpha_2}{1-\alpha_1}} \tag{11}$$

The function $M(\bar{\sigma})$ is deduced from a linear dependency between $\bar{\sigma}$ and the fatigue limit.

This cumulative damage equation allows a very good description of multilevel fatigue tests (Chaboche, 1974). In a certain sense it includes in a continuous way the microinitiation and micropropagation phases as discussed by Manson (1979), Cailletaud and Levaillant (1984), and Miller and Zachariah (1977). Moreover, by a convenient variable change, equation (10) can be incorporated into Continuum Damage Mechanics, with the effective stress concept, as shown by damage measurements described by Lemaître and Chaboche (1978, 1985). The microcrack measurements made by Cailletaud and Levaillant (1984), Hua and Socie (1984), and Socie et al. (1983), for instance, then show the possible equivalence between: (*a*) the definition by the effective stress concept; (*b*) the definition in terms of the remaining life concepts (subjacent in equation (10)); and (*c*) the quantification of physical damage, in terms of microcracking.

In the case of Low-Cycle Fatigue, the conventional parametrization of the life is written in terms of the plastic (or total) strain range. Provided the existence of a one-to-one relation between σ_M and $\Delta\epsilon_p$ (the cyclic curve) equation (10) can still be used and contains independently the influence of mean-stress.

The generalization to multiaxial loading conditions is a difficult problem. At least two parameters have to be considered: (*a*) an equivalent shear-stress amplitude, and (*b*) a mean (or maximum) hydrostatic stress. Experiments near the fatigue limit show the independency in the mean shear-stress and in the hydrostatic range (Dang Van, 1973). A possible form to generalize equation (10) has been proposed by Chaboche (1978). Additional studies are required to generalize such models in the case of nonproportional loading conditions.

2.3 Creep-Fatigue Interaction. One advantage of the CDM approach in creep and fatigue is to allow a natural way to predict creep-fatigue interaction (Chrzanowski, 1976). The simplest hypothesis consists in a direct summation of creep and fatigue damages, which leads to (Chaboche, 1980):

$$dD = f_c(\sigma, D)dt + f_F(\sigma_M, \bar{\sigma}, D)dN \tag{12}$$

where f_c is, for instance, deduced from equation (1) and f_F from equation (10). These two functions can be determined from pure tensile creep tests and pure fatigue tests (high frequency). The conditions at low-frequency or with hold times are then predicted by integrating numerically equation (12). This approach has given good results for several materials (Lemaître and Chaboche, 1985; Plumtree and Lemaître, 1979; Del Puglia and Vitale, 1982).

Let us note that the additive hypothesis does not correspond to the direct addition of physical damages of different natures. For instance, microcracks and cavities are not added. Only their mechanical effects are added, in the framework of the effective stress concept. The mechanical effects are obtained by the above mentioned one-to-one mappings between the physical damage (cracks or cavities) and the corresponding macroscopic variable D (see Sections 2.1 and 2.2).

2.4 Oxidation-Fatigue-Creep Interactions. At high temperature, the influence of time is often increased by the superposition of oxidation processes (Rezai-Aria and Rémy, 1986). The oxidation may enhance both the fatigue and creep mechanisms, contributing to both damage nucleation (in the form of surface cracks, preferential grain boundary oxidation,

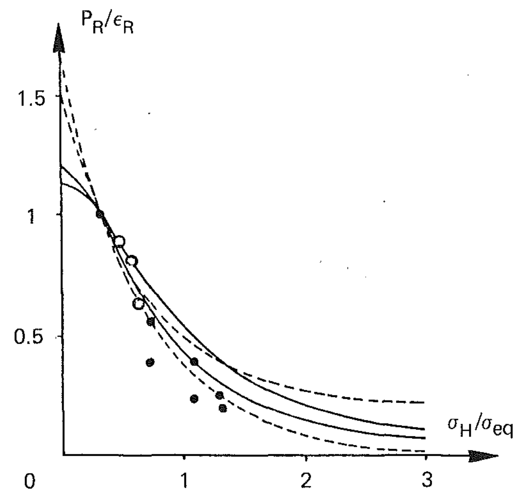


Fig. 2 Influence of triaxiality on fracture strain. • •: A508 steel; o o: H steel; = = = = =: domain covered by McClintock/Rice-Tracey models; ═══: domain covered by the present model.

or internal voids) and damage growth (crack tip deterioration, void growth from particles).

Such additional factors have been considered as acting on the fatigue damage process in terms of a fatigue damage model including both an initiation and a propagation period (Cailletaud and Levaillant, 1984). The incorporation of oxidation in the creep damage equations can be considered as implicit (on a conceptual point of view).

2.5 Ductile Plastic Damage. Ductile damage in metals is essentially the initiation and growth of cavities due to large deformations. Experiments of ductile rupture show that the dissipative potential ϕ^* can be expressed, in the framework of time-independent plasticity (Lemaître and Chaboche, 1985):

$$\phi^* = \tilde{\sigma}_{eq} - R - k + \frac{S}{2}\frac{1}{1-D}\left(-\frac{Y}{S}\right)^2 \tag{13}$$

$$\dot{\epsilon}_p = \lambda \frac{\partial \phi}{\partial \sigma}$$

$$\dot{r} = -\lambda \frac{\partial \phi^*}{\partial R} = \left(\frac{2}{3}\dot{\epsilon}_p:\dot{\epsilon}_p\right)^{1/2}(1-D) = \dot{p}(1-D) \tag{14}$$

$$\dot{D} = -\lambda \frac{\partial \phi^*}{\partial Y} = \frac{-Y}{S}\dot{p} \tag{15}$$

Using the relation (14) of Part I (Chaboche, 1987a), between σ_{eq} and $-Y$, and a particular choice for hardening:

$$\sigma_{eq} = R + k + K\dot{p}^{1/M} \tag{16}$$

K and M being coefficients, which are material dependent, one obtains the following differential constitutive equation for ductile plastic damage (Lemaître, 1985):

$$\dot{D} = \frac{K^2}{2ES}\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right]p^{2/M}\dot{p} \tag{17}$$

In the case of radial loading, when the principal directions of stresses do not vary, the triaxiality ratio σ_H/σ_{eq} is constant and this expression may be integrated using the conditions:

$p < p_D$ (damage threshold) $\rightarrow D = 0$

$p = p_R$ (strain to rupture) $\rightarrow D = D_c$.

Neglecting elastic strain in the calculation of p and using $p_D/p_R = \epsilon_D/\epsilon_R$, the equation for damage evolution may be written in terms of the one-dimensional threshold ϵ_D and one-dimensional strain to rupture ϵ_R:

$$D = \frac{D_c}{\epsilon_R - \epsilon_D}\left\langle p\left[\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right] - \epsilon_D\right\rangle \tag{18}$$

Identification of such a model consists in the quantitative evaluation of the coefficients ϵ_D, ϵ_R, and D_c (Poisson's ratio being known from elasticity), which can be done from experimental measurements such as those mentioned in Section 2.4 of Part I.

As shown in Fig. 2, the present model gives the influence of triaxiality on the strain to failure (Lemaître, 1985). It compares fairly well with the McClintock (1968) or Rice and Tracey (1969) models and can be used to predict the fracture limits of metal forming (Lemaître, 1984).

Let us note that the modified thermodynamic framework developed by Rousselier (1981) leads to similar results. It fits directly the Rice and Tracey model for a special choice of free energy. This theory allows a correct prediction of plastic instabilities and can be used to predict ductile fracture by means of local approaches (Rousselier et al., 1985; Rousselier, 1986). Let us mention also some work based on homogenization concepts which, in the case of ductile fracture, gives information about the damage evolution equations (Suquet, 1982; Dragon and Chihab, 1985).

2.6 Brittle Damage and Elastic Behavior. The failure of some brittle materials can be treated simply by considering the coupling between damage and the elastic behavior. Such theories have been developed for concrete by Krajcinovic and Fonseka (1981), Mazars (1985), and Marigo (1985). The possibilities can be illustrated by means of a very simple damage growth equation (Lemaître and Chaboche, 1985), written here for pure tension:

$$dD = \begin{cases} \left(\dfrac{\epsilon}{\epsilon_o}\right)^s d\epsilon & \text{if} \quad \epsilon = \xi \quad \text{and} \quad d\epsilon = d\xi > 0 \\ 0 & \text{if} \quad \epsilon < \xi \quad \text{or} \quad d\epsilon < 0 \end{cases} \tag{19}$$

where ϵ_o and s are material constants and ξ is a variable threshold ($d\xi = d\epsilon$ when $\epsilon = \xi$). With the initial conditions $D = \xi = \epsilon = 0$, integrating (19) and using the elastic behavior (equation (11) of Part I) leads to:

$$D = \left(\frac{\epsilon}{\epsilon_R}\right)^{s+1} \qquad \sigma = E\,\epsilon\left[1 - \left(\frac{\epsilon}{\epsilon_R}\right)^{s+1}\right] \tag{20}$$

where $\epsilon_R = [(s+1)\,\epsilon_o^s]^{1/s+1}$ is the rupture strain, when $D = 1$.

Figure 3 illustrates the stress-strain evolution obtained with $s = 2$ and the corresponding load-unload behavior for the damaged elastic material. As shown in Fig. 3(*b*), such a simple model correctly predicts the observed behavior of concrete under pure compression (from Krajcinovic and Fonseka, 1981).

2.6 Damage Anisotropy. The directional nature of damage is clear in many situations. In creep, for instance, microstructural observations have identified two classes of metallic materials (Leckie and Hayhurst, 1984; Hayhurst, 1983): (*a*) for copper, cavitation takes place on grain boundaries more or less perpendicular to the maximum principal stress, and (*b*) for aluminum alloys, grain boundary cavitation is much more isotropically distributed.

Clearly these observations relate to the anisotropy of the damage rate equation. In Continuum Damage Mechanics the problem of anisotropy concerns also the effect of damage on the constitutive behavior. Several levels of theories can then be considered, where the damage variable is considered as a scalar function (Leckie and Onat, 1980), vectors (Krajcinovic and Fonseka, 1981; Krajcinovic and Silva, 1982), or second (Kachanov, 1980) or fourth-order tensors (Chaboche, 1979; Chaboche, 1981). See also Krajcinovic (1984) for a more complete review.

The scalar measure of damage anisotropy is defined (in the creep range) by considering the grain boundaries orthogonal to the direction n and the fraction of boundaries which are cavitated. Then, to each direction n is associated a scalar valued function $V(n)$. It was shown by Leckie and Onat (1980) that a tensorial decomposition of such elementary damage measure leads to even-order tensors.

However, if one considers the damage as produced by small parallel cracks, it is possible to associate a vector to each crack direction (Krajcinovic and Fonseka, 1981). Such a theory, similar to the slip theory of plasticity, can be developed with a similar thermodynamic framework as the one presented in Part I, introducing, for instance, a vector as the dual variable to the present damage variable (instead of the scalar Y or of the fourth-order tensor $\mathbf{Y}$ defined by Lemaître and Chaboche, 1985). This kind of theory is very attractive and gives the direct distinction between active damage (open microcracks) and passive damage (closed microcracks). The main difficulty is the number of independent systems which have to be considered in a general case, but several useful applications have been obtained for concrete under special loading cases (Krajcinovic and Fonseka, 1981).

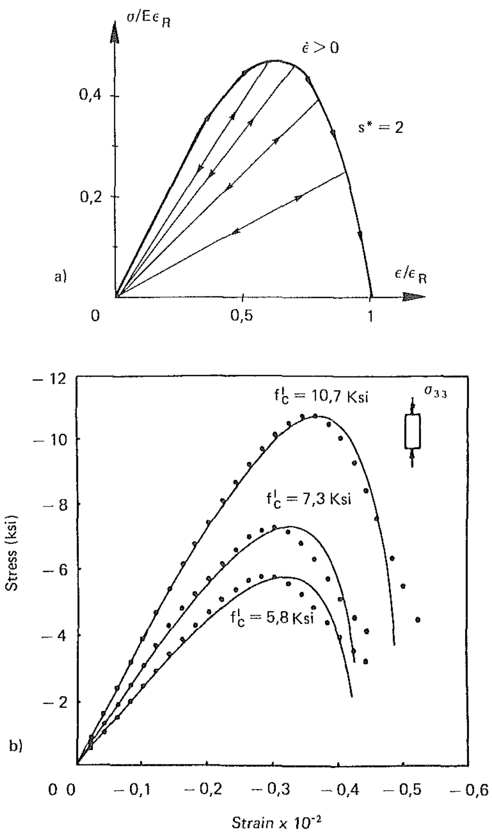


Fig. 3 (*a*) Stress-strain curve with the elastic behavior coupled with damage; (*b*) example of concrete under compression (from Krajcinovic and Fonseka, 1981)

Another method of representing the actual state of damage is to use a second-order symmetric damage tensor (Kachanov, 1980) as suggested by the relation (4) of Part I. The theory developed by Murakami and Ohno (1980) introduces the net stress concept in the damage growth equation and an effective stress tensor, through a fourth-order transformation, in the constitutive equation. The anisotropy of the damage growth equations is a linear combination of the isotropic case and a pure anisotropic one.

The damage state can be represented directly as a nonsymmetrical fourth-order tensor, in the framework of the effective stress concept for the constitutive behavior (Chaboche, 1979). Damage measurement follows directly from the methods mentioned in Section 2.4 of Part I. As in the previous theory the damage growth equation uses a linear combination of the isotropic case and a simple anisotropic one. Systematic comparisons between these two theories have shown their similarities and their capabilities to describe actual anisotropic damage states (Murakami and Imaizumi, 1982; Chaboche, 1984).

3 Life Prediction in Structures

3.1 Prediction of Crack Initiation. The life calculations of structures at high temperature incorporate two aspects treated independently or successively: (*a*) the macrocrack initiation is the important aspect for the design methodologies, and (*b*) the crack propagation prediction is used for design and maintenance through the concept of "Damage Tolerance."

Relative to the first step, the *classical approach* considers successively the calculation of the stress-strain state at each point of the structure and the prediction of crack initiation from the obtained stress or strain fields. In the case of Low-Cycle-Fatigue such calculations involve the full cyclic inelastic

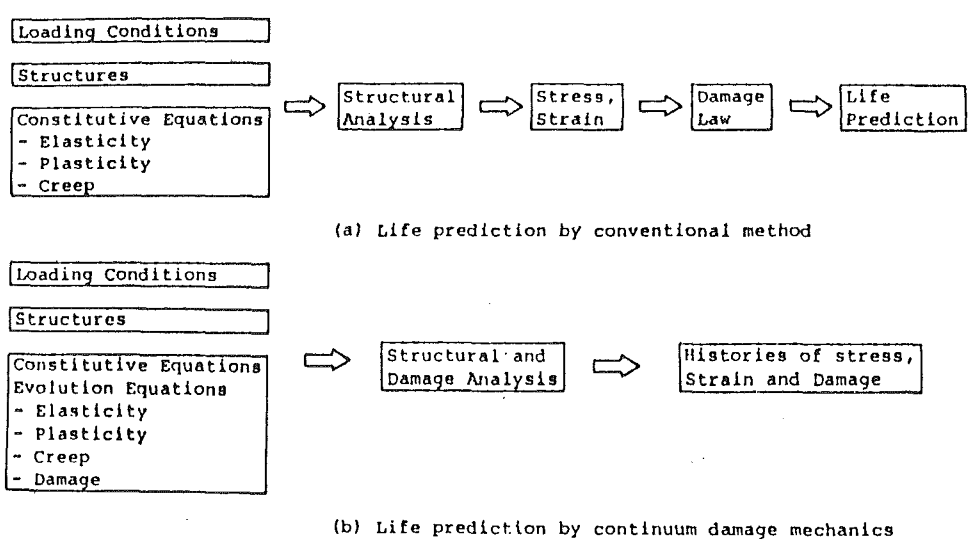


Fig. 4 Comparison between life predictions by conventional method (*a*) and that of continuum damage mechanics (*b*)

analysis to obtain relevant cyclic stabilized values. The difficulties associated with the stabilization of the solution were underlined elsewhere (Cailletaud and Chaboche, 1986).

One of the problems encountered in practical applications is the problem of fatigue damage nucleation and growth from initial stress concentrations. The macroscopic stress/strain gradients play an important role as compared to the microlevel, and the total volume of highly stressed material could be different. This difficulty is generally overcome by one of the two following ways: (*i*) taking into account the macro and micro-plasticity which induce stress redistribution, and (*ii*) introducing a critical distance, depending on the material.

Future approaches will consider the coupling phenomena between the progressive deterioration of the material and its stress-strain behavior, in the framework of Continuum Damage Mechanics, as shown schematically in Fig. 4, taken from Marquis et al. (1981). The existence of a stabilized cycle in the structure is no longer possible and the physical processes at each point of the structure have to be followed by the mathematical model during the whole life, that is for thousands of cycles.

Application of these concepts in fatigue, or more specifically in High-Temperature-Low-Cycle-Fatigue with hold periods, is not possible at the present time for actual structures, even with the most powerful computers. However, the increase of computer capacity and rapidity, the decrease of computing time cost, and the research of special algorithms to perform "cycle jumps" could allow some practical possibilities. Methods have been examined by Savalle and Culié (1978) and applied to the case of the tension-compression specimens.

In the creep situation, on the contrary, the coupling effect has been already considered in several applications (Marquis et al., 1981; Hayhurst, 1983) and permits the improvements of life predictions.

3.2 Crack Propagation by Fracture Mechanics Concepts. The classical approach to predict crack growth and fracture in structural components is based on Fracture Mechanics concepts. Crack growth laws and failure criteria are based on global parameters such as the stress intensity factor K, the elastic strain energy release rate G, the contour integral J. In many cases Linear Elastic Fracture Mechanics constitutes a practical tool to predict crack growth, especially in a situation of non-dissipative "small scale yielding" materials. There is no doubt about its utility and worthiness in the past and in the future.

However, in some situations this global approach presents some difficulties, even inconsistencies, especially when material nonlinearities play an important role, for example in ductile fracture or creep crack growth. Three developments for high temperature problems are briefly discussed below.

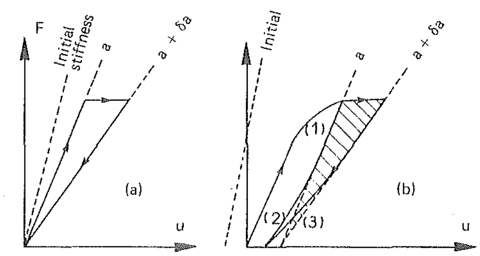
3.2.1 Creep-Fatigue Interaction for Brittle Materials. Analysis of crack propagation in the framework of linear fracture mechanics is possible under high temperature fatigue, even under conditions of creep-fatigue interaction, at least for materials with a low ductility. The case of IN 100, treated by Policella and Lesne (1986), is very representative. Under constant temperature conditions the following results have been obtained:

(1) The stress intensity factor K, calculated elastically or measured through the elastic strain energy release rate G (change in the measured stiffness of the specimen) is a useful parameter to correlate crack growth.

(2) Pure fatigue crack propagation (5 Hz frequency) is well correlated by a simple power law.

(3) Pure creep crack growth is described by a similar equation, using time instead of cycle.

(4) Fatigue with creep hold periods can be predicted from



(1) → Plastic flow during loading.
(2) → Actual unloadings before and after crack growth.
(3) → Fictitious elastic unloadings.

Fig. 5 Schematics of the elastic energy release rate: (*a*) elastic brittle solid; (*b*) elasto-plastic brittle solid

the addition of the two preceding contributions (with time integration of the creep term).

In the present case, the material has a low ductility but, due to the high temperature (1000°C) there are viscoplastic strains in the whole specimen. Linear Fracture Mechanics continues to apply because viscoplastic strain concentrates more at the crack tip. The concept of an elastic energy release, necessary to propagate the crack is justified by comparing the stiffness before and after the crack increment (Lemaître and Chaboche, 1985). In such an extension of Linear Fracture Mechanics to creeping materials, the stiffness is considered during an infinitely rapid unloading, inducing elastic changes in the whole structure (only compressive flow at the crack tip); by this way the conditions appear similar to that of "small scale yielding." Figure 5 illustrates schematically the definition of the elastic strain energy release (assuming here that the crack propagates at the maximum load). Let us note the consistency of using an "elastic" global parameter such as this with general thermodynamic concepts (Germain et al., 1983).

3.2.2 Application Under Thermal Fatigue. In thermal fatigue, the main loads applied to the structure are thermal ones. Linear Fracture Mechanics can still be used, defining properly the elastic energy release rate (De Lorenzi, 1982), especially for materials with a low ductility.

However, direct application of Linear Elastic Fracture Mechanics is incorrect. It must be applied *after* a preliminary elasto-plastic analysis (or elasto-viscoplastic) of the structure, without crack, in order to correctly take into account the global stress redistributions (external loading redistribution) induced by inelastic flow.

The example of the plate submitted to cyclic thermal gradients (Lesne and Chaboche, 1984), without any external mechanical loads, illustrates clearly the point that, due to compressive stresses in the heated region, where the crack propagates, the stress intensity factor would be negative if calculated directly with the elastic fields.

The procedure detailed by Policella and Lesne (1986) leads to a possible predictive way, using Linear Fracture Mechanics for a crack which has grown in the initially plastified structure (initial plastic strain field is obtained through the complete inelastic analysis of the structure without the crack). In this way, the crack tip plasticity is not included in the analysis, which clearly corresponds to the "small scale yielding" assumption and justifies the use of some crack growth equations determined with similar conditions (Policella and Lesne, 1986).

3.2.3 Application to Ductile Materials. For more ductile materials, difficulties are encountered in the application of the Linear Fracture Mechanics; there is no longer a one-to-one correlation between K and the crack growth rate, especially under creep conditions. Nonlinear Fracture Mechanics introduces different global parameters, such as J in ductile rup-

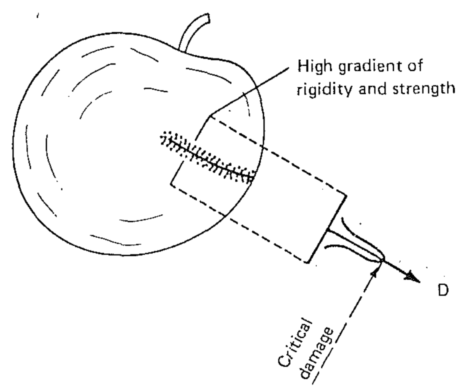


Fig. 6 Crack as a damaged zone

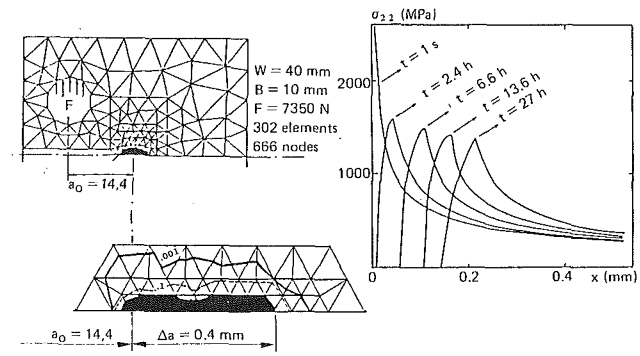


Fig. 7 Creep crack growth prediction by a local approach for a CT specimen in INCO 718 at 600°C

ture, ΔJ in fatigue (Low-Cycle-Fatigue), C^* in creep. These parameters are justified only for special inelastic constitutive behavior and the correlations are not significantly better (Bensussan et al., 1985).

An alternative global approach can be developed to treat ductile materials; continue to use a linear elastic fracture parameter such as the stress intensity factor (or the elastic energy release rate), extending the concept of "small scale yielding" for fictitious elastic unloadings, as shown above (Lemaître and Chaboche, 1985). This variable can be considered as the thermodynamic force associated to the crack length (Germain et al., 1983). In other words, its definition has nothing to do with the actual behavior of the material; K (or G) plays the role of the stress in classical plasticity. The influence of nonlinear behavior, of history effects, and of material processes at the crack tip will then be specified through the use of additional internal variables (as variable threshold parameters used by Pellas et al., 1977; Baudin and Robert, 1984) and additional growth equations:

$$\begin{aligned} da &= f(K, K_s, \ldots)dt \\ dK_s &= g(K, K_s, \ldots)dt \end{aligned} \tag{21}$$

In the case of fatigue crack growth it was shown by Lemaître and Chaboche (1985) that the dissipation is proportional to the effective stress intensity $K_{\text{eff}} = K_{\max} - K_s$, where K_s corresponds to the crack opening, chosen here as a growth rate threshold.

Let us remark that the present approach, considering only the linear elastic energy release rate, is a theoretical concept. It does not suppose the real measurement of the compliance, but considers simply the stiffness change induced by the geometry change when the crack grows (neglecting the possible influence of the overall inelastic deformation on the compliance). In other words, the elastic unloadings have to be considered as fictitious, allowing to define the same driving force as in the commonly accepted small scale yielding. Moreover, the crack closure effects, during tension or compression unloading periods, are taken into account by the phenomenological parameter K_s in equation (21).

3.3 Local Approaches to Fracture. As mentioned above some difficulties are encountered in applying global concepts of Fracture Mechanics to some complex situations including short cracks, history effects (overloads, warm-prestress, etc.), ductile fracture, and creep crack growth.

Even if some correlations or parametrizations have been found satisfactory in many cases, alternative methods, called "local approaches," have been under development for several years (Janson and Hult, 1977). They consider the actual behavior at the crack tip, trying to calculate as precisely as possible the local stress and strain fields and the corresponding deterioration. Local failure criterion allows a crack increment to be predicted, and eventually the crack instability.

In fact, two levels can be considered for local approachs:

• Numerical methods using discrete crack increments: One uses the techniques of node release to produce the crack growth when some critical value of a physical quantity is reached at some critical distance ahead of the crack tip (Newman, 1974; Hinnerirhs, 1980; Anquez, 1983; Devaux and Mottet, 1984). This physical quantity can be an equivalent stress or strain, energy, or some measure of local damage. In such situations the prediction of crack growth using this method will be mesh dependent (Newman, 1974), but the local mesh size is fixed by means of statistical considerations (Rousselier, 1986; Mudry, 1986). Simplified approaches can be developed, using analytical stress-strain fields near the crack tip (Maas and Pineau, 1985).

• Use of Continuum Damage Mechanics, including the progressive deterioration and the corresponding stiffness reduction. The crack (damaged zone) is then taken as the locus of material points with no rigidity, where damage has reached its critical value (Fig. 6). Conceptually, due to the stress redistribution associated with viscoplasticity and strength decrease, there is no need to introduce critical distance or node release techniques for crack growth simulation. After some maximum, the stresses in the plastic zone decrease continuously as damage increases, approaching the crack tip. This approach was studied first by Hayhurst et al. (1975) in the creep domain and is now under a large development in ductile fracture (Rousselier et al., 1985; Rousselier, 1986), creep crack growth (Saanouni et al., 1985; Hayhurst, 1985), fatigue (Ben Allal et al., 1984), and rupture of concrete (Mazars, 1985).

Among many others (Lemaître, 1985), several applications of local approaches have been done:

• From a theoretical point of view, the use of CDM with a discontinuous damage growth law (sudden change from 0 to 1) solves Rice's paradox, giving consistency to the energy dissipated at the crack tip. Moreover, such simplified damage laws allow analytical solutions for simple structures (Bui and Ehrlacher, 1985; Bui et al., 1986).

• The application of the first level of local approaches to fatigue can be considered now as classical (Newman, 1974; Anquez, 1983). For example, it gives correct predictions of crack opening or crack closure, even under complex loading conditions (multi levels, overloads, underloads, etc.).

• Fracture of brittle materials such as concrete has been simulated in various ways (Mazars, 1985; Bazant and Pijaudier-Cabot, 1987) and for several kinds of components, using continuum damage concepts and taking into account the coupling between damage and the elastic behavior, as mentioned in Section 2.6.

• In the case of ductile fracture two levels have been applied (Devaux and Mottet, 1984; Rousselier et al., 1985; Rousselier, 1986), introducing or not the notion of damage. The local mesh size is fixed through "critical distance" considerations or a statistics of defects in the volume element. Good predictions have been obtained with the two methods, comparing notched axisymmetric specimens, cracked specimens, and compact tension specimens.

• Creep crack growth has also been predicted using local approaches of both types (Saanouni et al., 1985; Hayhurst, 1985; Walker and Wilson, 1984; Tvergaard, 1986). In the case of using CDM concepts the main difficulties are associated with the time integration of the very stiff differential equations (Walker and Wilson, 1984). Special automatic time stepping has been developed for points approaching the rupture condition ($D = 1$). Figure 7 shows an application to the creep crack growth in a C.T. specimen in Inconel 718 at 600°C (Saanouni et al., 1985).

Such local approaches of fracture are very attractive, due to the possibility of implementing more physical rupture criteria and damage processes at the crack tip. They have already given good results in laboratory simulations and are beginning to be applied at an industrial level, especially in the case of cleavage and ductile fracture (in Devaux et al., 1986, for instance). Examples where local approaches have a benefit over classical fracture mechanics have been presented by Rousselier et al. (1985), Bensussan et al. (1985), Mudry (1985), and Devaux et al. (1986).

Several problems delay a more general use:

• *The cost of calculations.* However, the difference isn't so large if an inelastic analysis has also to be performed to apply Fracture Mechanics. Simplified procedures, using some analytical fields, could offer an intermediate, more practical, way.

• *The dependence on the finite element modeling, which seems a common feature of the various local approaches.* Both the "crack width", the crack growth rate, and the failure load depend on the chosen mesh size, even when using the complete elastic-plastic-damage coupling as by Saanouni et al. (1985). From a practical point of view the local mesh size has to be fixed in every application after checking a particular one. Another attractive way may be to introduce a nonlocal definition for damage growth, as already considered by Rousselier et al. (1985), Bazant and Pijaudier-Cabot (1987), Billardon (1986), and Saanouni and Lesne (1987).

4 Conclusion

Some practical damage rate equations have been briefly presented and discussed, in the framework of Continuum Damage Mechanics. Applications were shown on various kinds of materials, concerning: creep processes, fatigue processes, creep-fatigue interaction, ductile plastic damage of metals, and brittle damage, especially in concrete. In each case, uniaxial and multiaxial equations have been presented. Such damage growth theories can be used to predict crack initiation in structural components, as shown recently in various applications. Moreover, the use of CDM allows some interesting additional possibilities in the framework of local approaches to fracture.

This rational approach of damage proceeds as an extension of Continuum Mechanics. It allows many possibilities, introducing, for instance, some conncections between microstructure measurements and mechanical parameters. Moreover, this theory finds applications in many different situations (various loading conditions, various materials, various physical processes, etc.).

In that it concerns life predictions in structures, Damage Mechanics allows one to take into account the coupling effects between deterioration processes and mechanical behavior. It will certainly be developed further and extensively used in the future as a complementary tool, between Continuum Mechanics and Fracture Mechanics.

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