2013

Advanced Foundation Engineering

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CHAPTER 4: EARTH PRESSURE

Chapter 4

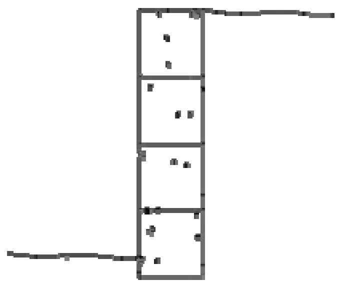
Earth Pressure

4.1 Introduction

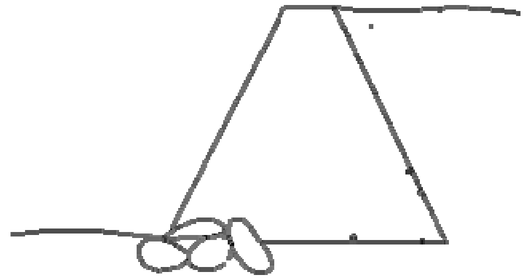
Steep soil slopes will not remain stable for a long period of time; therefore, in order to maintain a nearly vertical face, some support must be provided. The most common permanent form of support is the retaining wall; different types of retaining walls are shown in Fig 4.1. These walls are considered to be rigid and the design of these structures requires an estimate of the earth pressures that act on the structure. The induced earth pressures are caused by the weight of the wall, the weight of the backfill, and if present, by external loads acting on the wall or the backfill.

Ideally the base of a retaining wall should be below the ground surface in order to provide resistance against sliding, frost action, and to increase bearing capacity. Preferably, the backfill should consist of free-draining soils and a drainage system should be provided to prevent the build-up of water pressure and ice lenses. The selection of construction materials and the size of the components should be optimized to provide a safe structure at minimum cost.

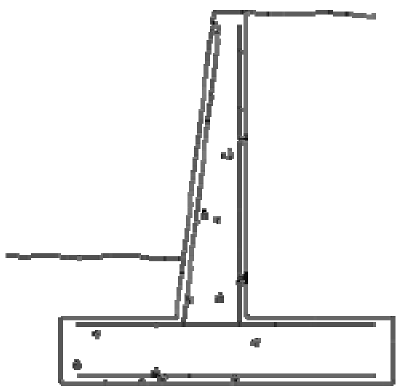
A common type of retaining wall used by many homeowners is shown in Fig 4.1(a). This type of wall is usually unsatisfactory since it has minimal resistance to overturning. Although the gravity wall [Fig 4.1(b)] is satisfactory, it is not as economical as a cantilever wall [Fig 4.1(c)]. Relatively massive crib walls [Fig 4.1(d)] are occasionally used on the side of major highways. Rock-filled wire gabions [Fig 4.1(e)] are frequently used for earth support and for erosion protection. Their popularity stems partly from the fact that they are relatively easy to construct, they require minimal engineering, and they are relatively flexible which means that significant differential settlements can be tolerated. A reinforced earth wall [Fig 4.1(f)] is a relatively new type of wall and has been used in many forms including bridge abutments. The reinforcing elements are typically strips of galvanized steel but other materials such as plastics have been introduced. The facing units can consist of steel, reinforced concrete, plastic, etc. and if the expense can be justified, decorative facing units may be selected. Basement walls [Fig 4.1(g)] are a special form of retaining wall; with these walls, it is assumed that there is no lateral movement of the wall or the backfill.



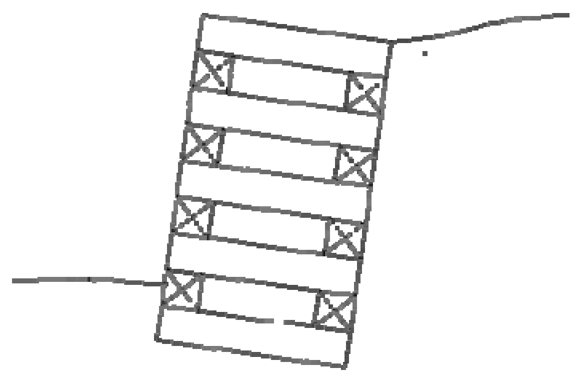
(a) Concrete block wall
(not recommended)



(b) Gravity wall
field stone, ledge stone
or mass concrete



(c) Reinforced concrete
cantilever wall



(d) Crib wall
structural members consist of timbers, steel,
or precast concrete - usually filled with rock

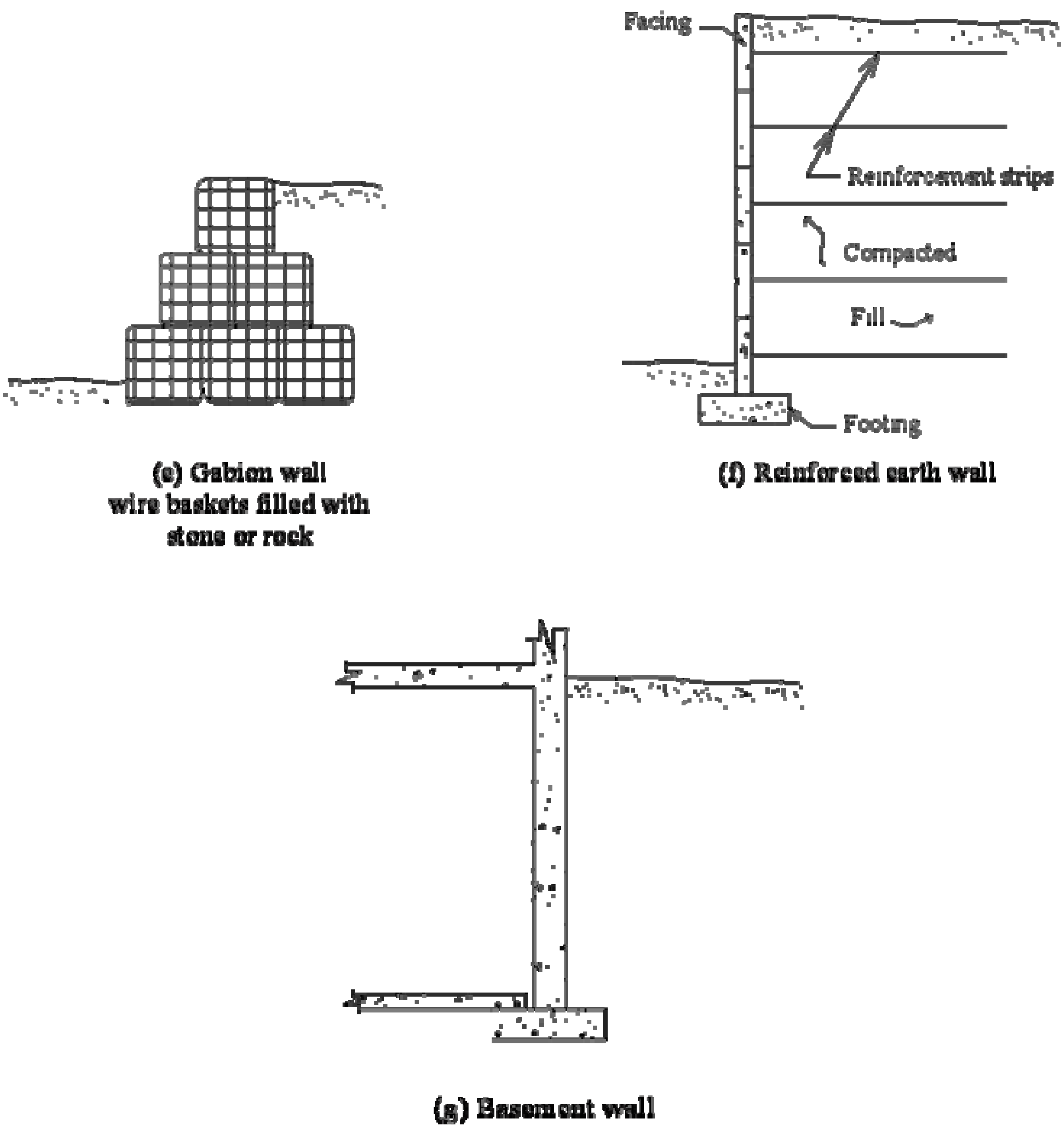


Fig 4.1 Common types of retaining walls

Computation of lateral earth pressure is required in the analysis and design of rigid retaining walls, sheet pile walls and other earth retaining structures. In this chapter the determination of lateral earth pressure on rigid retaining walls is dealt with.

A rigid retaining wall can be either of R.C.C or masonry, and is used to maintain difference in levels of ground surfaces on either side of it. The soil retained on the back side of wall is referred to as backfill. The problem of determining the lateral earth pressure has been investigated both theoretically and by experimental work since as far back as 1860, when Rankine proposed his theory considering plastic equilibrium of soil. To quantify the lateral earth pressure, three types of earth pressure depending on three possible conditions have to be considered. They are

1. Earth pressure at rest (e.g. bridge abutments)
2. Active earth pressure and (e.g. wall moves away from the soil)
3. Passive earth pressure (e.g. wall moves towards the soil)

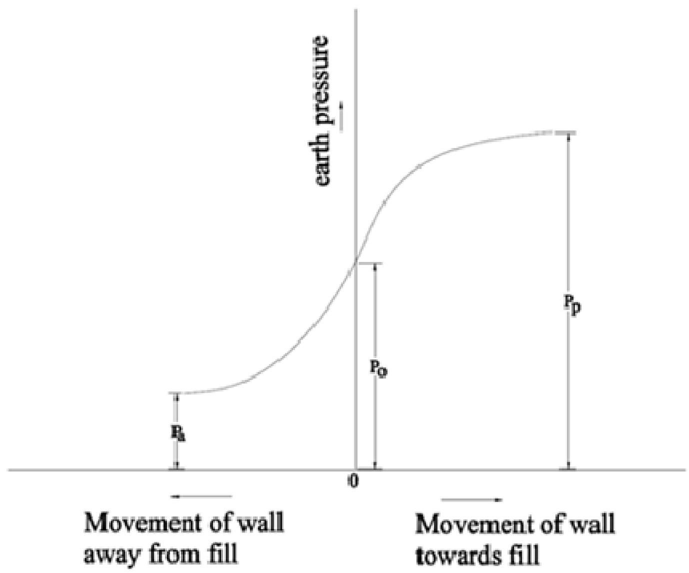


Fig 4.2 Three types of earth pressure

When the base of the wallis assumed to be rigidly fixed and the wall does not move, the pressure exerted by backfill on the back of the wall is referred to as earth pressure at rest (p_0). When the base of the wall yields and the wall moves away from the fill, the earth pressure decreases to a certain minimum value. This minimum value of earth pressure is referred to as active earth pressure (p_a). When the base of the wall yields and the wall moves towards the fill, the earth pressure increases to a certain maximum value. This maximum value of earth pressure is referred to as passive earth pressure (p_p).

4.2 Earth Pressure at Rest

The earth pressure at rest exerted on the back of a rigid retaining wall can be determined using theory of elasticity assuming the backfill soil to be elastic, homogenous, isotropic and semi-infinite.

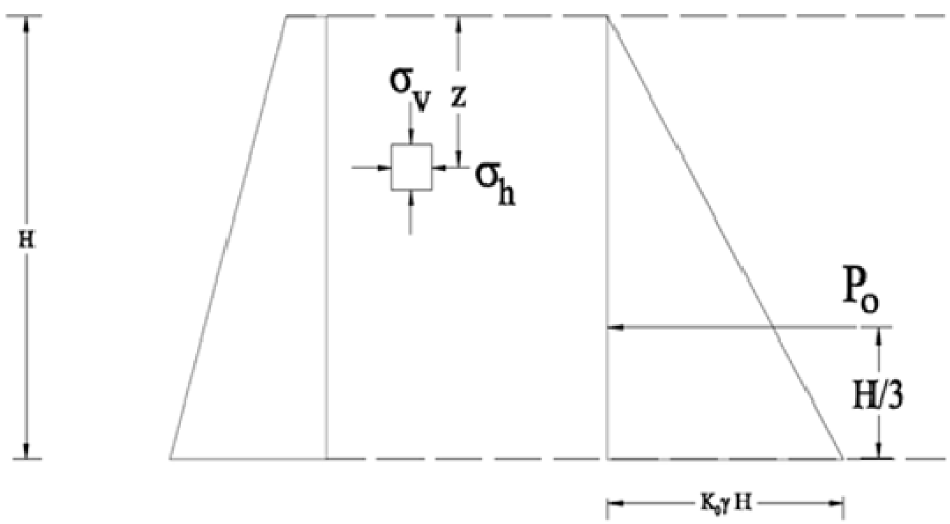


Fig 4.3 Earth pressure at rest

Consider an element at depth z below the surface of the backfill. Let σ_v and σ_h denote the vertical and horizontal stresses acting on the element, as shown in Fig 4.3. In the absence of shear stresses on the planes of σ_v and σ_h, the lateral strain ϵ_h in the horizontal direction is given by

$$\epsilon_h = \frac{1}{E}[\,\sigma_h - \mu(\sigma_h + \sigma_v)]$$

Forthe condition of earth pressure at rest, $\epsilon_h = 0$.

$$\therefore \sigma_h - \mu(\sigma_h + \sigma_v) = 0$$

i.e., $\sigma_h = \left(\frac{\mu}{1-\mu}\right)\sigma_v$

\ or $\sigma_h = K_0 \sigma_V$ Eq 4.1

where $K_0 = \frac{\mu}{1-\mu}$ Eq 4.2

K_0 is called the coefficient of earth pressure at rest.

Substituting p_0 for σ_h and γ_z for σ_V in equation Eq 4.1

we can write $p_0 = K_0\gamma z$ Eq 4.3

p_0 denotes the intensity of earth pressure at rest at any depth z.

At $z = 0$, $p_0 = 0$

At $z = H$, $p_0 = K_0\gamma H$.

The distribution of earth pressure at rest behind the wall is shown in Fig 4.3.

If we denote the resultant earth pressure per unit length perpendicular to plane of figure by P_0, then we have

P_0 = area of earth pressure distribution diagram

$= \frac{1}{2}(K_0\gamma H)H$

$= \frac{1}{2} K_0\gamma H^2$

Alternatively P_0 can be obtained as shown below.

$$P_0 = \int_0^H K_0\gamma z.dz = K_0\gamma\int_0^H z.dz = K_0\gamma\left[\frac{z^2}{2}\right]_0^H$$

$$= \frac{1}{2} K_0\gamma H^2.$$

Since soils are not perfectly elastic materials, they do not have well defined values of Poisson's ratio.

4.3 Rankine Active and Passive States of Plastic Equilibrium

Rankine (1860) investigated the plastic state of equilibrium. A mass of soil is said to be in plastic equilibrium if every point of it is on the verge of failure. The Mohr Circle which is widely used in the study of elastic equilibrium state can also be used to analyze the stress conditions in plasticequilibrium state.

In Fig 4.4(a) is shown Mohr circle for the active state of plastic equilibrium and in Fig 4.4(b) the corresponding slip planes.

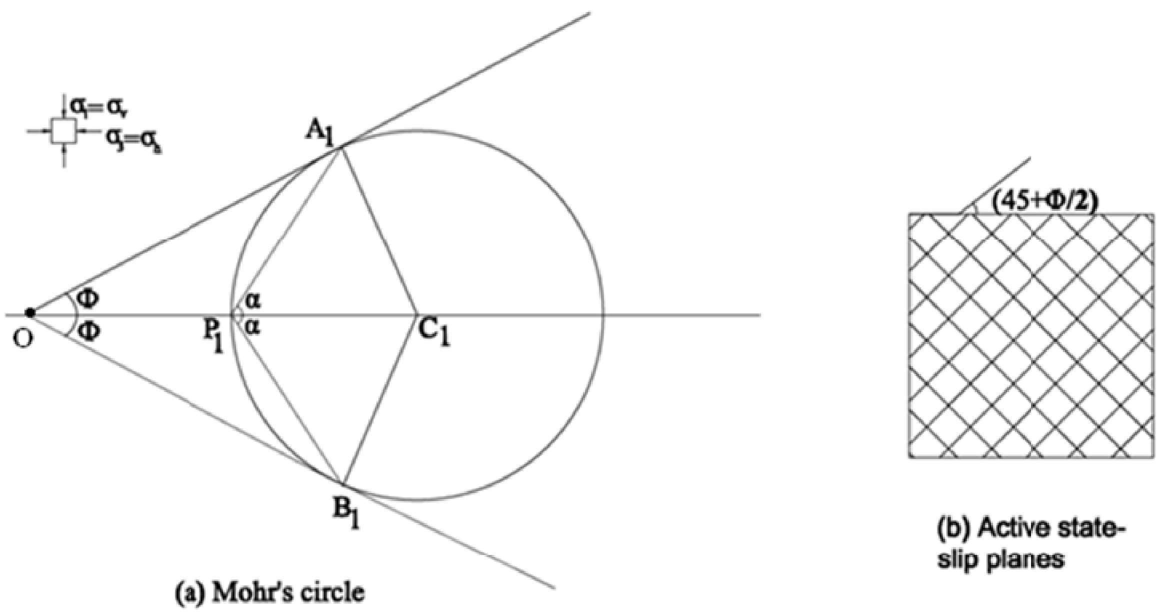


Fig 4.4 Rankine – Active state of plastic equilibrium

In the active state of plastic equilibrium $\sigma_3 = \sigma_h$ and $\sigma_1 = \sigma_v$, P_1 is the pole. P_1A_1 and P_1B_1 represent the slip planes. Clearly the slip planes are inclined to the horizontal at $\alpha = \left(45° + \frac{\emptyset}{2}\right)$.

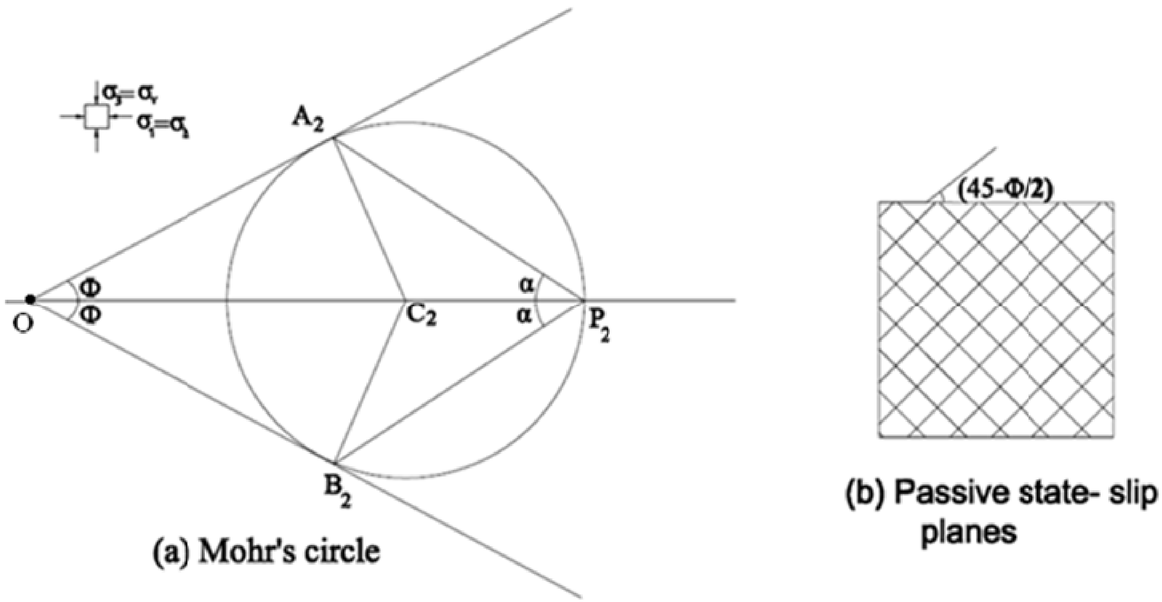


Fig 4.5 Rankine-Passive state of plastic equilibrium

In Fig 4.5(a) is drawn the Mohr Circle and in Fig 4.5(b) the slip planes for the passive state of plastic equilibrium, in which condition $\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_v$. P_2 is the pole. P_2A_2 and P_2B_2 represent the slip planes. Clearly the slip planes are inclined to the horizontal at $\alpha = \left(45° - \frac{\emptyset}{2}\right)$.

4.3.1 Active Earth Pressure of Cohesionless Soil by Rankine's Theory

Following are the assumptions made in the originally proposed Rankine's theory.

1. The soil mass is homogenous and semi-infinite.
2. The soil mass is cohesionless and dry.
3. The surface of soil is a plane which may be horizontal or inclined.
4. The back of the wall is vertical.
5. The back of the wall is smooth, so that there will be no shearing stresses between the wall and soil. Because of this assumption the stress relationship for any element adjacent to the wall is the same as that for any other element far away from the wall.
6. The wall yields about the base and thus satisfies the deformation condition for plastic equilibrium.

Because of the assumption that there is no friction between the soil and wall, the resultant earth pressure must be parallel to the surface of backfill. However in practise the back of the retaining walls constructed of masonry or concrete will never be smooth and the resultant active earth pressure will be inclined to the normal to the back of the wall at an angle equal to the angle of friction between the soil and back of wall.

In the following discussion the originally proposed Rankine's theory has been used to derive expression for coefficient of active earth pressure and then extended to fully submerged, partially submerged and stratified soil deposits.

Case 1: Dry or moist backfill with no surcharge

Consider an element at depth z below the surface of backfill. Let σ_v and σ_h denote the vertical and horizontal stresses acting on this element. In the active state of plastic equilibrium we have,

$\sigma_3 = \sigma_h$ and $\sigma_1 = \sigma_v$ Eq 4.4

The relationship between principal stresses in the plastic equilibrium condition is

$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$ where $\alpha = 45° + \frac{\emptyset}{2}$.

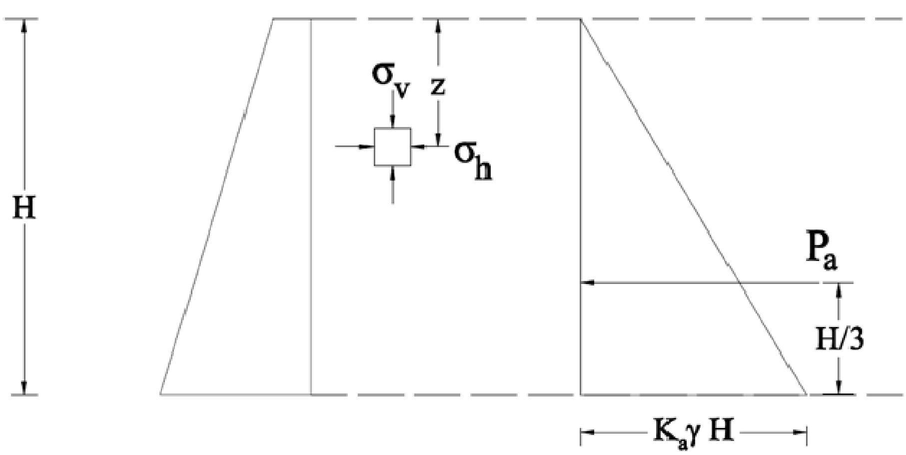


Fig 4.6 Active earth pressure due to moist back fill

For cohesionless soil c=0.

Therefore $\sigma_1 = \sigma_3 \tan^2\alpha$ Eq 4.5

Substituting Eqn 4.4 in Eqn 4.5 we get

$\sigma_v = \sigma_h \tan^2\alpha$

$\therefore \quad \sigma_h = \sigma_v \cot^2\alpha$

We write $\sigma_h = K_a \sigma_v$ Eq 4.6

where $K_a = \cot^2\alpha = \frac{1-\sin\emptyset}{1+\sin\emptyset}$

K_a is called Rankine's coefficient of active earth pressure.

Substituting p_a for σ_h and γz for σ_v in Eqn 4.6 we get the intensity of active earth pressure at depth z as

$p_a = K_a\gamma z$ Eq 4.7

At z=0, $p_a = 0$

At z=H, $p_a = K_a\gamma H$

The earth pressure distribution diagram is shown in Fig 4.6.

The resultant or total active earth pressure per unit length perpendicular to plane of figure is equal to the area of earth pressure distribution diagram.

$P_a = \frac{1}{2}(K_a\gamma H)(H) = \frac{1}{2}K_a\gamma H^2$

P_a acts at distance $\frac{H}{3}$ above base.

P_a can also be obtained as shown below.

$P_a = \int_0^H K_a\gamma z.dz = K_a\gamma\int_0^H z.dz = K_a\gamma\left[\frac{z^2}{2}\right]_0^H$

$= \frac{1}{2}K_a\gamma H^2$

Case 2: Backfill with surcharge

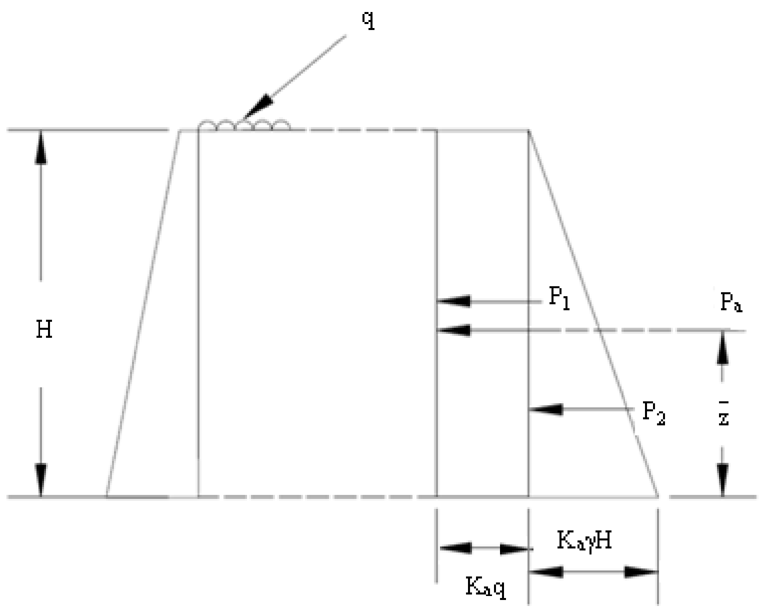


Fig 4.7 Backfill with surcharge

In the Fig 4.7 is shown a backfill with a surcharge q per unit area acting on its surface. At any depth z, intensity of active earth pressure,

$p_a = K_a\gamma z + K_a q$

At z=0, $p_a = K_a q$

At z=H, $p_a = K_a\gamma H + K_a q$

The active earth pressure distribution is as shown in Fig 4.7. Let the resultant active earth pressure per unit length of wall act at distance $\bar{z}$ above base.

We have,

$P_1 = K_a qH$ acting at distance $\frac{H}{2}$ above base.

$P_2 = \frac{1}{2}K_a\gamma H^2$ acting at distance $\frac{H}{3}$ above base.

Resultant active earth pressure $P_a = P_1 + P_2$

i.e $P_a = K_a qH + \frac{1}{2}K_a\gamma H^2$

Taking moments about the base, we get

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_a}$$

Case 3: Fully submerged backfill

In Fig 4.8 is shown a fully submerged backfill. At any depth z , we have

$p_a = Ka\gamma^1 z + \gamma_w z$

At $z=0$, $p_a = 0$

At $z=H$, $p_a = K_a\gamma^1 H + \gamma_w H$

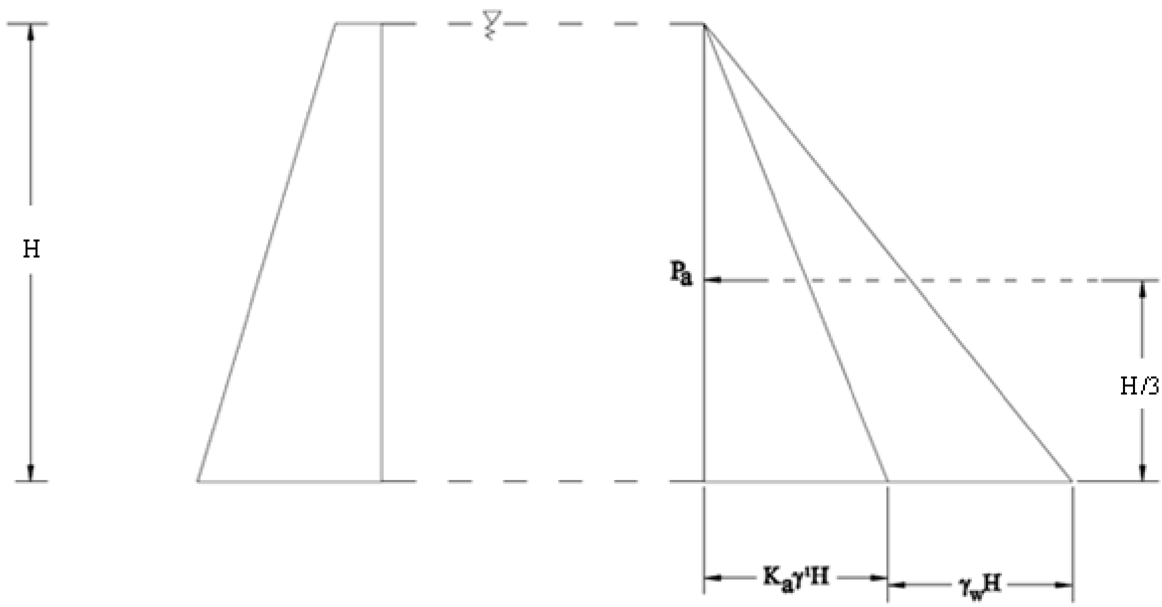


Fig 4.8 Fully submerged backfill

The active earth pressure distribution, is shown in Fig 4.8. The resultant active earth pressure is given by the area of pressure distribution diagram.

$$P_a = \frac{1}{2}(K_a\gamma^1 H + \gamma_w H)H = \frac{1}{2} K_a\gamma^1 H^2 + \gamma_w H^2$$

acting at distance $\frac{H}{3}$ above base.

Case 4: Partially submerged backfill

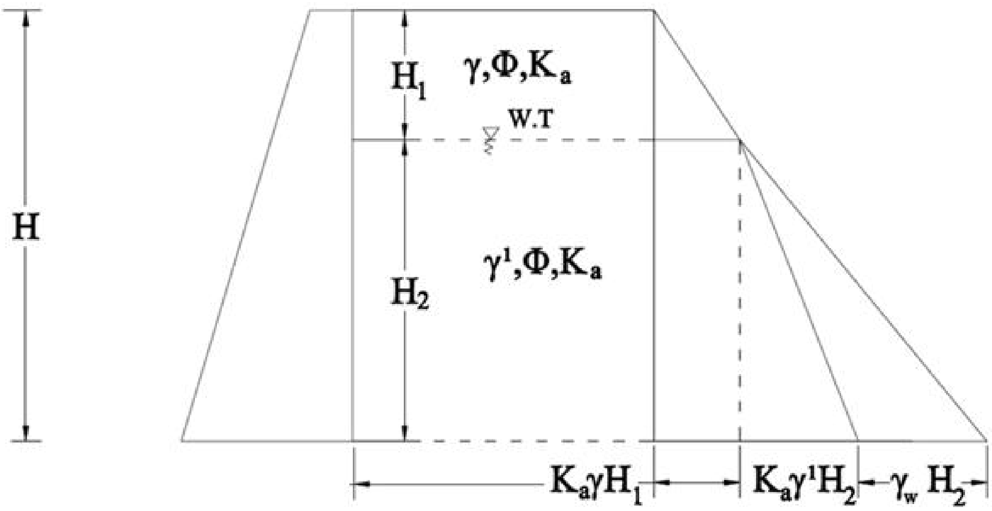


Fig 4.9 Partially submerged backfill

In Fig 4.9 is shown a partially submerged backfill with no change in ∅ on submergence.

For $0 \le z \le H_1$,

$p_a = K_a \gamma z$

At $z = 0$, $p_a = 0$

At $z = H_1$, $p_a = K_a \gamma H_1$

For $H_1 \le z \le H$,

$p_a = K_a \gamma H_1 + K_a \gamma^1 (z - H_1) + \gamma_w (z - H_1)$

At $z = H_1$, $p_a = K_a \gamma H_1$

At $z = H$, $p_a = K_a \gamma H_1 + K_a \gamma^1 (H - H_1) + \gamma_w (H - H_1)$

$= K_a \gamma H_1 + K_a \gamma^1 H_2 + + \gamma_w H_2$

The active earth pressure distribution is shown in Fig 4.9

Case 5: Partially submerged backfill taking into account reduction in ∅ onsubmergence

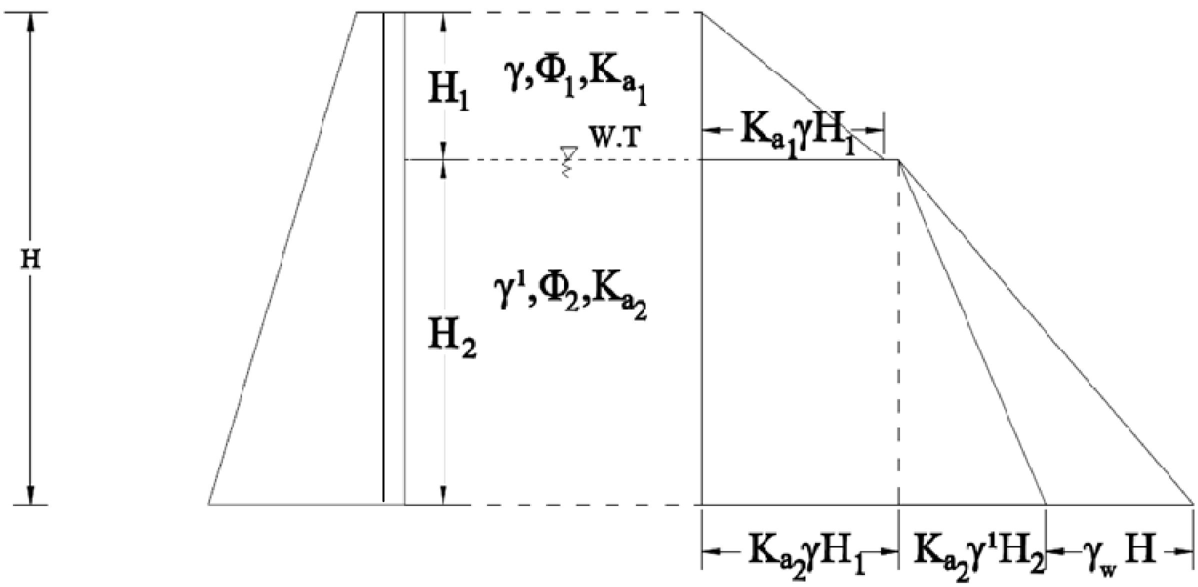


Fig 4.10 Partially submerged backfill

In the Fig 4.10 is shown partially submerged backfill in which ∅ is shown reduced on submergence from $\emptyset_1$ to $\emptyset_2$. Since $\emptyset_2 < \emptyset_1$, we have $K_{a2} > K_{a1}$.

For $0 \leq z \leq H_1$,

$p_a = K_{a1}\gamma z$

At $z = 0$, $p_a = 0$

At $z = H_1$, $p_a = K_{a1}\gamma H_1$

For $H_1 \leq z \leq H$,

$p_a = K_{a2}\gamma H_1 + K_{a2}\gamma^1(z - H_1) + \gamma_w(z - H_1)$

At $z = H_1$, $p_a = K_{a2}\gamma H_1$

At $z = H$, $p_a = K_{a2}\gamma H_1 + K_{a2}\gamma^1(H - H_1) + \gamma_w(H - H_1)$

$= K_{a2}\gamma H_1 + K_{a2}\gamma^1 H_2 + \gamma_w H_2$

The earth pressure distribution diagram is as shown in Fig 4.10

Case 6: Backfill with sloping surface

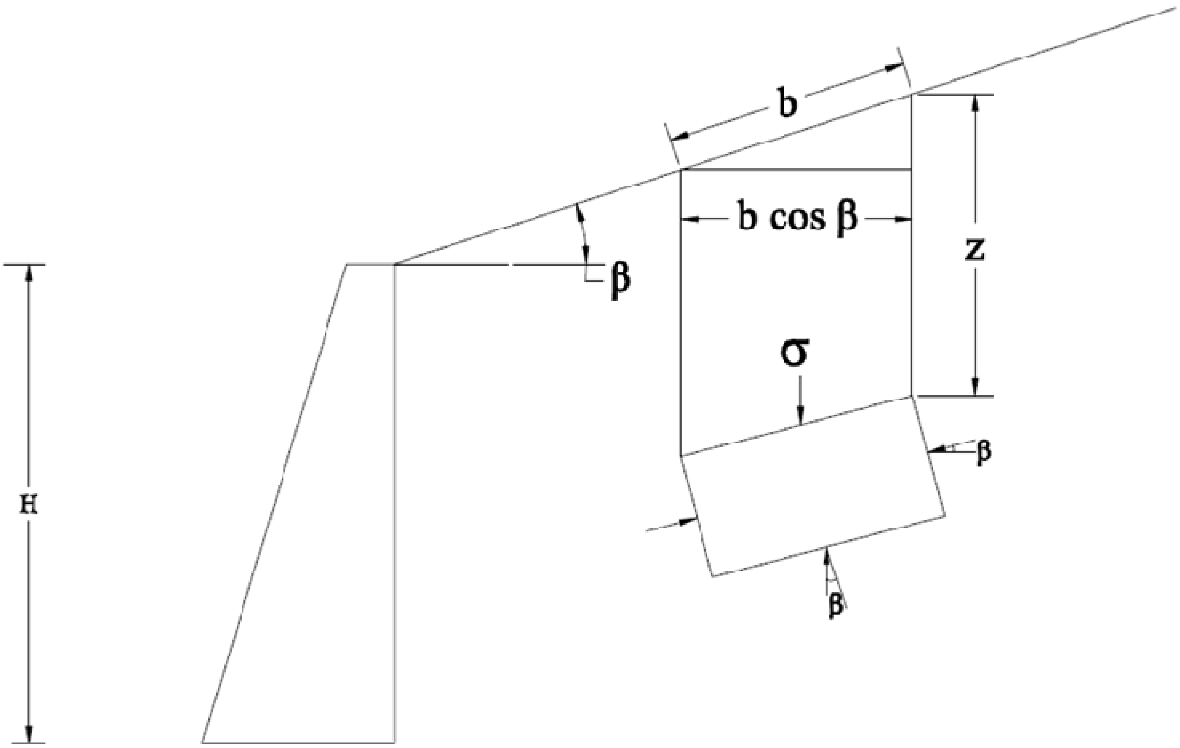


Fig 4.11 Backfill with sloping surface.

In Fig 4.11 is shown a backfill with its surface inclined to the horizontal at an angle β. The angle β is referred to as surcharge angle. For finding out the active earth pressure in this case by Rankine's theory, we consider an element at depth z as shown in Fig 4.11 such that the planes of the element are conjugate and the stresses acting on them, i.e the vertical stress σ and lateral stress p are conjugate stresses. Note stress p acts parallel to the sloping surface. p and σ are resultant stresses on the two conjugate planes and have the same angle of obliquity β. The relationship between princpal stresses σ_1 and σ_3 at failure is

$$\frac{(\sigma1-\sigma3)}{2} = \frac{(\sigma1+\sigma3)}{2} sin\emptyset + c\ cos\emptyset$$

For cohesionless soil c=0.

$\therefore\ (\sigma_1 - \sigma_3) = (\sigma_1 + \sigma_3)\sin\emptyset$ Eq 4.8

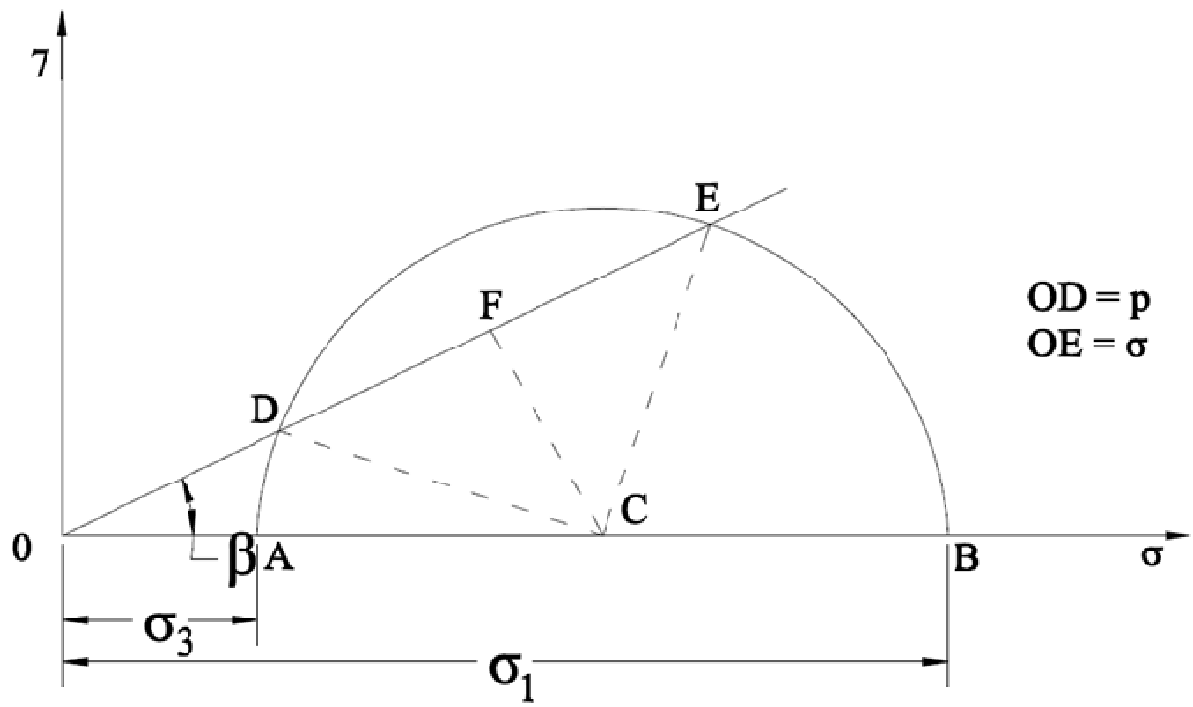


Fig 4.12 Mohr circle – To find relationship between p and σ

Fig 4.12 shows the Mohr's Circle of stresses for point of element considered in Fig 4.11.

The line ODE is drawn making an angle β with the σ- axis. Then OD and OE will represent the resultant stresses on the two conjugate planes i.e OD=p and OE= σ. CD and CE are joined and CF is drawn perpendicular to DE.

In Δ^{le} OFC, OF = OCcosβ = $\frac{(\sigma1+\sigma3)}{2}\cos\beta$

FC = OCsin β = $\frac{(\sigma1+\sigma3)}{2}\sin\beta$

In Δ^{le} DFC, $DC^2 = DF^2 + FC^2$

$$\therefore\ DF = \sqrt{(DC^2 - FC^2)} = \sqrt{\left(\frac{\sigma1-\sigma3}{2}\right)^2 - \left(\frac{\sigma1+\sigma3}{2}\right)^2 sin^2\beta}$$

But $(\sigma_1 - \sigma_3) = (\sigma_1+\sigma_3)\sin\emptyset$ from Eq 4.8

$$\therefore\ DF = \sqrt{\left(\frac{\sigma1+\sigma3}{2}\right)^2 sin^2\emptyset - \left(\frac{\sigma1+\sigma3}{2}\right)^2 sin^2\beta}$$

$$DF = FE = \frac{(\sigma 1 + \sigma 3)}{2}\sqrt{\sin^2\emptyset - \sin^2\beta}$$

$$\sigma = OE = OF{+}FE = \left(\frac{\sigma_1 + \sigma_3}{2}\right)\cos\beta + \left(\frac{\sigma_1 + \sigma_3}{2}\right)\sqrt{\sin^2\emptyset - \sin^2\beta} \qquad \text{Eq 4.9}$$

$$p = OD = OF{-}DF = \left(\frac{\sigma_1 + \sigma_3}{2}\right)\cos\beta - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\sqrt{(\sin^2\emptyset - \sin^2\beta)} \qquad \text{Eq 4.10}$$

Dividing Eqn 4.9 by Eqn 4.10, we get

$$\frac{p}{\sigma} = \frac{\cos\beta - \sqrt{\sin^2\emptyset - \sin^2\beta}}{\cos\beta + \sqrt{\sin^2\emptyset - \sin^2\beta}}$$

we write,

$$\frac{p}{\sigma} = K = \frac{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\emptyset)}}{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\emptyset)}}$$

K is referred to as Rankine's lateral pressure ratio (or conjugate ratio) for the case of backfill with sloping surface.

Referring to Fig 4.11 we have

$$\sigma = \frac{(\gamma z)(b\cos\beta)}{b} = \gamma z\cos\beta$$

If we denote the lateral active earth pressure by p_a, we get

$$p_a = \sigma K = \gamma z\cos\beta\left[\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset}}\right] \qquad \text{Eq 4.11}$$

we can also write,

$$p_a = K_a\gamma z$$

$$\text{where } K_a = \cos\beta\left[\frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\emptyset}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\emptyset}}\right] \qquad \text{Eq 4.12}$$

When surface of backfill is horizontal, β=0 and Eq 4.12 reduces to $K_a= \frac{1-sin\emptyset}{1+sin\emptyset}$ as obtained earlier.

In Fig 4.13 is shown the active earth pressure distribution in this case. The resultant active earth pressure acts at a distance $\frac{H}{3}$ above base and is parallel to the sloping surface.

It is important to note that if the backfill is submerged, only the lateral pressure due to submerged weight of soil will act parallel to sloping surface. The lateral pressure due to water will act normal to the back of wall.

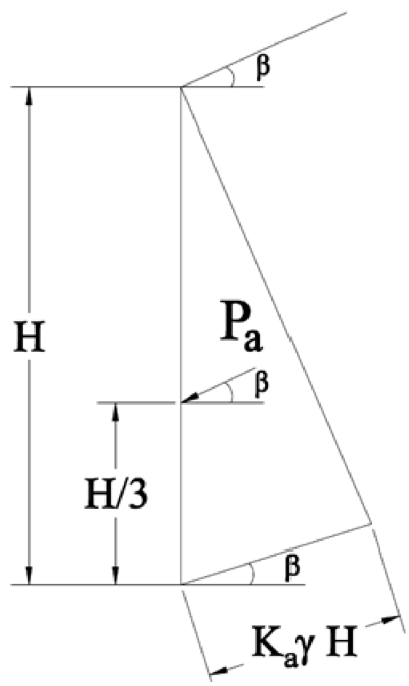


Fig 4.13 Active earth pressure distribution diagram

Case 7: Wall with inclined back and backfill with horizontal surface

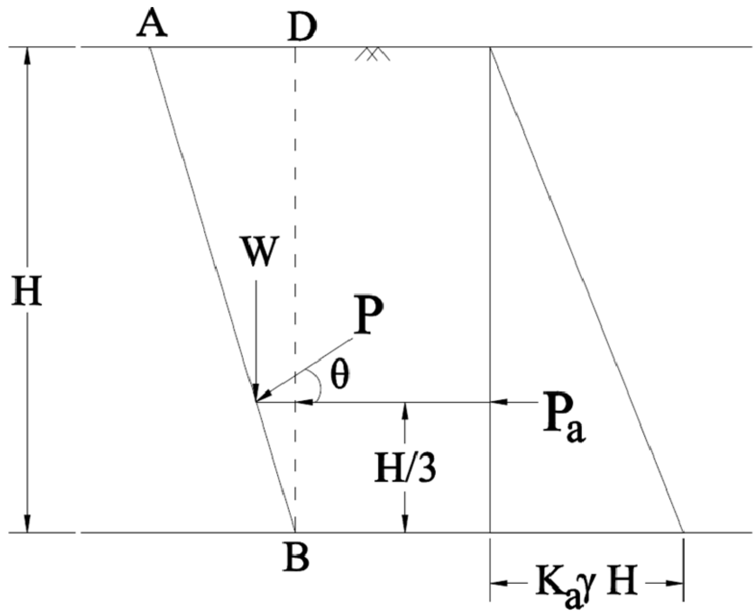


Fig 4.14 Inclined back with horizontal backfill surface

In Fig 4.14 the retaining wall has inclined back AB and the backfill has horizontal surface. Through heel B vertical line BD is drawn. The resultant active earth pressure P_a on vertical plane BD and the weight W of the soil wedge ABD are found. The total pressure on the back AB is then the vector sum of P_a and W.

$P = \sqrt{P_a^2 + W^2}$ where $P_a = \frac{1}{2} K_a \gamma H^2$

$\theta = \tan^{-1}\frac{W}{P_a}$

Case 8: Wall with inclined back and backfill with sloping surface

In Fig 4.15 the wall has inclined back AB and the backfill has sloping surface with surcharge angle β. Vertical line is drawn through heel B intersecting the sloping surface at D. The resultant active earth pressure P_a on the vertical plane BD and the weight W of the soil wedge ABD are found. The total pressure on the back AB is then the vector sum of P_a and W.

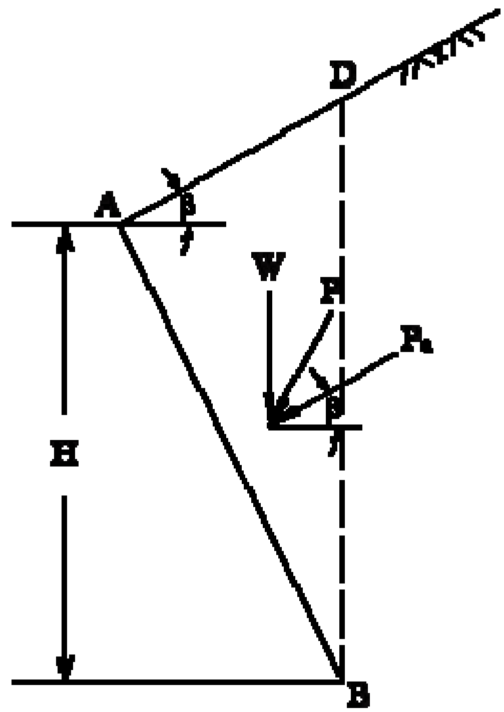


Fig 4.15 Inclined back and sloping backfill surface.

4.3.2 Active Earth Pressure of Cohesive Soil by Extension of Rankine's Theory

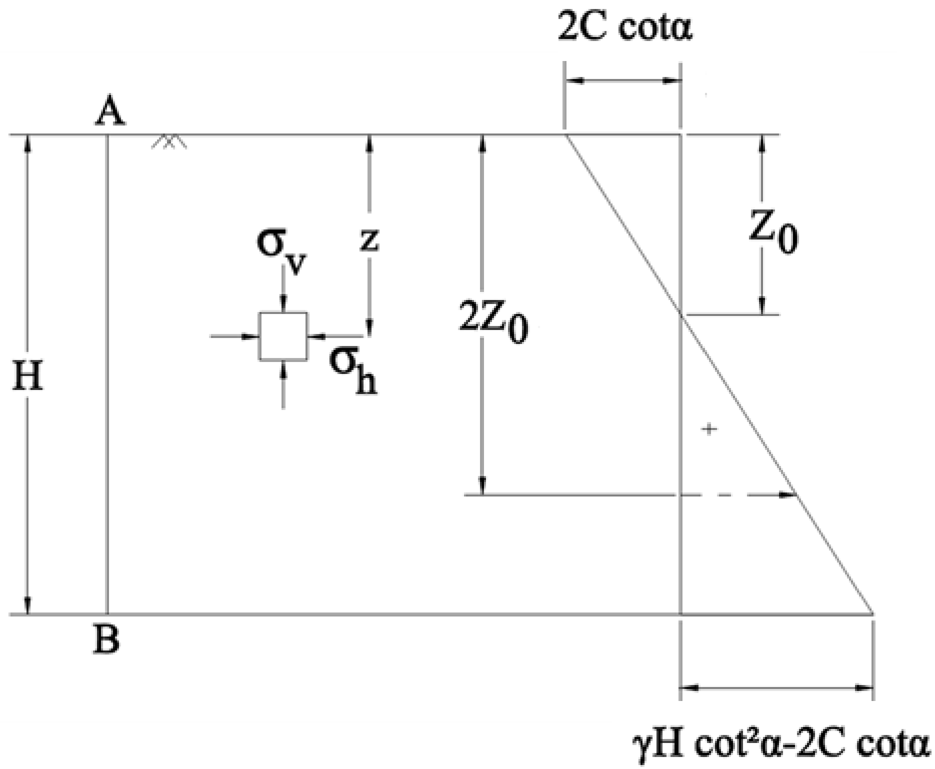


Fig 4.16 Active earth pressure of cohesive soil.

Consider an element at depth z. Let σ_v and σ_h denote the vertical and horizontal stresses acting on this element, as shown in Fig 4.16. In the active state of plastic equilibrium we have

$\sigma_3 = \sigma_h$ and $\sigma_1 = \sigma_v$ Eq 4.13

The relationship between principal stresses at failure is

$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$ Eq 4.14

where $\alpha = 45° + \frac{\phi}{2}$.

It may be recalled that in originally proposed Rankine's theory soil is considered as cohesionless. To extend Rankine's theory to cohesive soil c must be considered. Substituting Eqn 4.13 in Eqn 4.14, we get

$\sigma_v = \sigma_h \tan^2\alpha + 2c \tan\alpha$

or $\sigma_h = \sigma_v \cot^2\alpha - 2c \cot\alpha$ Eq 4.15

Further we substitute p_a for σ_h and γz for σ_v in Eqn 4.15. Then

$p_a = \gamma z \cot^2\alpha - 2c \cot\alpha$ Eq 4.16

Eqn 4.16 is known as Bell's equation. It shows that the effect of cohesion is to reduce the intensity of active earth pressure by 2c cotα at all depths z.

At z = 0, $p_a = -2c \cot\alpha$

At z = H, $p_a = \gamma H \cot^2\alpha - 2c \cot\alpha$

As the value of p_a changes from negative to a positive value, p_a becomes zero at some depth. Let p_a=0 at $z = z_0$.

Then $0 = \gamma z_0 \cot^2\alpha - 2c \cot\alpha$

$\therefore \quad z_0 = \frac{2c}{\gamma} \tan\alpha$ Eq 4.17

The active earth pressure distribution diagram is plotted in Fig 4.16. Since soils are weak in tension, due to negative earth pressure crack develops upto depth z_0. The total active earth pressure is then given by

$$P_a = \int_{z_0}^{H} [\gamma z \cot^2\alpha - 2c \cot\alpha] dz$$

$$= \left[\frac{\gamma z^2}{2} cot^2\alpha - 2c\, z\cot\alpha\right]_{z_0}^{H}$$

$$= \frac{1}{2}\gamma(H^2 - z_0^2)\cot^2\alpha - 2c(H - z_0)\cot\alpha.$$

If it is assumed that the crack does not develop then the net total active earth pressure is given by

$$P_a = \int_{0}^{H} [\gamma z \cot^2\alpha - 2c \cot\alpha] dz$$

$$= \left[\frac{\gamma z^2}{2} cot^2\alpha - 2c\, z\cot\alpha\right]_{0}^{H}$$

$$= \frac{1}{2}\gamma H^2\cot^2\alpha - 2cH\cot\alpha.$$

It may be noted that for a depth $2z_0$, the net total active earth pressure is zero. This is the maximum depth upto which a vertical cut can be made in the soil without any lateral support. It is called critical depth of excavation and is denoted by H_c.

Thus, $H_c = 2z_0 = \frac{4c}{\gamma} \tan\alpha$.

Other Cases:

1. **Backfill with uniform surcharge**

   If the backfill carries a surcharge of intensity q per unit area, then we have, at any depth z,

   $p_a = \gamma z\cot^2\alpha + q\cot^2\alpha - 2c \cot \alpha.$

   At $z = 0$, $p_a = q\cot^2\alpha - 2c \cot \alpha$

   At $z = H$, $p_a = \gamma H\cot^2\alpha + q\cot^2\alpha - 2c \cot \alpha$

   Let $z = z_0$ at $p_a = 0$.

Then $0 = \gamma z_0 \cot^2\alpha + q \cot^2\alpha - 2c \cot \alpha$

$\therefore\ z_0 = — \tan\alpha - –$

2. Submerged backfill

If water table exists at a depth H_1 below the surface of backfill then we have

For $0 \le z \le H_1$ $\quad p_a = \gamma z \cot^2\alpha - 2c \cot\alpha$

For $H_1 \le z \le H$ $\quad p_a = [\gamma H_1 \cot^2\alpha + \gamma^1 (z - H_1)\cot^2\alpha + \gamma_W (Z - H_1) - 2c\cot\alpha]$

3. Backfill of intact saturated clay

For computation of active earth pressure of intact saturated clays, for temporary works or immediately after construction, we can take $\emptyset = \emptyset_u = 0$ so that

$\alpha = 45° + — = 45°$ and $\cot\alpha = \cot 45° = 1$.

Then, at any depth z,

$$p_a = \gamma_{sat} z \cot^2\alpha - 2c_u \cot \alpha$$
$$= \gamma_{sat} z - 2c_u.$$

4.3.3 Passive Earth Pressure of Cohesionless Soil – by Method Based on Rankine's Theory

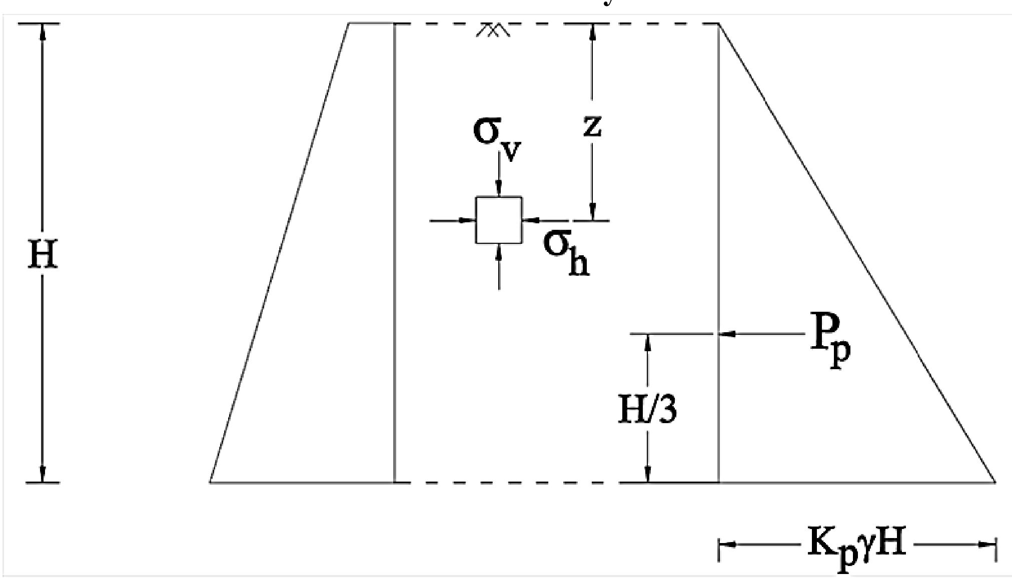


Fig 4.17 Passive earth pressure of cohesionless soil

Consider an element at depth z below the surface of backfill. Let σ_v and σ_h denote the vertical and horizontal stresses acting on this element. In the passive state of plastic equilibrium we have

$\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_v$

Eq 4.18

The relationship between principal stresses at failure is

$$\sigma_1 = \sigma_3 \tan^2\alpha + 2c \tan\alpha$$

For cohesionless soil c=0.

$$\therefore \sigma_1 = \sigma_3 \tan^2\alpha \qquad \text{Eq 4.19}$$

Substituting Eq 4.18 in Eq 4.19, we get

$$\sigma_h = \sigma_v \tan^2\alpha$$

we can write $\sigma_h = K_p \sigma_v$ Eq 4.20

where, $K_p = \tan^2\alpha = \frac{1+sin\emptyset}{1-sin\emptyset}$

K_p is called Rankine's coefficient of passive earth pressure.

Substituting p_h for σ_h and γz for σ_v we get intensity of passive earth pressure at any depth z as

$$p_p = K_p \gamma z \qquad \text{Eq 4.21}$$

At $z = 0$, $p_p = 0$

At $z = H$, $p_p = K_p \gamma H$.

The passive earth pressure distribution is shown in Fig 4.17.

The total passive earth pressure, $P_p = \frac{1}{2} K_p \gamma H^2$ and acts at distance $\frac{H}{3}$ above base.

Other cases:

1. **Backfill with uniform surcharge**

   If the backfill carries a uniform surcharge load q per unit area, then at any depth z,

   $p_p = K_p \gamma z + K_p q + 2c \tan\alpha$

2. **Submerged backfill**

If water table exists at any depth H_1 below surface of backfill, then assuming no change in $\emptyset$ after submergence,

For $0 \leq z \leq H_1$

$p_p = K_p \gamma z$

For $H_1 \leq z \leq H$

$$p_p = K_p \gamma H_1 + K_p \gamma^1 (z - H_1) + \gamma_w (z - H_1) + 2c\tan\alpha$$

3. **Backfill with sloping surface**

If β is the surcharge angle, then at any depth z

$$p_p = K_p \gamma z$$

$$\text{where } K_p = \cos\beta \frac{\cos\beta + \sqrt{(\cos^2\beta - \cos^2\emptyset)}}{\cos\beta - \sqrt{(\cos^2\beta - \cos^2\emptyset)}}$$

4.3.4 Passive Earth Pressure of Cohesive Soil- by Method Based on Rankine's Theory

Consider an element at depth z below the surface of backfill. Let σ_v and σ_h denote the vertical and horizontal stresses acting on this element. In the passive state of plastic equilibrium we have

$\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_v$ Eq 4.22

The relationship between principal stresses at failure is

$\sigma_1 = \sigma_3 \tan^2\alpha + 2c\tan\alpha$ Eq 4.23

where $\alpha = 45° + \frac{\emptyset}{2}$

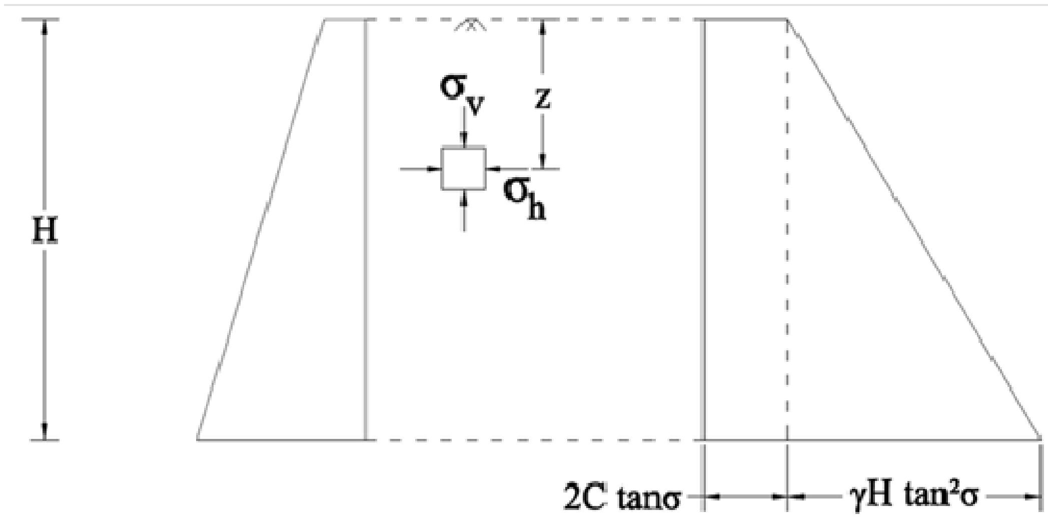


Fig 4.18Passive earth pressure of cohesive soil

Substituting Eqn 4.22 in Eqn 4.23 we get

$$\sigma_h = \sigma_v \tan^2\alpha + 2c\tan\alpha$$

Substituting p_p for σ_h and γz for σ_v, we get intensity of passive earth pressure at any depth z as

$$p_p = \gamma z \tan^2\alpha + 2c\tan\alpha \qquad \text{Eq 4.24}$$

At $z = 0$, $p_p = 2c\tan\alpha$

At $z = H$, $p_p = \gamma H \tan^2\alpha + 2c\tan\alpha$.

The passive earth pressure distribution is shown in Fig 4.18. The area of this diagram gives the resultant passive earth pressure $P_{p.}$

It may be noted that the effect of cohesion is to increase the passive earth pressure intensity at all depth by 2ctanα.

Other cases:

1. **Backfill with uniform surcharge**

   If the backfill carries a uniform surcharge load of q per unit area then we have

$$p_p = \gamma z \tan^2\alpha + q\tan^2\alpha + 2c\tan\alpha$$

2. **Submerged backfill**

If the water table exists at depth H_1 below the surface of backfill, then we have

For $0 \leq z \leq H_1$

$p_a = \gamma z\tan^2\alpha + 2c\tan\alpha$

For $H_1 \leq z \leq H$

$$p_a = [\gamma H_1\tan^2\alpha + \gamma^1(z-H_1)\tan^2\alpha + q\tan^2\alpha + \gamma_w(z-H_1) + 2c\tan\alpha]$$

4.4 Coulomb's Wedge Theory:

Rankine (1860) in his theory of earth pressure considered the stresses acting on an element and their relationship in the plastic equilibrium state. Earlier to this Coulomb (1776) proposed the wedge theory in which he assumed that a portion of soil mass adjacent to the retaining wall breaks away from the rest of the soil mass. By considering the forces acting on this soil wedge in the limiting equilibrium condition the lateral earth pressure is computed.

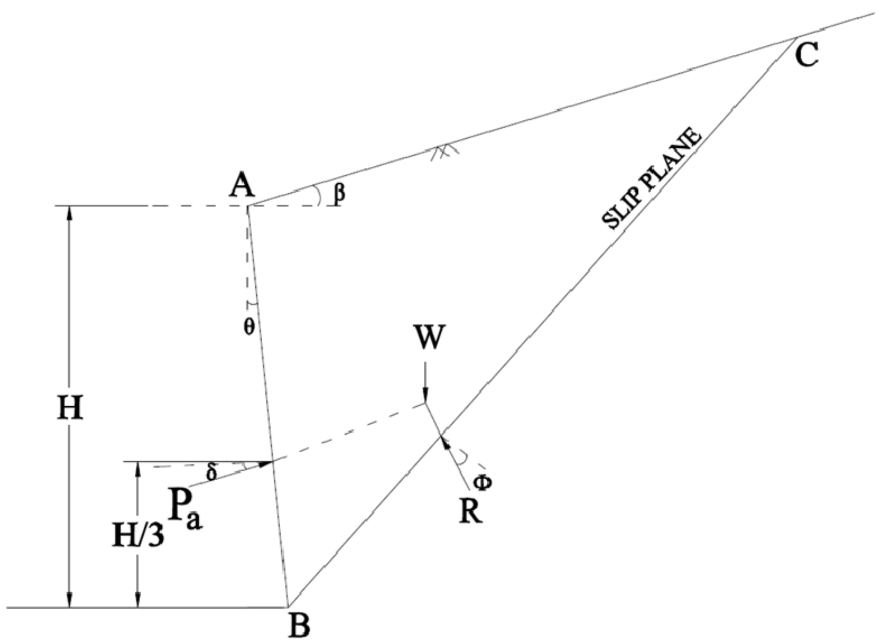


Fig 4.19 Free body diagram of sliding wedge

In Fig 4.19 ABC is the sliding wedge. Coulomb assumed that active earth pressure is caused when the wall tends to move downward and outward. On the other hand passive earth pressure is caused when the wall moves upward and inward. Fig 4.19 is the free body diagram of the sliding wedge in the limiting equilibrium condition for the active state.

Assumptions made in Coulomb's theory:

1. The backfill is cohesionless, dry, homogenous, isotropic and elastically undeformable but breakable.
2. The slip surface is a plane which passes through the heel of wall.
3. The sliding wedge behaves like a rigid body and the earth pressure can be computed by considering the limiting equilibrium of the wedge as a whole.
4. The back of the wall is rough.
5. The position and direction of the resultant earth pressure are known. It acts at distance one-third the height of the wall above base and is inclined at an angle δ to the normal to the back of wall, where δ is the angle of wall friction.
6. In the limiting equilibrium condition the sliding wedge is acted upon by three forces as shown in Fig 4.19.
    - (i) Weight W of the sliding wedge acting vertically through its centre of gravity.
    - (ii) The resultant active earth pressure P_a acting at distance $\frac{H}{3}$ above base and inclined at an angle δ to the normal to the back of wall.
    - (iii) The resultant reaction R inclined at an angle $\emptyset$ to the normal to the slip plane and passing through the point of intersection of the other two forces.

For the condition of yield of the base of wall and wall movement away from fill, the most dangerous or the critical slip plane is that for which the wall reaction is maximum. The active earth pressure is computed as the maximum lateral pressure which the wall must resist before it moves away from the fill.

4.5 Condition for Maximum Pressure from Sliding Wedge

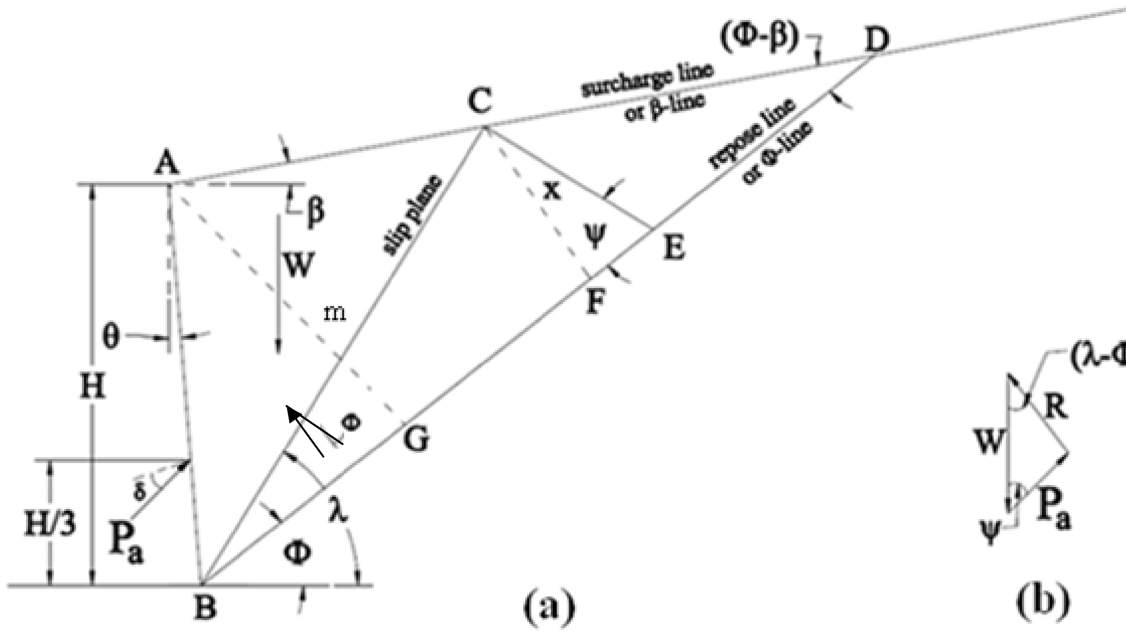


Fig 4.20 Condition for maximum pressure from sliding wedge - P_a

In Fig 4.20(a) AB is the back of the wall with positive batter angle θ. AD is surface of backfill inclined at an angle β to horizontal, and referred to as surcharge line. BD is inclined at angle ∅ to the horizontal and is called repose line as it is the slope with which soil rests without any lateral support. Let BC be the slip plane or rupture plane inclined at angle □ to the horizontal. We have to determine the position of slip plane for which the sliding wedge exerts maximum pressure on the wall. □ is referred to as critical slip angle. It is clear from Fig 4.20(a) that the critical slip plane lies between repose line (□=∅) and back of wall (□=90°+θ). Further we observe that P_a is inclined to the vertical at an angle (90-θ-δ) which is denoted by φ. The reaction R is inclined to the vertical at (□-∅). The triangle of forces is shown in Fig 4.20(b). In Fig 4.20(a) CE is drawn making angle φ with the ∅- line. Let x and m be the length of perpendiculars drawn from C and A to BD. Let BD be n. The triangle BCE and triangle of forces are similar. Therefore, we have

$$\frac{P_a}{CE} = \frac{W}{BE}.$$

i.e., $\frac{P_a}{CE}$ = W.$\left(\frac{CE}{BE}\right)$ Eq 4.25

From Δ^{le} CFE , $\sin\varphi = \frac{x}{CE}$

$\therefore$ $CE = \frac{x}{\sin\varphi} = x \operatorname{cosec}\varphi$

or $CE = A_1 x$ Eq 4.26

where $A_1 = \operatorname{cosec}\varphi$

BE =BD – FD + FE

From Δ^{le} CFD, $\tan(\emptyset-\beta) = \frac{x}{FD}$

$\therefore$ $FD = x \cot(\emptyset-\beta)$

From Δ^{le} CFE, $\tan\varphi = \frac{x}{FE}$

$\therefore$ $FE = x \cot\varphi$

Hence, $BE = n - x[\cot(\emptyset-\beta) - \cot\varphi]$

or $BE = n - A_2 x$ Eq 4.27

where $A_2 = [\cot(\emptyset-\beta) - \cot\varphi]$

$W = \gamma(\Delta\ ABC) = \gamma[\Delta\ ABD - \Delta\ BCD]$

i.e $W = \frac{1}{2}\gamma(m-x)n$ Eq 4.28

Substituting Eqns 4.26, 4.27 and 4.28 in Eqn 4.25, we get

$$P_a = \frac{1}{2}\gamma(m-x)n \ \frac{A_1 x}{n-A_2 x} = \left(\frac{1}{2}\ \gamma n\ A_1\right)\left(\frac{mx-x^2}{n-A_2 x}\right)$$

In the last equation x is the only variable which depends on the position of slip plane.

For maxima $\frac{dP_a}{dx} = 0$

$$\therefore \frac{dP_a}{dx} = \left(\frac{1}{2}\ \gamma n\ A_1\right)\frac{(m-2x)(n-A_2 x)-(-A_2)(mx-x^2)}{(n-A_2 x)^2} = 0$$

$\therefore (m - 2x)(n - A_2 x) = -A_2(mx - x^2)$

mn - A_2mx – 2nx + 2A_2x^2 = -A_2 mx + A_2x^2

mn – 2xn = - A_2x^2

Rearranging,

mn – xn = xn - A_2x^2 = x(n - A_2x) = $x \times BE$

We can write

$\frac{mn}{2} - \frac{xn}{2} = x\frac{BE}{2}$

or Δ ABD - Δ BCD = Δ BCE

i.e., Δ ABC = Δ BCE Eq 4.29

Thus the condition for the sliding wedge ABC to exert maximum pressure (P_a) on wall is that the slip plane BC is located such that triangles ABC and BCE are equal in area. Rebhann (1871) is credited to have presented this proof.

4.6 Rebhann's Graphical Method for Active Earth Pressure of Cohesionless Soil

Rebhann (1871) gave this graphical procedure for locating the slip plane and determining the total active earth pressure according to Coulomb's wedge theory.

Referring to Fig 4.21 the steps involved in the graphical procedure is

1. Given the height H and batter angle θ the back AB of the wall is constructed.
2. Through A , surcharge line or β-line is drawn inclined at an angle β to the horizontal.
3. Through B, repose line or ∅-line is drawn inclined at an angle ∅ to the horizontal, intersecting the β-line at D.

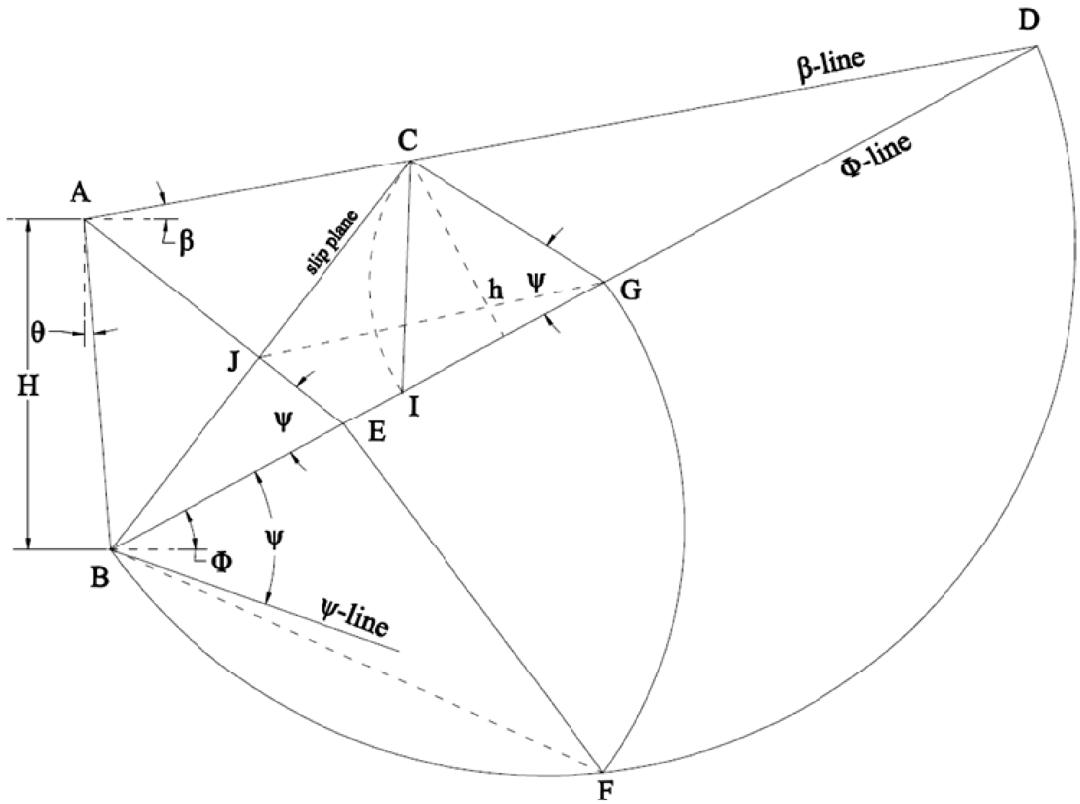


Fig 4.21 Rebhann's graphical method for active earth pressure

4. Through B, earth pressure line or ψ-line is drawn inclined at an angle ψ to the ∅-line. $\psi=90°-\theta-\delta$, where δ = angle of wall friction.
5. On BD as diameter a semi-circle is drawn.
6. From A, line AE is drawn parallel to ψ-line meeting BD at E.
7. At E a perpendicular to BD is erected to intersect the semicircle at F.
8. With B as centre and BF as radius an arc is drawn to intersect BD at G.
9. From G, GC is drawn parallel to ψ-line to intersect β-line at C.
10. With G as centre and GC as radius an arc is drawn to intersect BD at I. CI is joined and area of triangle CIG is computed. The total active earth pressure is calculated as

$$P_a = \gamma\,(\Delta CIG) = \frac{1}{2}\gamma h\,(IG) \qquad \text{Eq 4.30}$$

where γ = unit weight of soil

and h = height of ΔCIG with IG as base.

In the preceding graphical procedure BC is the slip plane and Eqn 4.30 gives the value of P_a. The proof for this is as follows:

From the properties of intersecting chords of a circle, we have

$$(BE)(ED) = EF^2$$

Adding BE^2 to both sides,

$$BE(ED) + BE^2 = EF^2 + BE^2$$

$$BE(ED+BE) = BF^2$$

$$(BE)\,(BD) = BF^2 = BG^2$$

or $$\frac{BE}{BG} = \frac{BG}{BD} \qquad \text{Eq 4.31}$$

Let AE and BC intersect at J and JG be joined.

Since in triangle BCG, JE is parallel to CG,

we have $$\frac{BE}{BG} = \frac{BJ}{BC} \qquad \text{Eq 4.32}$$

From Eqns 4.31 and 4.32, we get

$$\frac{BG}{BD} = \frac{BJ}{BC}$$

Hence in triangle BCD, JG is parallel to CD. Clearly, AJGC is a parallelogram. Therefore perpendicular distances of A and G from JC are equal. From this it follows that the two triangles ABC and BCG which have common base BC have also corresponding equal heights.

Hence Δ ABC = Δ BCG

This shows that BC is the slip plane.

As shown earlier

$\frac{P_a}{W} = \frac{CG}{BG}$ where W = weight of soil wedge ABC.

or $P_a = W.\ \frac{CG}{BG}$

$= \gamma(\Delta\ ABC).\ \frac{CG}{BG}$

$= \gamma(\Delta\ BCG)\ .\ \frac{CG}{BG}$

$= \gamma(\frac{1}{2} \times h \times BG)\ .\ \frac{CG}{BG}$

$= \gamma\ (\frac{1}{2} \times h \times CG)$

$= \gamma(\frac{1}{2} \times h \times IG)$

i.e $P_a = \gamma\ (\Delta\ CIG)$

4.6.1 Analytical Expression for Coulomb's K_a and hence P_a

For the purpose of obtaining P_a analytically, expression for Coulomb's coefficient of active earth pressure of cohesionless soil can be derived with the help of above proof.

$$P_a = \gamma\ (\Delta\ CIG) = \gamma\ (\frac{1}{2} \times h \times IG)$$

or $P_a = \frac{1}{2}\gamma h\ (CG)$ Eq 4.33

Since $h = CG \sin\psi$, $P_a = \frac{1}{2}\gamma(CG)^2 \sin\psi$

It is possible to express CG in terms of height H and angles θ, β, ∅ and δ.

Referring to Fig.4.21, from similar triangles AED and CGD

we have $\frac{CG}{AE} = \frac{CD}{AD}$

or $CG = AE\ \frac{CD}{AD}$ Eq 4.34

From triangle ABE

$$\frac{AE}{sinA\hat{B}E} = \frac{AB}{sinA\hat{E}B}$$

i.e., $\frac{AE}{\sin\ (90+\theta-\emptyset)} = \frac{Hsec\theta}{sin\psi}$

∴ AE = $\frac{H sec\theta . \cos(\emptyset - \theta)}{sin\psi}$ Eq 4.35

Further, $\frac{AD}{CD} = \frac{AC+CD}{CD} = \frac{AC}{CD} + 1 = \frac{JG}{CD} + 1$ (Since AC = JG)

From triangles BCG and BCD, $\frac{JG}{CD} = \frac{BG}{BD}$

∴$\frac{AD}{CD} = \frac{BG}{BD} + 1$ Eq 4.36

From the property of intersecting chords

we have (BE)(ED) = EF^2

Adding BE^2 to both sides,

(BE)(ED) + BE^2 = EF^2 + BE^2

BE(ED + BE) = BF^2

(BE)(BD) = BF^2 = BG^2

Dividing both sides by BD^2 and rearranging,

$\frac{BG}{BD} = \sqrt{\frac{BE}{BD}}$ Eq 4.37

By substitution in Eqn 4.36

$\frac{AD}{CD} = \sqrt{\frac{BE}{BD}} + 1$ Eq 4.38

From triangle ABE,

$\frac{BE}{\sin(\emptyset + \delta)} = \frac{AB}{sin\psi} = \frac{H\ sec\theta}{sin\psi}$

∴ BE = $\frac{H sec\theta\ . \sin(\emptyset + \delta)}{sin\psi}$

From triangle ABD,

$$\frac{BD}{\sin(90°+\beta-\theta)} = \frac{AB}{\sin(\emptyset-\beta)} = \frac{H\sec\theta}{\sin(\emptyset-\beta)}$$

$$\therefore \quad BD = \frac{H\sec\theta \, .\cos(\theta-\beta)}{\sin(\emptyset-\beta)}$$

By substitution in Eq 4.37,

$$\frac{BG}{BD} = \sqrt{\frac{H\sec\theta \; . \; \sin(\emptyset+\delta)\sin(\emptyset-\delta)}{\sin\psi \, . \, H\sec\theta \; \cos(\emptyset+\delta)}}$$

$$= \sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin\psi \; \cos(\theta-\beta)}}$$

By substitution in Eqn 4.36.

$$\frac{AD}{CD} = 1 + \sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin\psi \; \cos(\theta-\beta)}} \qquad \text{Eq 4.36a}$$

Substitute, for AE and $\frac{CD}{AD}$ from Eqns 4.35 and 4.36a in Eqn 4.34 we have,

$$CG = \frac{H\sec\theta\cos(\emptyset-\theta)}{\sin\psi\left[1+\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin\psi\cos(\theta-\beta)}}\right]}$$

Substitute the above expression for CG in Eqn 4.33 and noting that $\psi = 90° - \theta - \delta$, we obtain

$$P_a = \frac{1}{2}\gamma H^2\left[\frac{\sec\theta\cos(\emptyset-\theta)}{\sin\psi\left[1+\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\sin\psi\cos(\theta-\beta)}}\right]}\right]^2 . \sin\psi$$

$$= \frac{1}{2}\gamma H^2\left[\frac{\sec\theta\cos(\emptyset-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\cos(\theta-\beta)}}}\right]^2$$

Or $P_a = \frac{1}{2} K_a \gamma H^2$

where $K_a = \left[\frac{\sec\theta\cos(\emptyset-\theta)}{\sqrt{\cos(\theta+\delta)}+\sqrt{\frac{\sin(\emptyset+\delta)\sin(\emptyset-\beta)}{\cos(\theta-\beta)}}}\right]$

Note 1: It is difficult to correctly estimate the value of δ. However, the following guidelines may be used in practice.

i. For slightly rough walls, $\delta = \frac{1}{3}\phi$
ii. For fairly rough walls, $\delta = \frac{2}{3}\phi$
iii. For rough walls with well drained backfill, $\delta = \frac{3}{4}\phi$
iv. For backfill subjected to vibration, $\delta = 0$

Note 2: Rebhann's method is said to be based on the method earlier proposed by Poncelet. It is, therefore, also referred to as Poncelet construction.

Special Cases:

<u>Case (i) β is nearly equal to ∅.</u>

When β is nearly equal to ∅, the surcharge line and repose line may not meet within the sheet. In such a case the following procedure can be used.

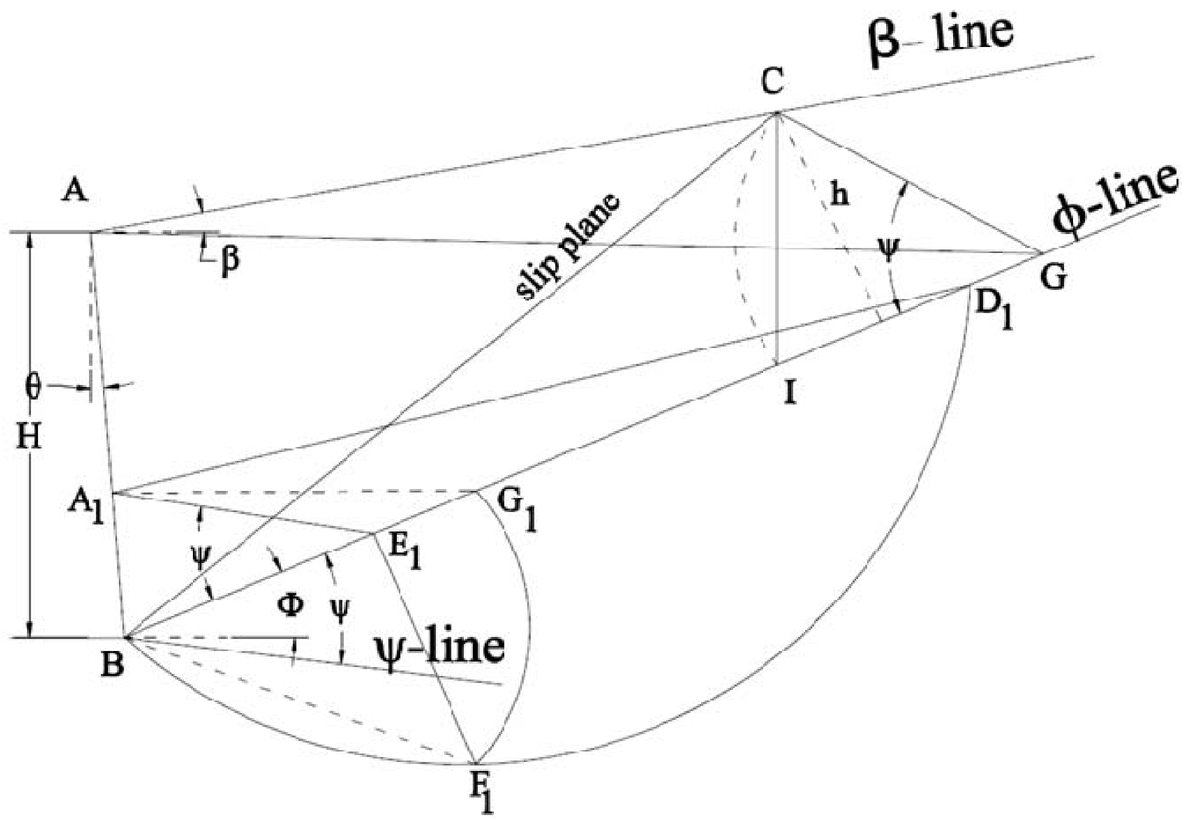


Fig 4.22 Rebhann's graphical method β is nearly equal to ∅

1. With height H and batter angle θ, the back of the wall, AB is constructed.
2. Through A, β-line is drawn inclined at angle β to the horizontal.
3. Through B, ∅-line is drawn inclined at angle ∅ to the horizontal.
4. Through B, ψ-line is drawn inclined at angle ψ to the ∅-line ($\psi = 90° - \theta - \delta$).
5. In this case β-line and ∅-line do not intersect within the sheet. Hence an auxiliary point D_1 is selected on the ∅-line. On BD_1 as diameter a semicircle is drawn.
6. Through D_1, A_1D_1 is drawn parallel to β-line, intersecting the back of the wall at A_1.
7. Through A_1, A_1E_1 is drawn parallel to ψ-line, intersecting the ∅-line at E_1.
8. At E, a perpendicular to BD_1 is erected intersecting the semicircle at F_1.
9. With B as centre, BF_1 as radius an arc is drawn, intersecting BD_1 at G_1.
10. A_1G_1 is joined and AG is drawn parallel to A_1G_1, intersecting ∅-line at G.
11. Through G, GC is drawn parallel to ψ-line intersecting β-line at C. BC is the slip plane.
12. With G as centre, GC as radius an arc is drawn to cut ∅ line at I. The area of triangle CIG is computed. The total active earth pressure is given by

$P_a = \gamma(\Delta CIG) = \frac{1}{2}\gamma h(IG)$

where γ = unit weight of soil in wedge ABC.

h= height of triangle CIG with IG as base.

Case (ii) β is equal to ∅

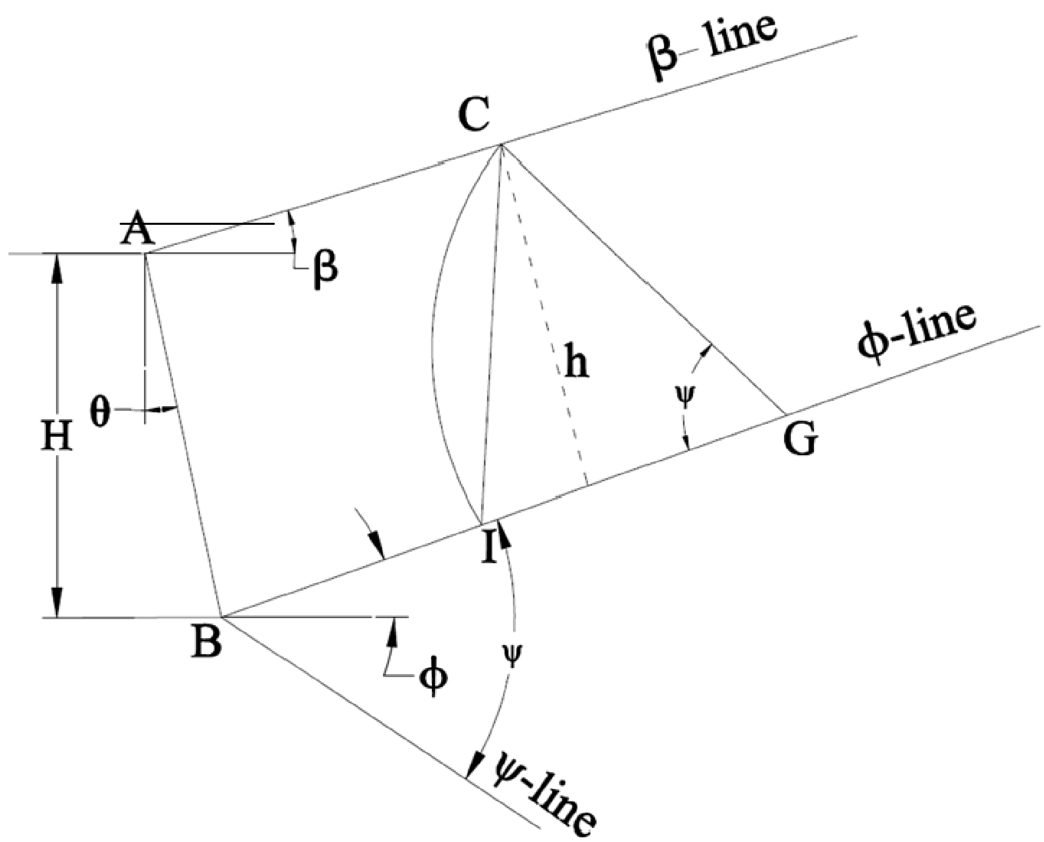


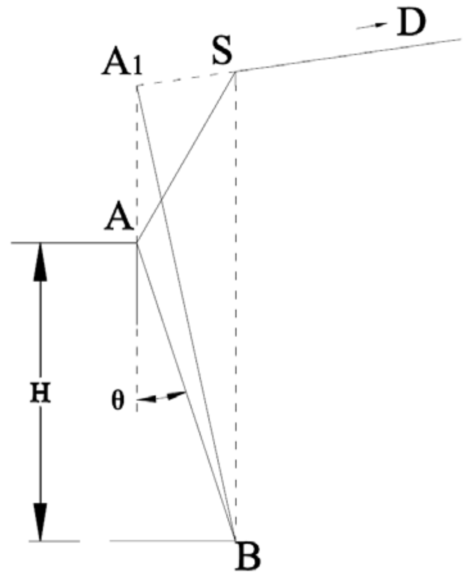
Fig 4.23 Rebhann's graphical method - β is equal to ∅

When β is equal to ∅, ∅-line will be parallel to β-line. Since height of triangle CIG remains same wherever it is located between β-line and ∅-line, point G is chosen at convenient point on ∅-line. Through G, GC is drawn parallel to ψ-line, intersecting β-line at C. With G as centre, GC as radius an arc is drawn cutting ∅-line at I. CI is joined. The area of triangle CIG is computed. The total active earth pressure is then given by

$P_a = \gamma(\Delta\ CIG) = \frac{1}{2}\gamma h(IG)$

Case (iii) Backfill with broken surface

In Fig 4.24, S denotes a break in the surface of backfill. To eliminate this break BS is joined.



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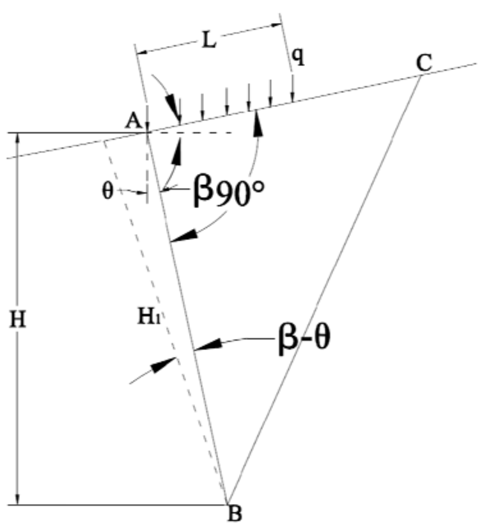
Through A, a line is drawn parallel to BS to intersect DS produced at A_1. BA_1 is joined. The resultant active earth pressure is determined treating A_1B as the back of wall.

Case (iv) Backfill with uniform surcharge load.

In Fig 4.25, uniform surcharge load of intensity q is shown acting on the surface of backfill, over a distance L. If γ is the unit weight of soil in the sliding wedge ABC, the equivalent unit weight γ_e of soil wedge with the surcharge load included in it is found as follows



$$\gamma_e = \frac{\gamma(\Delta ABC) + Lq}{\Delta ABC} = \gamma + \frac{Lq}{\Delta ABC}$$

Fig 4.25 Backfill with surcharge load.

Considering unit length normal to plane of figure,

If H_1 be the height of perpendicular drawn from B to ground surface, then

$$\Delta\, ABC = \frac{1}{2} H_1 (AC)$$

H_1 can be measured or computed as shown below

$\frac{H}{AB} = \cos\theta$

$\therefore \quad AB = \frac{H}{\cos\theta}$

$\frac{H_1}{AB} = \cos(\beta - \theta)$

$\therefore \quad H_1 = AB\cos(\beta - \theta) = \frac{H\cos(\beta-\theta)}{\cos\theta}$

The effect of surcharge load is taken into account by replacing γ by γ_e while computing P_a.

Note: Ifthe surcharge load extends beyond C, the value of L in (qL) should be taken equal to AC.

4.7 Culmann's Graphical Method for Active Earth Pressure of Cohesionless Soil Based on Coulomb's Wedge Theory

This graphical method given by Culmann (1886) is more general than Rebhann's method and is very convenient to use in the case of layered backfill, backfill with breaks at surface and different types of surcharge load.

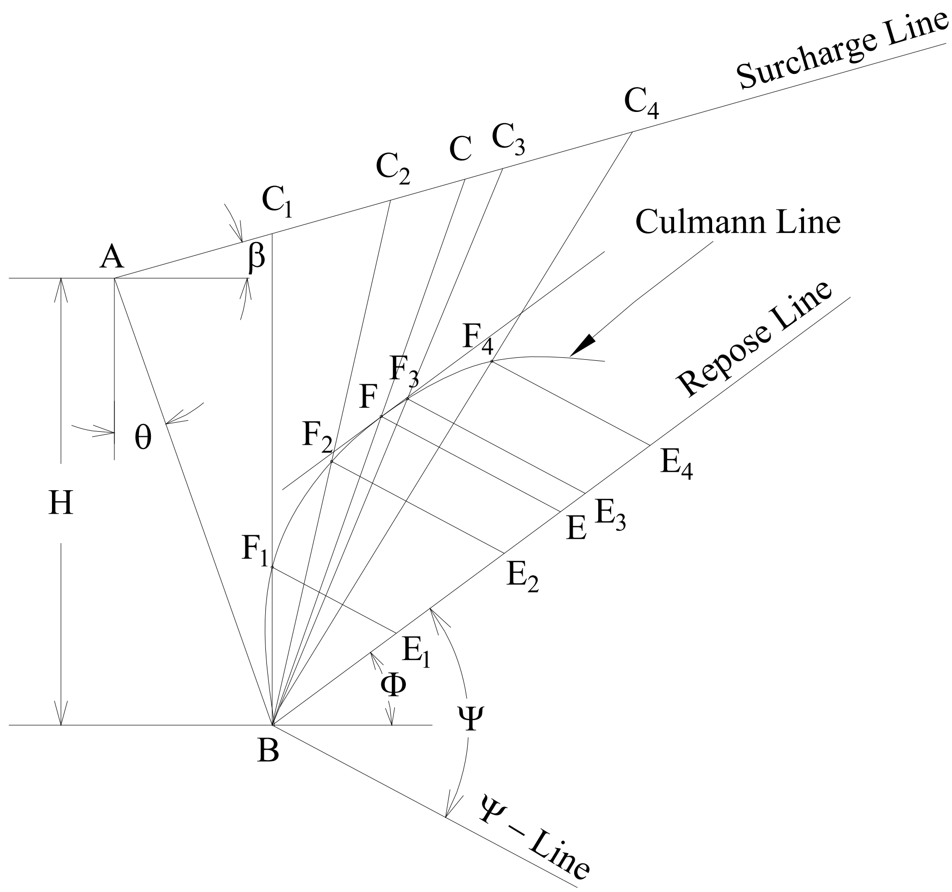


Fig 4.26 Culmann's graphical method

The steps involved in the Culmann's method are as follows:

1. Given height H and batter angle θ , the back AB of the wall is constructed.
2. Through A, the surcharge line (β-line) is drawn inclined at angle β to the horizontal.
3. Through B, the repose line (∅-line) is drawn inclined at an angle ∅ to the horizontal.
4. Again through B, the ψ-line is drawn inclined at an angle ψ to the ∅-line ($\psi = 90° - \theta - \delta$).
5. Trial slip planes BC_1, BC_2…… are drawn. The weights of the wedges ABC_1, ABC_2…… are calculated and plotted to scale as BE_1,BE_2,…… on the ∅-line.
6. Through E_1, E_2,…. lines are drawn parallel to ψ-line, intersecting BC_1, BC_2,……. at F_1, F_2..... respectively.
7. A smooth curveis drawn through points B, F_1, F_2…… This curve is called Culmann line.

8. A line is drawn parallel to Ø-line and tangential to Culmann line. Let it touch Culmann line at F. BF is joined and produced to intersect the β-line at C. Then BC is the critical slip plane.
9. Through F, line FE is drawn parallel to ψ-line, intersecting Ø-line at E.
10. The weight W of the wedge ABC is calculated. The resultant active earth pressure P_a is given by

$$\frac{P_a}{W} = \frac{FE}{BE}$$

$$\therefore \; P_a = W.\left(\frac{FE}{BE}\right) \qquad \text{Eq 4.39}$$

Special cases:

(i) Backfill with uniform surcharge load.

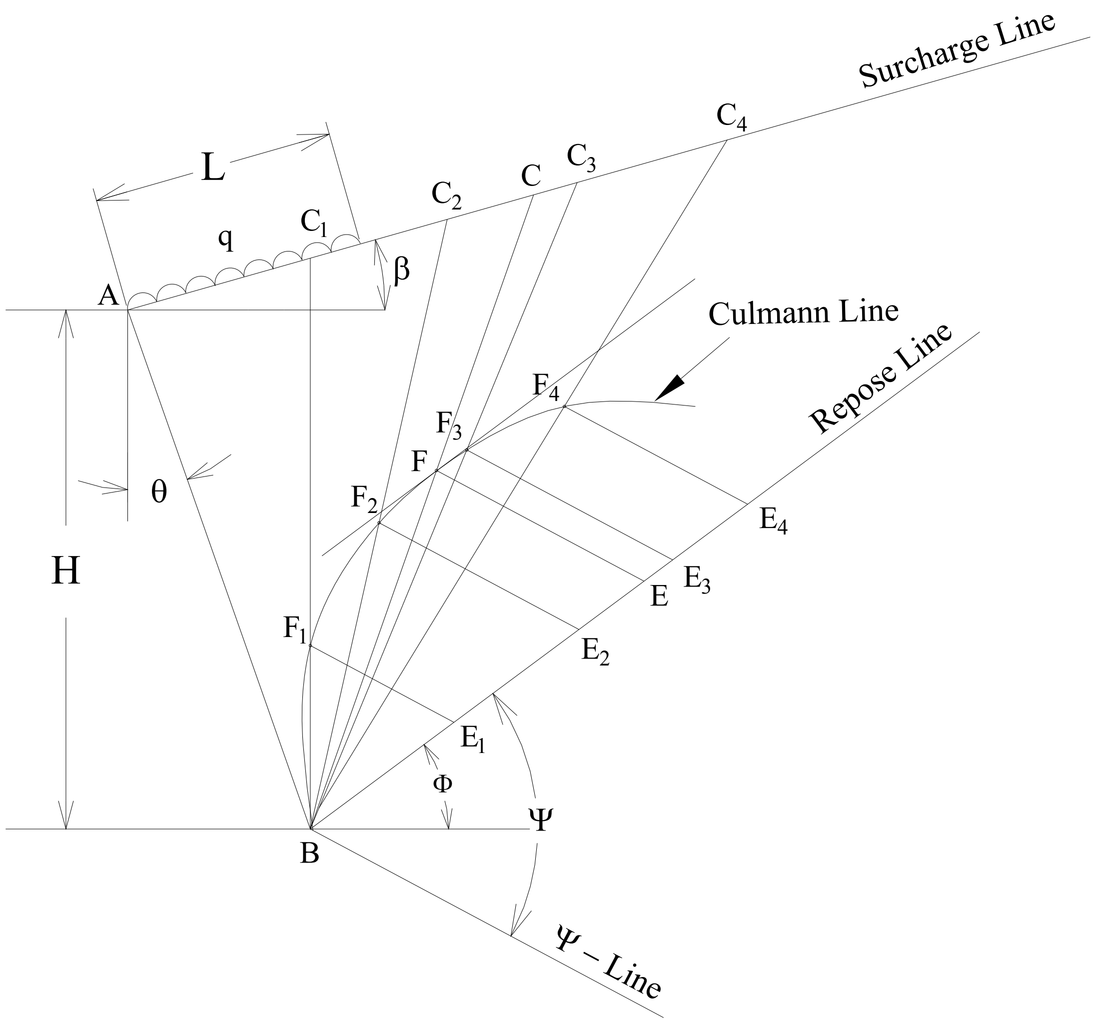


Fig 4.27 Culmann's method – Backfill with uniform surcharge load

As an illustration consider Fig 4.27 in which uniformly distributed surcharge load of intensity q is shown acting over a length L. The procedure is similar to the previous case but for the following changes.

i. BE_1 represent the sum of weight of wedge ABC_1 and surcharge load $q(AC_1)$.
ii. BE_2 represents the sum of weight of wedge ABC_2 and surcharge load qL. Similarly BE_3, BE_4 represent the sum of respective sliding wedges and surcharge load Lq.
iii. The resultant active earth pressure is given by

$$\frac{P_a}{W} = \frac{FE}{BE}$$

$$\therefore\ P_a = W.\left(\frac{FE}{BE}\right)$$

where W = (weight of wedge ABC) + (qL).

ii) Backfill with line load.

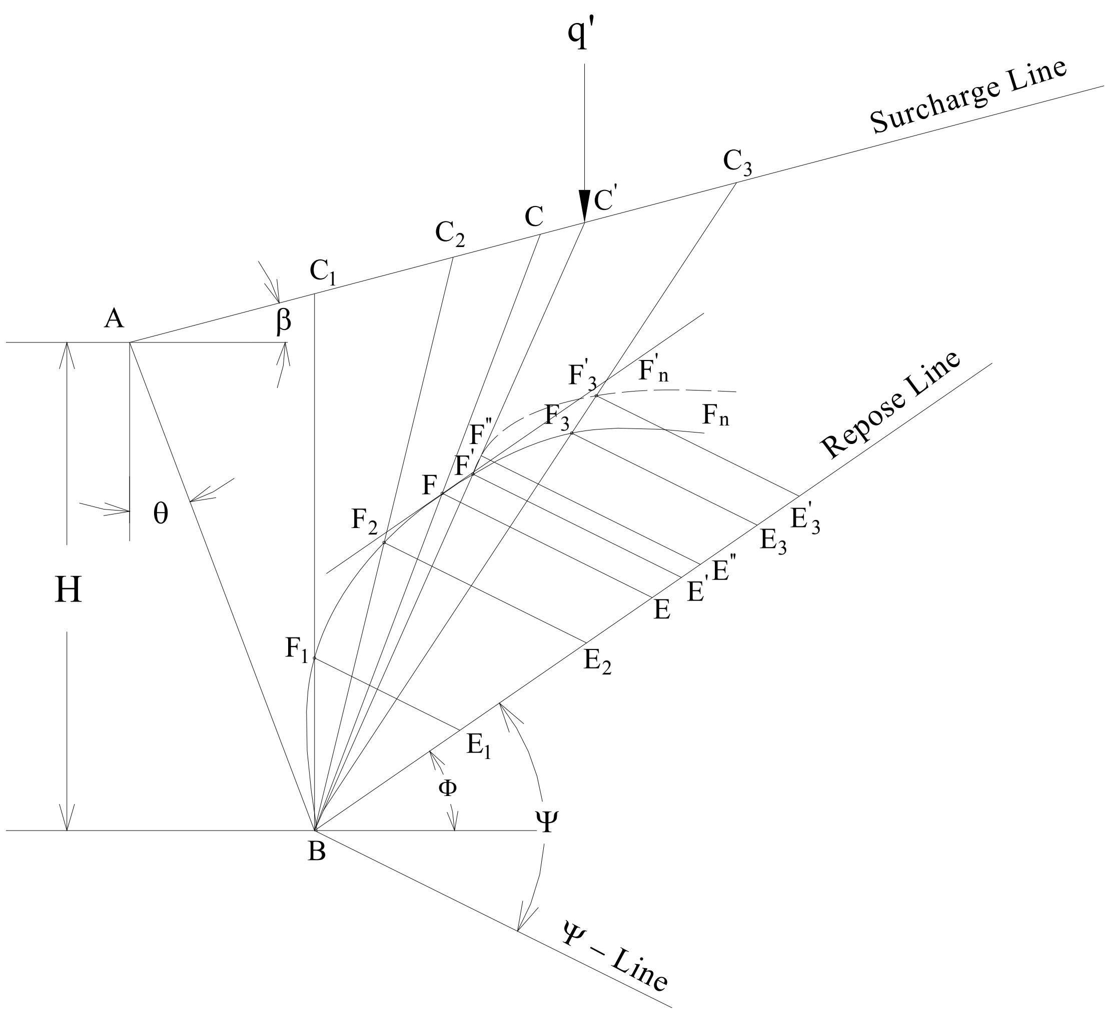


Fig 4.28 Culmann's method – effect of line load

As an illustration, consider Fig 4.28 in which a line load of intensity q^1 (per unit run) acts at distance d from top of wall. Example of line load is load due to any wall or a railway track running parallel to retaining wall. In the Fig 4.27, B, $F_{1,}$ F_2…… F_n is Culmann line obtained without considering line load. BC then represents the critical slip plane and the resultant activepressure is given by

$\therefore\ P_a = W.\left(\frac{FE}{BE}\right)$ Eq 4.40

where W = weight of wedge ABC.

If we consider the line load then E′, E_3….. will get shifted to E′′, E_3'… with E′E′′ = E_3E_3' = ….. q. There is an abrupt change in the Culmann line from F_1 to F′ and $BF_1F_2F'F''$…..F_n'

represents Culmann line obtained considering the line load. If $E''F''$ is greater than EF, slip occurs along BC' and the resultant active earth pressure is given by

$$P_a = W'.\left(\frac{F"E"}{BE"}\right) \qquad \text{Eq 4.41}$$

where W' = (weight of wedge ABC') + q. On the other hand if $E''F''$ is less than EF, slip occurs along BC and P_a is given by Eqn 4.40.

Culmann's method can also be used to find the minimum safe distance from top of retaining wall at which the line load can be placed without causing increase in P_a.

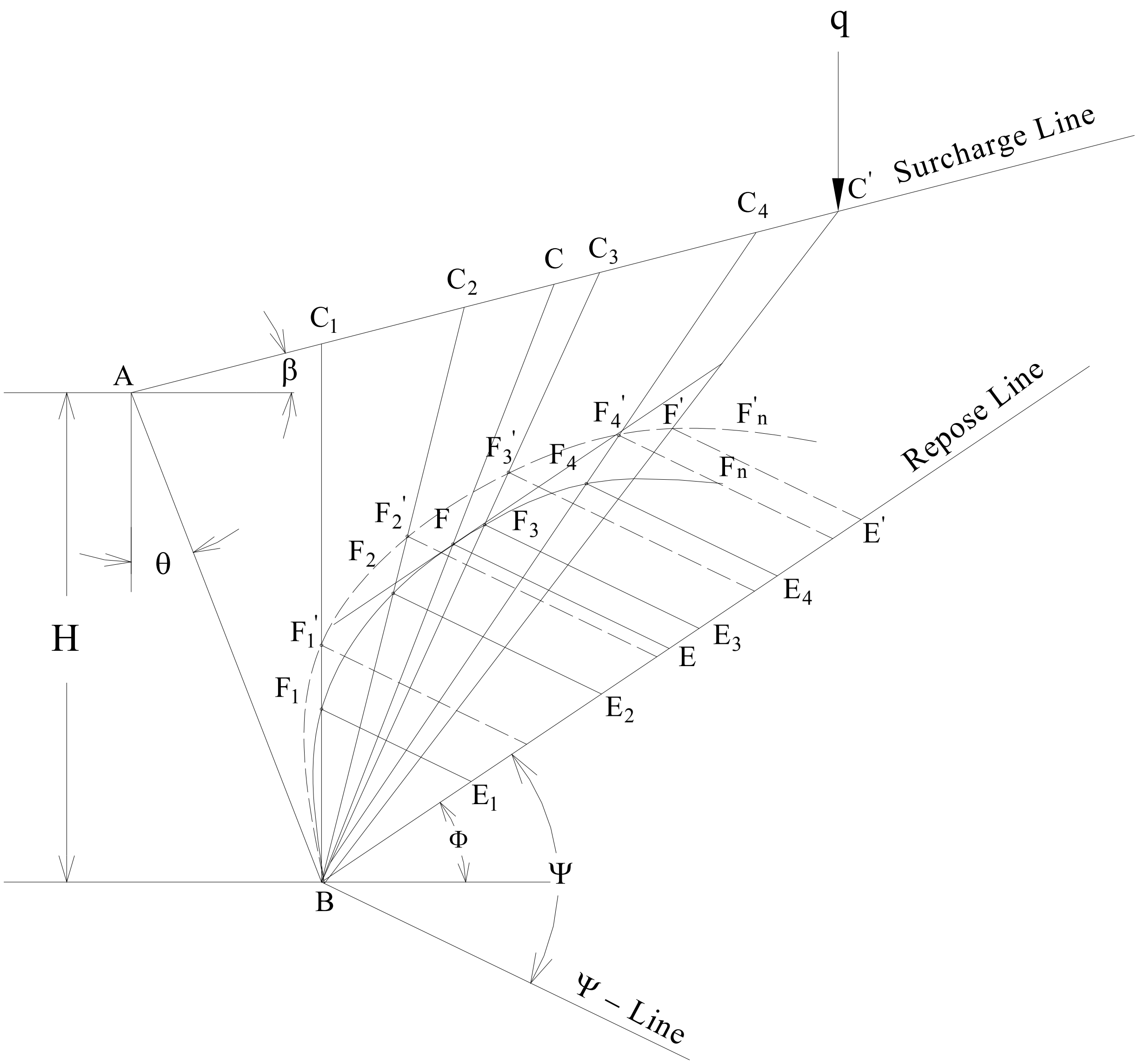


Fig 4.28a Safe location of line load

In Fig 4.28a, BF_1F_2……F_n represent Culmann line obtained without considering line load. $BF_1^1F_2^1$…..F_n^1 represents Culmann line obtained by placing line load q successively at C_1, C_2……… The line drawn tangential to Culmann line BF_1F_2……F_n and parallel to ∅-line touches the Culmann line at F and BC represents the critical slip plane when line load is not considered. In that case the resultant active earth pressure is given by

$$P_a = W\left(\frac{FE}{BE}\right) \qquad \text{Eq 4.42}$$

where W= weight of wedge ABC. The tangent drawn as described above is produced to cut the Culmann line $BF_1^1F_2^1.....F_n^1$ at F^1. BF^1 is joined and produced to intersect ground line at C^1. Then AC^1 represent the minimum safe distance at which q can be located without causing increase in P_a given by Eqn 4.42

4.8 Design of Gravity Retaining Wall

Gravity retaining walls are constructed of mass concrete, brick masonry or stone masonry. A gravity retaining wall resists the lateral earth pressure by virtue of its weight. Hence it is thicker in section compared to a cantilever or counterfort R.C.Cretaining wall which resists the lateral earth pressure by virtue of its resistance to bending.

The criteria of design of gravity retaining walls are:

1. The base width of the soil must be such that the maximum pressure exerted at base on soil does not exceed the safe bearing capacity of soil.
2. No tension should develop anywhere in the base.
3. The wall must be safe against sliding.
4. The wall must be safe against overturning.

Analysis:

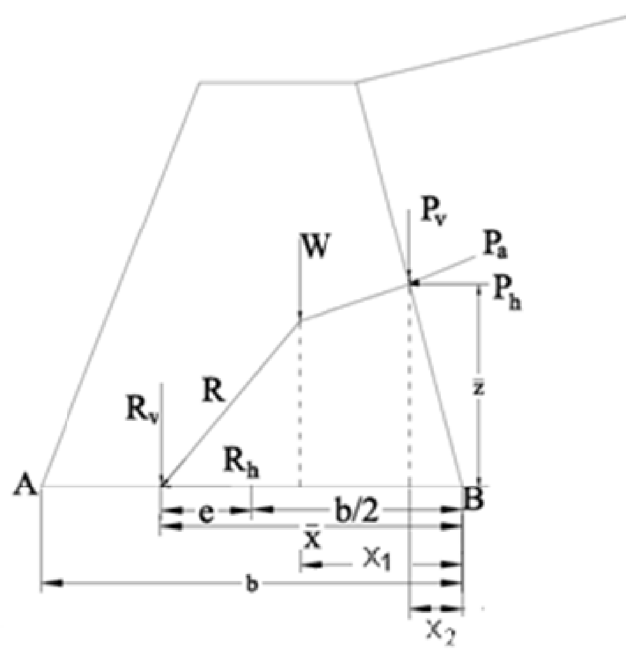


Fig 4.29 Forces on gravity retaining wall

In Fig 4.29, P_a is the resultant active earth pressure. Let P_v and P_h be the vertical and horizontal components of P_a . W is the weight of the wall. Both P_a and W are calculated per unit length of wall perpendicular to plane of figure. Let the resultant R of P_a and W intersect the base at a point with eccentricity e as shown in Fig4.29. If R_v and R_h denote the vertical and horizontal components of R

we have

$R_v = W + P_v$ Eq 4.43

$R_h = P_h$ Eq 4.44

Taking moments about B and applying Varignon's theorem we have

$$R_v \bar{x} = Wx_1 + P_v x_2 + P_h \bar{z}$$

$\therefore \bar{x} = \frac{Wx_1 + P_v x_2 + P_h \bar{z}}{R_v}$ Eq 4.45

Since R_v acts eccentrically on the base the pressure at base on soil will be combinations of direct and bending stresses.

Assuming a linear distribution of pressure, we have

Pressure at A, $p_1 = \frac{R_v}{b}\left(1 + \frac{6e}{b}\right)$

Pressure at B, $p_2 = \frac{R_v}{b}\left(1 - \frac{6e}{b}\right)$

In Fig 4.30 (a), (b) and (c) the pressure distribution diagrams for the three cases, that is when R strikes the base

(i) within middle one-third

(ii) at the outer edge of middle one-third and

(iii) outside middle one-third are shown clearly when $e < \frac{b}{6}$, both p_1 and p_2 are compressive. When $e = \frac{b}{6}$ or $\bar{x} = \frac{2b}{3}$ p_1 is compressive but P_2 becomes zero. When $e > \frac{b}{6}$, p_1 is compressive but p_2 becomes tensile. Since soil is generally incapable of resisting tension, the pressure is likely to be redistributed along the

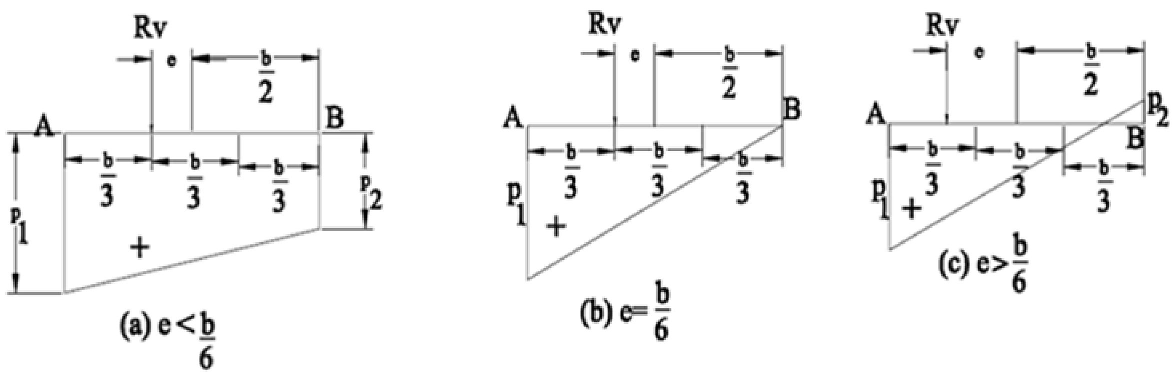


Fig 4.30 Pressure distribution at base of wall

base still in contact with soil, for a width of $3b^1$ where b^1 is the distance of point of application of R_v from A, i.e., $b^1 = \left(\frac{b}{2} - e\right)$. In such a case, we have

$$p_1 = \frac{2R_v}{3b^1} = \frac{2R_v}{3\left(\frac{b}{2} - e\right)}$$

To satisfy the design criteria, already outlined we have the following conditions.

1. The maximum pressure on soil p_1 should not exceed the safe bearing capacity of soil.
2. To satisfy the condition that no tension should develop, $e \leq \frac{b}{6}$ or $\bar{x} \leq \frac{2b}{3}$
3. For no sliding to occur,

   $R_h < \mu . R_v$

   where $\mu = \tan\delta$ and δ is the limiting angle of friction between soil and base of wall. The factor of safety against sliding is given by

   $$F = \frac{\mu R_v}{R_h}$$

   The minimum value of F expected in practice is 1.5.
4. For the wall to be safe against overturning, the resultant R must strike the base within the base width. It is easily observed that if the condition for no tension is satisfied then the safety against overturning is automatically ensured.