9.1 Specific Force

A short horizontal reach of a prismatic channel is considered. Further, the external frictional force and the effect of weight component of water can be considered as negligible. Then

$$\frac{\gamma Q}{g}\left(\beta_2 \overline{V}_2 - \beta_1 \overline{V}_1\right) = P_1 - P_2 + W\sin\theta - P_f$$

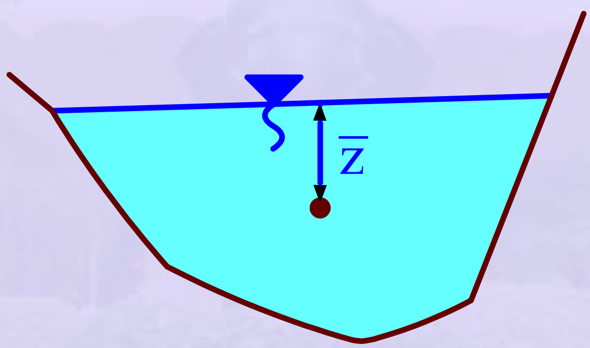
If $\theta = 0$, and $P_f = 0$ and also if $\beta_1 = \beta_2 = 1$, then the momentum equation simplifies can be written as

$$\frac{\gamma Q}{g}\left(\overline{V}_2 - \overline{V}_1\right) = P_1 - P_2$$

The hydrostatic pressure forces P_1 and P_2 are respectively

$$P_1 = \gamma \overline{z}_1 A_1 \;\; and \;\; P_2 = \gamma \overline{z}_2 A_2$$

in which $\overline{z}_1$ and $\overline{z}_2$ are the distances to the centroids below the surface of flow of the respective water flow areas (A_1 and A_2).



centroid from free surface

Also, $\overline{V}_1 = \frac{Q}{A_1}$ and $\overline{V}_2 = \frac{Q}{A_2}$.

Then, the momentum equation reduces to

$$\frac{Q^2}{gA_1} + \overline{z}_1 A_1 = \frac{Q^2}{gA_2} + \overline{z}_2 A_2$$

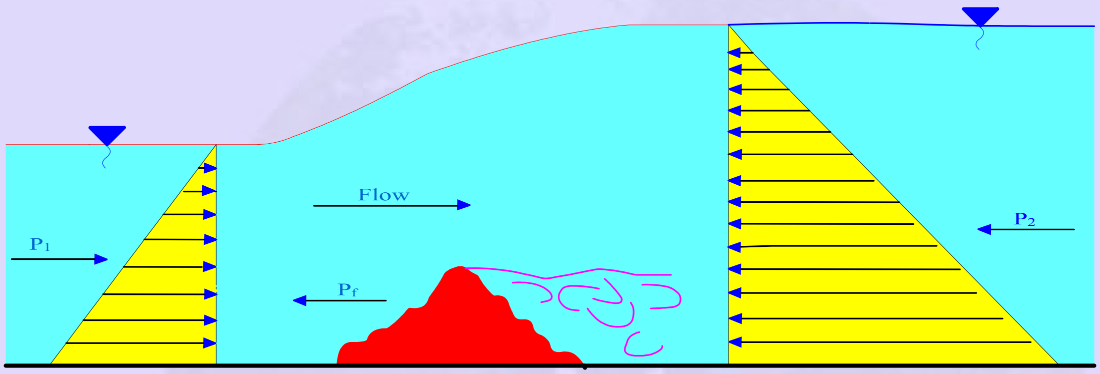
The two sides of the above equation are analogous and, hence, may be generally expressed for any channel geometry as

$$\boxed{M = \frac{Q^2}{gA} + \overline{z}A}$$

The first term is the rate of change of momentum of the flow passing through the channel section per unit weight of water, and the second term is the force per unit weight of water. Since both terms are essentially force per unit weight of water, their sum is known as the specific force indicated as M. Accordingly, it may be expressed as $M_1 = M_2$. This means that the specific forces of sections 1 and 2 are equal, provided that the external forces and the weight effect of water in the reach between the two sections can be ignored.

9.1.1 The momentum Function - Rectangular channels

The general situation is shown in Figure in which there may or may not be an energy loss between sections 1 and 2, and there may or may not be some obstacle on which there is a drag force P_f. In Figure the direction of P_f is that of the force exerted by the obstacle on the flow. It is this force (not the drag on the obstacle) which is to be considered in the momentum equation.



Definition Sketch - Momentum Equation

If there are any bluff body offering resistance force (P_f) to flow then

$$M_1 - M_2 = \frac{P_f}{\gamma}$$

The force P_f should include the frictional resistance due to boundary surface, and weight of the bluff body.

The following are some of the particular cases that occur in the field

1. Energy loss $\Delta E = 0, P_f \neq 0$ (the sluice gate)

2. $\Delta E \neq 0, P_f = 0$ (the simple hydraulic jump)

3. $\Delta E \neq 0, P_f \neq 0$ (the hydraulic jump with its formation assisted by some obstructions in the flow such as dentated sill (Forced hydraulic jump)

Sequent depths of Normal Hydraulic jump

If $P_f = 0$ then the specific force equations can be simplified as

$$\frac{q^2}{g}\left(\frac{1}{y_1}-\frac{1}{y_2}\right)=\frac{1}{2}\left(y_1{}^2-y_2{}^2\right)$$

$$i.e., \quad \frac{q^2}{gy_1y_2}=\frac{1}{2}\left(y_2+y_1\right)$$

`The substitution $q = v_1\ y_1$ leads to

$$\frac{v_1{}^2}{g}=\frac{1}{2}\frac{y_2}{y_1}\left(y_2+y_1\right)$$

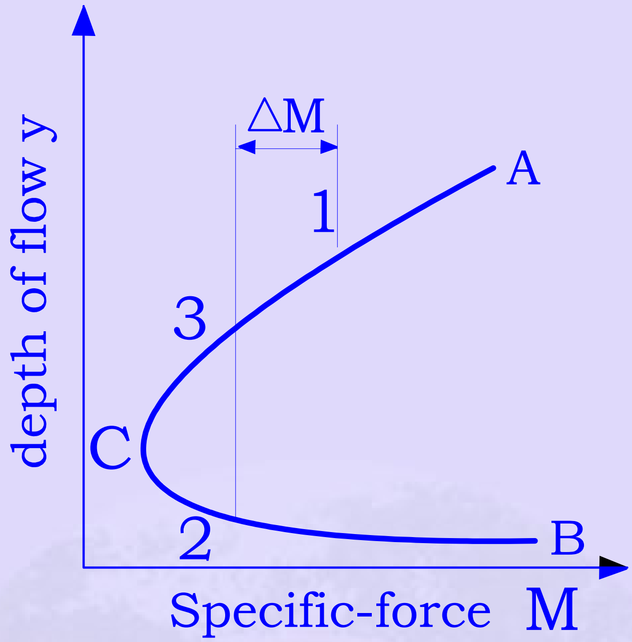
$$or \quad \frac{v_1{}^2}{gy_1}=F_1^2=\frac{1}{2}\frac{y_2}{y_1}\left(\frac{y_2}{y_1}+1\right)$$

which is the well- known equation of the normal hyraulic jump (NHJ). The Froude number F plays a key role. The above equation is quadratic in y_2 / y_1, whose solution is given by $\frac{y_2}{y_1}=\frac{1}{2}\left[\sqrt{1+8F_1^2}-1\right]$ and $\frac{y_1}{y_2}=\frac{1}{2}\left[\sqrt{1+8F_2^2}-1\right]$

In general, there are three independent quantities, and knowing two of them initially third one can be calculated. The downstream control can create appropriate conditions to form the jump. The corresponding depths y_1 and y_2 are known as conjugate or sequent depths.

9.1.2 Specific Force Diagram

The diagram shows the variation of the depth against the specific force for a given channel section and discharge, is called specific - force diagram. This curve has two limbs AC and BC. The limb BC approaches the horizontal axis asymptotically toward the right. The limb CA rises upward and extends indefinitely to the right. For a given value of the specific force, the curve has two possible depths y_1 and y_2. These two depths constitute the initial and sequent depths of a hydraulic jump (see box). At point C the specific force is minimum at the critical depth (see box).



Specific-force diagram

C is the point of minimum specific force for a given discharge – This corresponds to critical depth, AC is the sub critical limb, BC is the super critical limb. For a given specific energy there are two depths (Points 2, and 3 respectively) known as sequent depths. The difference between points 1 and 3 represent ΔM =specific force at point 1 minus the specific force at point 3.

The phenomenon of the hydraulic jump occurs when flow changes from supercritical to sub critical flow.

Minimum value of specific force:

The specific force to be of a minimum value then the first derivation of M with respect to y should be zero, i.e. $\frac{dM}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d(\bar{z}A)}{dy} = 0$

For a change die in the depth, the corresponding change $d(\bar{z}A)$click in the static moment of the water area becomes $d(\bar{z}A) \approx Ady$. Then the above equation simplifies as

$$\frac{dM}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + A = 0$$

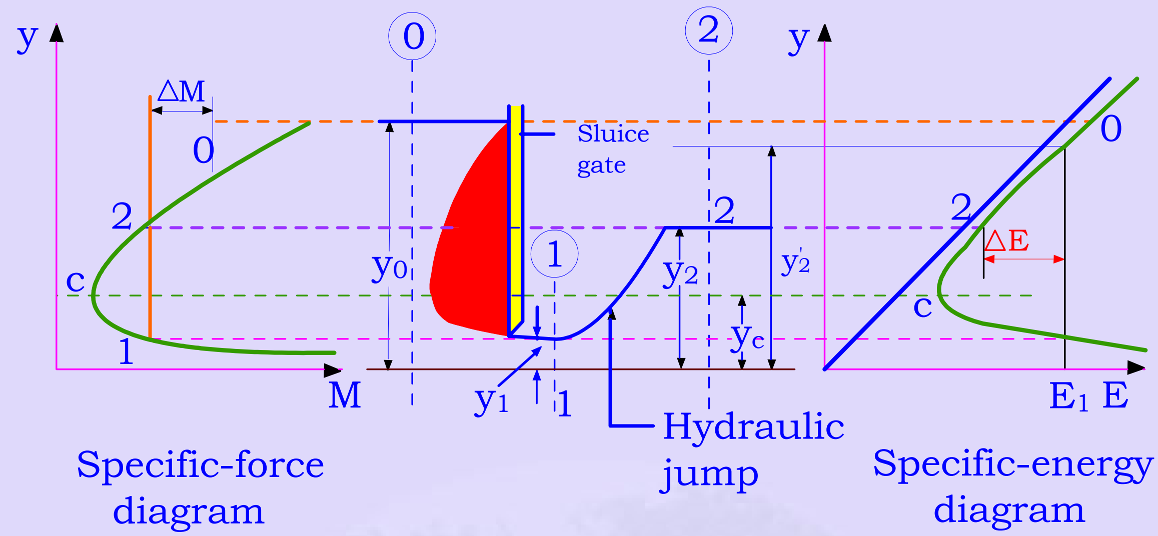
Since, $dA/dy = T$, $Q/A = \bar{V}$, and $A/T = D$. the above equation reduces to

$$\frac{\bar{V}^2}{2g} = \frac{D}{2}$$

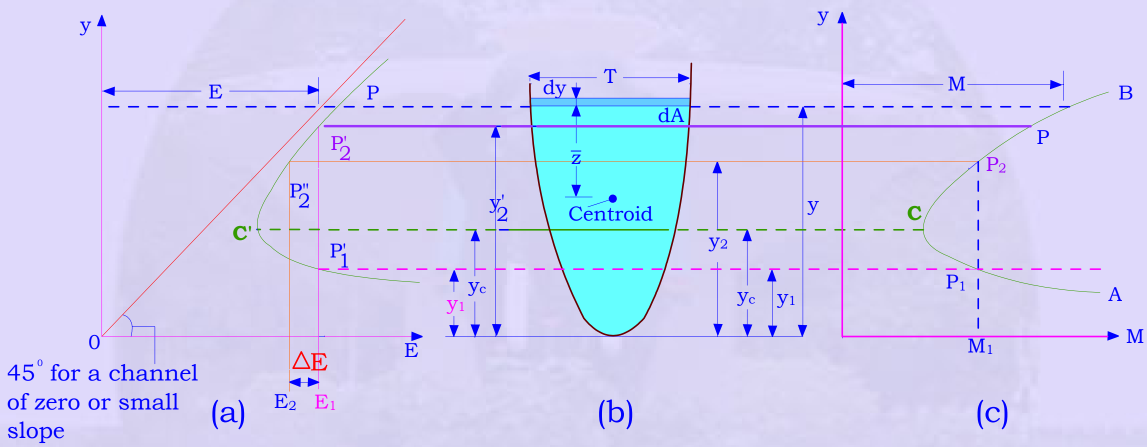
This is the criterion for the critical flow condition (Froude number =1). Therefore, the depth at the minimum value of the specific force is the critical depth. In other words the specific force is minimum for the given discharge at the critical state of flow.

9.1.3 Comparison between specific force and specific energy

For a given specific energy E_1, the specific - energy curve indicates two possible depths, namely, a low stage y_1 in the supercritical flow region and a high stage y_2 in the sub critical flow region. For a given value of M_1, the specific-force curve also indicates two possible depths, namely, an initial depth y_1 in the supercritical region and a sequent depth y_2 in the sub critical flow region. If the low stage and the initial depth are both equal to y_1. Then the sequent depth y_2 is always less than the high stage y'_2. Furthermore, the energy content E_2 for the depth y_2 is less than the energy content E_1 for the depth y_2. Hence, in order to maintain a constant value of M_1, the depth of flow may be changed from y_1 to y_2 which results in loss of specific energy is $\Delta E = E_1 - E_2$.



Hydraulic jump at sluice gate outlet



Specific-force curve supplemented with specific-energy curve.
(a) Specific-energy curve; (b) channel section; (c) specific-force curve

Note:

Specfic energy diagram	Specific force diagram
1. Given E_1 as initial depth y_1 (point P_1'). Initial depth y_1 is super critical depth. 2. Corresponding to E_1 the alternate depth y_2' on sub critical limb P_2' 3. The sequent depth due to hydraulic jump is y_2 and the corresponding specific	1. Corresponding to initial depth y_1 specific force is M_1 (Point P1). Initial depth y_1 is super critical depth. 2. Corresponding to alternate depth y_2' the specific force is point P. 3. The corresponding specific force for the sequent depth is M_1. In other words for Normal Hydraulic Jump, the upstream and downstream specific forces are the same. 4. The specific force corresponding to sequent depth is indicated by the point P_2.

In Hydraulic jump energy loss takes place. The depth corresponding to given E_1 at high stage is known as alternate depth to y_1 and vice versa. Whereas the depths due to jump are known as sequent depths.

y_1, y_2 are sequent depths.

y_1, y_2' are alternate depths.