

16.4 History of Uniform Flow Velocity and Resistance Factor

The design of the cross-section of the [Roman aqueducts](#) was based on structural rather than hydraulic requirements. Though the importance of the downward slope of the channel was realized, the aqueducts were laid at slopes governed by the topographic considerations alone.

HERO of Greece (after 150 B.C.) has clearly indicated that the rate of flow depended upon the overall change in the elevation on one hand, and upon the velocity as well as the cross-sectional area on the other.

LEONARDO DA VINCI (1452-1519):

“The water of straight rivers is the swifter the farther away it is from the walls, because of resistances.

Water has higher speed on the surface than at the bottom. This happens because water on the surface borders on air which is of little resistance, because lighter than water, and water at the bottom is touching the earth which is of higher resistance, because heavier than water and not moving. From this follows that the part which is more distant from the bottom has less resistance than that below”.

As regards the basic law of continuity of flow, he has clearly stated that:

“A river in each part of its length in an equal time gives passage to an equal quantity of water, whatever the width, the depth, the slope, the roughness, the tortuosity.

Each movement of water of equal surface width will run the swifter the smaller the depth”.

The law of continuity was explained in more certain terms and popularized by BENEDETTO CASTELLI (c 1577 – c 1644), became widely known in Italy as Castelli 's law.

The 18th century witnessed the advent of hydrodynamics – LEONHARD EULER (1707 – 1783) giving the equations of motion of an ideal fluid and DANIEL BERNOULLI (1700 – 1782) enunciating the famous energy equation which goes by his name.

HENRI DE PITOT (1695 – 1771) devised the velocity measuring device which carries his name – the Pitot tube.

CORNELIUS VELSON (1749), at Amsterdam, came to the conclusion that the velocity of flow should be proportional to the square root of the slope.

ALBERT BRAHMS, in 1757, considered the resistance thus set up to be proportional to the area of cross-section divided by the length of the wetted perimeter. Thus, resulted the expression $R = A / P$ where R is the hydraulic radius; A the area of cross section of the flow, and P the wetted perimeter.

16.4.1 Development of the Empirical Formulae

Credit for the first as well as the most lasting equation of resistance in uniform open channel flow goes to ANTOINE CHEZY, (1718 – 1798), a French Hydraulician, who was assigned the project of determining the cross-section of a canal to supply water to the city of Paris from the river Yvette.

Chezy put forth that \bar{V}^2 / RS_0 would be the same for all streams having similar characteristics; where \bar{V} is the mean velocity of flow and S the bed slope of the channel. Chezy, however, did not assume that the value \bar{V}^2 / RS_0 was a constant for all streams, as he found this value to vary from one stream to another.

The present-day- Chezy-formula is written as $\bar{V} = C\sqrt{RS}$ where C is known as 'Chezy Coefficient'. On the basis of a few observations of the flow made on an earthen channel, the Courpalet Canal and the Seine River, Chezy arrived at the value of C equal to 31. However, it should be noted that this formula, empirical in nature, is not dimensionally homogeneous. The Chezy coefficient C is not a pure number, but has a dimension of $[L]^{1/2} [T]^{-1}$, where $[L]$ and $[T]$ are units of length and time of any measuring system.

PIERRE LOUIS GEORGES DU BUAT (1734-1809): He proposed a formula for average velocity.

$$\bar{V} = \frac{48.85 \sqrt{R} - 0.8}{\sqrt{1/S_0} - \ln \sqrt{(1/S_0) + 1.6}} - 0.05\sqrt{R} \quad [\text{in metric units}].$$

The surface roughness of the boundaries was ignored in the formulation of the above equation.

JOHANN ALBERT EYTELWEIN (1764-1848), published, at Berlin in 1801, a formula for open channel flow, namely $\bar{V} = 50.9 \sqrt{RS_0}$ (in metric units)

A firm proponent of non dimensional quantities in the analysis of any problem, JULIUS WEISBACH (1806-1871), was the first to write a formula for resistance to flow through

closed pipes as
$$h_L = f \frac{L}{d_0} \frac{\bar{V}^2}{2g}$$

in which f is a non dimensional friction coefficient, which is commonly known at present as the Darcy-Weisbach friction factor, h_L the head lost due to the frictional resistance, L the length of pipe in which the head loss h_L has occurred and d_0 the diameter of the pipe. Weisbach reported that f is a function of the Reynolds number R_e and the relative roughness, for a given shape of cross-section.

By this period, the general form of the resistance equation for the uniform flow in rigid bed open channels was accepted to be given by the triple-factor formula $\bar{V} = CR^x S_0^y$ which represented the interdependence between the mean velocity of flow, hydraulic radius and the slope of the channel. Values of the coefficient C and the exponents x and y were chosen to make the formula conform to the experimental data obtained by each investigator. The various investigators, in choosing different values of C , x and y which they believed to be the most probable values, have deduced a large number of empirical flow formulae.

The first systematic and extensive series of experiments on open channel flow, to discover how the coefficient C varied with different kinds of roughness of the boundaries, were first begun by HENRY PHILIBERT GASPARD D'ARCY (1803-1858) in 1855 in France, and were continued after his death by his worthy assistant HENRI EMILE BAZIN (1829-1917).

D'ARCY conducted his studies in a wooden flume, 600 m long, drawing its supply from the Bourgogne Canal through a specially constructed head reservoir and discharging into the river Ouche. The flume was 2 m wide and 1 meter deep and has the feasibility of its inclination and cross-section could be changed. Rectangular, trapezoidal, triangular and semi-circular cross-sections were tested. The different surfaces tested

included cement, wood, brick, fine and coarse gravel, rock, and surface with artificial roughness in the form of wooden strips fixed transverse to the flow. Measurements on some earthen channels, which formed branches of the Bourgogne Canal, were also made.

Bazin observed that the value of C increased with an increase in slope, but concluded that this increase is of too small moment to be provided for in the equation.

Two Swiss engineers, EMILE OSCAR GANGUILLET (1818-1894) and WILHELM RUDOLPH KUTTER (1818-1888) concluded that the two formulae proposed by Bazin stood for two extreme conditions, and none of the two could be applied for general application. They published results in 1869.

$$C = \frac{a + \frac{l}{n} + \frac{\bar{m}}{S_0}}{1 + \left(a + \frac{\bar{m}}{S_0}\right) \frac{n}{\sqrt{R}}}$$

A detailed account of the development of the above formula was given by LINDQUIST.

The values of the constants a , l and \bar{m} arrived at by GANGUILLET AND KUTTER from the analysis of their data were

Constants	in metric units
a	23.00
l	1.00
\bar{m}	0.00155

PHILIPPE-GASPARD GAUCKLER (1826-1905) made a proposal of two formulae for use in different slope ranges, as follows:

$$\bar{V} = \lambda_1 R^{4/3} S_0 \quad \text{or} \quad C = \lambda_1 R^{5/6} S_0^{1/2} \quad \text{for } S_0 \text{ less than } 0.0007$$

$$\text{and } \bar{V} = \lambda_2 R^{2/3} S_0^{1/2} \quad \text{or} \quad C = \lambda_2 R^{1/6} \quad \text{for } S_0 \text{ greater than } 0.0007$$

in which λ_1 and λ_2 are coefficients to be determined experimentally.

In 1889, ROBERT MANNING (1816-1897), an Irish engineer, presented a paper containing several formulae for the velocity of flow in open channels, at a meeting of the Institution of Civil Engineers of Ireland. This paper was later published in the Transactions of the above Institution in 1891. In this paper, Manning proposed an

equation similar to the above equation to be in better agreement with the available experimental data of flow in open channels than any other formula used till that time. Manning found that the average value of the exponent of R varied from 0.6499 to 0.8395 on the basis of the experiments on artificial channels by D'ARCY and Bazin. He adapted an approximate value of 2 / 3 for this exponent. MANNING finally proposed, for earth channels in good condition, the formula.

$$\text{In metric units, } \bar{V} = 34 S_0^{1/2} \left(R^2 + \frac{R}{4} - 0.03 \right)$$

Or

$$C = 34 \left(1 + \frac{\sqrt{R}}{4} - \frac{0.03}{\sqrt{R}} \right)$$

The chronology of the present day Manning formula is given in detail in the discussions made by KING, CHOW, ROUSE, ROBERTSON, DOOGE, POWELL, POSEY.

By 1889, it was discovered that the reciprocal of λ , expressed in metric units, corresponded very closely to the roughness coefficient n associated with Ganguillet-Kutter formula. Thus, in 1891, FLAMANT gave the formula $\bar{V} = \frac{1}{n} R^{2/3} S_0^{1/2}$ (in metric

units) as Manning equation.

Later in 1923, STRICKLER supported the same formula, independently and chiefly based on his own observations in Switzerland. His analysis resulted in the equation.

$$\bar{V} = MR^{2/3} S_0^{1/2}$$

Manning formula reads as $\bar{V} = \frac{1}{n} R^{2/3} S_0^{1/2}$

and the coefficient C turns out to be $C = \frac{1}{n} R^{1/6}$

It is to be noted that the same numerical value of n can be used both in English and metric systems.

The coefficient C has one and the same value for all channels of very large dimensions.

Thus, Bazin proposed a new formula

$$\frac{RS}{\bar{V}^2} = \left[0.0115 \left(1 + \frac{Y_B}{\sqrt{R}} \right) \right]^2 \quad (\text{in metric units})$$

which can be reduced to the form

$$C = \frac{86.96}{1 + \frac{Y_B}{\sqrt{R}}} \text{ (in metric units)}$$

The term Y_B in the above equation is a roughness factor. However, Bazin's Y_B exhibits a thirty fold variation for a threefold variation in Kutter's n .

As the slope of the channel is, once again, not considered in the above equation, Bazin's C is considered to be a function of R alone and not S_0 .

Another empirical formula for the Chezy coefficient C was given by PAVLOVSKII, in 1925.

$$\text{The formula is } C = \frac{R_i}{n} \text{ (in metric units)}$$

$$\text{in which } i = 2.5\sqrt{n} - 0.13 - 0.75\sqrt{R}(\sqrt{n} - 0.10)$$

The values of n in the above formula are the same as those in the case of Manning formula. The use of this formula is limited to the ranges of hydraulic radius between 0.10 and 3.0 m and n between 0.011 and 0.040. For practical purposes, PAVLOVSKII also offered two approximate formulae for the exponent i , VIZ.,

$$i = 1.5\sqrt{n} \text{ for } R \text{ less than 1 meter and}$$

$$i = 1.3\sqrt{n} \text{ for } R \text{ greater than 1 meter.}$$

But it is the original formula of PAVLOVSKII which, in spite of its cumbersome form, is generally used in preference to the above simplified formulae.

1. CHEZY FORMULA (1775):

$$\bar{V} = C\sqrt{RS}$$

2. DU BUAT FORMULA (1779):

$$\bar{V} = \frac{48.85 \sqrt{R} - 0.8}{\sqrt{\frac{1}{s_0} - \ln \sqrt{\frac{1}{s_0} + 1.6}}} - 0.05\sqrt{R}$$

3. GIRARD FORMULA (1803):

4. DE PRONY FORMULA (1804):

$$RS_0 = 0.00004445 \bar{V} + 0.00030931 \bar{V}^2 \text{ (in metric units)}$$

5. EYTELWEIN FORMULA (1814-1815):

$$RS_0 = 0.0000243 \bar{V} + 0.000336 \bar{V}^2 \quad (\text{in metric units})$$

6. LAHMEYER FORMULA (1845)

This is based on 616 gaugings on the river Weser in Germany, and takes into consideration the effect due to bends in a river.

$$\frac{RS_0}{\bar{V}\sqrt{\bar{V}}} = 0.0004021 + 0.0002881 \frac{W}{r_c} \quad (\text{in metric units})$$

in which W is the width of the river and R_c the radius of curvature of the river. For a straight reach of the river, the term containing R_c should be dropped out. It is to be noted that the term W/r_c is reported under the root sign by LELIAVSKY.

7. ST. VENANT FORMULA (1851):

$$\bar{V} = 60 (RS_0)^{11/21} \quad (\text{in metric units})$$

8. TADINI FORMULA (1850): (in metric units)

$$\bar{V} = 50 \sqrt{RS_0}$$

The same formula is also attributed to COURTOIS.

9. HUMPHREYS and ABBOT FORMULA (1861)

$$\bar{V} = \left[\sqrt{\frac{0.0025 * 0.933}{\sqrt{R} + 0.457}} + \sqrt{68.72 R \sqrt{S_0}} - 0.05 \sqrt{\frac{0.933}{\sqrt{R} + 0.457}} \right]^2 \quad (\text{in metric units})$$

10. GANGUILLET AND KUTTER FORMULA (1869):

In metric units, C is given by

$$C = \frac{23.0 + \frac{1}{n} + \frac{0.00155}{S_0}}{1 + \left(23.0 + \frac{0.00155}{S_0} \right) \frac{n}{\sqrt{R}}}$$

11. 'REDUCED' FORM OF GANGUILLET – KUTTER FORMULA:

$$C = \frac{100\sqrt{R}}{n + \sqrt{R}} \quad (\text{in metric units})$$

12. GIBSON FORMULA:

$$C = \frac{24.55 + \frac{1}{n}}{1 + 24.55 \frac{n}{\sqrt{R}}} \text{ (in metric units)}$$

13. MANNING FORMULA (1889):

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \text{ (in metric units)}$$

14. BAZIN FORMULA (1897):

$$C = \frac{86.96}{1 + \frac{\gamma_B}{\sqrt{R}}} \text{ (in metric units)}$$

15. SIEDEK FORMULA (1901):

This formula was given, in metric units, for the case of natural streams and rivers.

$$\bar{V} = \frac{y_{\text{mean}} \sqrt{1000 S_0}}{(W)^{1/20}}$$

where y_{mean} is the mean depth of flow. This formula was stated to be applicable to “normal” channels was classified, with the corresponding correction to the basic formula given above and is expressed in terms of tables and involved formula.

16. VELLUT FORMULA (1902):

$$C = \frac{23.0 + \frac{1}{\gamma_V}}{1 + \frac{25.0 \gamma_V}{\sqrt{R}}} \text{ (in metric units)}$$

Where γ_V is the roughness coefficient.

17. HERMANEK FORMULA (1905): (in metric units)

This formula is proposed for rivers and streams. Forcheimer, modified the formula and presented the same as follows.

- (i) $\bar{V} = 30.7 y S^{0.5}$ for $y_{\text{mean}} < 1.5 \text{ m}$
- (ii) $\bar{V} = 34.0 y^{0.75} S^{0.5}$ for $1.5 < y_{\text{mean}} < 6 \text{ m}$
- (iii) $\bar{V} = 44.5 y^{0.60} S^{0.5}$ for $y_{\text{mean}} > 6 \text{ m}$

18. MATAKIEWICZ FORMULA (1911): (in metric units) reported by STRICKLER

$$(i) \bar{V} = 35.4 y^{0.7} S^{0.493 + 10S}$$

$$(ii) \bar{V} = 35.4 R^{0.7} S^\beta$$

in which β is a variable exponent dependant on boundary roughness

19. KOCHLIN FORMULA (1913): (in metric units)

$$\bar{V} = C_K (1 + 0.6 \sqrt{R}) \sqrt{RS}$$

where C_K is the roughness parameter.

20. BARNES FORMULA (1916):

$$\bar{V} = C R^\alpha S^\beta$$

in which C , α and β vary depending on the type of the channel boundaries

21. STRICKLER FORMULA (1923):

$$\bar{V} = M R^{2/3} S^{1/2}$$

22. FORCHHEIMER FORMULA (1923):

$$\bar{V} = C R^{0.7} S^{0.5}$$

where the value of the coefficient C varied from 143 to 43 (in English units)

23. PAVLOVSKII FORMULA (1925): (in metric units)

$$C = \frac{1}{n} R^i$$

in which

$$i = 2.5 \sqrt{n} - 0.13 - 0.75\sqrt{R} (\sqrt{n} - 0.10)$$

Manning formula has the main advantage of being simple, easily remembered and least laborious in computations. Also, it was found from the analysis of several tests under wide ranges of flow conditions as regards roughnesses of the boundaries, and shape, size and types of channels, that this formula yields results accurate enough for all practical purposes, when the values of roughness coefficient "n" already standardized for Ganguillet- Kutter formula themselves were adapted. This formula was more accurate for small slopes. The change over to the use of Manning formula was thus made convenient for there was no need to get familiarized with a new set of roughness coefficients.

Another advantage of the simple form of Manning formula is that a very simple relation exists between any given value of n and the corresponding value of velocity or slope. If a certain error be made in selecting n , then the computed value of velocity, and also the discharge in its turn, will involve the same percentage error but in the opposite direction. Likewise the value of slope computed to give a certain velocity will contain twice the same percentage error. The importance of this knowledge is of immense help to the designers.

REYNOLDS who, by his classical experiments with dyes, demonstrated clearly the difference between the two types of flows viz; laminar and turbulent and indicated the presence of a critical velocity. REYNOLDS also showed the physical significance of his dimensionless number.

$$R_e = \frac{\bar{V} L \rho}{\mu} = \frac{V L}{\nu}$$

he showed that a corresponding change in the law of resistance

occurred with the change in the type of motion.

By this time, the Darcy-Weisbach equation for head loss through circular pipes

$$h_f = f \frac{L}{d} \frac{\bar{V}^2}{2g}$$

was well established.

A set of very comprehensive and carefully conducted tests on the flow of water in circular pipes of different materials and of different diameters, by DARCY, revealed the following important phenomena.

(a) The coefficient of friction f is dependent on the Reynolds number R_e and the relative roughness of the pipe $\frac{k}{d_0}$, where k is the average depth of pipe wall roughness and d_0

is the diameter of the pipe.

(b) The coefficient f decreases with an increasing Reynolds number, the rate of decrease being smaller for greater relative roughness.

(c) The coefficient f is independent of the Reynolds number for certain relative roughness, and

(d) The coefficient f increases with an increasing relative roughness for any particular value of Reynolds number.

From dimensional analysis also, the same result is obtained, $f = f(R_e, k / d_o)$

In 1932 - 33, NIKURADSE conducted a series of well-planned tests on flow through circular pipes, artificially roughening the inside walls of the pipes by cementing layers of sand grains of uniform diameter.

Together with the theoretical work of PRANDTL and von KARMAN, Nikuradse's experimental findings have led to the establishment of semi rational formulae for velocity distribution and hydraulic resistance for turbulent flows in circular pipes.

The Hagen-Poiseuille equation can be written as

$$h_f = \frac{32\gamma \mu \bar{V} L}{d_0^2}$$

Where γ_v is the specific weight of the liquid.

In 1913, BLASIUS, drawing on the boundary layer theory, developed an empirical expression for the coefficient of friction f

$$f = \frac{0.3164}{(\bar{V} d_0 / \nu)^{0.25}} = \frac{0.3164}{R_e^{0.25}}$$

This result was based on the experimental data of flow in smooth circular pipes with the Reynolds numbers up to 100,000.

For the range, $4,000 \leq R_e \leq 100,000$, an almost perfect agreement between this equation and the experimental curve of NIKURADSE was observed. However, BLASIUS equation deviated considerably from the experimental curve when the Reynolds number exceeded 1,00,000.

COLEBROOK and WHITE carried out their investigations using commercial pipes and found significant difference in the value of f from those of NIKURADSE in the transition region from smooth turbulent to completely rough flow.

$$\frac{1}{\sqrt{f}} = 1.74 - 2.0 \log \left[\left(\frac{k_s}{r_0} \right) + \frac{18.7}{R_e \sqrt{f}} \right]$$

MOODY has plotted the above equation to appear in the form of a family of

$\log f$ vs $\log R_e$ curves for various $\frac{1}{\sqrt{f}} = 1.74 - 2.0 \log \left[\left(\frac{k_s}{r_0} \right) + \frac{18.7}{R_e \sqrt{f}} \right]$ values.

Application of the semi-rational formulae to open channel flows:

Analysing Bazins experimental data in this connection, KEULEGAN arrived at the equation $1/\sqrt{f} = 2.034 \log (R/k_s) + 2.211$ in the case of turbulent flow in rough-walled channels.

Thus, the logarithmic formulae for rough walled channels were expressed as follows:

$\frac{V}{v_*} = \bar{A}_S + 5.75 \log \left(\frac{Rv_*}{v} \right)$ for smooth channels and $\frac{V}{v_*} = \bar{A}_R + 5.75 \log \left(\frac{R}{k_s} \right)$ for rough

channels.

in which the characteristics \bar{A}_S and \bar{A}_R are functions of the Froude number.

16.4.2 Exponential Formulae

STRICKLER expressed the Manning 'n' in terms of roughness k_s as $n = 0.00106 k_s^{1/6}$ (k_s in cm)

But he started with the numerical value of 1.476 instead of 1.486 in the Manning formula. Strickler's formula for n is given by $n = 0.0342 d_m^{1/6}$

in which d_m is the median sieve size of the sand grains and in feet.

$$n = 0.02789 d_m^{1/6}$$

in which d_m is in "m".

WILLIAMSON from his experimental data and also with some suggested corrections to Nikuradse's data, gave the formula

$$n = 0.031 k_s^{1/6} \quad (\text{in English units})$$

Bretting stated that the logarithmic equation for the rough turbulent flow could be replaced by three exponential formulae each valid for a particular range of values of relative roughness. He found that exponential law equivalent to Manning formula was valid when $4.32 < R/k_s < 276$ requires Manning formula to be as given below.

$$n = 0.0387 k_s^{1/6} \quad (k_s \text{ in meters})$$

Manning formula is an exponential equation applicable to a particular range. In the first place, Manning's formula, in which V is associated with square root of S_0 , is there by limited in its application to the fully developed rough turbulent flow.

$$C = \sqrt{8g/f} = R^{1/6}/n \quad \text{or} \quad n = R^{1/6} \sqrt{f/8g}$$

For fully developed flow at high Reynolds number, f is found to be independent of Reynolds number, and nearly proportional to $1/R^{1/3}$. Thus, in the fully developed regions of flow, a nearly constant value of n is realized.

HENDERSON gives the criterion, for the satisfactory application of Manning equation, to be

$$(n^6) \sqrt{RS_f} \geq 3.0755 * 10^{-14} \quad \text{with the assumption } n = 0.03795 d^{1/6}$$

$$\nu = 1.01 * 10^{-6} \text{ m}^2/\text{s} \quad \text{and } g = 9.81 \text{ m/s/s}$$

Significant differences were observed between the discharge computed using a constant value of n and the actual discharge in the case of channels which gradually closed at the top, during the experimental investigations.

Moreover, it has also been observed that the value of the coefficient ' n ' varies considerably, even in prismatic channels (without gradually closing tops, (i) with age; (ii) in the presence of algae and vegetation and (iii) when the water carries sediment. A deposit of slimy silt on the bottom and sides of the channel was found to greatly reduce the frictional resistance to flow.

In the case of silt carrying waters, the lower layers of the moving water which are heavily silt-laden will form a kind of slurry which produces a lubricating effect in damping the vortices created at the surface of contact between the boundaries of the channels and the flowing water. The presence of large boulders on the bed also contributes to the varying nature of the coefficient ' n ' with the stage of flow.

The variation of the Manning coefficient ' n ' with the curvature of the channel was investigated by EDDY and SCOBNEY. The results, in general, indicated that while relatively low values of n were obtained for channels having smooth curvature with large

radius, sharp curvatures of the channels resulted in increased values of n . The effect of channel irregularity, non-linear alignment of the channel and obstructions to the flow on the flow characteristics and the roughness coefficient. Further, the value of Manning n was observed to vary with the stage and discharge in the natural streams and rivers, depending upon the existing conditions of the particular channel. In 1956, COWAN developed a procedure to select the value of n applicable to natural streams, floodways and similar channels. This method involved the selection of the basic n'_0 value for a straight, uniform, smooth channel in the natural material and of the modifying values for each of the five primary affecting factors; viz.

- (i) n'_1 due to the surface irregularities;
- (ii) n'_2 due to the variation in the shape and size of the channel cross-sections;
- (iii) n'_3 due to the presence of obstructions in the flow;
- (iv) n'_4 because of growth of vegetation, algae or weeds; and,
- (v) n'_5 due to the meandering of the channel.

COWAN presented the values of the correction factors for various conditions. The value of n may be computed by the equation, $n = (n'_0 + n'_1 + n'_2 + n'_3 + n'_4) n'_5$

The factors affecting the Manning coefficient are summarized in an excellent manner by CHOW and he has stated that there is no evidence about the size and shape of a channel as an important factor affecting the value of n .