5.5 Axially compressed column

As pointed out in the last chapter, local buckling under compressive loading is an extremely important feature of thin walled sections. It has been shown that a compressed plate element with an edge free to deflect does not perform as satisfactorily when compared with a similar element supported along the two opposite edges. Methods of evaluating the effective widths for both edge support conditions were presented and discussed.

In analysing column behaviour, the first step is to determine the effective area (A_{eff}) of the cross section by summing up the total values of effective areas for all the individual elements.

The ultimate load (or squash load) of a short strut is obtained from

$$P_{cs} = A_{eff} \cdot f_{yd} = Q. A. f_{yd} \qquad \textbf{(5.15)}$$

Where

P_{cs} = ultimate load of a short strut

A_{eff} = sum of the effective areas of all the individual plate elements

Q = the ratio of the effective area to the total area of cross section at yield stress

In a long column with doubly - symmetric cross section, the failure load (P_c) is dependent on Euler buckling resistance (P_{EY}) and the imperfections present. The method of analysis presented here follows the Perry-Robertson

approach presented in the chapter on "Introduction to Column Buckling". Following that approach, the failure load is evaluated from

$$P_c = \frac{1}{2}\left\{\left[P_{cs} + (1+\eta)P_{Ey}\right] - \sqrt{\left[P_{cs} + (1+\eta)P_{Ey}\right]^2 - 4P_{cs}.P_{Ey}}\right\}$$

$$\text{Where } \eta = 0.002\left(\frac{l_e}{r_y} - 20\right), \quad \text{for } \frac{l_e}{r_y} > 20 \qquad \textbf{(5.16)}$$

$$\eta = 0, \quad \text{for } \frac{l_e}{r_y} > 20$$

P_{EY} = the minimum buckling load of column = $\frac{\pi^2 E I_{min}}{l_e^2}$

and r_y = radius of gyration corresponding to P_{EY}.

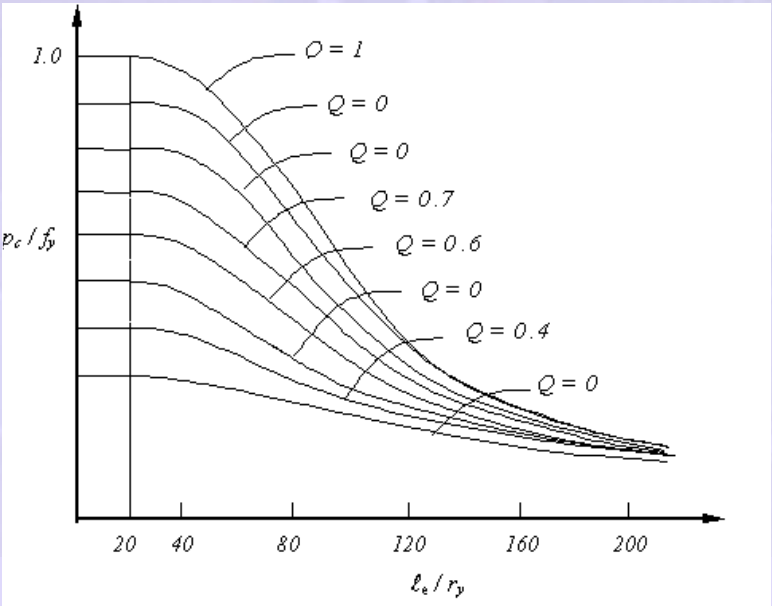


Fig.5.16 Column Strength (non- dimensional) for different Q factors

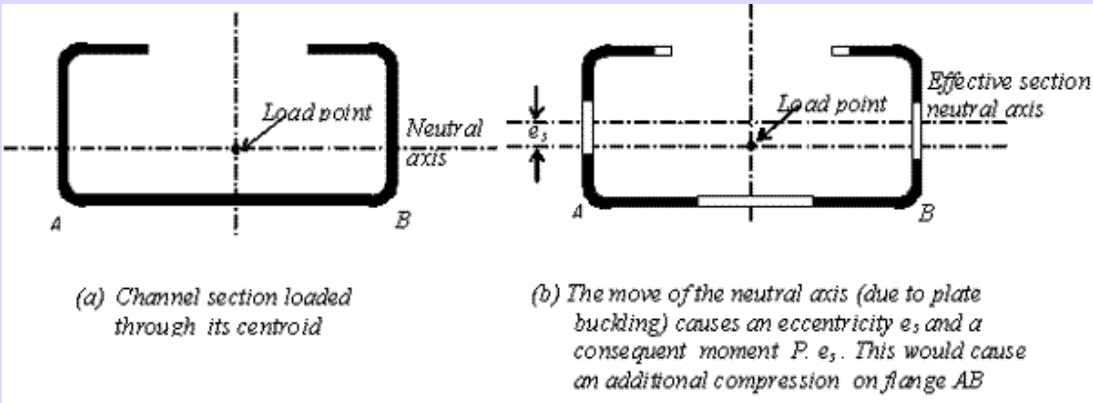


Fig.5.17 Effective shift in the loading axis in an axially compressed column

Fig. 5.16 shows the mean stress at failure ($p_c = P_c$ / cross sectional area) obtained for columns with variation of l_e / r_y for a number of "Q" factors. (The y-axis is non dimensionalised using the yield stress, f_y and "Q" factor is the ratio of effective cross sectional area to full cross sectional area). Plots such as Fig.7.16 can be employed directly for doubly symmetric sections.

5.5.1 Effective shift of loading axis

If a section is not doubly symmetric (see Fig. 5.17) and has a large reduction of effective widths of elements, then the effective section may be changed position of centroid. This would induce bending on an initially concentrically loaded section, as shown in Fig.5.17. To allow for this behaviour, the movement of effective neutral axis (e_s) from the geometric neutral axis of the cross section must be first determined by comparing the gross and effective section properties. The ultimate load is evaluated by allowing for the interaction of bending and compression using the following equation:

$$P_{ult} = \frac{P_c . M_c}{M_c + P_c . e_5} \qquad \textbf{(5.17)}$$

Where P_c is obtained from equation (5.16) and M_c is the bending resistance of the section for moments acting in the direction corresponding to the movement of neutral axis; e_s is the distance between the effective centroid and actual centroid of the cross section.

5.5.2 Torsional - flexural buckling

Singly symmetric columns may fail either (a) by Euler buckling about an axis perpendicular to the line of symmetry (as detailed in 5.5.1 above) or (b) by a combination of bending about the axis of symmetry and a twist as shown in Fig.5.18. This latter type of behaviour is known as Torsional-flexural behaviour. Purely torsional and purely flexural failure does not occur in a general case.

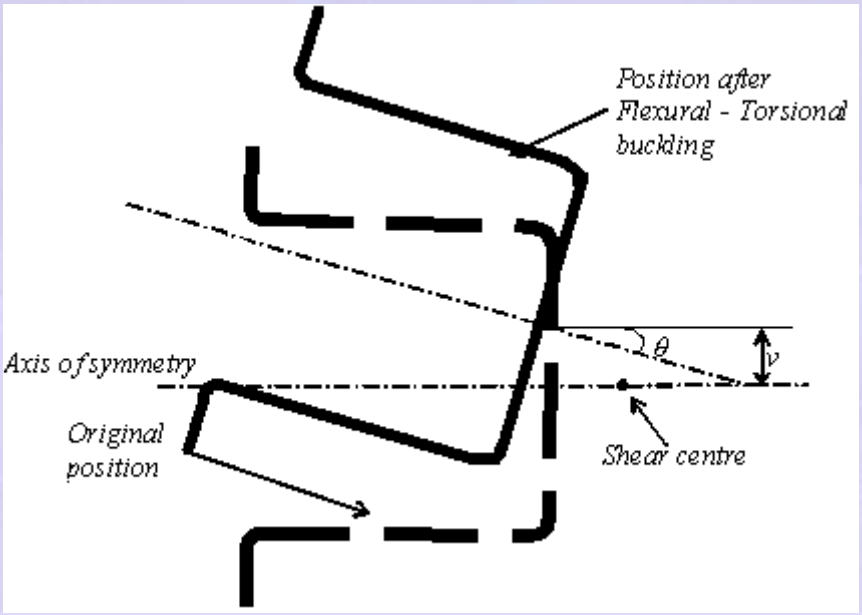


Fig.5.18 Column displacements during Flexural - Torsional buckling

Theoretical methods for the analysis of this problem was described in the chapters on Beam Columns. Analysis of torsional-flexural behaviour of cold formed sections is tedious and time consuming for practical design. Codes deal

with this problem by simplified design methods or by empirical methods based on experimental data.

As an illustration, the following design procedure, suggested in BS5950, Part 5 is detailed below as being suitable for sections with at least one axis of symmetry (say x - axis) and subjected to flexural torsional buckling.

Effective length multiplication factors (known as α factors) are tabulated for a number of section geometries. These α factors are employed to obtain increased effective lengths, which together with the design analysis prescribed in 5.5.1 above can be used to obtain torsional buckling resistance of a column.

$$\text{For} \quad P_{EY} \le P_{TF}, \qquad \alpha = 1$$

$$\text{For} \quad P_{EY} > P_{TF}, \qquad \alpha = \sqrt{\frac{P_{EY}}{P_{TF}}} \qquad \textbf{(5.18)}$$

α Values can be computed as follows:

Where P_{EY} is the elastic flexural buckling load (in Newton's) for a column about the y- axis, i.e.

$$\frac{\pi^2 EI_y}{l_e^{\,2}}$$

le = effective length (in mm) corresponding to the minimum radius of gyration

P_{TF} = torsional flexural buckling load (in Newtons) of a column given by

$$P_{TF} = \frac{1}{2\beta}\left[(P_{EX} + P_T) - \left\{ (P_{EX} + P_T)^2 - 4\beta P_{EX} P_T \right\}^{1/2} \right] \qquad \textbf{(5.19)}$$

where P_{EX} = Elastic flexural buckling load of the column (in Newton's) about the x- axis given by

$$\frac{\pi^2 EI_y}{l_e^2}$$

P_T = Torsional buckling load of a column (In Newton's) given by

$$P_T = \frac{1}{r_0^2}\left(GJ + \frac{2\pi^2 . E T}{l_e^2}\right) \qquad \textbf{(5.20)}$$

β is a constant given by $\beta = 1 - \left(\frac{x_0}{r_0}\right)$ ***(5.21)***

In these equations,

r_o = polar radius of gyration about the shear centre (in mm) given by

$$r_0 = \left(r^2_x + r^2_y + x^2_0\right)^{1/2} \qquad \textbf{(5.22)}$$

Where

r_x, r_y are the radii of gyration (in mm) about the x and y- axis

G is the shear modulus (N/mm^2)

x_0 is the distance from shear centre to the centroid measured along the x axis (mm)

J St Venants' Torsion constant (mm^4) which may be taken as $\sum \frac{bt^3}{3}$ summed up for all elements,

Where b = flat width of the element and t = thickness (both of them measure in mm)

I_x the moment of inertia about the x axis (mm^4)

Γ Warping constant for all section.

5.5.3 Torsion behaviour

Cold formed sections are mainly formed with "open" sections and do not have high resistance to torsion. Hence the application of load which would cause torsion should be avoided where possible. Generally speaking, by adjusting the method of load application, it is possible to restrain twisting so that torsion does not occur to any significant extent.

In general, when examining torsional behaviour of thin walled sections, the total torsion may be regarded as being made up of two effects:

- St. Venant's Torsion or Pure Torsion
- Warping torsion.

St.Venant's torsion produces shear stresses, which vary linearly through the material thickness. Warping torsion produces in-plane bending of the elements of a cross section, thus inducing direct (i.e. normal) stresses and the angle of twist increases linearly.

Since cold formed sections are thin walled, they have very little resistance to St. Venant's Torsion and will twist substantially. The extent of warping torsion in a thin walled beam is very much dependent on the warping restraint afforded by the supports as well as the loading conditions and the type of section.

If the beam ends are restrained from warping, then short beams exhibit high resistance to warping torsion and the total torque acting on such a beam will be almost completely devoted to overcoming warping resistance, the St Venant's Torsion being negligible. Conversely, the resistance to warping torsion becomes low for long beams and warping stresses and degrees of twist become very large.

A detailed theoretical treatment of beams subject to bending and torsion is given in another chapter. As stated previously, particular care and attention should be paid to the detailing of the connections and the method of load application so that the design for torsion does not pose a serious problem.