## Finite Element Analysis QUIZ II (Closed Book)

## Answer all questions.

Maximum marks: 20
All questions carry equal marks.
Time: 120 minutes

## Question 1:

Analyze the space truss shown in the figure below. The truss is composed of four nodes, whose coordinates (in meters) are shown in the figure, and three elements, whose crosssectional areas are all $10 \times 10^{-4} \mathrm{~m}^{2}$. The modulus of elasticity $E=210 \mathrm{GPa}$ for all the elements. A load of 20 kN is applied at node 1 in the global $x$-direction. Nodes 2 to 4 are pin supported and thus constrained from movement in the $x, y$, and $z$ directions.


## Question 2:

A beam is clamped at the left end and on a spring at the right end as shown in the figure below. A force $F=3,000 \mathrm{~N}$ acts downward at the right end as shown. The spring stiffness $k=3,000 \mathrm{~N} / \mathrm{m}$. The beam properties are $L=1 \mathrm{~m}, E I=1,000 \mathrm{Nm}^{2}$. Determine the deflection curve $v(x)$ and bending moment curve $M(x)$.


## Question 3:

For the four-noded bar element shown in the figure below, show that the Jacobian determinant is $|\underline{J}|=L / 2$. Also determine the shape functions $N_{1}$ to $N_{4}$ and the strain/displacement matrix $\underline{B}$.


## Question 4:

Evaluate the integrals (a) $I=\int_{-1}^{1}\left[x^{2}+\cos (x / 2)\right] d x$ and (b) $I=\int_{-1}^{1}\left[3^{x}-x\right] d x$ using threepoint Gaussian quadrature. $x_{1}, x_{3}= \pm 0.77459666924148, x_{2}=0, w_{1}, w_{3}=5 / 9, w_{2}=8 / 9$.

## Question 5:

Consider two triangular elements shown in figure below. The nodal displacements are given as $\left\{u_{1}, v_{1}, u_{2}, v_{2}, u_{3}, v_{3}, u_{4}, v_{4}\right\}=\{-0.1,0,0.1,0,-0.1,0,0.1,0\}$. Calculate displacements $\{u, v\}$ and strains $\{\partial u / \partial x, \partial v / \partial y, \partial u / \partial y+\partial v / \partial x\}$ in both elements.


