Finite Element Analysis QUIZ I (Closed Book)

Answer all questions.   Maximum marks: 20
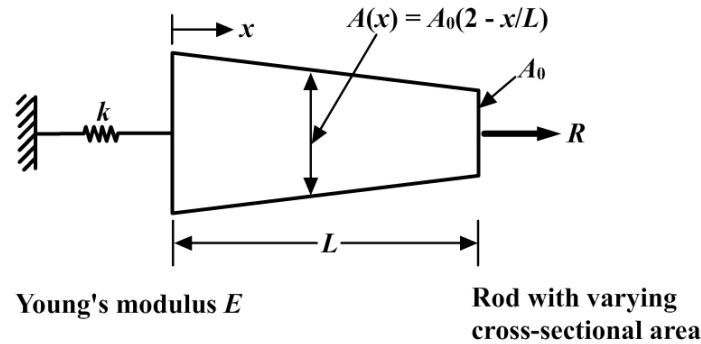
All questions carry equal marks.   Time: 120 minutes

Question 1:

Establish the differential equation of equilibrium of the problem shown in the figure below and the (geometric and force) boundary conditions.

Young's modulus E   **Rod with varying cross-sectional area**
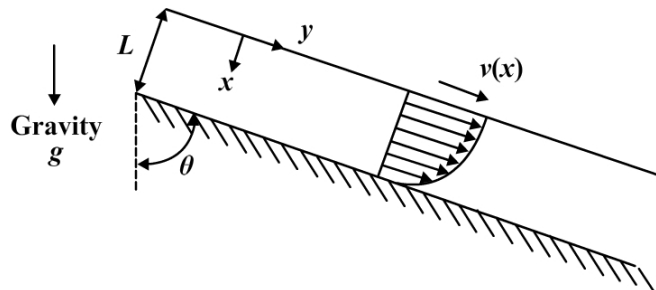
Question 2:

The governing equation for a fully developed steady laminar flow of a Newtonian viscous fluid on an inclined flat surface (see the figure below) is given by

$$\mu \frac{d^2 v}{dx^2} + \rho g\, \cos\theta = 0 \quad 0 \le x \le L$$

where μ = coefficient of viscosity, v = fluid velocity, ρ = density, g = acceleration due to gravity, θ = angle between the inclined surface and the vertical. The boundary conditions are given by

$$\frac{dv}{dx}\bigg|_{x=0} = 0 \text{ (zero shear stress)}, \quad v(L) = 0 \text{ (no slip)}.$$

Find the velocity distribution $v(x)$ using the least squares weighted residual method, assuming trial solution to be $v(x) = a_0 + a_1 x + a_2 x^2$.

Question 3:

Use Galerkin's method of weighted residuals to obtain a one-term approximation to the solution of the differential equation

$$\frac{d^2 y}{dx^2} + y = 4x \quad 0 \le x \le 1$$

with boundary conditions $y(0) = 0$, $y(1) = 1$. Hint: Assume trial solution to be $y = a_1 x(x-1) + x$.
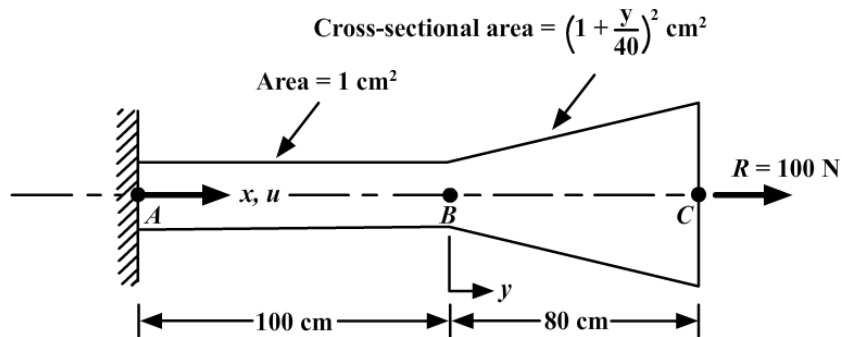
Question 4:

Consider a simple bar fixed at one end ($x = 0$) and subjected to a concentrated force at the other end ($x = 180$) as shown in the figure below. Using the notation given in the figure, the total potential of the structure is

$$\Pi = \int_0^{180} \frac{1}{2} EA \left(\frac{du}{dx} \right)^2 dx - 100 u \Big|_{x=180}$$

and the essential boundary condition is $u\big|_{x=0} = 0$. Calculate the displacement and stress distributions using the variational method with the following displacement assumptions:

$$u = \frac{x u_B}{100}, \qquad\qquad 0 \le x \le 100$$

$$u = \left(1 - \frac{x-100}{80} \right) u_B + \left(\frac{x-100}{80} \right) u_C, \quad 100 \le x \le 180$$

where u_B and u_C are the displacements at points B and C.

Cross-sectional area = $\left(1 + \frac{y}{40}\right)^2$ cm²

Area = 1 cm²

R = 100 N

x, u

A   B   C

y

|←——100 cm——→|←——80 cm——→|

Question 5:

The plane truss shown in the figure below consists of three members connected to each other and to the walls by pin joints. The members make equal angles with each other, and Element 2 is vertical. The members are identical to each other with properties: Young's modulus $E = 206$ GPa, cross-sectional area $A = 1 \times 10^{-4}$ m^2, and length $L = 1$ m. An inclined force $F = 20,000$ N is applied at Node 1. Solve for the displacements at Node 1 and stresses in the three elements.