Lesson 6

1. What are the sources of errors in numerical solutions of linear systems?

In practical solutions of linear systems (Ax = b), elements in both the coefficient matrix (A) and the right hand side vector (b) are approximate. These are potential sources of errors. However in addition to errors in x due to discrepancies in A and b, additional round off errors accrue during numerical solution of x = A^{-1}b. Bounds on these errors too are desirable. One simple way to get an estimate of the error during the numerical solution of Ax = b is to compute the residual vector r, i.e. after solving for x we compute r = x – Ab. The smaller the value of r, the more accurate the solution.

2. Why is the condition number important?

If the condition number is large, then small relative perturbations in the components of A will produce large changes in the solution x of the linear system Ax = b. Hence knowing the condition number of the coefficient matrix gives some idea about how stable the solution of a linear system is. However it is not straightforward to use the definition $\mathrm{K}(A) = \|A^{-1}\| \|A\|$ to find the condition number since the inverse of the matrix A has to be found prior to calculating the norm.

3.How can the solution obtained using an ill-conditioned coefficient matrix be improved?

When the coefficient matrix A is moderately ill conditioned, but not excessively so, the Gauss elimination solution can be improved (errors reduced) by performing iterations.

4. What are the two types of bounds on round off error and how do they differ?

Bounds on round off error can be a-prior bounds and a-posteriori bounds. A-priori bounds on the error can be found prior to the solution procedure. A-posteriori bounds can only be evaluated by observing how the components of the A matrix and b vector evolve during the solution process.

5. Can scaling the components of the coefficient matrix or of the unknowns reduce round-off errors and improve the accuracy of the solution?

Scaling of the unknowns or of the coefficient matrix has no effect on accuracy of the computed solution except by affecting the choice of pivots. Thus a suitable scaling, by increasing the size of the pivots may improve the stability of a solution, but it cannot improve its accuracy by reducing round off errors.