Lesson 5

1. What are special storage schemes and why are they required?

In some applications of numerical analysis e.g. for finite element applications, we often encounter symmetric positive definite matrices where the location of non-zero elements differ greatly from row to row. In order to save storage it is desirable that only the non-zero elements be stored. Thus special storage schemes have been devised for such matrices e.g. various profile storage schemes. They have 2 goals: (a) Reducing storage (b) Allowing fast access (with minimum computational cost) to the elements of the matrix during computations.

2. What is the storage scheme for the skyline solver?

The skyline solver is an example of a profile storage scheme. The upper triangle of each symmetric matrix A (n x n) is stored column by column in a vector, say ‘a’. For each column only the elements from the first non-zero entry to the diagonal are stored including the zeroes contained between the first non-zero entry and the diagonal. The location of the diagonal elements in ‘a’ are stored in a separate pointer vector.

3. What is sparseness preservation and how can it be accomplished?

Even if a compact storage scheme is adopted, it is of limited usefulness if successive stages of a numerical algorithm introduce a large number of non-zero elements. Hence it is desirable to use a solver algorithm which is both numerically stable and sparseness preserving. Partial pivoting does not preserve sparseness. Instead an alternative pivoting scheme known as “threshold” pivoting can be adopted to preserve sparseness.

4. How does one define the L_2 norm of a vector and a matrix?

The L_2 norm of a matrix x and a vector A are defined as:

$$\|x\|_2 = \sqrt{x.x} \quad \|A\|_2 = \sqrt{a_{ij}a_{ij}}$$

5. How is the inifinite norm of a vector and matrix defined?

The infinite norms are given by:

$$\|x\|_\infty = \max_{1\le i\le n}|x_i| \quad \|A\|_\infty = \max_{1\le i\le n}\sum_{i=1}^{n} |a_{ij}|$$

6. When are the matrix and vector norms consistent?

In general, if matrix and vector norms satisfy the requirement that $\|Ax\| \leq \|A\|\,\|x\|$ for any A and x, then the norm is said to be consistent.

7. How is the condition number of a matrix A defined?

The condition number of a matrix A is defined by K(A) where K(A) is:

$$\mathrm{K}(A) = \|A\|\|A\|^{-1}$$

where the norm considered is a consistent norm.