Lesson 37

1. What is an integral equation?

An integral equation is an equation in which the unknown function appears under an integral sign. If in addition the equation includes only linear functions of the unknown function it is a linear integral equation, otherwise it is a nonlinear integral equation.

2. What is a Fredholm equation?

A linear integral equation of the following form is known as a Fredholm equation:

$$\alpha(x)y(x) = F(x) + \lambda\int_a^b K(x,\xi)y(\xi)\,d\xi \quad (*)$$

Here $y(x)$ is the unknown function while $K(x,\xi)$, $\alpha(x)$ and $F(x)$ are known functions while λ, a and b are known constants. The function $K(x,\xi)$ which depends on the independent variable x as well as the auxiliary variable ξ is known as the kernel of the integral equation.

3. What is a Volterra Equation?

If the upper limit of the integral in (*) is the independent variable x rather than the constant 'b', the integral equation is known as Volterra's equation:

$$\alpha(x)y(x) = F(x) + \lambda\int_a^x K(x,\xi)y(\xi)\,d\xi \quad (**)$$

4. What is an integral equation of the first kind?

It is clear from both the Fredholm equation and the Volterra equations that when the function α is zero, the unknown function y appears only under the integral sign. In that case, the integral equations are known as integral equations of the first kind.

5. What is an integral equation of the second kind?

If $\alpha(x)$ is not zero, for both the Fredholm equation and the Volterra equation, by a suitable manipulation and change of variables, the integral equation can be written in a form such that $\alpha = 1$. Integral equations with $\alpha = 1$ are known as integral equations of the second kind.

6 Why do integral equations contain a complete formulation of the problem?

In general, an integral equations contain the complete formulation of a problem, including initial and boundary conditions. Integral equation formulations are therefore different from differential equations where initial and boundary conditions have to be specified separately