

## Lesson 35

1. What are spline functions?

A spline function of order  $p$  with 'nodes' at points:  $x_0, x_1, x_2, \dots, x_m$  with  $x_i \in [a, b]$  is a function such that on each subinterval  $[x_i, x_{i+1}]$  it would be represented by a polynomial of order  $p$ . In addition the  $p^{\text{th}}$  order spline function has all derivatives upto order  $p - 1$  continuous everywhere in the interval  $[a, b]$ .

2. What is the advantage of using spline functions?

Unlike equidistant interpolation with higher order polynomials often result in ill - conditioned interpolations interpolation with spline functions involves equidistant interpolation but always gives well - conditioned approximation functions.

3. Why are two additional constraint equations necessary for cubic splines?

The constraints are necessary to ensure continuity in the derivatives at the end points of the interval over which the spline interpolation is defined.

If a cubic spline is used to approximate a function  $f(x)$  whose second derivatives are not zero at the end points of the interval, then a good fit is still obtained near the centre of the interval (error =  $o(h^4)$ ) while the error near the boundaries of the interval are larger.