Lesson 31

1. When can a linear interpolation to an unknown function be preferable to a polynomial interpolation?

   Suppose the function values at $x_0, \ldots x_m$ are known and it is required to evaluate the function at x where $x_{k-1} < x < x_k$. Then instead of fitting a m^{th} order polynomial through the points $x_0, \ldots x_m$ a simpler solution may be to calculate $f(x)$ by linearly interpolating between $f(x_k)$ and $f(x_{k-1})$ which are function values evaluated at x_k and x_{k-1}.

   It can be shown that in a table of equidistant, correctly rounded function values, if the second difference $\Delta^2 f$ calculated from the function values satisfies the condition $|\Delta^2 f| \leq 4 \times 10^{-t}$ everywhere in the interval, then the total error in linear interpolation can only slightly exceed 10^{-t} in magnitude, where 10^{-t} is one unit in the last digit in the function values.

2. What are the components of the total error due to linear interpolation?

   The total error consists of several contributions. These include (a) the round off error in the known function values (b) truncation error, due to approximating the unknown function using a linear polynomial between the grid points (c) round off errors made during the computations involved in finding the linear interpolation. The bound on the error due to linear interpolation does not include the contribution due to (c), since it is assumed to be negligible compared to (a) and (b).

3. What is the disadvantage of using linear interpolation?

   In order to satisfy the requirement that the second difference $\Delta^2 f | \leq 4 \times 10^{-t}$ everywhere in the interval, the points $x_0, x_1 \ldots x_m$ must be sufficiently closely spaced. Given a fixed interval, this requires knowledge of the function values over a narrowly spaced grid. This increases the computational cost. On the other hand if a higher order interpolant is used, comparatively fewer grid points are required.

4. What is the Lagrange interpolation formula and why is it important?

   The Lagrange interpolation formula is widely used to construct interpolation functions (shape functions) in finite element computations. It is stated as:

$$Q(x) = \sum_{i=0}^{m} f_i \delta_i(x)$$

where δ_i is the polynomial of degree m and satisfies the relation:

$$\delta_i(x_j) = 1 \quad \text{if} \quad j = i$$

$$= 0 \quad \text{if} \quad j \neq i \qquad \forall \quad j = 0,1,2\ldots m$$

Hence $\delta_i(x)$ has the form:

$$\delta_i(x) = \frac{(x - x_0)(x - x_1)\ldots(x - x_{i-1})(x - x_{i+1})}{(x_i - x_0)(x_i - x_1)\ldots(x_i - x_{i-1})(x_i - x_{i+1})}$$

$$= \prod_{\substack{j=0 \\ j \neq i}}^{m} \frac{(x - x_j)}{(x_i - x_j)}$$

Given the form of $\delta_i(x)$, by definition $Q(x_j) = f(j), \qquad j = 0,1,\ldots m$

5. <u>What are Chebyshev polynomials?</u>

Chebyshev polynomials are an important class of orthogonal polynomials. They possess several desirable properties which make them well suited as interpolation functions.