Lesson 23

1. What are the initial and boundary conditions for the heat flow problem?

   In order to get unique solutions we have to specify initial conditions $T(x,t)|_{t=0} = T^0(x)$ as well as boundary conditions.

   In the heat flow problem we may specify the temperatures at the two ends of the bar:

   $$T(x,t)|_{x=0} = T_1(t) \text{ and } T(x,t)|_{x=l} = T_2(t)$$

   The most general boundary conditions are again the Robin boundary conditions where we prescribe a combination of the temperature and its spatial derivatives at the ends:

   $$T(0,t) + \alpha T_x(0,t) = T_1(t)$$
   $$T(l,t) + \beta T_x(l,t) = T_2(t)$$

2. What could be two possible methods for solving the diffusion problem analytically?

   Two possible methods include the method of eigen functions while the second approach involves the use of Laplace transforms.

3. How are the time dependent parts of the Fourier solutions for the wave equation and the diffusion equation different?

   Unlike the time dependent coefficients in the Fourier solution of the wave equation, the time dependent exponents in case of the diffusion equation decay at a fixed exponential rate $e^{-\kappa(2\pi n)^2 t}$. For the wave equation, the time dependent coefficients are harmonic functions and hence oscillatory in nature.

   The difference is related to fundamental distinctions in the physical behavior modeled, since the heat diffusion phenomenon is inherently decaying in nature.

4. What is the Laplace Transform?

   The Laplace transform is a means to transform differential operators to equivalent algebraic operators. Using the Laplace transform, a differential equation in the spatial and time domains can be transformed into an equivalent algebraic equation. Consider a partial differential equation with unknown variable *u*, which is a function of spatial variables ***x*** and time *t*. Successive Laplace transforms in the spatial and time domains result in an algebraic equation in the transformed space, *s*. Solving this algebraic equation one can obtain u as a function of s. Subsequent use of the inverse Laplace transform results in a solution for the partial differential equation: an expression for the unknown variable u in terms of spatial variables $\boldsymbol{x}$ and time t.