Lesson 16

1. What are the possible convergence problems that a Newton Raphson procedure may encounter?

   If the slope undergoes large changes along the equilibrium path, it is necessary that (a) the orientation of the path be correct (b) step sizes along the path be controlled. Otherwise the Newton Raphson procedure may not converge. In addition, once a limit point has been exceeded, there may be a need to change the direction of the load incrementation, otherwise the Newton Raphson procedure may diverge.

2. What are arc length methods?

   Instead of taking the full displacement increment $\Delta\mathbf{u}^*$ obtained using $\Delta\mathbf{u}^* = \mathbf{J}^{*-1}\Delta\mathbf{f}^*$ it may be necessary to scale the load and displacement increment in order to obtain convergence. Arc length methods are based on this idea: that the combined displacement-load increment should be controlled during equilibrium iterations, if convergence is to be ensured once a limit point in the load displacement curve has been exceeded.

   After the initial load and displacement increment from an equilibrium state, the equilibrium iterations are restricted to a constrained surface in the combined displacement-load space. The next equilibrium state occurs at the intersection of the equilibrium path with the constrained surface.

3. When are gradient based iterative methods useful?

   If the Jacobian is is a full matrix, then direct solution of this system by Gauss elimination or its variants is most economical. However if the Jacobian is sparse, then it may be more efficient computationally to consider iterative methods. Iterative methods such as Jacobi, Gauss-Seidel. SOR or alternatively, gradient based methods are widely used for sparse systems

4. What is the steepest descent method?

   The steepest descent method is a method used to find the minimum of a quadratic form, which is a scalar quadratic function of a *n*-dimensional vector **x:**

   $$f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{A}\mathbf{x} - \mathbf{b}^T\mathbf{x} + c$$

   where **A** is a matrix, **b** a vector and c a scalar constant

5. What is the difference between a global minimum and a local minimum? Is the steepest descent method always guaranteed to find the global minimum?

Non linear functions in general possess a large number of local minima. Ordinarily for a n-dimensional nonlinear equation there is no numerical method that will assuredly converge to a global minimum. Convergence to a particular local minimum can be assured so long as the starting point is "close" to that local minimum. Non linear functions which are quadratic forms are exceptions to this general rule. This is because quadratic forms with a positive definite coefficient matrix **A** have the property that the minimum **x** obtained by solving **Ax** = **b** is not only a local minimum of the function but also a global minimum. Thus the steepest descent method is guaranteed to find the global minimum of a quadratic form.

6. Can the steepest descent method find the global minimum of a non-quadratic form? What are the convergence criteria for the steepest descent method?

The steepest descent method is not guaranteed to find the global minimum for a non-quadratic form. However if during the line searches that are part of the steepest descent procedure, the Armejo condition is always satisfied, the steepest descent method is guaranteed to converge – if not to a global minimum at least to a local minimum.

The condition number of the coefficient matrix **A** determines the rate of convergence of the steepest descent method. Convergence is best for condition numbers close to one. Poor rates of convergence result when the coefficient matrix has a large condition number.