

## Lesson 15

1. When is an engineering problem load controlled and when is it displacement controlled?

If the independent variables of the problem are loads then the problem is known as a load-controlled problem. If the independent variables are displacements then we have a displacement controlled problem.

2. What is the residual vector? Why is it zero for linear problems?

Typically in a non linear problem we start with an initial guess for  $\tilde{\mathbf{u}}_{n+1}$ , and compute the new value of the internal force  $\mathbf{g}(\tilde{\mathbf{u}}_{n+1})$ . However if problem is nonlinear usually  $\mathbf{g}(\tilde{\mathbf{u}}_{n+1}) \neq \mathbf{f}_{n+1}$ . The difference  $\mathbf{f}_{n+1} - \mathbf{g}(\tilde{\mathbf{u}}_{n+1})$  is denoted as the residual vector  $\mathbf{r}_{n+1}$ . It is the part of the load that is not balanced by the internal forces. The purpose of the iterative scheme is to minimize the residual vector  $\mathbf{r}_{n+1}$ . However for a linear problem  $\mathbf{g}(\tilde{\mathbf{u}}_{n+1}) = \mathbf{f}_{n+1}$  and thus the residual vector is always zero.

3. What is meant by convergence of a nonlinear iteration? What is the convergence criteria for a nonlinear iterative scheme?

In a nonlinear iterative scheme the iteration process must be continued until the residual becomes near zero. Once the residual has become near zero, equilibrium has been reached for load step  $n$  and a new load increment is applied. The process of the residual becoming zero is known as convergence. The load increment is supposed to have converged if for small values of  $\varepsilon$ ,  $\|\mathbf{r}_{n+1}\| < \varepsilon \|\mathbf{f}_{n+1}\|$

4. What is the advantage of using the modified Newton method in multi-dimensions?

In the full Newton method, the Jacobian matrix has to be formed and inverted for each iteration. Computing the derivatives each iteration is expensive. In the one-dimensional Newton iteration it was found that it may not be necessary to compute the derivative at each iteration: the resultant loss of accuracy will only marginally affect the rate of convergence. In multi-dimensions this is potentially extremely advantageous since the cost of computing the tangent stiffness matrix is high.

Moreover, at each iteration we need to invert the tangent stiffness matrix to obtain the update to the solution. Thus if we can avoid having to compute the tangent stiffness at each iteration we can save the cost of inverting it every iteration, provided we store the last inverted tangent stiffness. For large systems this can lead to huge savings but unlike full Newton – it does not lead to quadratic convergence.

5. What is the difference between the modified Newton method and the Quasi-Newton method?

The modified Newton method preserves the tangent stiffness that is computed during the initial iteration of any load increment and uses it for all the iterations of that increment. However during an iteration, as soon as a displacement update has been determined, the corresponding internal forces can be evaluated. The updated internal forces provide additional information on the stiffness of the system. This information can be used to construct modifications to the stiffness matrix to yield improved convergence properties with modest computational effort.