Lesson 10

1. What is meant by a similarity transformation?

   If A is a n x n matrix, and P is any non-singular matrix with the same dimensions as A, then $A^* = PAP^{-1}$ is a similarity transformation of the matrix A. A similarity transformation preserves eigen values but linearly transforms the eigen vectors. Thus A^* and A have the same eigen values, but the eigen vectors of A^* are the eigen vectors of A premultiplied by the matrix P.

2. Can similarity transformations be used to obtain iteratively the eigen values and eigen vectors of a matrix?

   There is a broad class of methods for extracting the eigen values and eigen vectors of a matrix using similarity transformations. Typically they involve a sequence of similarity based transformations involving an orthogonal matrix. The sequence of transformations preserve symmetry, since if A is a symmetric matrix, A^* will also be symmetric.

3. How are the transformation matrices of a similarity based method for extracting the eigen values and eigen vectors chosen?

   The orthogonal transformations used in practice are mainly of two types – plane rotations and reflections. The aim of both types of transformations is reduce the initial matrix to as close to a diagonal form as possible – with the eigen values occupying the diagonal locations.

4. What is Jacobi's method?

   Jacobi's method is one of the most widely used similarity transformations for obtaining the eigen values and eigen vectors of a similarity matrix. It involves repeated similarity transformations with plane rotation matrix Q_k so that A_k tends to a diagonal matrix. Repeated use of this scheme leads to annihilation of all off-diagonal elements and finally leads to a diagonal matrix with the eigen values of A comprising the diagonal elements.

5. Is it always possible to reduce a matrix to diagonal form by performing similarity transformations?

   In general it is not possible to reduce a matrix to diagonal form by a finite sequence of similarity transformations. Thus many methods for eigen value extraction attempt to transform the matrix A to some other compact form which can be attained in a finite number of steps. Such forms include the symmetric tridiagonal form for symmetric matrices and the Hessenberg form for non-symmetric matrices. The eigen values and eigen vectors of tridiagonal and Hessenberg matrices can then be calculated relatively simply, by equation solving.