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Minimum Stages Calculation For Multicomponent Distillation Column

Keywords: Minimum No. of stages, Minimum reflex ratio, Actual no. of stages

For a specified separation between two key components of a multicomponent mixture, an exact expression is easily developed for the required minimum number of equilibrium stages, which corresponds to total reflux.

Consider the figure in which no feed is adding and no product is taking out.

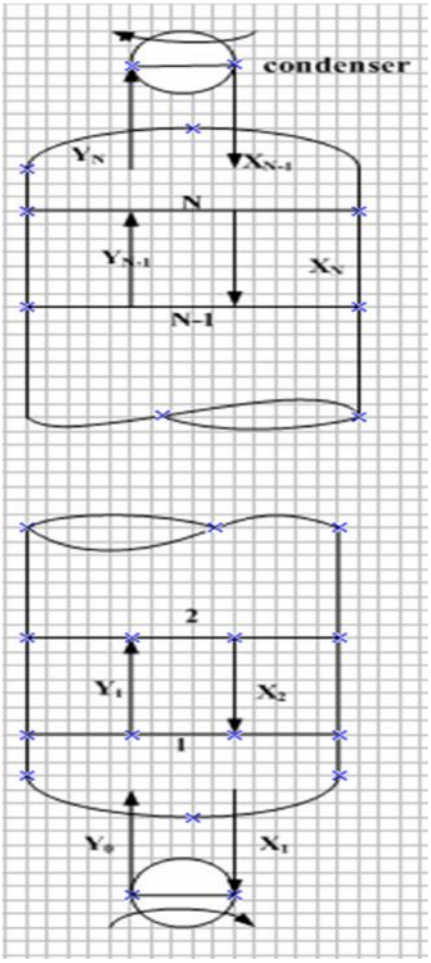


Fig. 28.1: Distillation Column Under Total reflex Condition

For steady state operation within the column, heat input to the reboiler and heat output from the condenser are made equal consuming (no heat losses).

By material balance, vapour and liquid streams passing between any pair of stages have equal flow rates and composition,

e.g. $V_{N-1}=L_N$ *and* $Y_{1N-1}=X_{1N}$.

However, vapour and liquid flow rates will change from stage to stage unless the assumption of const. molal over flow is valid.

For component **i** at stage **1**

$$Y_{i1}=K_{i1}X_{i1} \quad \ldots 28.1$$

But for passing streams

$$Y_{i1}=X_{i2} \quad \ldots 28.2$$

From equation (28.1) and equation (28.2)

$$X_{i2}=K_{i1}X_{i1} \quad \ldots 28.3$$

Similarly for stage 2

$$Y_{i2}=K_{i2}X_{i2} \quad \ldots 28.4$$

Combining equation (28.3) and (28.4)

$$Y_{i2}=K_{i2}K_{i1}X_{i1} \quad \ldots 28.5$$

Similarly for ith component

$$Y_{iN}=K_{iN}K_{iN-1}K_{iN-2}\ldots\ldots\ldots K_{i1}X_{i1} \quad \ldots 28.6$$

Similarly for jth component

$$Y_{iN}=K_{In}K_{iN-1}K_{iN-2}\ldots\ldots\ldots K_{i1}X_{i1} \quad \ldots 28.7$$

From equation (28.6) and (28.7)

$$(Y_{iN}/Y_{jN})=\alpha_N\,\alpha_{N-1}\ldots\ldots\ldots\ldots\,\alpha_2\,\alpha_1\,(X_{i1}/X_{j1}) \quad \ldots 28.8$$

Or

$$(X_{iN+1}/X_{i1})\,(X_{j1}/X_{jN+1}) = \prod_{k=1}^{Nmin}\alpha k \quad \ldots 28.9$$

Where $\alpha_k = (K_{ik}/K_{jk})$, the relative volatility between component i & j. Equation (28.9) relates the relative enrichments of any two components i & j over a cascade of N stages to the

relative volatilities between the two components.

Assume constant relative volatility

$$(X_{iN+1}/X_{i1})\ (X_{ji}/X_{jN+1})=\alpha^{N} \qquad \ldots 28.10$$

Or

$$Nmin = \frac{log\left\{\left[\frac{Xi,N+1}{Xi1}\right]\left[\frac{Xji}{XjN+1}\right]\right\}}{log\alpha ij} \qquad \ldots 28.11$$

Equation (28.11) is known as the Fenske equation where i=LK and j=HK, the minimum number of equilibrium plots is influenced by the components only by their effect on the value of the relative volatility between the key components.

Equation (28.11) also a rapid estimation of minimum equilibrium stages. A more convenient form of equation (28.11) is obtained by replacing the product of mole-fraction ratios by the equivalent product of mole-distribution ratios in terms of component distillate and bottom flow rates d & b, respectively, and replacing the relative volatility by a geometric mean of the top-stage and bottom stage values. Thus;

$$Nmin = \frac{log\left\{\left[\frac{di}{dj}\right]\left[\frac{bj}{bi}\right]\right\}}{log\ \alpha m} \qquad \ldots 28.12$$

Where ; $$\alpha_m=[(\alpha_{ij})_N\ (\alpha_{ij})_1]^{1/2} \qquad \ldots 28.13$$

Thus , the minimum number of equilibrium stages depends on the degree of separation of the two key components and their relative volatility, but is independent of feed-phase condition. Equation(28.12) is combination with (28.13) is exat for two minimum stages. For stage one, it is equivalent to the equilibrium flash equation. In practices, distribution columns are designed for separation corresponding to as many as 150 minimum equilibrium stages.

Distribution of non key components at Total Reflux

The Fenske equation is restricted to the two key components. Once $\mathbf{N_{min}}$ is known, equation (28.12) can be used to calculate mole fraction X_{N+1} and X_1 for all non key components. These values provide a first approximation to the actual product distribution when more than the minimum number of stages is employed.

Let i= a non key component and

J= the heavy key or reference component denoted by r, then equation (28.12) becomes

$$(d_i/b_i) = (d_r/b_r)(\alpha_{ir})_m^{Nmin} \quad \ldots 28.14$$

Substituting $f_i = d_i + b_i$ *in equation (28.14) since*

$$b_i = f_i / [1 + (d_r/b_r)(\alpha_{ir})_m^{Nmin}] \quad \ldots 28.15$$

$$d_i = f_i (d_r/b_r)(\alpha_{ir})_m^{Nmin} / [1 + (d_r/b_r)(\alpha_{ir})_m^{Nmin}] \quad \ldots 28.16$$

Equation (28.15) & (28.16) give the distribution of non key components at total reflux as predicted by Fenske equation.

Minimum reflux from Underwood equation:

Minimum reflux is based on the specifications for the degree of separation between two key components. The minimum reflux is finite and feed, product with drawls are permitted. However, a column cannot operate under this condition because of the requirement of infinite stages. But it is useful limiting condition. Shiras, Hanson, and Gibson classified multi component systems as having one or two pinch points on classes. For class-1 separations, all components in the feed distribute to both the distillate and bottoms products. Class – 1 separation can occur with narrow boiling range mixtures or when the degree of separation between the key components is not sharp.

For class – 2 separations, one or more of the components appear in only one of the products. If neither the distillate nor the bottoms product contains all the feed components, two pinch points occur away from the feed stage. Stages between the feed stage and rectifying section pinch point remove. Heavy components that do not appear in the distillate. Light components that do not appear in the bottoms are removed by the stages between the feed stage and the stripping section between pinch point.

Underwood equation:

$$\sum \frac{\sim_i x_{i,d}}{\sim_i - \theta} = R_m + 1$$

Where

$\sim_i$ = the relative volatility of component i with respect some ref. component, usually the heavy component.

R_m = the minimum reflux ratio.

x_{id} = concentration of component i in the tops at minimum reflux.

and $\bar{}$ is the root of the equation

$$\sum \frac{\tilde{}_i x_{i,f}}{\tilde{}_i \bar{}} = 1 - q$$

$x_{i,f}$ = the concentration of component i in the feed.

q = condition of feed = $\frac{heat\ to\ vaporize\ one\ mol\ of\ feed}{molar\ latent\ heat\ of\ feed}$

Gilliland correlation for actual reflux ratio:

$$Y = \frac{N - N_{\min}}{N + 1} = 1 - \exp\left[\left(\frac{1 + 54.4X}{11 + 117.0}\right)\left(\frac{X - 1}{X^{0.5}}\right) \right]$$

$$X = \frac{R - R_{\min}}{R + 1}$$

R = (1.1 – 1.5) $R_{\min}$

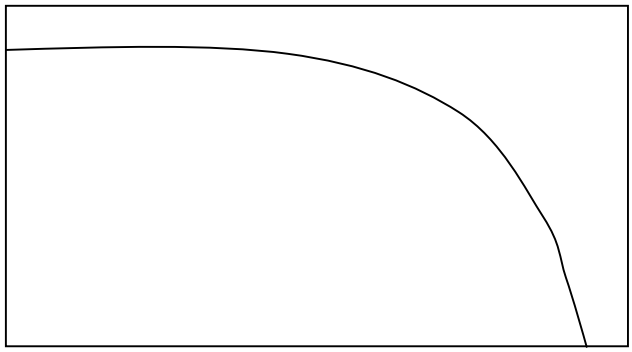


Fig. 28.2: Gilliland Plot

Feed stage location:

Kirkbridge correlation:

$$\frac{N_R}{N_S} = \left[\left(\frac{Z_{HK,F}}{Z_{LK,F}} \right) \left(\frac{x_{LK,B}}{x_{HK,D}} \right)^2 \left(\frac{B}{D} \right) \right]^{0.206}$$

where N_R= number of stages above the feed, including any partial condenser,

Ns = number of stages below the feed, including the reboiler,

B = molar flow bottom product,

D = molar flow top product,

$Z_{HK,F,}$ — concentration of the heavy key in the feed,

$Z_{LK,F}$ = concentration of the light key in the feed,

$X_{HK,d}$ = concentration of the heavy key in the top product,

$X_{LK,b}$ = concentration of the light key if in the bottom product.