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Modelling Of Multiple Effect Evaporators With Boiling Point Rise

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System that exhibit boiling point elevations generally possess liquid enthalpies that depend on both temperature and composition. In the following analysis, the general case is considered wherein both variations are involved.

Forward Feed :

Equation of energy balance ,heat transfer rate and boiling point rise of i^{th} effect evaporator for forward feed arrangement is as follows.

Solute material balance:

$$FX_f = L_i X_i \quad \dots 24.1$$

Overall material balance:

$$L_{i-1} = V_i + L_i \quad \dots 24.2$$

Energy balance:

$$L_{i-1}h(\bar{T}_{i-1}, X_{i-1}) + V_{i-1}H(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i H(P_i, \bar{T}_i) + L'_{i-1} h(P_{i-1})$$

Since $L'_{i-1} = V_{i-1}$

$$L_{i-1}h(\bar{T}_{i-1}, X_{i-1}) + V_{i-1}\lambda(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i H(P_i, \bar{T}_i) \quad \dots 24.3$$

Heat transfer rate:

$$V_{i-1}H(P_{i-1}) - L'_{i-1}h(P_{i-1}) = U_i A_i (T_{i-1} - \bar{T}_i)$$

Since $L'_{i-1} = V_{i-1}$

$$V_{i-1}\lambda(P_{i-1}) = U_i A_i (T_{i-1} - \bar{T}_i) \quad \dots 24.4$$

Boiling point rise:

$$\bar{T}_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3 \quad \dots 24.5$$

Substituting equation (24.1),(24.2),(24.3) in (24.4) we get:-

$$L_{i-1}h(\bar{T}_{i-1}, X_{i-1}) + (L_{i-2} - L_{i-1})\lambda(P_{i-1}) = L_i h(\bar{T}_i, X_i) + (L_{i-1} - L_i)H(P_i, \bar{T}_i) \quad \dots 24.6$$

$$(L_{i-2} - L_{i-1})\lambda(P_{i-1}) = U_i A_i (T_{i-1} - \bar{T}_i) \quad \dots 24.7$$

Equations (24.1),(24.5),(24.6),(24.7) represent the model for forward feed arrangement in an N-

effect evaporator. Systems. However, it cannot be used in its present form as it contains quantities like enthalpy of liquid, h ; enthalpy of vapour, H ; latent heat of vaporization of steam, λ ; and overall heat transfer coefficient, U whose direct measurement is difficult. The functional relationship of these quantities with temperature and concentration of the liquid streams and the pressure of the steam fed to the evaporator have been given by various equations.

The models for aqueous **sugar solution** in an N-effect evaporator is:-

For sugar solution correlation for h , H , λ , U are:-

$$h = (4.182 - 2.2403X_{i-1})(T_{i-1} - T_r)$$

$$H = 4.154(T_i - T_r) + 2.0125 \times 10^{-4}(T_i^2 - T_r^2) + 1.62(T_i - T_r) + 2.0285 \times 10^{-4}(T_i^2 - T_r^2) - 0.3747 \times 10^{-7}(T_i^3 - T_r^3) + \lambda$$

$$\lambda = -80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{.5} - 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04 \times 10^{-4}T_{i-1}^3 + 8597.953$$

$$U = 18.083(T_i/X_i)$$

Solute material balance:

$$FX_f = L_i X_i \quad \dots 24.8$$

Energy Balance:

$$\begin{aligned} L_{i-1}(4.182 - 2.2403X_{i-1})(T_{i-1} - T_r) + (L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{.5} - 4213.519\ln T_{i-1} \\ + 0.0918T_{i-1}^2 - 1.04 \times 10^{-4}T_{i-1}^3 + 8597.953] = L_i(4.182 - 2.2403X_{i-1})(T_i - T_r) + (L_{i-1} - L_i)[4.154(T_i - T_r) \\ + 2.0125 \times 10^{-4}(T_i^2 - T_r^2) + 1.62(T_i - T_r) + 2.0285 \times 10^{-4}(T_i^2 - T_r^2) - 0.3747 \times 10^{-7}(T_i^3 - T_r^3) - 80.345T_i \\ - 21035.87/T_i + 2049.123T_i^{.5} - 4213.519\ln T_i + 0.0918T_i^2 - 1.04 \times 10^{-4}T_i^3 + 8597.953] \end{aligned} \quad \dots 24.9$$

Heat transfer rate:

$$\begin{aligned} (L_{i-2} - L_{i-1})[-80.345T_{i-1} - 21035.87/T_{i-1} + 2049.123T_{i-1}^{.5} - 4213.519\ln T_{i-1} + 0.0918T_{i-1}^2 - 1.04 \times 10^{-4}T_{i-1}^3 \\ + 8597.953] = 18.083(T_i/X_i) A_i(T_{i-1} - T_i) \end{aligned} \quad \dots 24.10$$

Boiling point rise:

$$T_i = T_r + 7.2X_i - 11X_i^2 + 29.50X_i^3 \quad \dots 24.11$$

The solution of these equation can be done as explained in previous lecture.

Equation for N- effect for other type of feed arrangements are as follows:-

Backward feed:

For backward feed arrangement, equations of material balance, energy balance, heat transfer rate, and boiling point rise about i^{th} effect are as follows:

Overall material balance:

$$L_{i+1} = V_i + L_i \quad \dots 24.12$$

Energy balance:

$$L_{i+1}h(\bar{T}_{i+1}, X_{i+1}) + V_{i-1} H(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i h(P_i, \bar{T}_i) + L'_{i-1} h(P_{i-1})$$

$$\text{Since } L'_{i-1} = V_{i-1}$$

$$L_{i+1}h(\bar{T}_{i+1}, X_{i+1}) + V_{i-1} \lambda(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i H(P_i, \bar{T}_i) \quad \dots 24.13$$

Heat transfer rate

$$V_{i-1}H(P_{i-1}) - L'_{i-1} h(P_{i-1}) = U_i A_i(T_{i-1} - \bar{T}_i)$$

$$\text{Since } L'_{i-1} = V_{i-1}$$

$$V_{i-1}\lambda(P_{i-1}) = U_i A_i(T_{i-1} - \bar{T}_i) \quad \dots 24.14$$

Boiling point rise:

$$\bar{T}_i = T_i + (\gamma + nT_i)X_i + \alpha X_i^2 + \beta X_i^3 \quad \dots 24.15$$

Substituting equation (1) in (2) and (3) we get:-

$$L_{i+1}h(\bar{T}_{i+1}, X_{i+1}) + (L_i - L_{i-1})\lambda(P_{i-1}) = L_i h(\bar{T}_i, X_i) + (L_{i+1} - L_i)H(P_i, \bar{T}_i) \quad \dots 24.16$$

$$(L_i - L_{i-1})\lambda(P_{i-1}) = U_i A_i(T_{i-1} - \bar{T}_i) \quad \dots 24.17$$

Equations (24.4), (24.5) and (24.6) represent a model for the evaporation of aqueous solution in an N-effect evaporator having backward feed arrangement. The terms h, H, λ , and U contained in equations (24.5) and (24.6) are substituted by their respective equations the resulting equations describe the model of N-effect evaporator under backward feed arrangement.

Mixed feed:

In this type of feed arrangement the liquid stream entering to the i^{th} effect has been taken as L_k , the equations of material balance, energy balance, heat transfer rate, and boiling point rise about i^{th} effect are as follows:

Overall material balance:

$$L_K = V_i + L_i \quad \dots 24.18$$

Energy balance:

$$L_k h(\bar{T}_k, X_k) + V_{i-1} H(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i H(P_i, \bar{T}_i) + L'_{i-1} h(P_{i-1})$$

$$\text{Since } L'_{i-1} = V_{i-1}$$

$$L_k h(\bar{T}_k, X_k) + V_{i-1} \lambda(P_{i-1}) = L_i h(\bar{T}_i, X_i) + V_i H(P_i, \bar{T}_i) \quad \dots 24.19$$

Heat transfer rate:

$$V_{i-1}H(P_{i-1}, T_{i-1}) - L'_{i-1} h(P_{i-1}) = U_i A_i(T_{i-1} - \bar{T}_i)$$

$$\text{Since } L'_{i-1} = V_{i-1}$$

$$V_{i-1}\lambda(P_{i-1}) = U_i A_i(T_{i-1} - \bar{T}_i) \quad \dots 24.20$$

Boiling point rise:

$$\bar{T}_i = T_i + (\gamma + nT_i) X_i + \alpha X_i^2 + \beta X_i^3 \quad \dots 24.21$$

Substituting equation (1) to (2) and (3) we get:-

$$L_k h(\Phi_k, X_k) + (L_{k-1} - L_{i-1})(P_{i-1}) = L_i h(\Phi_i, X_i) + (L_k - L_i) H(P_i, \Phi_i) \quad \dots 24.22$$

$$(L_{k-i} - L_{i-1})(P_{i-1}) = U_i A_i (T_{i-1} - \Phi_i) \quad \dots 24.23$$

It is important to mention here that when i^{th} effect corresponds to the feed-introduction effect, $L_k = F$.
for the case when i^{th} effect is located after the feed-introduction effect, $L_k = L_{i-1}$; whereas $L_k = L_N$
when i^{th} effect is just before the feed-introduction effect; and for all other situations, $L_k = L_{i+1}$.