Module No: 02 Fundamental concepts
Lecture-03: Fundamental concepts related to heat integration

Key words: Thermodynamics, Carnot cycle, Quality of heat, Work potential, Entropy, Convective heat transfer

This lecture deals with the fundamental concepts required to understand the concept of heat integration using the principles of process integration and more specifically the "Pinch Technology"

2.0 Laws of thermodynamics

Heat integration using Pinch Technology employs extensively the laws of thermodynamics. Thus a discussion on the first two laws of thermodynamics is included.

2.1 First law of thermodynamics (FLT)

The first law of thermodynamics may be expressed by several forms of the fundamental thermodynamic relation [3]:

> *Increase in internal energy of a system = heat supplied to the system + work done on the system*
>
> *For a thermodynamic cycle the net heat supplied to the system equals the net work done by the system.*
>
> The net change in internal energy is the energy that flows in as heat minus the energy that flows out as the work that the system performs on its environment. Work and heat are not defined as separately conserved quantities; they refer only to processes of exchange of energy.
>
> These statements entail that the energy obeys the principle of conservation of energy. The principle of conservation of energy may be stated in several ways:
>
> Energy can be neither created nor destroyed. It can only change forms.

However, the concept of energy in the first law does not account for the observation that natural processes have a preferred direction of progress. For example, spontaneously, heat always flows to regions of lower temperature, never to regions of higher temperature without external work being performed on the system. The first law is completely symmetrical with respect to the initial and final states of an evolving system.

2.2 The *second law of thermodynamics* (SLT)

The second law of thermodynamics distinguishes between reversible and irreversible physical processes. It shows the existence of a quantity called the entropy of a system, and its change during a process expresses the irreversibility of actual physical processes by the statement that the entropy of an isolated macroscopic system never decreases.

The second law of thermodynamics places constraints upon the direction of heat transfer and the attainable efficiencies of heat engines. In doing so, it goes beyond the limitations imposed by the first law of thermodynamics. The maximum efficiency which can be achieved is the Carnot efficiency.

The second law of thermodynamics is a profound principle of nature which affects the way energy can be used. There are several approaches to stating this principle qualitatively. Here are some corollaries of SLT.

1. It is impossible for a system to transfer heat from a lower temperature reservoir to a higher temperature without any external work. Simply, heat transfer can only occur spontaneously in the direction of temperature decrease.
2. Any system which is free of external influences becomes more disordered with time. This disorder can be expressed in terms of the quantity called entropy. The entropy of a system is a measure of the amount of molecular disorder within the system. A system can only generate, not destroy, entropy. The entropy of a system can be increased or decreased by energy transports across the system boundary.
3. It is impossible for a system to receive a given amount of heat from a high-temperature reservoir and provide an equal amount of work output. While a system converting work to an equivalent energy transfer as heat is possible, a device converting heat to an equivalent energy transfer as work is impossible. For example, we cannot build a heat engine that has a thermal efficiency of 100%.
4. A basic corollary of the SLT is the statement that the sum of the entropy changes of a system and that of its surroundings must always be positive, that is, the universe (the sum of all systems and surroundings) is constrained to become forever more disordered and to Proceed towards thermodynamic equilibrium with some absolute maximum value of entropy.

The natural direction of a change in state of a system is from a state of low probability to one of higher probability and disordered states are more probable than ordered ones. Thus the natural direction of change of state of a system is from order to disorder.

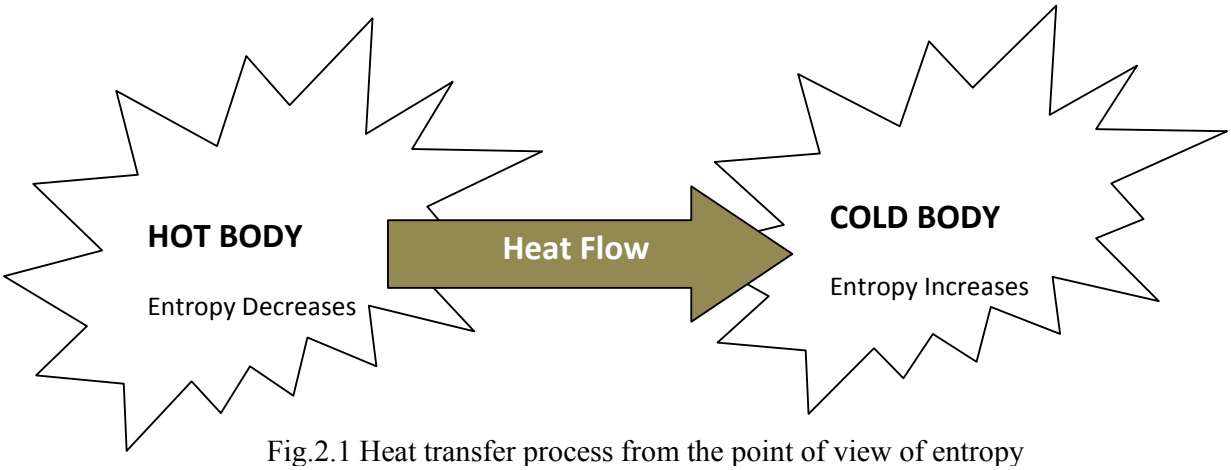


Fig.2.1 Heat transfer process from the point of view of entropy

Figure 2.1 illustrates a heat transfer process from the entropy point of view. That during a heat transfer process, the net entropy increases. The SLT requires that the increase in entropy of the cold body be greater than the decrease in entropy of the hot body. As a matter of fact, the increase in entropy of the cold body is more than the decrease in the entropy of the hot body and thus overall entropy increases. The conclusion is that processes can occur only in the direction of increased overall entropy or molecular disorder.[2]

The SLT is a valuable guide to energy policy as it establishes the relationship between entropy and the usefulness of energy. Energy is most useful to us when we can get it to flow from one substance to another, e.g., to warm a house and we can use it to do work.

2.2.1 Carnot Cycle

The purpose of a heat engine,Fig.2.2, is to take heat(Q_H) from a high-temperature source(T_H), convert some of this to mechanical work, and then reject the rest of the heat energy(Qc) to a lower-temperature source(sink at Tc). Carnot described heat engines in a crude analogy to waterwheels. The energy available for conversion in a waterwheel is the gravitational energy contained in water as it flows from some height (from a dam or mountain lake) down through the wheel. The amount of energy available depends on the difference in height-the "head" between the source and the pool below the wheel. The energy available to a heat engine depends on the "temperature head." Just as a high dam can provide more energy than a low one, a large temperature difference can provide more energy to be converted by a heat engine than can a small temperature difference. In the example of a heat engine, the high-temperature reservoir would be the hot steam coming from the power plant boiler. The low-temperature reservoir to which the engine (in this case steam turbine) rejects the unconverted energy is the condenser cooled by circulating water. [2]

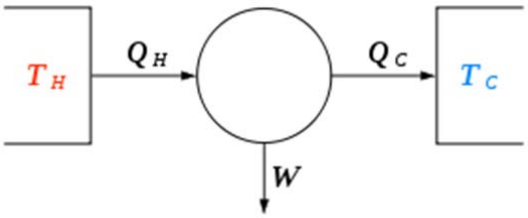


Fig.2.2 Carnot Cycle

$$\eta = W/Q_H = 1 - T_C/T_H \qquad \ldots. (2.1)$$

Carnot efficiency (η) is the maximum theoretical efficiency a friction-free heat engine could have. It's always less than 100% and for real heat engines may be less than 80%. Add friction and other losses and the actual efficiency is typically less than 50%.

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2.3 Quality of heat energy at different temperatures

The cost of a certain amount of energy is a function of temperature. For example 2000 kJ of thermal energy at 100 °C costs less than same amount of energy at 200°C. The readers can find the logic behind the above statement in thermodynamics. It is a well-known fact and can be proved through Carnot Cycle Efficiency, Entropy and work potential.

Since the second law of thermodynamics states that not all supplied heat in a heat engine can be used to do work, the Carnot efficiency limits the fraction of heat that can be used.

The Carnot efficiency, defined as the fraction of the ingoing heat energy that is converted to available work, is expressed as:

$$\mu_C = (T_i - T_o) / T_i \qquad \ldots. (2.2)$$

Where;

μ_C = efficiency of the Carnot cycle

T_i = temperature at the engine inlet (K)

T_o = temperature at engine exhaust (K)

The wider is the range of temperature between T_i and T_o, the more efficient the cycle is. The lowest temperature is limited by the temperature of the sink of heat - it can be the atmosphere or the ocean, river or whatever available [1].

2.3.1 Explanation based on Carnot efficiency

Let us consider that the sink temperature for Carnot cycle is at 298 K(ambient temp. 25°C).

Then the Carnot efficiency for the 100°C (373 K) hot stream will be equal to 0.201((373-298)/373) whereas, Carnot efficiency for 200°C (473 K) will be 0.37.

Thus, heat from high temperature heat source (200°C) can be converted to work more efficiently than from the source of 100°C.

2.3.2 Explanation based on Entropy

Consider an equal amount of heat energy equal to 2000 kJ is transferred from both the heat sources having temperatures 100°C and 200°C. Then the entropy change for the heat source as a result of transfer of 2000 kJ of heat from the reservoir can be computed as:

ΔS, Entropy for 100°C heat sources = Q/T = 2000/(100+273) = 5.361 kJ/K

Similarly

ΔS, Entropy for 200°C heat sources = Q/T = 2000/(200+273)= 4.228 kJ/K

As the entropy of the 200°C heat source is less than the 100°C heat source the quality of heat source having 200°C is better than 100°C.

2.3.3 Explanation based on Work potential

Work potential is the potential of the heat to deliver work output. In other words, if Q is the quantity of heat supplied at a given temperature T then what is the maximum work that this heat can produce is the work potential. The maximum work (work potential) is computed by multiplying Q with Carnot efficiency.

For 100 °C sources if Q is 2000 kJ then work potential is = 2000x0.201 = 402 kJ

For 200°C source is Q=2000kJ then work potential is = 2000x0.37 = 740 kJ

The heat source which has a higher work potential is termed as better quality heat source. In this case the 200°C heat source is a better quality heat source than a 100°C heat source.

Thus, for high temperature sources one always pays more than that for a heat source whose temperature is lower as high temperature sources are of better quality than the comparative low temperature sources. Thus with rise in temperature the quality of heat energy improves as can be seen from Carnot Efficiency, entropy and work potential.

However, it is reverse with refrigeration system as shown in Fig.2.3. For example creation of a -10°C cold utility is difficult than creating the -5°C cold utility. In other words the more we go below the ambient temperature, the cost of the cold utility increases. The above facts are described below:

Heat at -10°C has got better work potential as compared to that of heat at -5°C. To explain it let us consider a Carnot Cycle operating between ambient temperature (25 °C) and the refrigerated region temperature -10°C as well as -5°C.

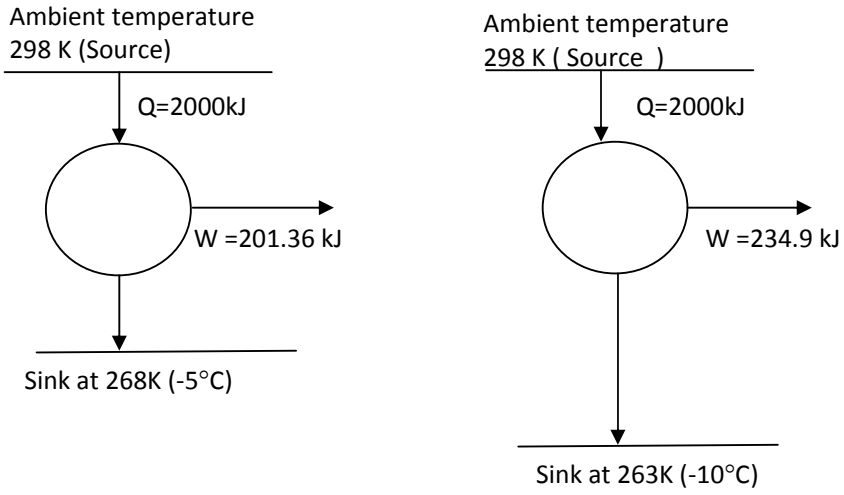


Fig.2.3 Work potential for different low temperature sinks

Work potential for (-5°C sink) = 2000kJx (1- 268/298) =2000x (1-0.89932) = 201.36

Work potential for (-10°C sink) = 2000kJx (1- 263/298) =2000x(1-0.88255) = 234.9

As per the second law of thermodynamics heat can't be transferred from a low temperature to high temperature naturally without work. Thus to create cold utility of 2000 kJ through refrigeration one has to reverse the cycle and the minimum work required to create a (-10 °C cold utility of 2000 kJ) will be 234.9 kJ where the same for (-5°C cold utility of 2000 kJ) will be 201.36. Thus one has to spend more work to create a -10°C cold utility of same amount (2000kJ) than that at -5°C as can be seen from Fig.2.4.

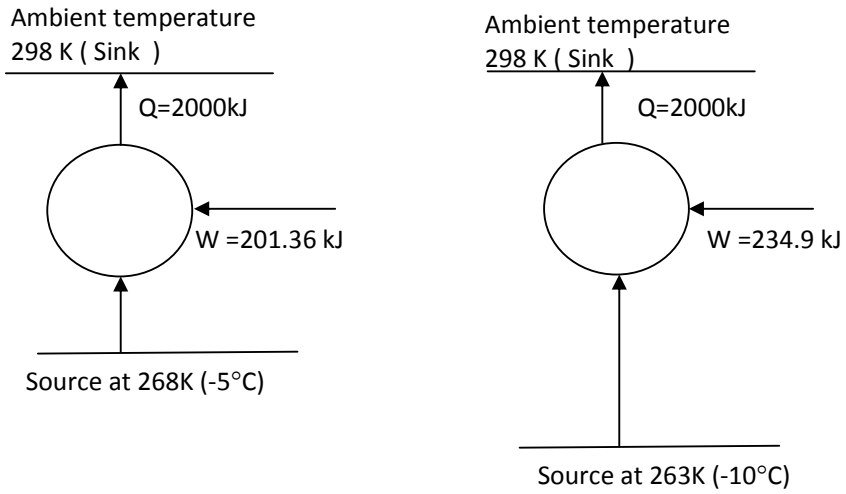


Fig.2.4 Minimum work required to transfer 2000 kJ from different low temperature sources

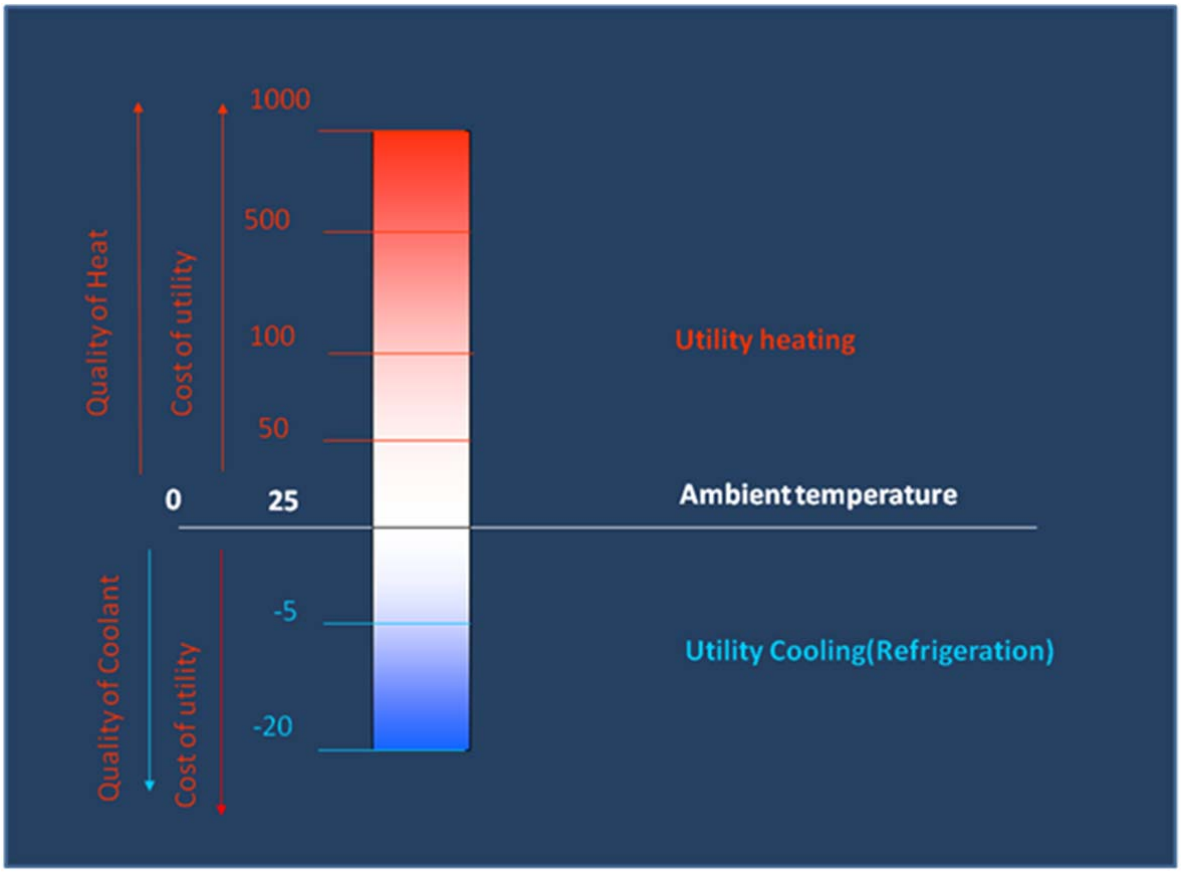


Fig.2.5 Quality of heat energy sources

Fig.2.5 shows that the quality of heat source increases if the temperature of the heat source increases from ambient temperature. Due to rise in quality of heat source the cost of heat source also increases for a given amount of heat duty.

However, for refrigerated cold utilities (below ambient temperature) lower is the temperature greater is the cost of the utility.

Therefore, one should always select the lowest temperature hot utility amongst different hot utilities available which can meet the requirement. Examples of hot utilities are steam, exhaust steam, hot oil, etc. For cold utilities (below ambient temperature) highest temperature cold utility should be selected amongst different cold utilities if it can serve the purpose.

2.4 Heat Transfer across a temperature difference (ΔT)

Selection of a particular temperature difference (ΔT) to transfer a given amount of heat, Q, out of several available ΔT values is crucial in heat integration. High value of ΔT in general decreases the fixed cost of the heat transferring equipment such as heat exchanger but at the same time degrades the heat to a considerable extent. A discussion in this regard is given below:

Example

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K through a conductor (metal wall as in the case of heat exchanger). Determine which heat transfer process is more irreversible [2]. In other words in which process the energy is degraded less.

Solution: A sketch of the reservoirs is shown in Fig. 2.6(a) & (b). Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system. The problem statement gives the impression that the two reservoirs are in direct contact during the heat transfer process. But this cannot be the case since the temperature at a point can have only one value, and thus it cannot be 800 K on one side of the point of contact and 500 K on the other side. In other words, the temperature function cannot have a jump or discontinuity. Therefore, it is reasonable to assume that the two reservoirs are separated by a partition through which the temperature drops from 800 K on one side to 500 K (or 750 K) on the other side.

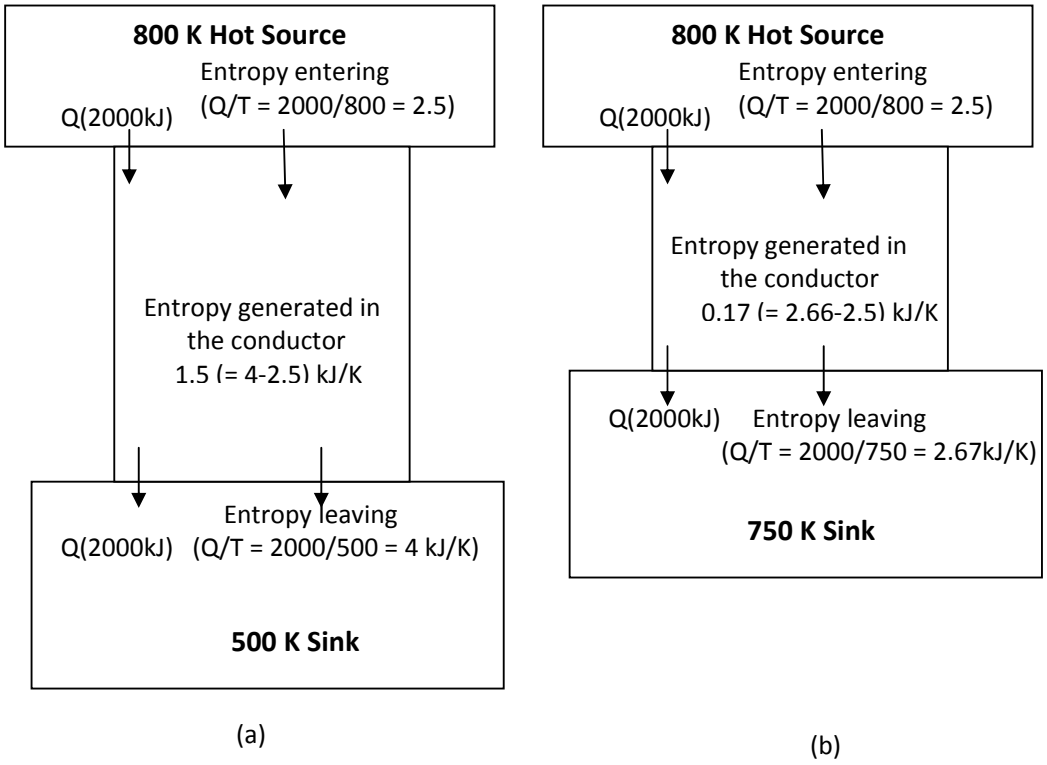


Fig.2.6 (a) and (b) Heat source and sink diagrams for the example

The total entropy change for the process in part "b" is smaller, and therefore this process is associated with smaller irreversibility. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility. In other words, quality of energy is degraded less in case "b" in comparison to "a". Therefore, it can be inferred that if a process can be carried out under a lower temperature difference then one should not employ high temperature

difference to carry out the same process. By doing so, one can save in terms of degradation of energy.

However, from the point of view of design of heat exchangers, if a process is operating under higher temperature difference (Large value of ΔT) it will require less heat transfer area to pass a given amount of heat in comparison to lower value of ΔT. Thus operation at higher value of ΔT will decrease the fixed cost of heat exchanger at the cost of degradation of energy.

In a process industry the whole industry runs between two temperature levels defined by the hot utility (which is the hottest) and cold utility. Pictorially this is shown by Fig.2.7.

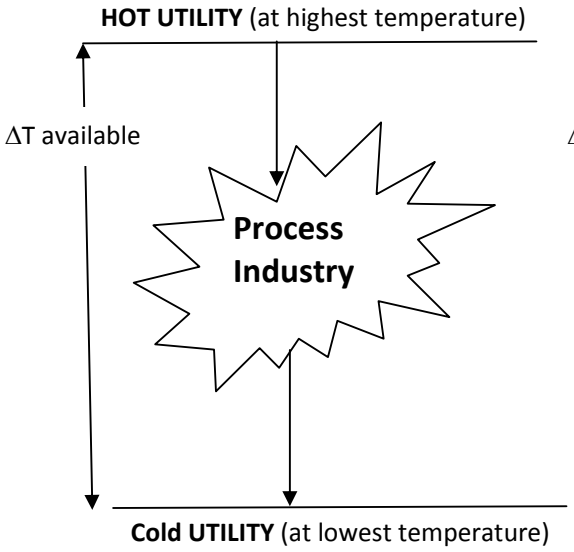


Fig.2.7 Process industry operating under a temperature differential

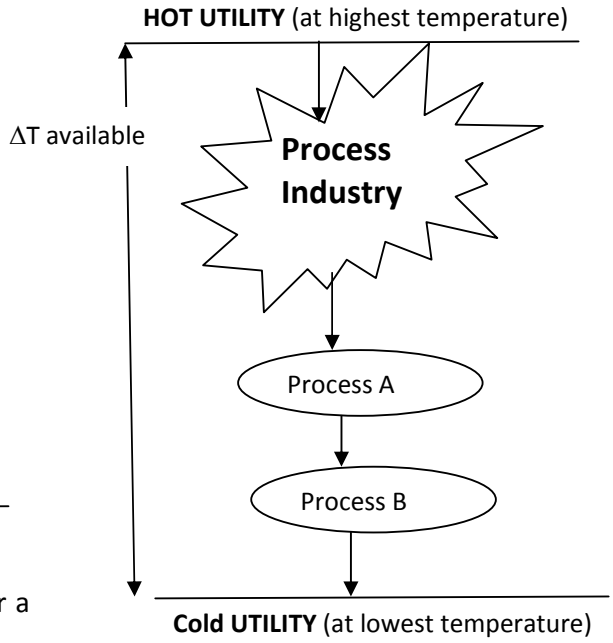


Fig.2.8 Process A & B are included in the heat flow path under the same ΔT

However, the ΔT available for industry is more than it is required, and then more processes can be included in the path of heat flow as shown in Fig.2.8. In such cases the additional processes will get a free ride. The process A will operate from the exhaust heat of Process Industry and process B will operate with the help of exhaust heat of Process A.

Hence, selection of proper ΔT under which a process will operate is very crucial for heat integration.

2.5 Convective Heat Transfer

In a large number of heat exchangers convective heat transfer takes place. The overall heat transfer coefficient in such exchangers is influenced by the thickness and thermal conductivity of the mediums through which heat is transferred. The larger is this coefficient, the easier heat is the transfer of heat from hot fluid to the cold fluid that is being heated. In a heat exchanger, the

relationship between the overall heat transfer coefficient (U) and the heat transfer rate (Q) can be demonstrated by the following equation:

$$Q = UA\Delta T_{LM} \quad \ldots (2.3)$$

Where,

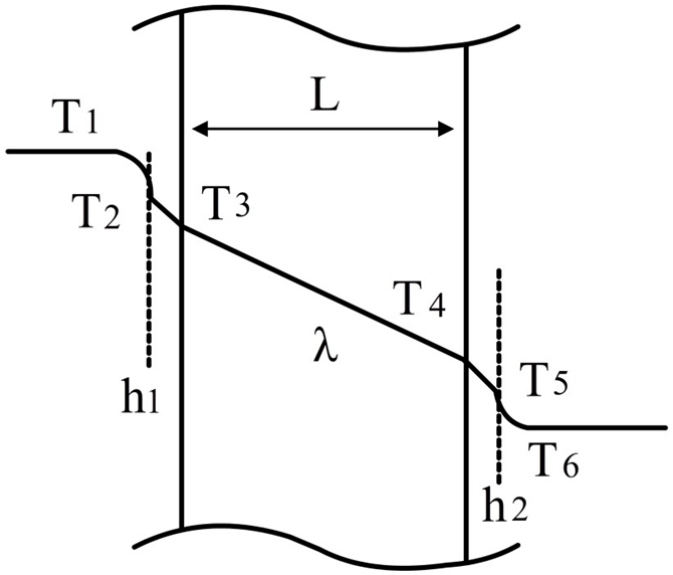
Q = heat transfer rate, W (J/s)
A = heat transfer surface area, m^2
U = overall heat transfer coefficient, $W/(m^2°C)$
ΔT_{LM} = logarithmic mean temperature difference, °C

From this equation we can see that the Q value is directly proportional to U. Assuming the heat transfer surface and temperature difference remain unchanged, the greater the U value, the greater the heat transfer rate. In other words, if Q and U are constant then A will decrease with rise in the value of ΔT_{LM}.

$$\frac{\quad}{\quad} \quad \ldots. (2.4)$$

Though it is true for a heat exchanger, it does not hold good in the case of heat exchanger network design during heat integration where Q also increases with ΔT_{min}. Thus one does not get the complete benefit of reduction in A due to rise in the value of ΔT_{min}.

2.5.1 Heat transfer through a metal wall



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Fig.2.9 Heat transfer across a plate when fluids are flowing on both sides of it

For Fig.2.9, the overall heat transfer coefficient, U, for such a system can be defined as:

$$\frac{1}{U} = \frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2} \qquad \ldots (2.5)$$

Where;

h_1 & h_2 = convective heat transfer coefficients, W/(m^2 °C)
L = thickness of the wall, m
k = thermal conductivity, W/(m °C)

The convective heat transfer coefficient (h), sometimes referred to as the film coefficient, is often used when calculating heat transfer between a fluid and a solid. In the case of a heat exchanger, heat transfer basically occurs from fluid 1 (source of heat) to solid (metal wall) to fluid 2 (product being heated).

2.5.2 Heat Transfer across a pipe when fluids are flowing within the pipe as well as over the outer surface of pipe.

This situation is more close to shell and tube heat exchangers which are used in Heat Integration as shown in Fig.2.10.

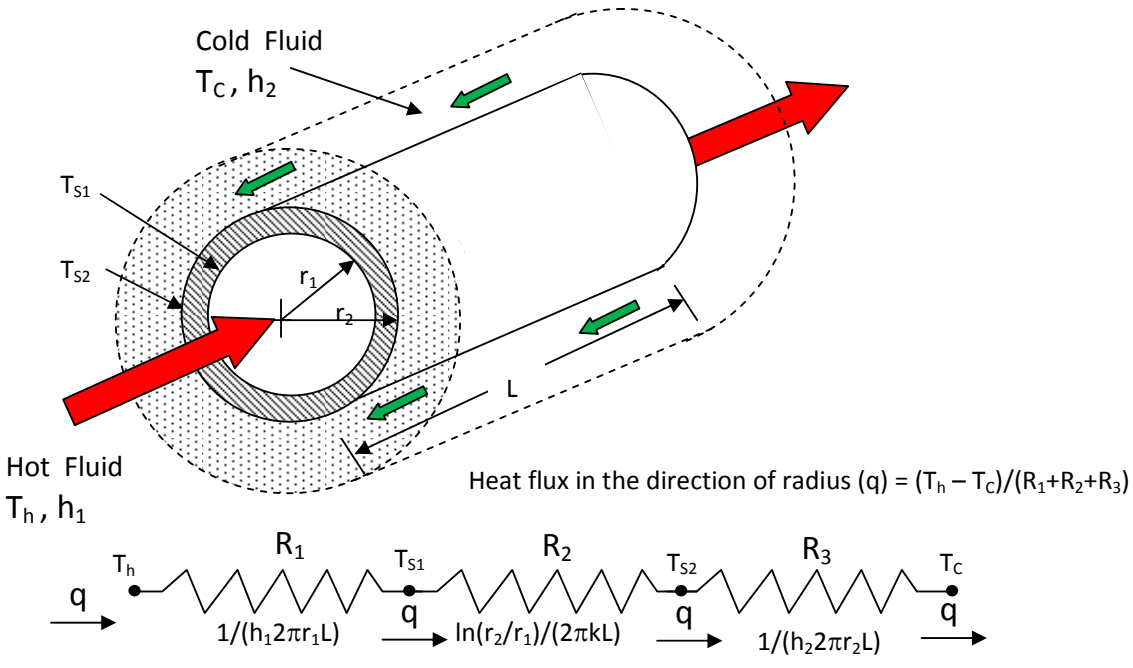


Fig.2.10: Shell & tube heat exchanger

In such a case the U can be defined based on inside area of the tube (U_i) as well as outside area of the Tube (Uo). For defining U_i and U_o two different equations are used as given below:

$$U_i = 1/ (1/h_1 + (r_2-r_1) A_1/kA_{ln} + A_1/ (A_o h_2)) \quad \ldots (2.6)$$

$$U_o = 1/ (A_2/ (A_1 h_1) + (r_2-r_1) A_2/kA_{ln} + 1/h_2) \quad \ldots (2.7)$$

Where;

A_1 is the inside area of the tube ($2\pi r_1 L$), m^2

A_2 is the outside area of the tube ($2\pi r_2 L$), m^2

k is the thermal conductivity of tube wall, W/(m°C)

A_{ln} is the log mean area of tube and is denoted by $(A_2-A_1)/ \ln (A_2/A_1)$

If r_2-r_1 is very small then A1 will be approximately equal to A2 and the above equation reduces to:

$$U_i = 1/ (1/h_1 + 1/h_2) \quad \ldots (2.8)$$

$$U_o = 1/ (1/h_1 + 1/h_2) \quad \ldots (2.9)$$

Thus, there will be no difference in U_i and U_o.

2.3 Thermal driving force in exchangers

Let us consider a heat exchanger in the form of two concentric pipes as shown in Fig. 2.11. The cold fluid is flowing inside the inner pipe and hot fluid is flowing in countercurrent mode through the annulus between the two pipes. It represents a true counter current flow.

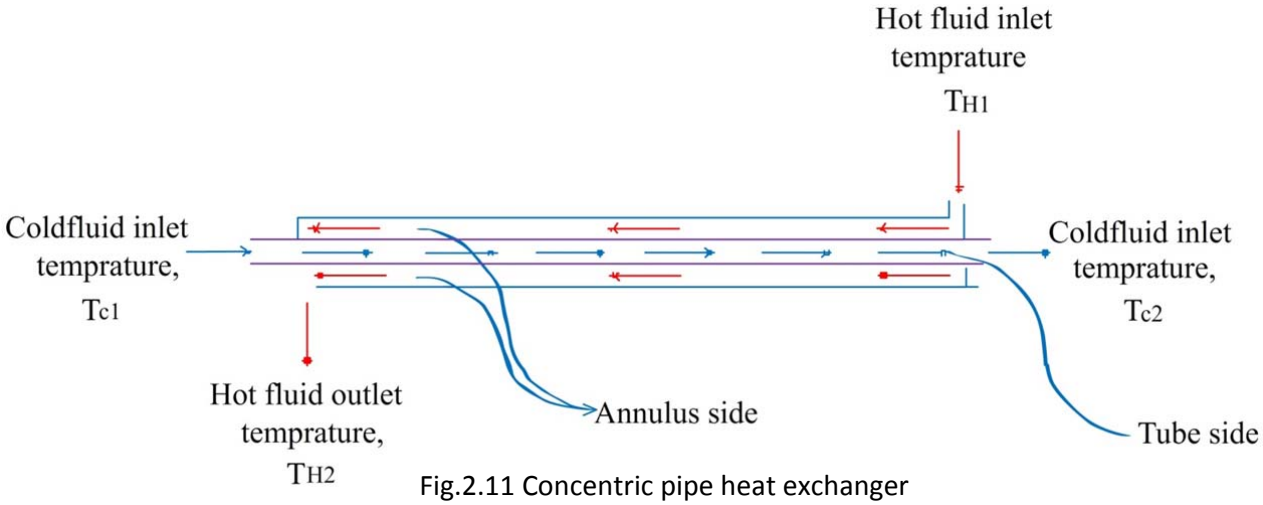


Fig.2.11 Concentric pipe heat exchanger

Follwowing assumptions are made for the better undersatnding of the heat transfer process inside the heat exchanger: -

a) There is no phase change in the system
b) Heat loss from the system to the surrounding is assumed to be negligible.
c) The process is assumed to be steady state.
d) Fluid properties as well as overall heat transfer coefficient U is considered as constant.

The temperature variation (along the length) of heat exchanger as a function of enthalpy picked up(along the length) is shown in Fig.2.12.

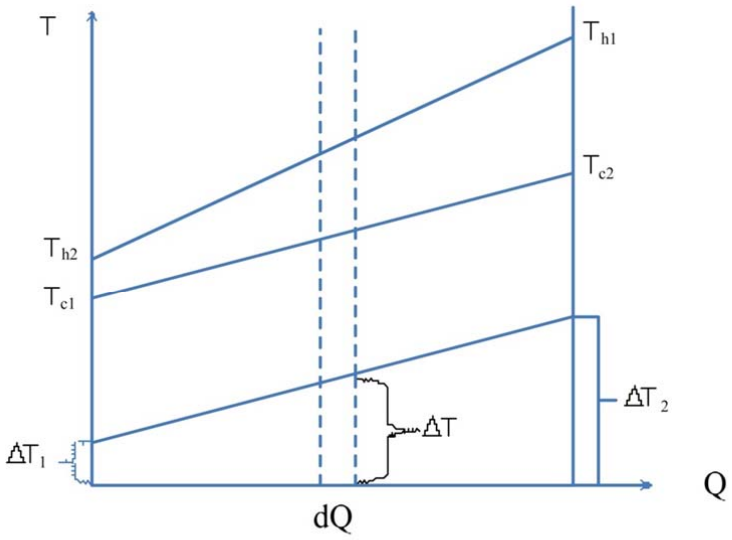


Fig.2.12 Heat transfer process in concentric pipe heat exchanger

Where;

$$\Delta T_1 = (T_{h2}-T_{c1}) \text{ and } \Delta T_2 = (T_{h1}-T_{c2.})$$

The slope of the temperature difference (driving force (ΔT)) line w.r.t. Q axis can be written as:

$$d(\Delta T) / dQ = [(\Delta T_2 - \Delta T_1) / Q] \qquad \ldots(2.10)$$

Now, for the differential element shown in the figure above, dQ can be written as: -

$$dQ = U * dA * \Delta T \quad \ldots (2.11)$$

Now, substituting value of dQ from equation (2.11) into equation (2.10) will give: -

$$\frac{d\,(\Delta T)}{U * dA * \Delta T} = \frac{(\Delta T_2 - \Delta T_1)}{Q} \quad \ldots (2.12)$$

On rearranging the above equation, we get

$$dA = \frac{Q}{(\Delta T_2 - \Delta T_1)} * \frac{1}{U} * \frac{d(\Delta T)}{\Delta T} \quad \ldots (2.13)$$

Now, on integrating the above equation on both sides, we get

$$\int_0^A dA = \frac{Q}{(\Delta T_2 - \Delta T_1)} * \frac{1}{U} * \int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} \quad \ldots (2.14)$$

Thus,

$$A = \frac{Q}{(\Delta T_2 - \Delta T_1)} * \frac{1}{U} * ln\frac{\Delta T_2}{\Delta T_1} \quad \ldots (2.15)$$

Or,

$$Q = U * A * \frac{(\Delta T_2 - \Delta T_1)}{ln\frac{\Delta T_2}{\Delta T_1}} \quad \ldots (2.16)$$

Or,

$$Q = U * A * \Delta T_{LM} \quad \ldots (2.17)$$

Where,

$$\Delta T_{LM} = \frac{(\Delta T_2 - \Delta T_1)}{ln\frac{\Delta T_2}{\Delta T_1}} \quad \ldots (2.18)$$

Equation 2.17 can also be arranged as follows to obtain the Area required by a individual heat exchanger or network for a given heat duty and heat transfer coefficient:

$$A = \frac{Q}{U * \Delta T_{LM}} \quad \ldots (2.19)$$

Thus, higher is the value of ΔT_{LM}, lower will be the area requirement for the same heat duty and overall heat transfer coefficient.

Now, Let us consider two concentric pipe heat exchangers namely HE1 and HE2. HE1 is a concurrent heat exchanger and HE2 is a countercurrent heat exchanger. Same hot and cold fluids are flowing in both the exchangers. The details of the two streams are given in table 1: -

Specifications	Hot Stream	Cold Stream
CP(mCp)	15 kW/ C	10 kW/C
Inlet Temperature	300 C	30 C
Heat Duty	1500 kW	1500 kW

*Assumption: - The fluid properties are not varying with temperature.

Therefore, the output temperature of the hot and cold streams is calculated to be:

Outlet Temperature of Hot stream = inlet temp. - (Q / CP) = 300 – (1500 / 15) = 300 – 100

= 200 C

Outlet temperature of cold stream = inlet temp. + (Q / CP) = 30 + (1500 / 10) = 30 + 150

= 180 C

Therefore, if we draw the temperature and heat duty diagram for the two heat exchangers, we get the Figs. 2.13 & 2.14:

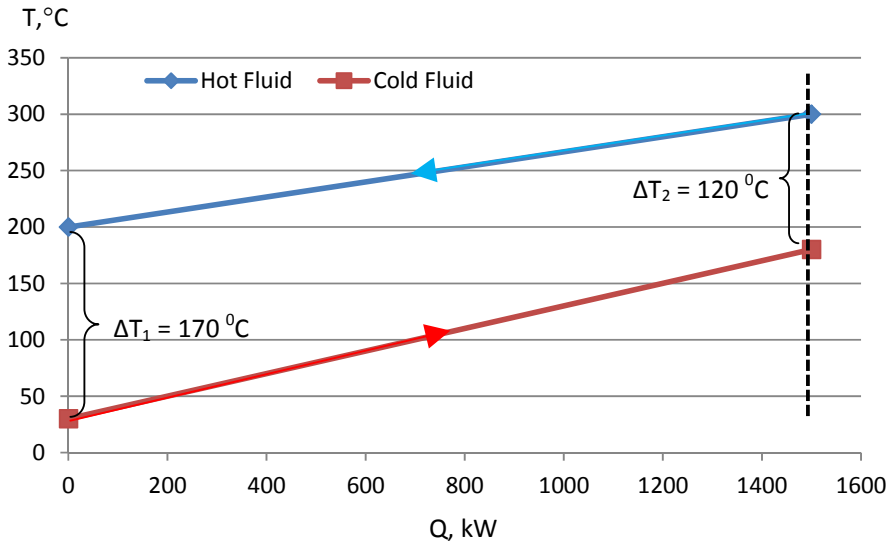


Fig. 2.13 Temperature vs. Heat duty diagram for countercurrent HE2

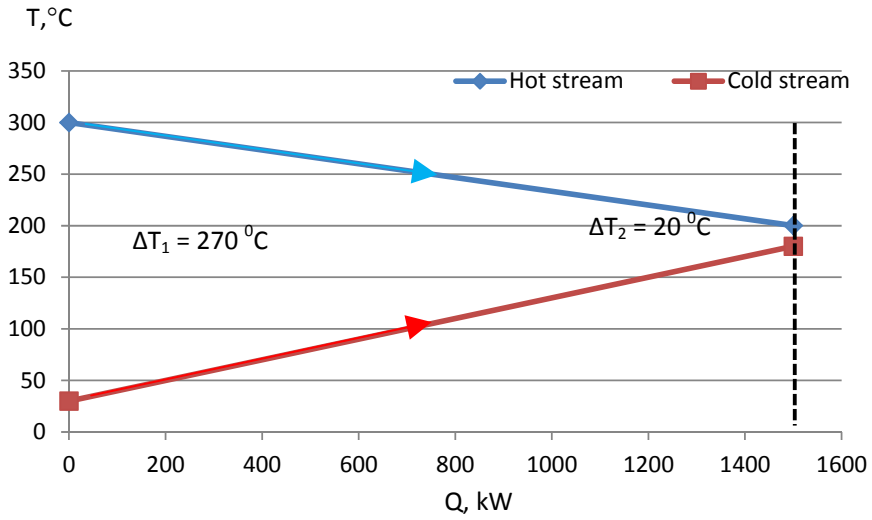


Fig. 2.14 Temperature vs. Heat duty diagram for co-current HE1

Thus, from the above diagram it is clear that in concurrent heat exchanger, the driving force or (ΔT) decreases along the length and it is very small at the outlet of the heat exchanger. This leads to a lower effective temperature difference (and hence higher area requirement compared to countercurrent heat exchanger where the driving force remains almost constant throughout the heat exchanger. This will become clear from the following calculations: -

For HE1,

$$= (270 - 20) / \ln (270/20)$$

$$= 250 / 2.6027 = 96.05 \text{ C}$$

Thus,

$$A = 1500 / (U^* 96.054) = 15.6162 / U \text{ m}^2$$

For HE2,

$$= (170 - 120) / \ln (170 / 120)$$

$$= 50 / \ln (17/ 12) = 143.55 \text{ C}$$

Thus, $A = 1500 / (U^* 143.55) = 10.45 / U \text{ m}^2$

Also, it should be noted that the outlet temperature of the cold fluid in a concurrent heat exchanger cannot be higher that the outlet temperature of the hot fluid as this will lead to temperature cross resulting in very high area requirement. However, in a countercurrent heat exchanger, the hot stream can be utilized to a temperature lower than the outlet temperature of the cold stream as evident from Fig. 2.15:

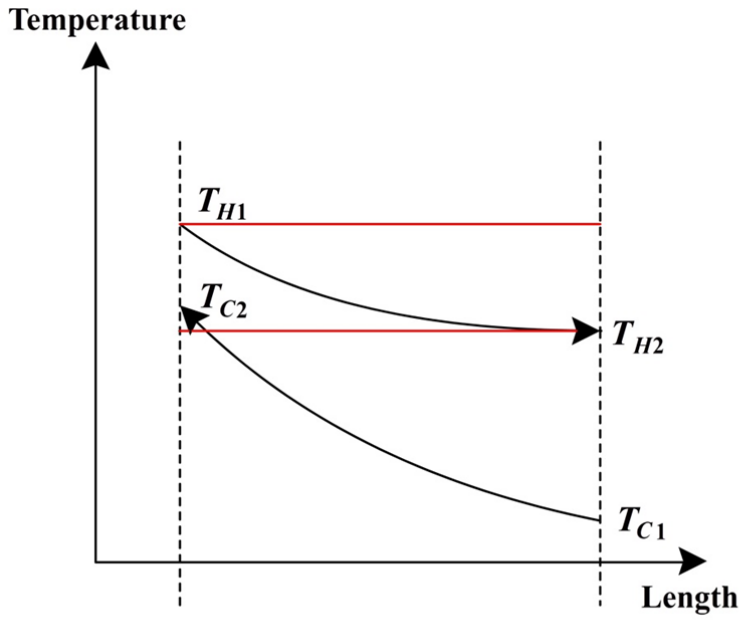


Fig. 2.15 temperature profile of a countercurrent heat exchanger

The above discussions corresponds to 1-1 (1shell pass- 1 tube pass) heat exchangers. Many other flow arrangements other than 1-1 exists, the most common of which is the 1-2 design as shown in Fig. 2.16 (a &b) along with its temperature profile:

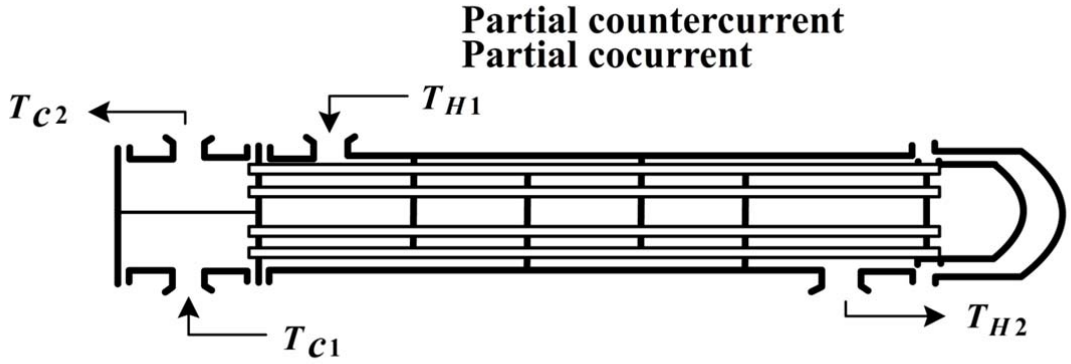


Fig. 2.16(a) 1-2 Shell and Tube heat exchanger

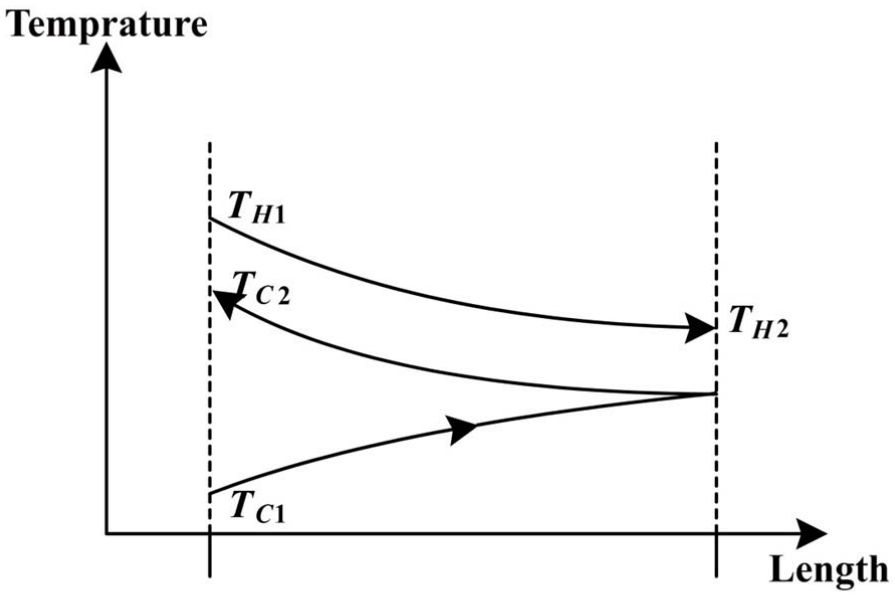


Fig. 2.16 (b) Temperature profiles of a 1-2 Shell and Tube heat exchanger

From the above diagram, we can see that flow arrangement inside a 1-2 shell and tube heat exchanger involves part countercurrent and part co-current flow. Therefore the effective temperature difference for heat exchange is reduced compared to pure counter current heat exchanger.

To accommodate this, F_T factor is used in the basic heat exchanger design equation as given below:

$$Q = U^* A^* \Delta T^* F_T \quad \text{... (2.20)}$$

Where,

$$F_T = f(R, P) < 1$$

$$R \text{ (Ratio of two heat capacity flow rate)} = \frac{CP_C}{CP_H} = \frac{Q / (TC_1 - TC_2)}{Q / (TH_1 - TH_2)}$$

$$= \frac{(TH_1 - TH_2)}{(TC_1 - TC_2)} \quad \text{... (2.21)}$$

$$P \text{ (Thermal effectiveness of the exchanger)} = \frac{(TC_2 - TC_1)}{(TH_1 - TC_1)} \quad \text{... (2.21}$$

From Eq. 2.20, we can clearly say that 1 – 2 design needs a larger area than the 1 – 1 design. However, 1 – 2 design is very common as it offers many practical advantages such as:

a) Allowance for thermal expansion
b) Easy mechanical cleaning
c) Good heat transfer coefficient on tube side due to higher velocity

Thermal Effectiveness of a heat exchanger: - The thermal effectiveness of the heat exchanger is defined as the ratio of the actual heat transfer to the maximum possible heat transfer.

$$\text{Thermal effectiveness, } P = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}} \quad \text{... (2.22)}$$

The actual heat transfer may be computed by calculating either the energy lost by the hot fluid or the energy gained by the cold fluid. Thus, for a heat exchanger, the actual heat transfer

$$q = CP_{hot} * (T_{H1} - T_{H2}) = CP_{cold} * (T_{C2} - T_{C1}) \quad \text{... (2.23)}$$

The maximum possible heat transfer from an exchanger, is possible if one of the fluids has to undergo a temperature difference equal to the maximum possible temperature difference inside the exchanger which is the difference between the entering temperatures of the hot and cold fluids. According to the energy balance equation, the fluid which might undergo this temperature difference must be the one having minimum value of CP(MCp) as the energy received by one fluid must be equal to the energy given by the other fluid. Therefore, maximum possible heat transfer is given by:

$$Q_{max} = CP_{min} * (T_{Hinlet} - T_{Cinlet}) \quad \text{... (2.24)}$$

Now, the fluid having minimum CP may either be a hot fluid or a cold fluid depending on the mass flow rate and specific heats. Thus, the thermal effectiveness(P) may be written as:

$$P_h = (CP_{hot} * (T_{H1} - T_{H2})) / (CP_{hot} * (T_{H1} - T_{C1}))$$

$= (T_{H1} - T_{H2}) / (T_{H1} - T_{C1})$ … (2.25)

$P_c = (CP_{cold} * (T_{C2} - T_{C1})) / (CP_{cold} * (T_{H1} - T_{C1}))$ … (2.26)

$= (T_{C2} - T_{C1}) / (T_{H1} - T_{C1})$ … (2.27)

Here, the subscripts on the effectiveness symbol designate the fluid which has minimum value of CP.

Expressions for F_T factor

The expression of F_T for 1-2 heat exchangers is as follows:

For $R \neq 1$,

$$F_T = \frac{\sqrt{R^2+1}\ln(\frac{1-P}{1-RP})}{(R-1)\ln\left(\frac{[2-P(R+1-\sqrt{R^2+1}]}{[2-P(R+1+\sqrt{R^2+1}]}\right)} \quad \ldots (2.28)$$

For R = 1

$$F_T = \frac{\frac{\sqrt{2}P}{(1-P)}}{\ln\left(\frac{[2-P(2-\sqrt{2}]}{[2-P(2+\sqrt{2}]}\right)} \quad \ldots (2.29)$$

The shape of these functions is shown in Fig.2.18

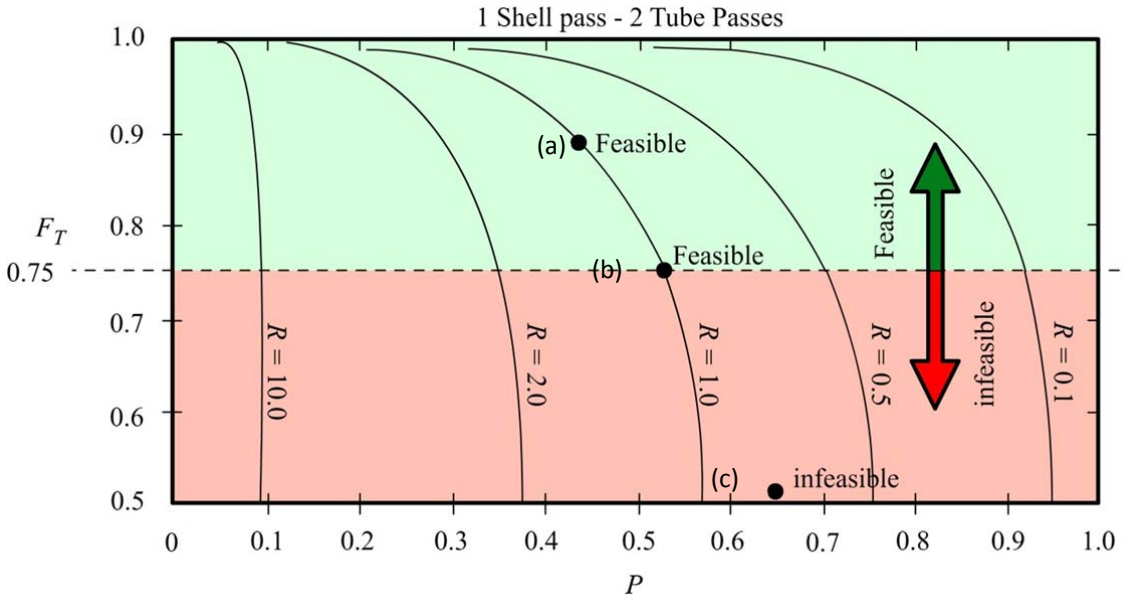


Fig.2.18 F_T curves for 1-2 S&T Heat Exchanger

From the above diagram, it can be seen that with the decrease in F_T, the slope of F_T curve for a given value of R becomes very steep and approaches to a certain value of P asymptotically.

Now, three basic situations can be encountered while using 1-2 heat exchangers:

a) **Temperature Approach**: - The final temperature of the hot stream is higher than the final temperature of the cold, shown in Figure 2.19 and represented as (a) in the F_T diagram (Fig. 2.18). This is called temperature approach as outlet temperature of hot stream approaches to the outlet temperature of cold stream. This situation is straightforward to design, since it can always be accommodated in a single 1 – 2 shell.

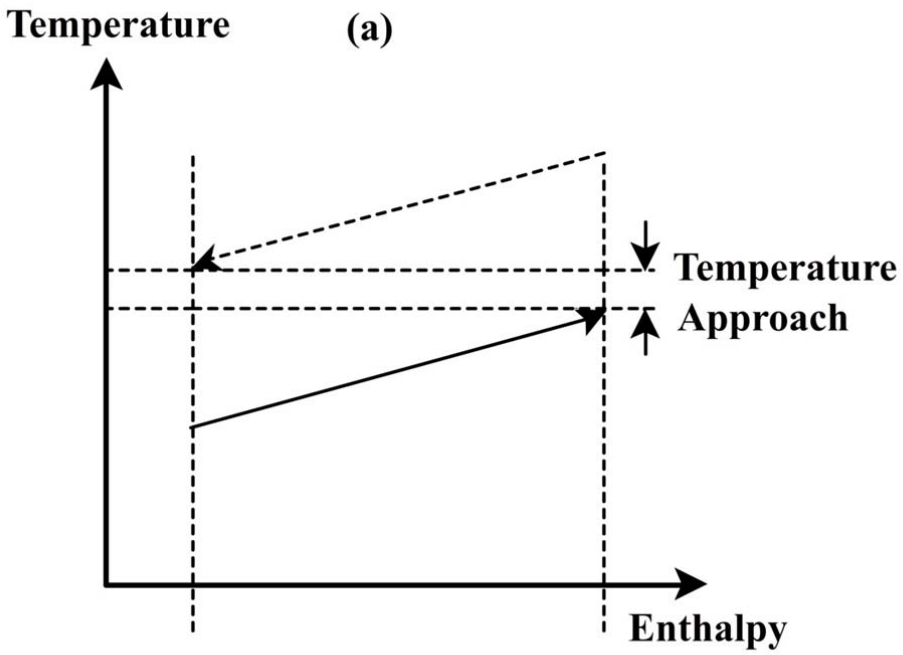


Fig.2.19 Temperature approach when final temperature of hot stream is higher than the final temperature of cold stream

b) **Temperature Cross**: - The outlet temperature of the hot stream is slightly lower than the outlet temperature of the cold stream, as illustrated in Fig. 2.20 and represented in F_T diagram as point (b) in Fig. 2.18. This is called temperature cross. This situation is usually straightforward to design, provided the temperature cross is small, as it can be accommodated in a single shell. However, the decrease in F_T which is a function of extent of temperature cross increases the heat transfer area requirements significantly.

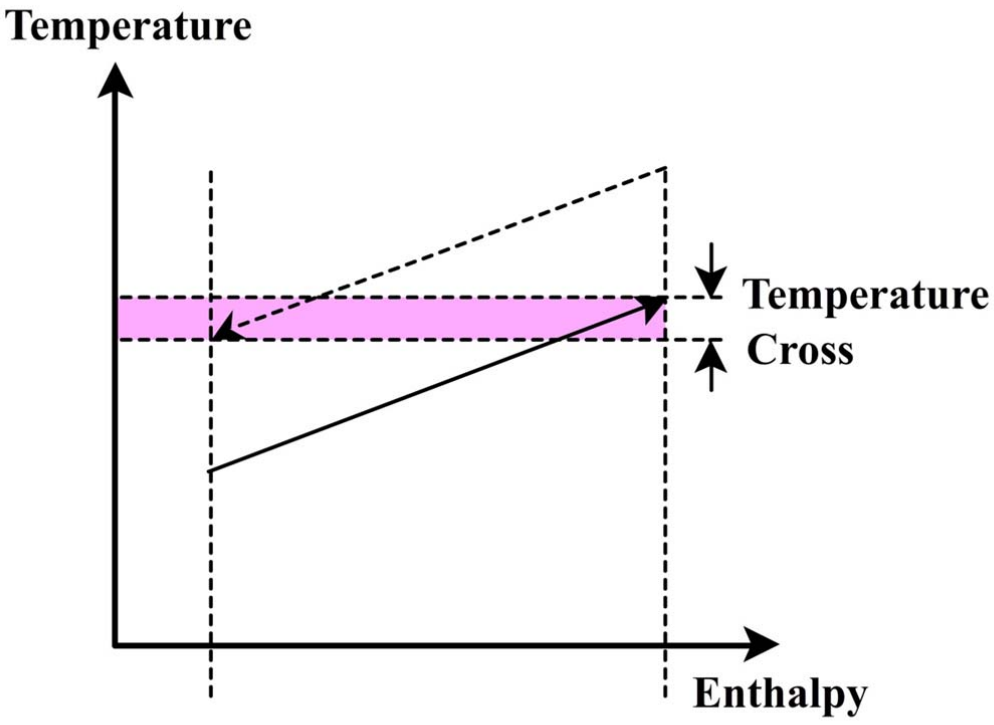


Fig.2.20 Temperature approach when final temperature of hot stream is less than the final temperature of cold stream showing a small temperature cross

c) **Large Temperature cross**: As the amount of temperature cross increases, problems are encountered as illustrated in Fig. 2.21 below and represented in the F_T diagram as point (c) in Fig. 2.18.

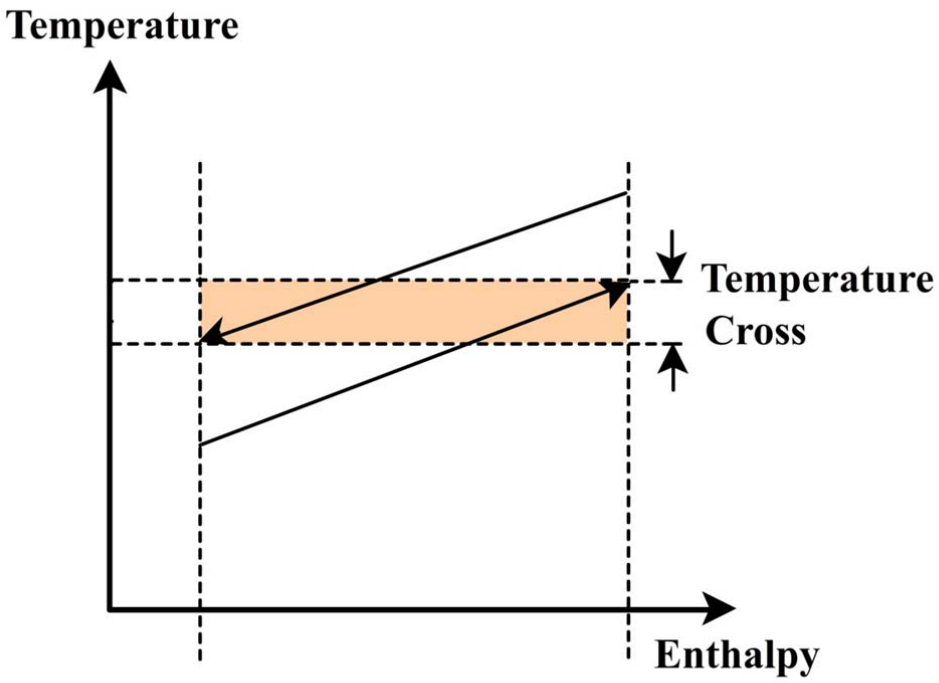


Fig.2.21 Temperature approach when final temperature of hot stream is less than the final temperature of cold stream showing a large temperature cross

In this case, the F_T decreases significantly, causing a dramatic increase in the heat transfer area requirement leading to an infeasible design. Local reversal of heat flow may also be encountered, which is wasteful in heat transfer area. Thus, for a given R, the design of the heat exchanger becomes less and less efficient as the asymptotic region of the F_T curve is reached.

The maximum temperature cross that can be tolerated is often set by rules of thumb, for example, $F_T > 0.75$. It is important to avoid low values of FT because:

1) Low values of F_T indicate ineffective use of the heat transfer area.
2) Any uncertainties or inaccuracies in design data will significantly affect the design of heat exchanger when design is carried out in an area where FT slopes are steep. Further, operation of Heat Exchanger is these area will be unpredictable.

Consequently, to be confident in a design, those parts of the F_T chart where slopes are steep should be avoided, even if $F_T > 0.75$. A simple method to achieve this is based upon the fact that for any value of R there is a maximum asymptotic value for P, say P_{max}, which is achieved when F_T tends to negative infinity as shown in Fig. 2.22 , and is obtained as follows:

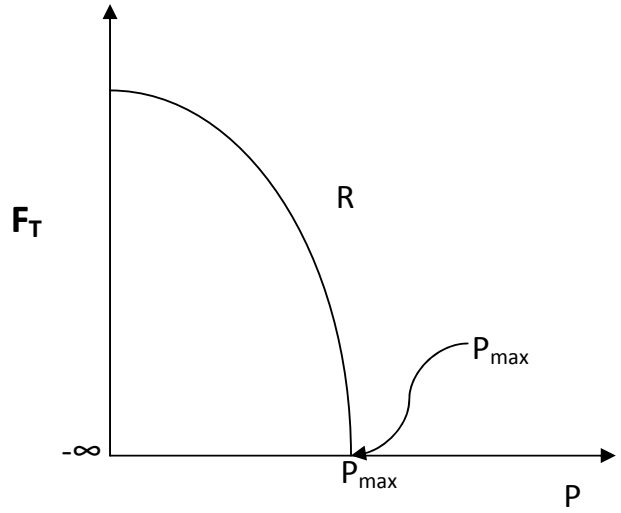


Fig. 2.22 Determination of P_{max}

According to Bowman et al (4):

For $R \neq 1$,

$$F_T = \frac{\sqrt{R^2+1}\ln(\frac{1-P}{1-RP})}{(R-1)\ln\left(\frac{[2-P(R+1-\sqrt{R^2+1}]}{[2-P(R+1+\sqrt{R^2+1}]}\right)} \quad \ldots.(2.30)$$

For R = 1

$$F_T = \frac{\frac{\sqrt{2}P}{(1-P)}}{\ln\left(\frac{[2-P(2-\sqrt{2}]}{[2-P(2+\sqrt{2}]}\right)} \quad \ldots.(2.31)$$

The maximum value of P, for any R, occurs as F_T tends to negative infinity. From the F_T functions above, for F_T to be determinate:-

1. $P < 1$
2. $RP < 1$

3. $\frac{[2-P(R+1-\sqrt{R^2+1}]}{[2-P(R+1+\sqrt{R^2+1}]} > 0$

Condition 3 applies to Equation 3.31 when R = 1. Both Conditions 1 and 2 are always true for a feasible heat exchange with positive temperature differences.

The fraction in condition 3 is greater than zero in two cases:

a) Either both numerator and denominator are greater than zero.

$$[2 - P(R + 1 - \sqrt{R^2 + 1}] > 0 \; and \; 2 - P(R + 1 + \sqrt{R^2 + 1}] > 0$$

Or, $$2 > P(R + 1 - \sqrt{R^2 + 1}] \; and \; 2 > P(R + 1 + \sqrt{R^2 + 1}]$$

Or, $$P < \frac{2}{R+1-\sqrt{R^2+1}} \text{ and } P < \frac{2}{R+1+\sqrt{R^2+1}}$$

b) Or both numerator and denominator are less than zero.

$$[2 - P(R + 1 - \sqrt{R^2 + 1}] < 0 \; and \; 2 - P(R + 1 + \sqrt{R^2 + 1}] < 0$$

Or, $$2 < P(R + 1 - \sqrt{R^2 + 1}] \; and \; 2 < P(R + 1 + \sqrt{R^2 + 1}]$$

Or, $$P > \frac{2}{R+1-\sqrt{R^2+1}} \text{ and } P > \frac{2}{R+1+\sqrt{R^2+1}}$$

For Condition 3, either “a” or “b” are true but not both. Let us consider condition “b” in more detail. For positive values of R, $R + 1 + \sqrt{R^2 + 1}$ is a continuously increasing function of R, and

- As R $\rightarrow$0, $(R + 1 - \sqrt{R2 + 1}) \rightarrow 0$
- As R $\rightarrow \infty$, $(R + 1 - \sqrt{R2 + 1}) \rightarrow 1$

For determination of limit as R$\rightarrow\infty$, multiply and also divide $(R + 1 - \sqrt{R2 + 1})$ by its conjugate $(R + 1 + \sqrt{R2 + 1})$,

$$\frac{\left(R + 1 - \sqrt{R2 + 1}\right) * (R + 1 + \sqrt{R2 + 1})}{(R + 1 + \sqrt{R2 + 1})}$$

$$= \frac{(R+1)^2-(\sqrt{R2+1}))^2}{(R + 1 + \sqrt{R2 + 1})}$$

$$= \frac{2R}{(R + 1 + \sqrt{R2 + 1})}$$

Now, dividing both the numerator and denominator by R, we get

$$\frac{2}{(1 + \frac{1}{R} + \sqrt{1 + \frac{1}{R^2}})}$$

Now, if R→∞, $(R + 1 - \sqrt{R2\ +\ 1}) \rightarrow \frac{2}{2} = 1$

Let us consider condition "b",

Since, we know that for positive value of R,

$$R + 1 + \sqrt{R^2 + 1} > R + 1 - \sqrt{R^2 + 1}$$

Thus,

$$\frac{2}{R + 1 - \sqrt{R^2 + 1}} > \frac{2}{R + 1 + \sqrt{R^2 + 1}}$$

Thus,

$$\text{If } P > \frac{2}{R+1-\sqrt{R^2+1}}, \text{ then } P \text{ will be} > \frac{2}{R+1+\sqrt{R^2+1}}$$

And for $P > \frac{2}{R+1-\sqrt{R^2+1}}$, for positive values of R (for Condition "b" to apply), P > 2

as, R+1 $-\sqrt{R^2 + 1}$ is always positive and less than 1 for positive values of R and thus $\frac{2}{R+1-\sqrt{R^2+1}}$ is always greater than 2.

However, P < 1 for feasible heat exchange. Thus, Condition b does not apply for the design of feasible heat exchangers.

Now let us consider Condition "a",

Again, we know that for positive value of R,

$$R + 1 + \sqrt{R^2 + 1} > R + 1 - \sqrt{R^2 + 1}$$

Thus,

$$\frac{2}{R + 1 - \sqrt{R^2 + 1}} > \frac{2}{R + 1 + \sqrt{R^2 + 1}}$$

Thus,

$$\text{If, } P < \frac{2}{R+1+\sqrt{R^2+1}}, \text{ then, } P, \text{ will be} < \frac{2}{R+1-\sqrt{R^2+1}}$$

Hence, both inequalities for Condition "a" are satisfied when

$$P < \frac{\quad}{\quad}$$

Thus, the maximum value of $P(P_{max})$ for any value of R, is given by:

$$P_{max} = \frac{\quad}{\quad} \qquad \ldots. (2.32)$$

However, practical designs will be limited to some fraction of P_{max}, that is:

$$P = X_P P_{max}, \quad \text{where, } 0 < X_P < 1$$

Where, X_P is a constant defined by the designer.

A line of constant X_P is compared with a line of constant F_T in the Fig. 2.22. It can be seen that the line of constant X_P avoids the regions of steep slope.

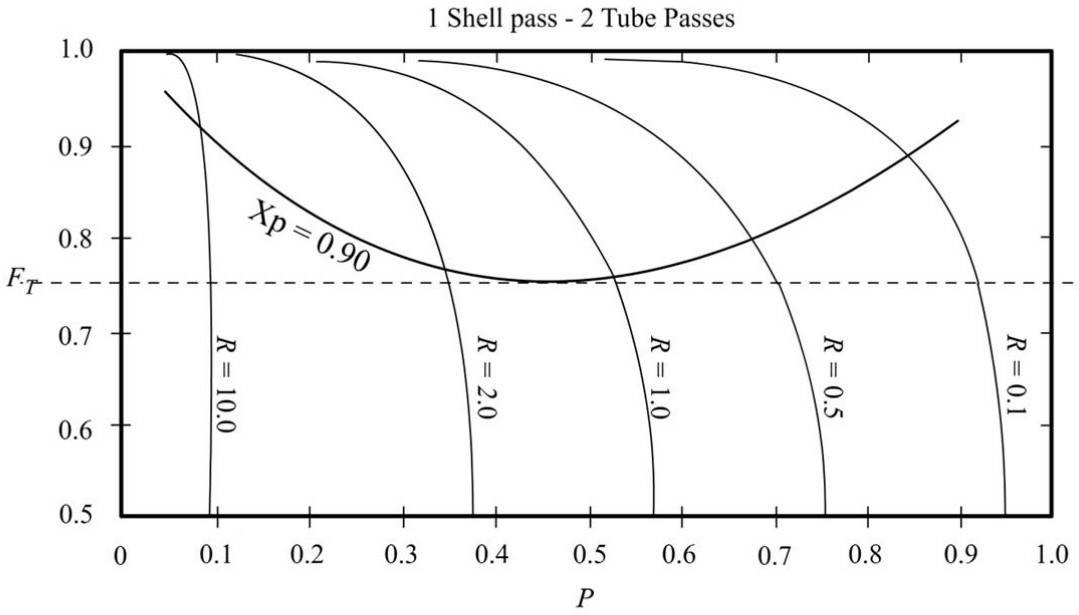


Fig. 2.23 Line of constant X_P in F_T diagram

Situations are often encountered where the design is infeasible in a 1 – 2 shell and tube exchanger, because the F_T is too low or the F_T slope too large. If this happens, either different types of shell or multiple shell arrangements must be considered. For example by using two 1 –

2 shells in series, the temperature cross in each individual shell is reduced below that for a single 1 – 2 shell for the same duty. This will become clearer from the Fig. 2.24 and Fig.2.25.

To illustrate this, let us consider a shell & tube (1- 2) heat exchanger shown below with its Temperature Enthalpy and Temperature- Length diagram.

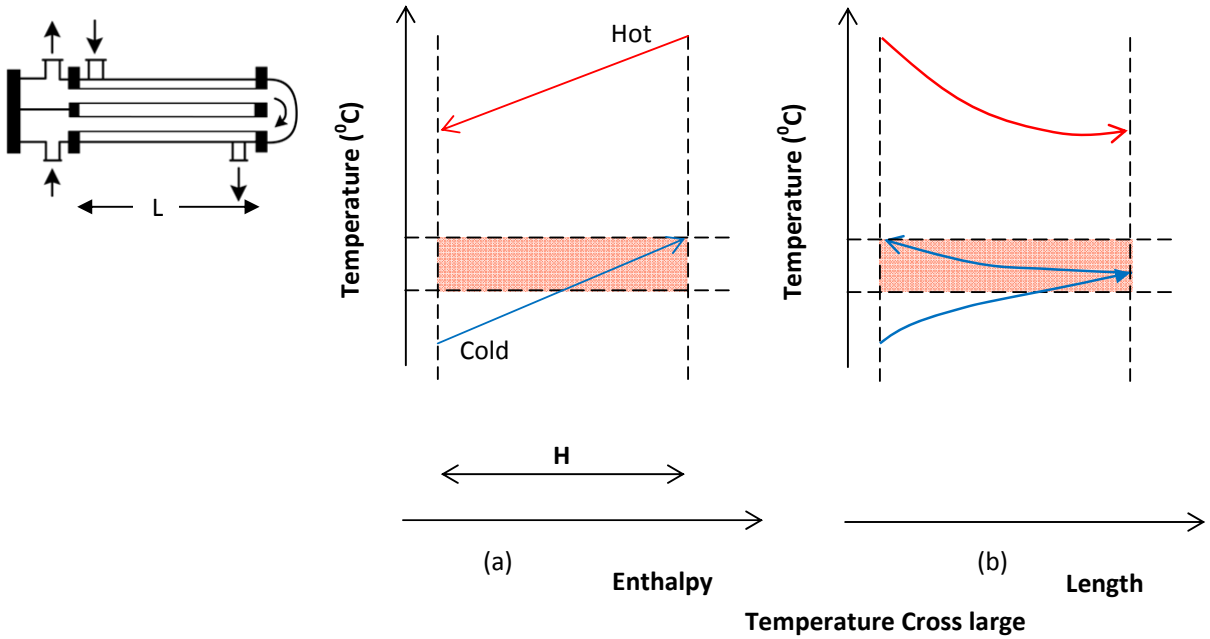


Fig.2.24 Temperature profile of a 1-2 S&T heat exchanger as a function of enthalpy(a) as well as length(b)

From the Temperature- Enthalpy diagram, it is clear that the outlet temperature of the cold stream is much higher than the outlet temperature of the hot stream. So, the temperature cross is large as evident from Temperature- Length diagram. Therefore, this design is not feasible.

Now, if we consider 2 numbers of shell & tube (1- 2) heat exchangers in series having Enthalpy H1 and H2 such that

H1 + H2 = H

This arrangement mimics a two shell pass and four tube pass(2-4) heat exchanger. The Temperature- Enthalpy and Temperature- Length diagram for such a system is shown below:

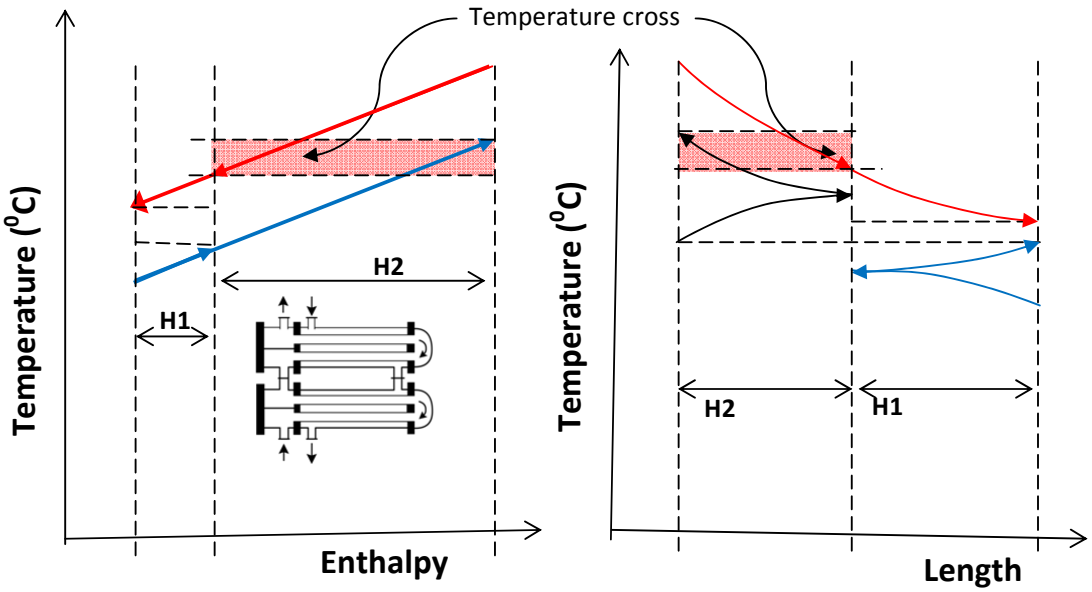


Fig.2.25 Temperature cross in two S&T(1-2) heat exchangers kept in series

Thus, from above Temperature Enthalpy diagram, it is clear that there is no temperature cross in Heat exchanger with duty H1 and also the temperature cross in heat exchanger with duty H2 is much smaller than the single heat exchanger with duty H shown earlier. Thus, we can safely say that putting shells in series reduces the temperature cross in individual exchangers. This is clearer from the Fig.2.25 the Temperature- Length diagram for two shells in series case.

The concept of HRAT and EMAT

The Double Temperature Approach (DTA) requires the selection of two approach temperatures for synthesis of HEN[9]. These temperatures are called the Heat Recovery Approach Temperature (HRAT) and the Exchanger Minimum Approach Temperature(EMAT). This approach differs from the work of Linnhoff and Flower[10] as well as similar works where authors have taken a single direct variable as ΔT_{min}.

Heat Recovery Approach Temperature(**HRAT**): The heat recovery approach temperature is the minimum approach temperature between the hot and cold composite curves as shown in Fig.2.26. For a given value of HRAT the minimum hot and cold utility consumption for the problem can be estimated either using composite curves or problem table algorithm.

Exchanger Minimum Approach Temperature(**EMAT**): Exchanger Minimum Approach Temperature is the minimum of the exchanger approach temperatures within the network of process exchangers. The exchanger approach temperature is the minimum temperature of approach within a single exchanger. This minimum value can take place at any place along the

length of the heat exchanger not invariably at the extreme ends of the heat exchanger as shown in Fig.2.27.

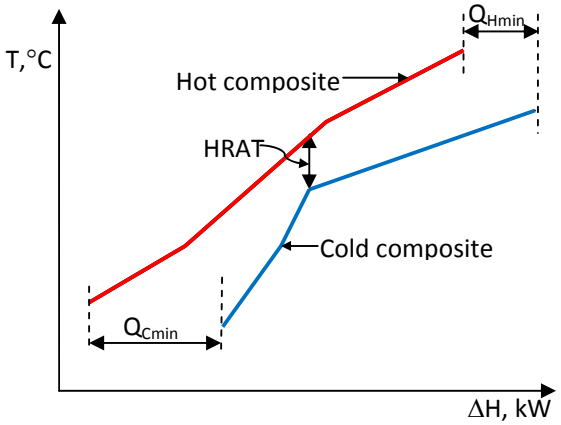


Fig.2.26 Hot and cold composite curves with HRAT

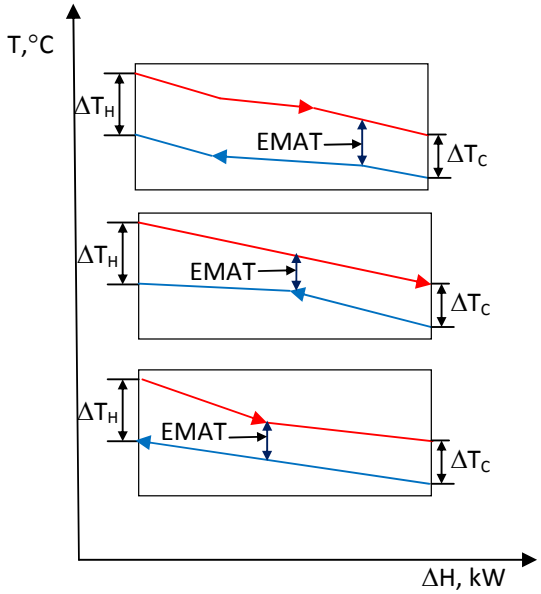


Fig.2.27 T-H diagram for exchangers with EMAT

The relationship between HRAT and EMAT is such that, for a given value of HRAT, a network cannot be designed in which EMAT is greater than HRAT. However, EMAT can be selected equal or less than HRAT for HEN synthesis.

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