

MODULE 3: MASS TRANSFER COEFFICIENTS

LECTURE NO. 6

3.5.2 Penetration theory

Most of the industrial processes of mass transfer is unsteady state process. In such cases, the contact time between phases is too short to achieve a stationary state. This non stationary phenomenon is not generally taken into account by the film model. In the absorption of gases from bubbles or absorption by wetted-wall columns, the mass transfer surface is formed instantaneously and transient diffusion of the material takes place. Figure 3.4 demonstrates the schematic of penetration model.

Basic assumptions of the penetration theory are as follows:

- 1) Unsteady state mass transfer occurs to a liquid element so long it is in contact with the bubbles or other phase
- 2) Equilibrium exists at gas-liquid interface
- 3) Each of liquid elements stays in contact with the gas for same period of time

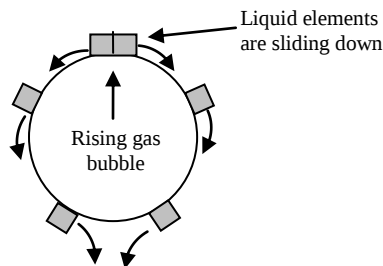


Figure 3.4: Schematic of penetration model.

Under these circumstances, the convective terms in the diffusion can be neglected and the unsteady state mass transfer of gas (penetration) to the liquid element can be written as:

$$\frac{\partial c}{\partial t} = D_{AB} \frac{\partial^2 c}{\partial Z^2} \quad (3.60)$$

The boundary conditions are: $t = 0, Z > 0 : c = c_{Ab}$ and $t > 0, Z = 0 : c = c_{Ai}$. The term c_{Ab} is the concentration of solute at infinite distance from the surface and c_{Ai} is the concentration of solute at the surface. The solution of the partial differential equation for the above boundary conditions is given by the following equation:

$$\frac{c_{Ai} - c}{c_{Ai} - c_{Ab}} = \operatorname{erf}\left(\frac{Z}{2\sqrt{D_{AB}t}}\right) \quad (3.61)$$

Where $\operatorname{erf}(x)$ is the error function defined by

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-Z^2) dZ \quad (3.62)$$

If the process of mass transfer is a unidirectional diffusion and the surface concentration is very low ($c_{Ab} \sim 0$), the mass flux of component A, N_A [$\text{kg m}^{-2} \text{s}^{-1}$], can be estimated by the following equation:

$$N_A = \frac{-\rho D_{AB}}{1 - c_{Ab}} \left(\frac{\partial c}{\partial Z} \right)_{Z=0} \approx -\rho \left(\frac{\partial c}{\partial Z} \right)_{Z=0} \quad (3.63)$$

Substituting Equation (3.61) into Equation (3.63), the rate of mass transfer at time t is given by the following equation:

$$N_A(t) = \rho \sqrt{\frac{D_{AB}}{\pi t}} (c_{Ai} - c_{Ab}) \quad (3.64)$$

Then the mass transfer coefficient is given by

$$k_L(t) = \sqrt{\frac{D_{AB}}{\pi t}} \quad (3.65)$$

The average mass transfer coefficient during a time interval t_c is then obtained by integrating Equation (3.61) as

$$\bar{k}_{L,av} = \frac{1}{t_c} \int_0^{t_c} k(t) dt = 2 \sqrt{\frac{D_{AB}}{\pi t_c}} \quad (3.66)$$

So from the above equation, the mass transfer coefficient is proportional to the square root of the diffusivity. This was first proposed by R. Higbie in 1935 and the theory is called Higbie's penetration theory.

3.5.3 Surface Renewal Theory

For the mass transfer in liquid phase, Danckwert (1951) modified the Higbie's penetration theory. He stated that a portion of the mass transfer surface is replaced with a new surface by the motion of eddies near the surface and proposed the following assumptions:

- 1) The liquid elements at the interface are being randomly swapped by fresh elements from bulk
- 2) At any moment, each of the liquid elements at the surface has the same probability of being substituted by fresh element
- 3) Unsteady state mass transfer takes place to an element during its stay at the interface.

Hence, average molar flux, $N_{A,av}$

$$N_{A,av} = (C_{Ai} - C_{Ab}) \sqrt{s \times D_{AB}} \quad (3.67)$$

Comparing Equation (3.67) with Equation (3.8) we get

$$k_{L,av} = \sqrt{s \times D_{AB}} \quad (3.68)$$

where s is fraction of the surface renewed in unit time, i.e., the rate of surface renewal [s^{-1}].

3.5.4 Boundary Layer Theory

Boundary layer theory takes into account the hydrodynamics/flow field that characterizes a system and gives a realistic picture of the way mass transfer at a phase boundary. A schematic of concentration boundary layer is shown in Figure 3.5.

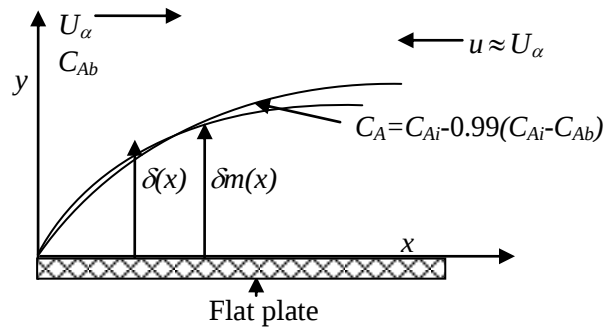


Figure 3.5: Schematic of concentration boundary layer.

When $\delta = \delta(x) \rightarrow u = U_\alpha$ and when $\delta = \delta_m(x) \rightarrow u = 0.99U_\alpha$ distance over which solute concentration drops by 99% of $(C_{Ai} - C_{Ab})$.

$$Sh_x = \frac{x \cdot k_{L,x}}{D_{AB}} = 0.332(Re)^{0.5} (Sc)^{0.33} \quad (3.69)$$

where, x is the distance of a point from the leading edge of the plate; $k_{L,x}$ is the local mass transfer coefficient.

$$Sh_{av} = \frac{l \cdot k_{L,x}}{D_{AB}} = 0.664(Re)^{0.5} (Sc)^{0.33} \quad (3.70)$$

where, l is the length of the plate.

Example problem 3.2: Zakauskas (*Adv. Heat Transfer*, 8, 93, 1972) proposed the following correlation for the heat-transfer coefficient in a staggered tube bank arrangement:

$$Nu = 0.453 Re^{0.568} Pr^{0.36}, \quad \text{where } 10 < Re (= \rho u_g d / \mu) < 10^6 \quad \text{and} \quad 0.7 < Pr < 500$$

Estimate the mass-transfer coefficient by using the mass and heat transfer analogy if to be expected for evaporation of n-propyl alcohol into carbon dioxide for the same geometrical arrangement of tube diameter (d) of 38 mm when the carbon dioxide flows at a maximum velocity (u_g) of 10 m/s at 300 K and 1 atm. Properties of dilute mixtures of propyl alcohol in carbon dioxide at 300 K and 1 atm are: Molecular weight (M) = 44 gm/mole, density (ρ) = 1.8 kg/m³, Viscosity (μ) = 1.49×10^{-5} kg/m.s, diffusivity (D_{AB}) = 7.6×10^{-6} m²/s and universal gas

constant (R) = 8.314 J/mole.K. Sherwood number is defined based on diameter of the tube as $k_G RTd / D_{AB}$

Solution 3.2: $Re = 1.8 \times 10 \times 0.038 / 1.49 \times 10^{-5} = 45906.04$

$Sc = (1.49 \times 10^{-5}) / (1.8 \times 7.6 \times 10^{-6}) = 1.08$

Analogy to heat transfer

$Sh = 0.453 Re^{0.568} Sc^{0.36} = 207.68$

So, $k_G RTd / D_{AB} = 207.68$

Therefore $k_G = (207.68 \times D_{AB}) / (RTd) = 1.665 \times 10^{-05} \text{ mole/m}^2 \cdot \text{s} \cdot \text{Pa}$