MODULE 2: DIFFUSION

LECTURE NO. 2

2.2 STEADY STATE MOLECULAR DIFFUSION IN FLUIDS UNDER STAGNANT AND LAMINAR FLOW CONDITIONS

2.2.1 Steady state diffusion through a constant area

Steady state diffusion through a stagnant gas film

Assume steady state diffusion in the Z direction without any chemical reaction in a binary gaseous mixture of species A and B. For one dimensional diffusion of species A, the Equation of molar flux can be written as

$$N_A = -CD_{AB}\frac{dy_A}{dZ} + y_A(N_A + N_B) \qquad (2.11)$$

Separating the variables in Equation (2.11), it can be expressed as

$$\frac{-dy_A}{N_A - y_A(N_A + N_B)} = \frac{dZ}{CD_{AB}} \qquad (2.12)$$

For the gaseous mixture, at constant pressure and temperature C and D_{AB} are constant, independent of position and composition. Also all the molar fluxes are constant in Equation (2.12). Therefore the Equation (2.12) can be integrated between two boundary conditions as follows:

at $Z = Z_1$, $y_A = y_{A1}$

at $Z = Z_2$, $y_A = y_{A2}$

where *1* indicates the start of the diffusion path and *2* indicates the end of the diffusion path. After integration with the above boundary conditions the Equation for diffusion for the said condition can be expressed as

$$N_A = \frac{N_A}{(N_A + N_B)}\frac{CD_{AB}}{Z_2 - Z_1}\ln\left[\frac{\frac{N_A}{(N_A + N_B)} - y_{A_2}}{\frac{N_A}{(N_A + N_B)} - y_{A_1}}\right] \qquad (2.13)$$

For steady state one dimensional diffusion of A through non-diffusing B, $N_B = 0$ and N_A = constant. Therefore $N_A/(N_A + N_B) = 1$. Hence Equation (2.13) becomes

$$N_A = \frac{CD_{AB}}{Z_2 - Z_1} \ln\left[\frac{1 - y_{A_2}}{1 - y_{A_1}}\right] \tag{2.14}$$

Since for an ideal gas $C = \frac{p}{RT}$ and for mixture of ideal gases $y_A = \frac{p_A}{P}$, the Equation (2.14) can be expressed in terms of partial pressures as

$$N_A = \frac{PD_{AB}}{(Z_2 - Z_1)RT} \ln\left[\frac{P - p_{A_2}}{P - p_{A_1}}\right] \tag{2.15}$$

Where P is the total pressure and p_{A1} and p_{A2} are the partial pressures of A at point 1 and 2 respectively. For diffusion under turbulent conditions, the flux is usually calculated based on linear driving force. For this purpose the Equation (2.13) can be manipulated to rewrite it in terms of a linear driving force. Since for the binary gas mixture of total pressure P, $P - p_{A_2} = p_{B_2}$; $P - p_{A_1} = p_{B_1}$; $p_{A_1} - p_{A_2} = p_{B_2} - p_{B_1}$. Then the Equation (2.15) can be written as

$$N_A = \frac{PD_{AB}}{(Z_2 - Z_1)RT} \left[\frac{p_{A_1} - p_{A_2}}{p_{B_2} - p_{B_1}}\right] \ln\left[\frac{p_{B_2}}{p_{B_1}}\right] \tag{2.16}$$

Or

$$N_A = \frac{PD_{AB}}{(Z_2 - Z_1)RTp_{B,M}} (p_{A_1} - p_{A_2}) \tag{2.17}$$

Where $p_{B,M}$ is called logarithmic mean partial pressure of species B which is defined as

$$p_{B,M} = \frac{p_{B_2} - p_{B_1}}{\ln\left(\frac{p_{B_2}}{p_{B_1}}\right)} \tag{2.18}$$

A schematic concentration profile for diffusion A through stagnant B is shown in Figure 2.1. The component A diffuses by concentration gradient, $-\frac{dy_A}{dZ}$. Here flux is inversely proportional to the distance through which diffusion occurs and the

concentration of the stagnant gas ($p_{B,M}$) because with increase in Z and $p_{B,M}$, resistance increases and flux decreases.

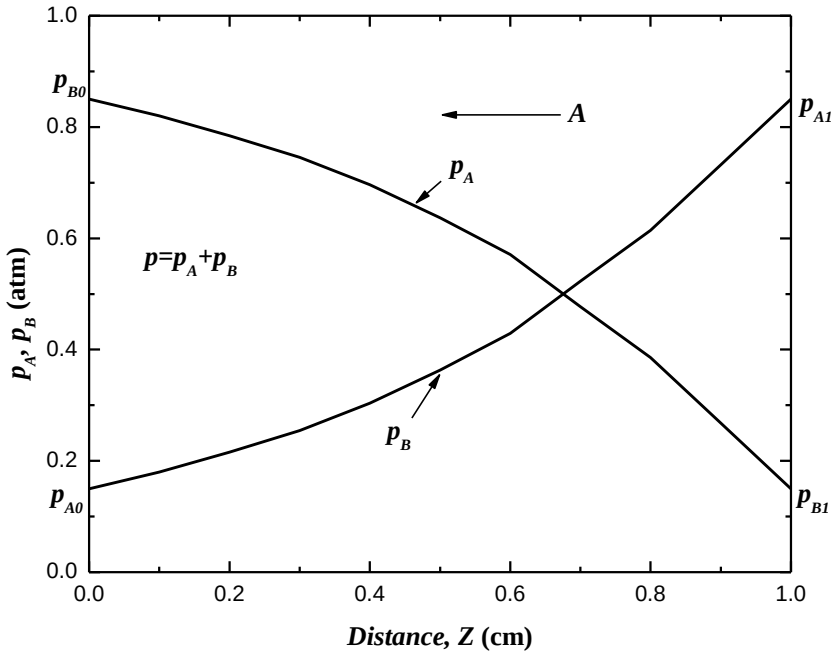

Figure 2.1: Partial pressure distribution of A in non-diffusing B

2.2.2 Steady state equimolar counter diffusion:

This is the case for the diffusion of two ideal gases, where an equal number of moles of the gases diffusing counter-current to each other. In this case $N_B = -N_A$ = constant and $N_A + N_B = 0$. The molar flux Equation (Equation (2.11)) at steady state can then be written as

$$N_A = -\frac{D_{AB} P}{RT} \frac{dy_A}{dZ} \qquad (2.19)$$

Integrating the Equation (2.19) with the boundary conditions: at $Z = Z_1$, $y_A = y_{A1}$; at $Z = Z_2$ $y_A = y_{A2}$, the Equation of molar diffusion for steady-state equimolar counter diffusion can be represented as

$$N_A = \frac{D_{AB}P}{RT(Z_2 - Z_1)}(y_{A_1} - y_{A_2}) = \frac{D_{AB}}{RT(Z_2 - Z_1)}(P_{A_1} - P_{A_2}) \quad (2.20)$$

It may be noted here also that molar latent heats of vaporization of *A* and *B* are equal. So, $\Delta H_A^V = \Delta H_B^V$, where, ΔH_A^V and ΔH_B^V are molar latent heats of vaporization of A and B, respectively. The concentration profile in terms of partial pressure is shown in Figure 2.2.

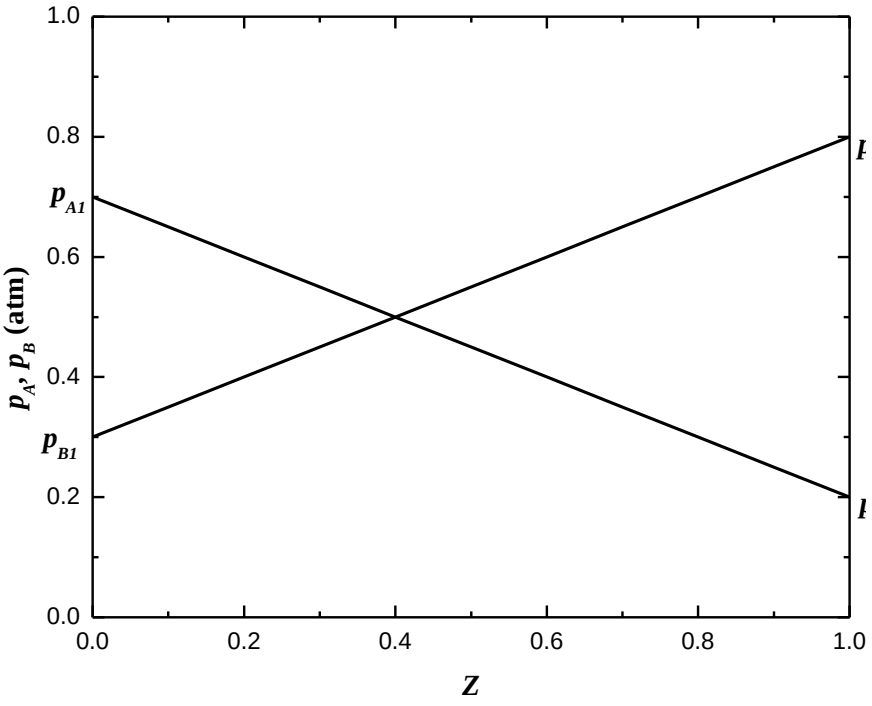

Figure 2.2: Equimolar counter diffusion of A and B: Partial pressure distribution with position

2.2.3 Non-equimolar counter diffusion

In some practical cases, A and B molecules diffuse in opposite directions at different molar velocities [1]. Let carbon monoxide is generated from the reaction between hot char and oxygen. The stoichiometry is as follows:

$$2C + O_2(A) \leftrightarrow 2CO(B) \qquad (2.21)$$

When one mole oxygen molecule diffuses towards char, two moles carbon monoxide molecules diffuse in opposite direction. Here, $N_A = -N_B/2$ and molar latent heats of vaporization are not equal. Hence,

$$N_A \Delta H_A^V = -N_B \Delta H_B^V \qquad (2.22)$$