Advanced Numerical Analysis for Chemical Engineering
Quiz -1 (1 hrs.)

1. Consider a continuously operated fermenter described by the following equations

$$\frac{dX}{dt} = -DX + \mu X \tag{1}$$

$$\frac{dS}{dt} = D(S_f - S) - \frac{1}{Y_{X/S}}\mu X \tag{2}$$

$$\frac{dP}{dt} = -DP + (\alpha\mu + \beta)X \tag{3}$$

$$\mu = \frac{\mu_m(1 - \frac{P}{P_m})S}{K_m + S + \frac{S^2}{K_i}} \tag{4}$$

where X : effluent cell-mass or biomass concentration, S: substrate concentration, P : product concentration, D : dilution rate, S_f : feed substrate concentration, $Y_{X/S}$: cell-mass yield, α and β :yield parameters for the product, μ_m: maximum specific growth rate, P_m : product saturation constant, K_m :substrate saturation constant, K_i : substrate inhibition constant.

(a) The vectors of state (dependent) variable ($\mathbf{x}$) and input (independent) variables ($\mathbf{u}$) for this problem are defined as (2 mark)

(a) $\mathbf{x} = \begin{bmatrix} X & S_f & P \end{bmatrix}^T$; $\mathbf{u} = \begin{bmatrix} D & S \end{bmatrix}^T$

(b) $\mathbf{x} = \begin{bmatrix} X & S & P \end{bmatrix}^T$; $\mathbf{u} = \begin{bmatrix} D & S_f \end{bmatrix}^T$

(c) $\mathbf{x} = \begin{bmatrix} \mu & S & P \end{bmatrix}^T$; $\mathbf{u} = \begin{bmatrix} X & S_f \end{bmatrix}^T$

(d) $\mathbf{x} = \begin{bmatrix} \mu & S & P \end{bmatrix}^T$; $\mathbf{u} = \begin{bmatrix} D & S_f \end{bmatrix}^T$

(b) Consider a situation where D and S_f are fixed at $D = 0.16$ and $S_f = 23$ and values of ($Y_{X/S}, \alpha,\ \beta,\ \mu_m,\ P_m, K_m,\ K_i$) are given. It is desired to estimate X, S and P when all the transients have vanished and the system has come to a steady state. The resulting set of equations can be classified as (1.5 marks)

(a) ODE-Initial Value Problem (b) ODE-Boundary Value Problem (c) Nonlinear Algebraic Equations (d) Differential Algebraic Equations (e) Linear Algebraic Equations

(c) Consider a situation where at $t = 0$ we have $X = 7.0, S = 25, P = 24$ and values of $(Y_{X/S}, \alpha,\ \beta,\ \mu_m,\ P_m, K_m,\ K_i)$ are given. From $t = 0$ to 100, we change $D(t)$ as $D(t) = 0.16 + 0.02sin(0.2t)$ while holding $S_f = 23$. It is desired to find time variation of $X(t), S(t)$ and $P(t)$ as a function of time over $[0, 100]$. The resulting set of equations can be classified as (1.5 marks)

(a) ODE-Initial Value Problem (b) ODE-Boundary Value Problem (c) Nonlinear Algebraic Equations (d) Differential Algebraic Equations (e) Linear Algebraic Equations

2. Let X represent set of continuous functions on interval $0 \leq t \leq 1$ with inner product defined as

$$\langle \mathbf{x}(t), \mathbf{y}(t) \rangle = \int_0^1 t(1-t)\mathbf{x}(t)\mathbf{y}(t)dt$$

Given a set of four linearly independent vectors

$$\mathbf{x}^{(1)}(t) = 1; \quad \mathbf{x}^{(2)}(t) = t$$

find orthonormal set of vectors $\mathbf{e}^{(1)}(t)$ and $\mathbf{e}^{(2)}(t)$. (5 marks)

3. Show that in $C[-1, 1]$ the following function

$$\langle f(t), g(t) \rangle = \max_t \ |x(t)y(t)|$$

cannot define an inner product. (5 marks)