Prof. Shankar
Lec-26 21/11/2012

Advanced Reaction Engineering

Gas Solid Reactions

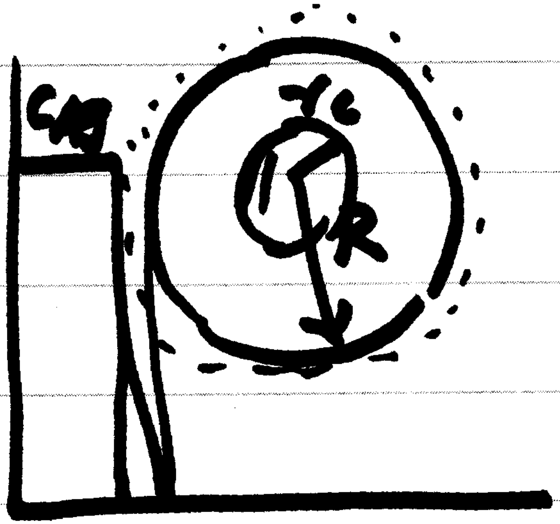
Wed.
21/Nov/12
1030-1120

$C + O_2 = CO_2 + \text{Energy}$

$\text{Wood} + O_2 = CO_2 + \text{Energy}$

Film Diffusion Control.

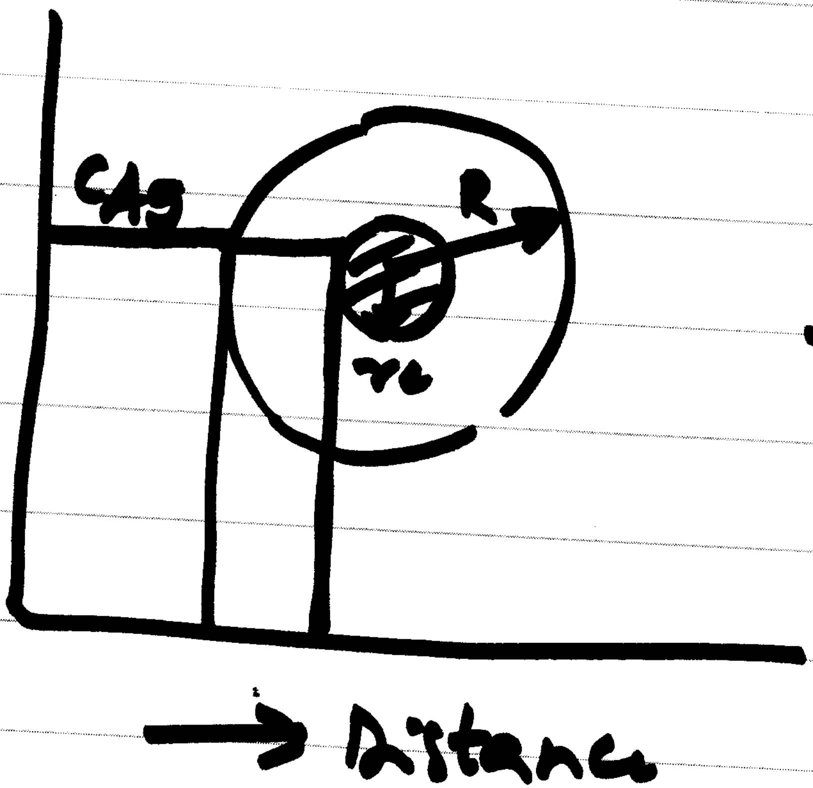
$$1 - X_B = \frac{\frac{4}{3}\pi r_c^3 \rho_B}{\frac{4}{3}\pi R^3 \rho_B}$$



$$\frac{t}{\tau_F} = 1 - \frac{r_c^3}{R^3}$$

$$= X_B$$

$$\tau_F = \rho_B R / 3 b k_g C_{Ag}$$

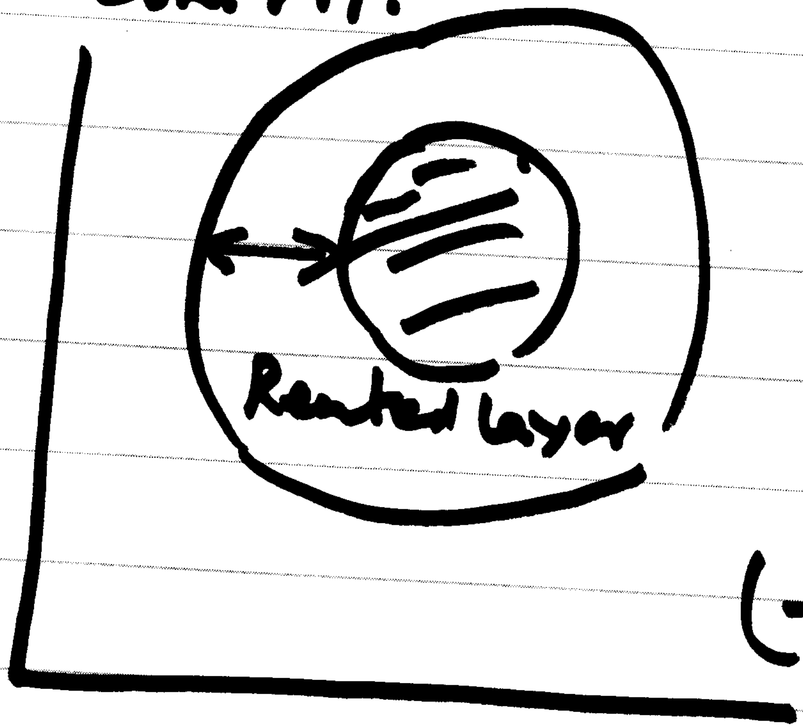


$$\frac{t}{\tau_R} = 1 - r_c/R$$

$$\tau_R = \frac{\rho_B R}{b k_s C_{Ag}}$$

$A(g) + b\,B(s) \rightarrow$ Products

Ash Layer Diffusion Control.

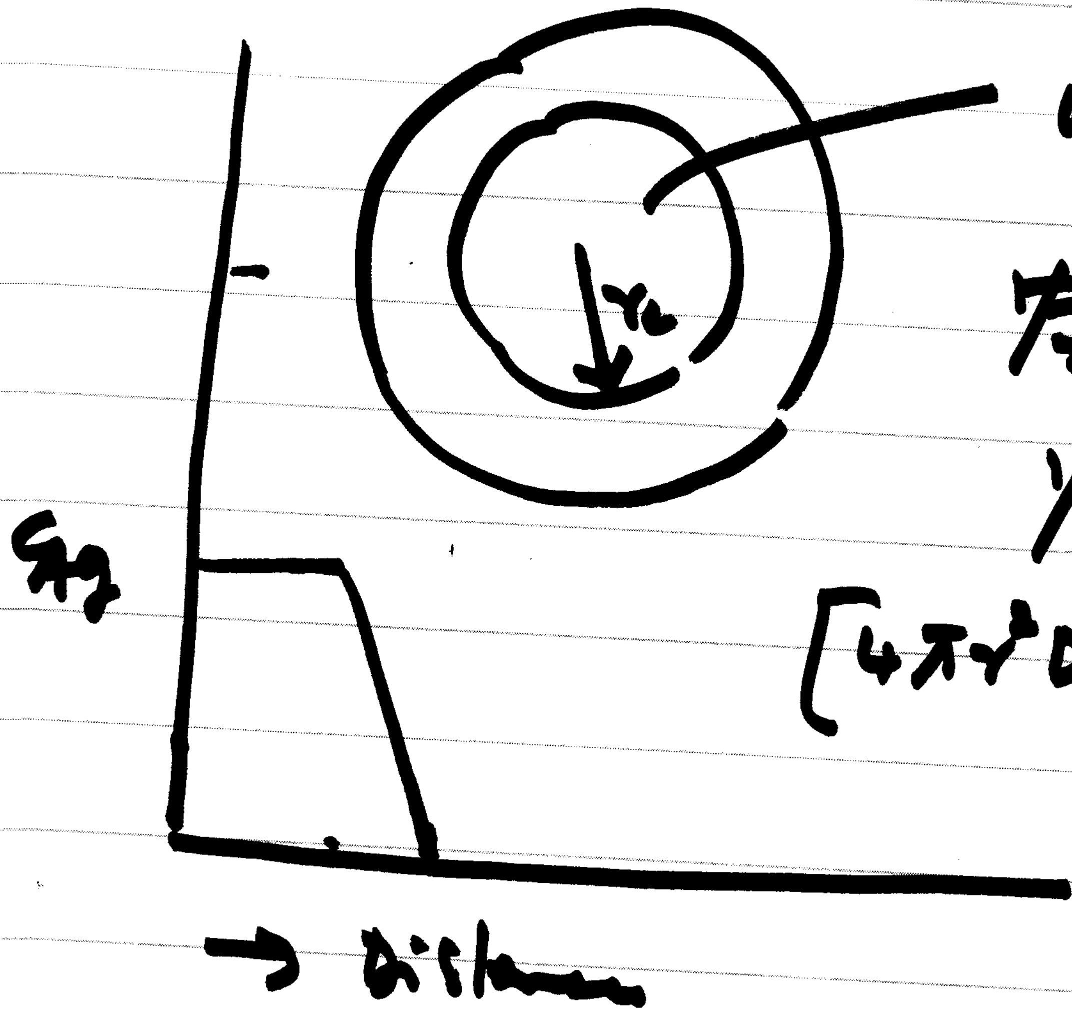


$$\frac{dN_B}{dt} = r_B S$$

$$(-r_A S) = \left[D\left(\frac{\partial C}{\partial r}\right) 4\pi r^2 \right]_{r_c}$$

$$(-r_B S) = b(-r_A S)$$

$$= \left[b\, D\left(\frac{\partial C}{\partial r}\right) 4\pi r^2 \right]_{r_c}$$



[illegible] +

I/p − O/p + Gen = Acc

$$\left[4\pi r^2 D \frac{\partial C}{\partial r}\right]_{r+\Delta r} - \left(4\pi r^2 D \frac{\partial C}{\partial r}\right)_r + 0$$

$$= \left(\frac{\partial C}{\partial t}\right) 4\pi r^2 dr \, \epsilon$$

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r} r^2 D \frac{\partial c}{\partial r}\right] = \epsilon \frac{\partial c}{\partial t}$$

$r = R \qquad c = c_0 = c_{Ag}$

$r = r_c \qquad c = 0 \quad$ (Rxn) irreversible

$y = r/R; \qquad \theta = t/\tau_D \qquad \psi = c/c_0$

$$\tau_D = \frac{\frac{4}{3}\pi R^3 \rho_B}{b\left(\frac{D c_0}{R/2}\right) 4\pi R^2} = \frac{\rho_B R^2}{6 b D c_0} \leftarrow c_{Ag}$$

$$\frac{1}{R^2}\frac{1}{y^2}\frac{\partial}{\partial y R}\left[R^2 y^2 \frac{\partial}{R}\frac{\partial \psi}{\partial y}\right]$$

$$= \frac{\epsilon_i}{\tau_D}\frac{\partial \psi_i}{\partial \theta}$$

$$\frac{1}{R^2 y^2}\frac{\partial}{\partial y}\left[y_D^2 \frac{\partial \psi}{\partial y}\right] = \frac{\epsilon}{\tau_D}\frac{\partial \psi_i}{\partial \theta}$$

$$\frac{1}{R^2 y^2}\frac{\partial}{\partial y}\left[y^2 \not{\rho} \frac{\partial \psi}{\partial y}\right] = \frac{\epsilon\, 6 b \not{\kappa}_0}{\rho_0 R^2}\frac{\partial \psi}{\partial \theta}$$

Small

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left[y^2\frac{\partial \psi_i}{\partial y}\right] = \left(\frac{6\,\epsilon\,C_0}{S_B}\right)\left(\frac{\partial \psi_i}{\partial \alpha}\right) \quad \text{QSSA}$$

$C_0 = 0.05$ gmol/L

$S_B = 10$ gmol/L

$\epsilon = 0.3$

y (Order =1 ψ (0 1)

$$\frac{(6)(0.3)(0.05)}{10}$$

$\underline{0.009}$

$O(1)$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial \psi}{\partial y}\right) = 0 \qquad \text{QSSA.}$$

$\psi = 1$ @ $y = 1$

$\psi = 0$ @ $y = y_c$

$$\frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial \psi}{\partial y}\right)=0$$

$$y^2\frac{\partial \psi}{\partial y} = A$$

$$\frac{\partial \psi}{\partial y} = \frac{A}{y^2} \; ; \qquad \psi = -\frac{A}{y} + B$$

$\psi = 1$ @ $y = 1$	$1 = -A + B$	$y = 1$
$\psi = 0$ @ $y = y_c$	$0 = -\frac{A}{y_c} + B$	$y = y_c$

$$A = \frac{1}{(1/y_c - 1)} \; ; \qquad B = A/y_c$$

$$\psi = \frac{1}{1-y_c}\left\{1 - \frac{y_c}{y}\right\}.$$

$$\psi = \frac{1}{(1-y_c)}\left\{1 - \frac{y_c}{y}\right\}.$$

$y_c = r_c/R$

$$\underbrace{\left[D\,\frac{\partial c}{\partial r}\,.\,4\pi r^2\right]_{r_c}}_{(\cdot/R)} = \left[D\,4\pi y^2 R^2\,\frac{c_0}{R}\,\frac{\partial\psi}{\partial y}\right]_{y_c}$$

$$= \left[4\pi D c_0\, y^2 R \left(\frac{\partial\psi}{\partial y}\right)\right]_{y = y_c}$$

$$\left[D \left(\frac{\partial C}{\partial r} \right) 4 \pi r^2 \right]_{r = r_c} = \left(-r_{BE}^{S} \right) = \frac{b \, 4 \pi D C_0}{\left(1/r_c - 1/R \right)}$$

$$\left(-r_{BE}^{S} \right) = \frac{b \, 4 \pi D C_0}{\left(1/r_c - 1/R \right)}$$

$$\left(-\gamma_B S\right) = \frac{6\,4\pi D C_0}{\left(1/r_c - 1/R\right)}$$

$$\frac{d}{dt} N_B = \left(\gamma_B S\right)$$

$$\frac{d}{dt}\left[\frac{4}{3}\pi r_c^3 \rho_B\right] = -\frac{6\,4\pi D C_0}{\left(\frac{1}{r_c} - 1/R\right)}$$

$$\cancel{4\pi} r_c^2 \rho_B \frac{dr_c}{dt} = \frac{-\cancel{4\pi} b D C_0}{\left(\frac{1}{r_c} - \frac{1}{R}\right)}$$

$$\rho_B \left(\frac{1}{r_c} - \frac{1}{R}\right) r_c^2 = -b D C_0 \, dt$$

$$\rho_B \left[\frac{r_c^2}{2} - \frac{r_c^3}{3R}\right]_R^{r_c} = - b D C_1 t$$

$$\rho_B \left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} \right]_R^{r_c} = -6DC_0 t$$

(sig → 6)

$$\rho_B \left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} - \frac{R^2}{2} + \frac{R^2}{3} \right] = -6DC_0 t$$

$$R^2 \rho_B \left[\frac{1}{2} - \frac{1}{3} - \frac{3r_c^2}{R} + \frac{2r_c^3}{R^3} \right] = 6DC_0 t$$

$R^2 \rho_B \; \frac{1}{6} -$

$$\rho_B\left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} - \frac{R^2}{2} + \frac{R^3}{3R}\right] = -bDC_0 t$$

$$\rho_B\left[\frac{r_c^2}{2} - \frac{r_c^3}{3R} - \frac{R^2}{6}\right] = -bDC_0 t$$

$$\frac{\rho_B}{6}\left[R^2 - 3r_c^2 + \frac{2r_c^3}{R}\right] = bDC_0 t$$

$$\frac{\rho_B R^2}{6}\left[1 - 3\frac{r_c^2}{R^2} + 2\frac{r_c^3}{R^3}\right] = D C_0 t$$

$$\boxed{1 - 3\frac{r_c^2}{R^2} + 2\frac{r_c^3}{R^3} = \frac{6 D C_0 t}{\rho_B R^2}}$$

$r_c = 0 \quad t = \tau_D$

$$\tau_D = \frac{\rho_B R^2}{6 D C}$$

$$\frac{r_c}{R} = (1 - X_B)^{1/3}$$

$$1 - \frac{3 r_c^2}{R^2} + \frac{2 r_c^3}{R^3} = t/\tau_D.$$

$$1 - X_B = \frac{\frac{4}{3}\pi r_c^3 \rho_B}{\frac{4}{3}\pi R^3 \rho_B}$$

$$(1 - X_B) = \frac{r_c^3}{R^3}$$

$$(1 - X_B)^{1/3} = \frac{r_c}{R}$$

$A(g) + b\,B(s) \rightarrow \text{Products}$

$$\frac{dN_B}{dt} = r_{BS} = b\, r_{AS}$$

$\tau_R = \rho_B R / b k_s C_{AS}$

$\tau_F = \rho_B / 3 b k_g C_{AS}$

$\tau_D = \rho_B R^2 / 6 D C_{AS}$

$r_{BS} = -\, b k_s C_{AS} \cdot 4\pi r_c^2$ (Rxn)

$= -\, b k_g C_{AS}\, 4\pi R^2$ (Film)

$$= \frac{-4\pi b D C_{AS}}{\left(\frac{1}{r_c} - \frac{1}{R}\right)} \text{ (Ash)}$$

Resistance = $\dfrac{\text{Potential}}{\text{Rate}}$
Ω

$= C_{AS} / r_{AS}$

$$\Omega_{RXN} = \frac{C_{AS}}{-b k_S C_{AS} \cdot 4\pi r_c^2} = -\frac{1}{b k_S \cdot 4\pi r_c^2}$$

$$\Omega_F = \frac{C_{AS}}{-b k_S \cdot C_{AS}\, 4\pi R^2} = -\frac{1}{b k_g\, 4\pi R^2}$$

$$\Omega_D = \frac{-C_{AS}\left[1/r_c - 1/R\right]}{4\pi b D C_{AS}} = \frac{-\left[\frac{1}{r_c} - \frac{1}{R}\right]}{4\pi b D}$$

$$\frac{dN_B}{dt} = r_{BS} = \frac{C_{Ag}}{\Omega_{Rm} + \Omega_F + \Omega_D}$$

$$\frac{d}{dt}\left[\frac{4}{3}\pi r_3^3 \rho_B\right] = \frac{C_{Ag}}{\Omega_R + \Omega_P + \Omega_D}$$

$$4\pi r_c^2 \rho_B \, dr_c \left[\qquad \qquad \qquad \right]$$

$$\int_{R}^{r_c} 4\pi r_c^2 \rho_D \left[\frac{1}{b k_g 4\pi r_c^2} + \frac{1}{b k_g 4\pi R^2} + \frac{1}{4\pi D_b}\left(\frac{1}{r_c} - \frac{1}{R}\right)\right] dr_c$$

$$= - dt \cdot C_{Ag}$$

$$\frac{\rho_B R}{6 k_s C_{AS}}\left(1-\frac{r_c}{R}\right)+\frac{\rho_B R}{3 b k_g C_{AS}}\left(1-\frac{r_c^3}{R^3}\right)$$

$$+\frac{\rho_B R^2}{6 D b C_{AS}}\left(1-\frac{3 r_c^2}{R^2}+\frac{2 r_c^3}{R^3}\right)=\cancel{C_{Ag}}\, t$$

$$\tau_R\left(1-\frac{r_c}{R}\right)+\tau_F\left(1-\frac{r_c^3}{R^3}\right)+\tau_D\left(1-\frac{3 r_c^2}{R^2}+\frac{2 r_c^3}{R^3}\right)$$

$$= t$$