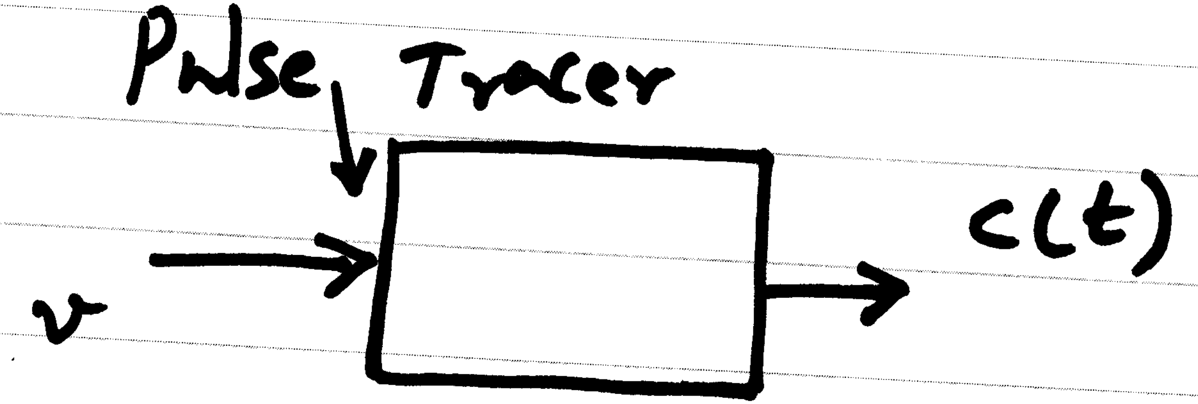
Prof. Shankar
Lec. 24
19/11/12

Advanced Reaction Engineering

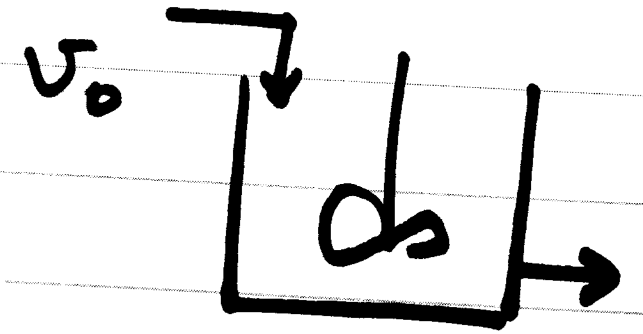
Residence Time Distribution

19 Nov 12
1515-1615



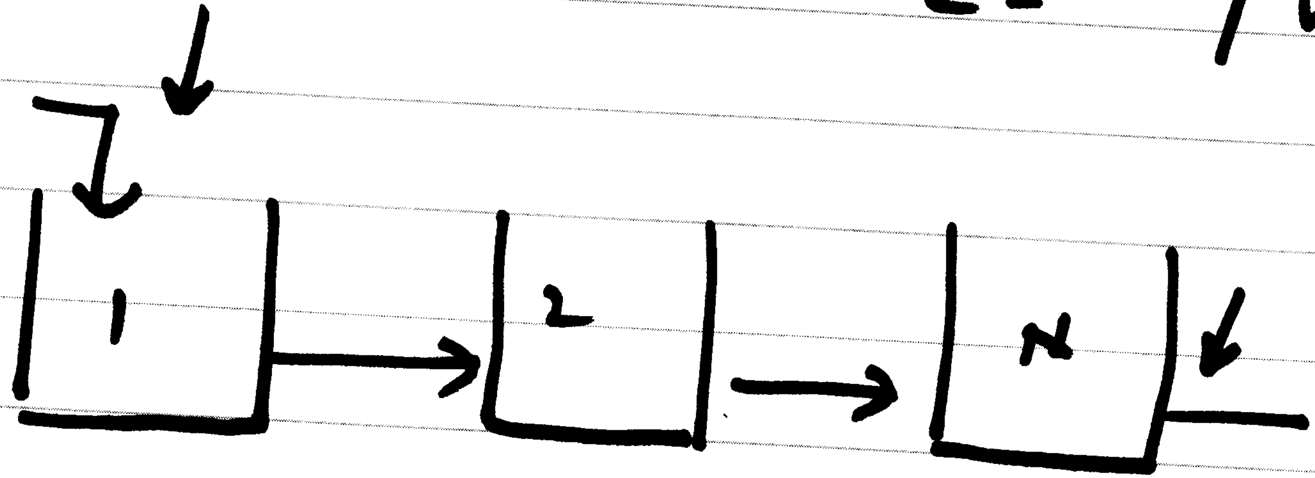
$$E(t)\,dt = \frac{(v\,c(t))}{M_0}$$

$$M_0 = \int_0^\infty v\,c(t)\,dt$$



$$E(t) = \frac{1}{\tau} e^{-t/\tau}$$

$$\tau = V/v_0$$

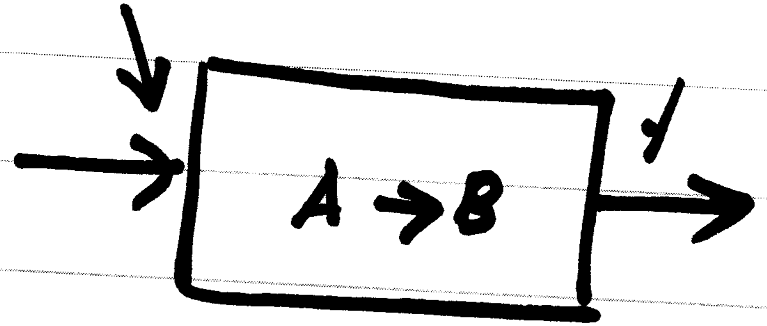


$$\tau_1 = \tau_2 = \tau_N$$

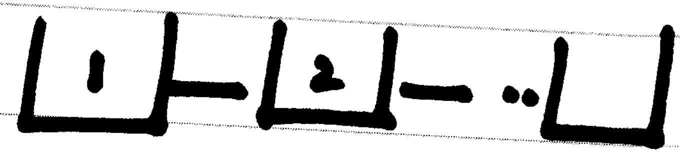
$$\tau = \frac{V}{v_0} = n\tau_i$$

$$E_n(t)\,dt = \frac{t^{n-1}}{(n-1)!\,\tau_i^{\,n}} e^{-t/\tau_i}$$

$$\tau_i = \frac{V_i}{v_0}$$



σ_n^2 , μ_n^2



$$C_1 = \frac{C_0}{(1+k\tau_i)}$$

i

$$n = \mu_n^2 / \sigma_n^2$$

$$\tau_i = \tau / N.$$

$$= \mu_n / N.$$

$$C_N = \frac{C_0}{(1+k\tau_i)^N}$$

$$N = \mu_n^2 / \sigma_n^2$$

$$1-X \quad \frac{C_N}{C_0} = \frac{1}{(1+k\tau_i)^N} =$$

Completely MicroMixed.

$$1 - X = \frac{C_N}{C_0} = \frac{1}{(1 + k\tau_i)^N}$$

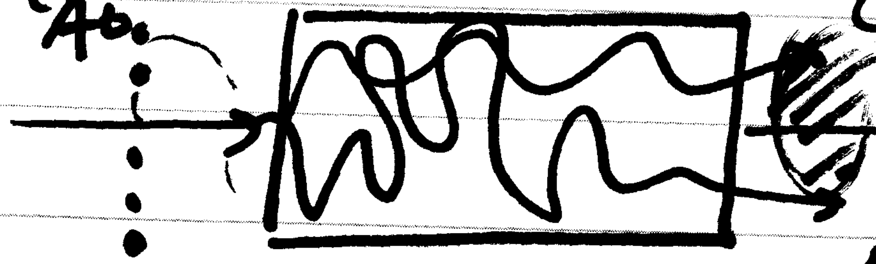
$$\boxed{N = \frac{\mu_n^2}{\sigma_n^2}}$$

$$= \frac{1}{\left(1 + k\,\frac{\tau}{N}\right)^N} = \frac{1}{\left[1 + k\,\frac{\mu_n \cdot \sigma_n^2}{\mu_n^2}\right]^{\frac{\mu_n^2}{\sigma_n^2}}}$$

$$1 - X = \frac{C_N}{C_0} = \frac{1}{\left[1 + k\ \sigma_n^2/\mu_n\right]^{\mu_n^2/\sigma_n^2}}$$

Conversions directly from RTD. (Completely Segregated Model)

C_{A0}



$C_{A1} \leftarrow t_1$

$C_{A2} - t_2$

$A \rightarrow B$

k [Late Mixing Model

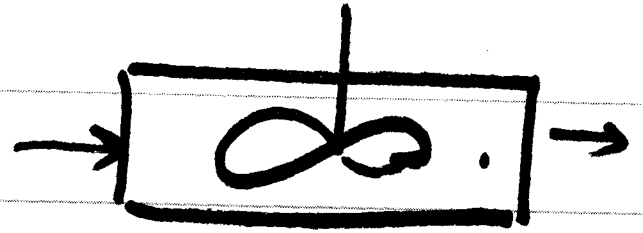
(1) $C_{A0} = \left[C_{A0}\, e^{-kt_1}\right] E(t_1)\,dt_1$

(2) $\left(C_{A0}\, e^{-kt_2}\right) E(t_2)\,dt_2$

$$C_{A,\ \text{exit}} = \int_0^{\infty} \underbrace{C_{A0}\, e^{-kt}}\ \underbrace{E(t)\,dt}$$

$$\bar{C}_A = \int_0^\infty \underbrace{\left(C_{A0}\, e^{-kt}\right)}\, \underline{E(t)}\, dt$$

Example



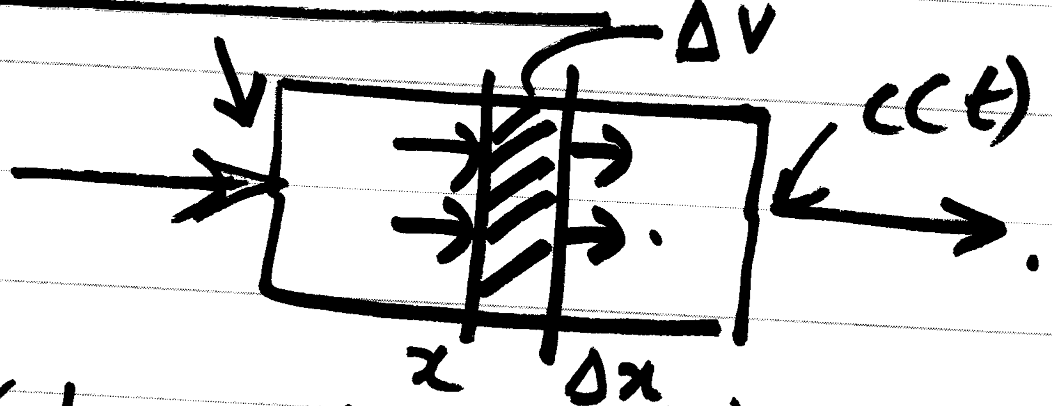
$$E(t) = \frac{1}{\tau_s}\, e^{-t/\tau}$$

$$\bar{C}_A = \int_0^\infty \left(C_{A0}\, e^{-kt}\right) \frac{1}{\tau_s}\, e^{-t/\tau}\, dt.$$

$$\boxed{\bar{C}_A = \frac{C_{A0}}{(1 + k\tau_s)}}$$

τ_s = residence time

Dispersion Model.



$$u\cancel{A}c\Big|_x - \left(u\cancel{A}c\right)\Big|_{x+\Delta x} + \left[-\cancel{A}D\frac{\partial c}{\partial x}\Big|_x - \left(-\cancel{A}D\frac{\partial c}{\partial x}\right)_{x+\Delta x}\right]$$

$$-kc\cancel{A}\cancel{\Delta x} = \cancel{A}\cancel{\Delta x}\frac{\partial c}{\partial t}$$

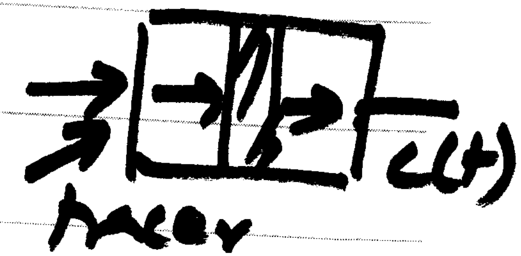
$$-u\frac{\partial c}{\partial x} + D\frac{\partial^2 c}{\partial x^2} = \cancel{\frac{\partial c}{\partial t}}$$

$$-kc$$

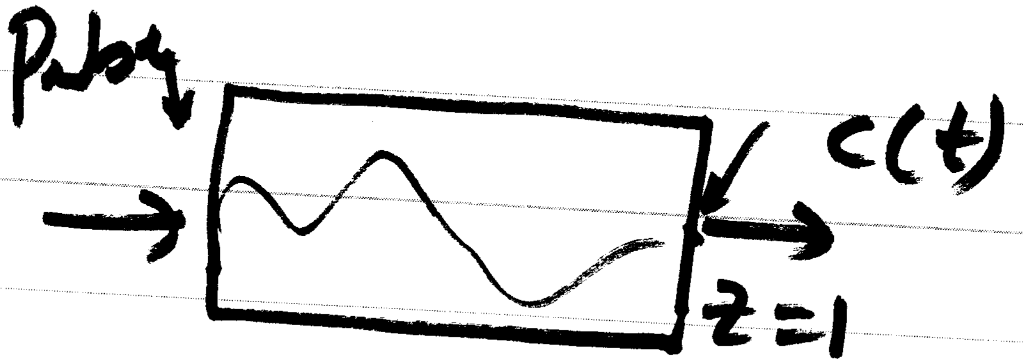
$\theta = t/\bar{t} \qquad \bar{t} = L/u, \quad Z = x/L$

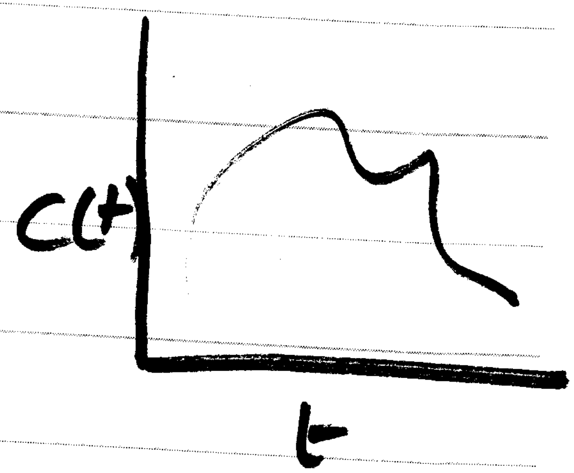
$$\bar{t}\,\frac{\partial c}{\partial \theta} = \frac{D}{L^2}\frac{\partial^2 c}{\partial Z^2} - \frac{u}{L}\cdot\frac{\partial c}{\partial Z} \qquad \bar{t} = \frac{L}{u}$$

$$\left(\frac{\partial c}{\partial \theta}\right) = \frac{\partial^2 c}{\partial Z^2}\frac{D}{L^2}\bar{t} - \cancel{\frac{u}{L}\bar{t}}\,\frac{\partial c}{\partial Z}$$



$$\boxed{\frac{\partial c}{\partial \theta} = \frac{D}{uL}\frac{\partial^2 c}{\partial Z^2} - \frac{\partial c}{\partial Z}}$$





$$\frac{\partial c}{\partial \theta} = \frac{D}{uL} \cdot \frac{\partial^2 c}{\partial z^2} - \frac{\partial c}{\partial z}.$$

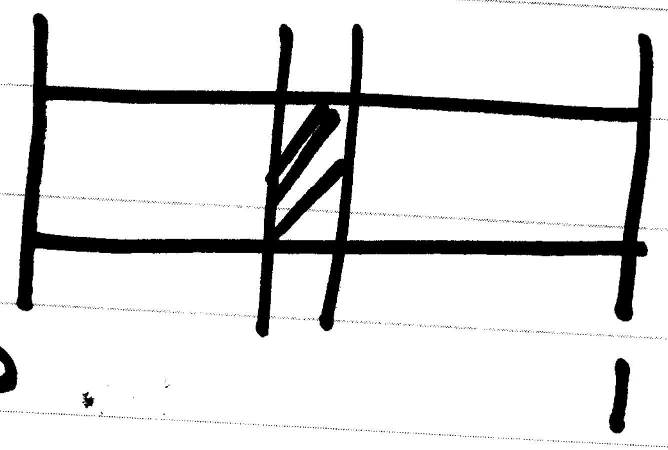
$\frac{D}{uL}$: Dispersion No ; $\frac{uL}{D}$ = Peclet No

$$\frac{\sigma^2}{\mu^2} = \frac{2}{Pe} - \frac{2}{Pe^2}\left(1 - e^{-Pe}\right)$$

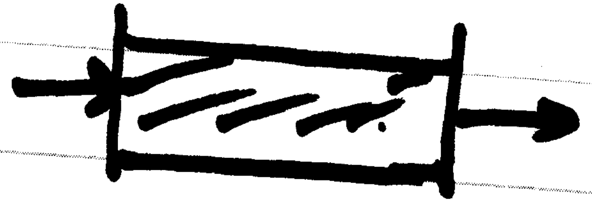
$$\boxed{D\frac{\partial^2 c}{\partial x^2} - u\frac{\partial c}{\partial x} - kc = 0.}$$

Tubular Axial Dispersion.

$$D\frac{d^2 c}{dx^2} - u\frac{dc}{dx} - kc = 0$$

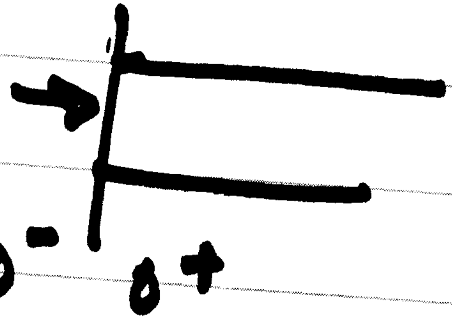


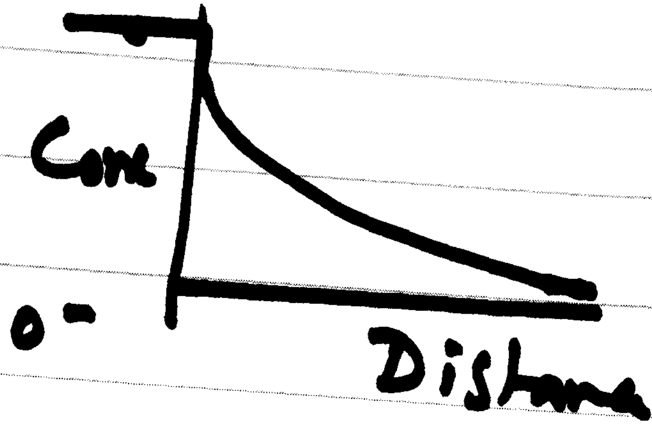
$z = x/L$



$x = 0$

$$(UC_A A)_{0^-} = \left[UC_A A - DA\frac{\partial c}{\partial x}\right]_{0^+}$$





$x = L$

$$\left[\cancel{UC_A A} - DA\frac{\partial c}{\partial x}\right]_{L^-} = \left[\cancel{UC_A A}\right]_{L^+}$$

$(\partial c/\partial x) = 0 \ @ \ x = L$

Tanks in Series — No of tanks

Dispersion Model $Pe\ \left(\frac{UL}{D}\right)$.

Recycle Reactor

$$\frac{k\bar{t}}{(R+1)} = \ln\left[\frac{1 - (R/R+1)\,x_s}{1 - x_s}\right]\left[R+1\right]$$

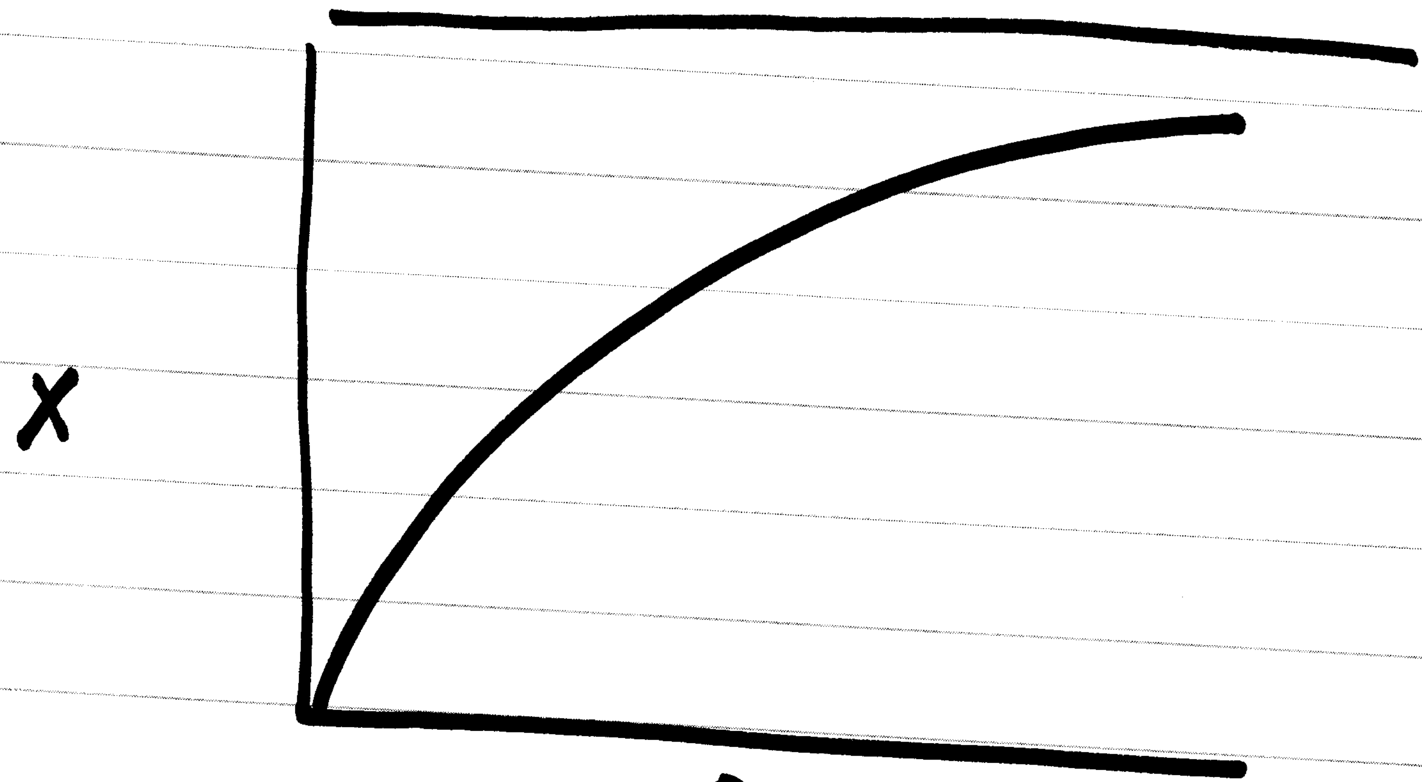
Dispersion

$$\psi = \frac{C_A(L)}{C_{A0}}$$

$$= \frac{4q \exp\left(Pe/2\right)}{(1+q)^2 \exp\left(\frac{Pe\, q}{2}\right) - (1-q)^2 \exp\left(-\frac{Pe\, q}{2}\right)}$$

$$\rightarrow q^2 = \left(1 + \frac{4k\bar{t}}{Pe}\right)$$

$$\rightarrow Pe = \frac{UL}{D}$$

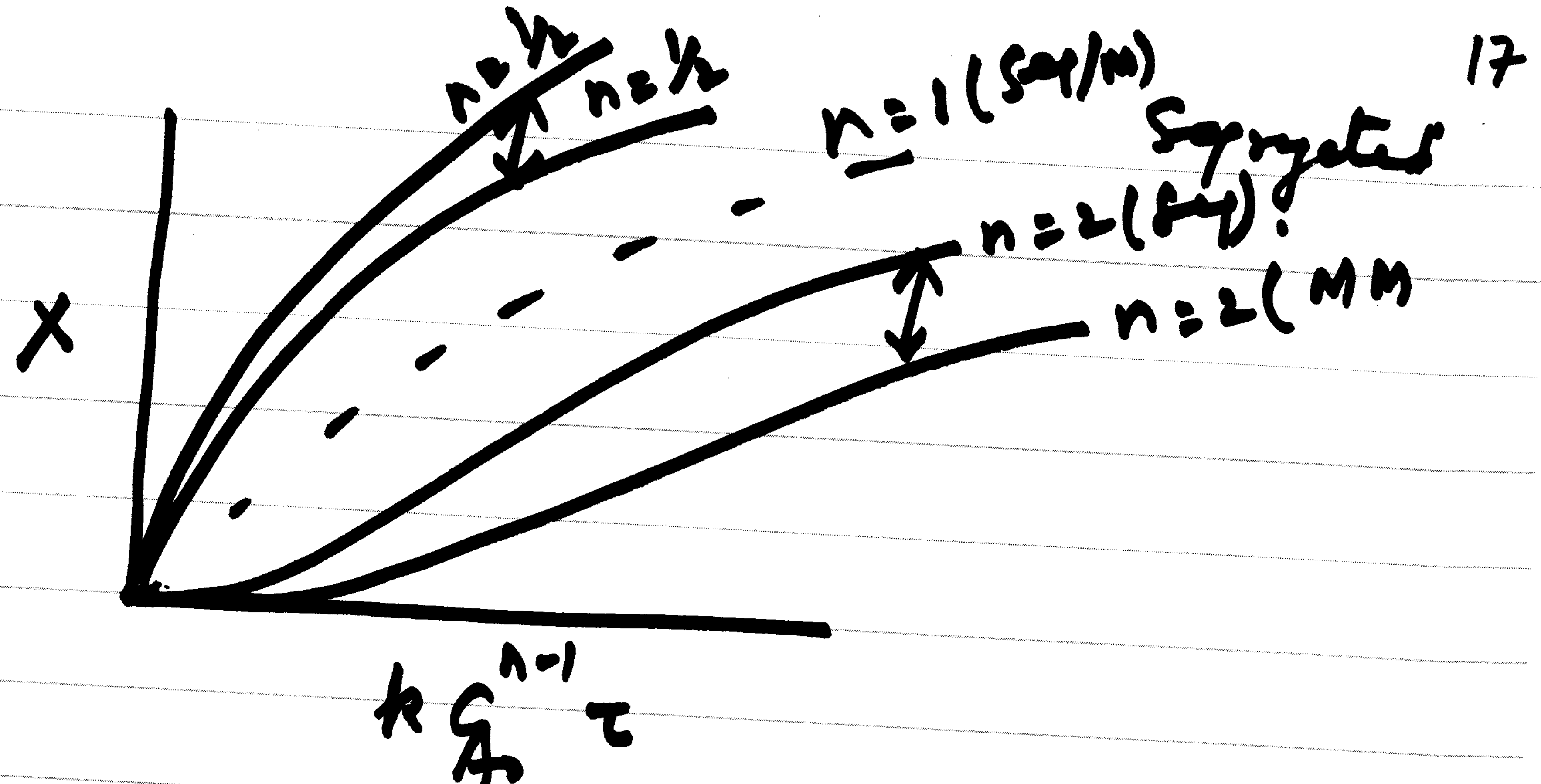


Pe → Plug Flow

← (R) N →

← R → Plug flow.

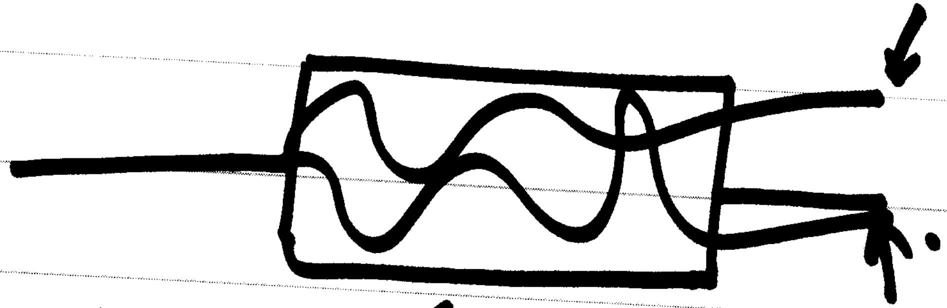
N → Plug flow



Completely Mixed mx.

$$C_{A0} - C_A - k C_{A0}^n \tau = 0$$

Completely Segregated Model.



$$C_A = C_{A0}\, e^{-kt} \quad \text{— (1)}$$

$$C_A^{1/2} = kt + 2C_{A0}^{0.5} \quad \text{— (2)}$$

$$\bar{C}_A = \int_0^{\infty} C_{A,\text{element}} \cdot E(t)\, dt$$

$C_{A,\text{element}}$:

$$\bar{C}_A = \int_0^{\infty}$$