Prof. Shankar
L-22 9/11/12
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Advanced Reaction Engineering

Stability Analysis of Exothermic CSTR

Friday.
9 Nov 12
1515-1615.

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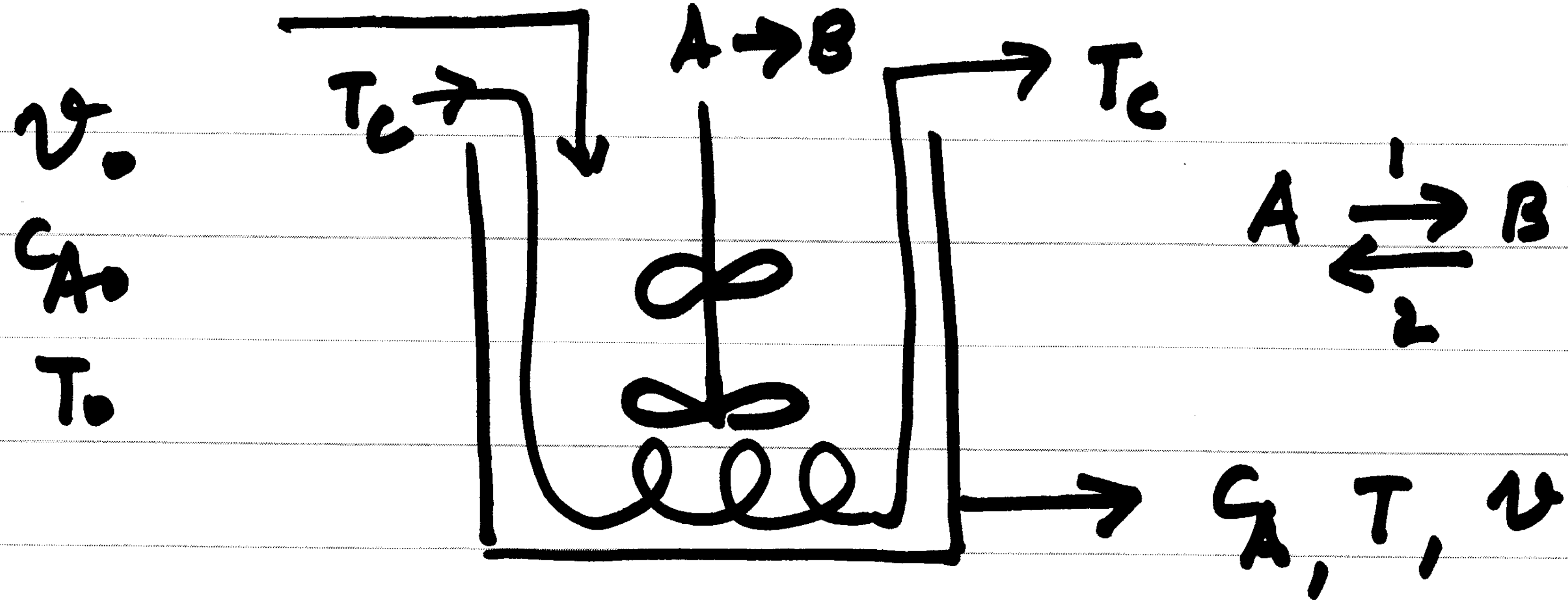


$$A \underset{2}{\overset{1}{\rightleftarrows}} B$$

Mole Balance

$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt}$$

$$X = \frac{C_{A0} - C_A}{C_A} \quad ; \quad r_A = (-1)\, r_1$$

if $v = v_0$

Mole Balance Eqn

$$v_0 C_{A0} - v_0 C_A - r_1 V = V \frac{d}{dt} C_A \quad [V: \text{constant}]$$

$$-\tau \frac{dx}{dt} = x - \frac{r_1 \tau}{C_{A0}}$$

$$\tau \frac{dx}{dt} = -x + \frac{r_1 \tau}{C_{A0}} \qquad -(1)$$

Energy Balance Eqn.

$$V \hat{C}_p \frac{dT}{dt} = v_0 \tilde{C}_p (T_0 - T) + r_1 V(-\Delta H_1^*) + Q - \cancel{W_s}$$

$$\frac{V \hat{C}_p}{v_0 \tilde{C}_p} \frac{dT}{dt} = (T_0 - T) + \frac{r_1 V(-\Delta H_1^*)}{v_0 \tilde{C}_p} + \frac{hA(T_c - T)}{v_0 \tilde{C}_p}$$

$$\tau \frac{dT}{dt} = (T_0 - T) + \beta (T_c - T) + r_1 \tau J_1$$

$$\frac{-\Delta H_1^*}{C_p} = J_1 \; ; \quad \beta = \frac{hA}{v_0 \tilde{C}_p}$$

$$\tau \frac{dT}{dt} = (1+\beta)(T_c^* - T) + \gamma_1 \tau J_1 \qquad (3)$$

where $(1+\beta)(T_c^*) = T_0 + \beta T_c$

At steady state

$$\tau \frac{dT_s}{dt} = 0 = (1+\beta)(T_c^* - T_s) + r_{1s}\,\tau J_1 \quad -(5)$$

$$\tau \frac{dx_s}{dt} = 0 \quad -X_s + \frac{r_{1s}\,\tau}{C_{A0}} \quad (4)$$

(1-4) and (3-5)

$$\tau \frac{d}{dt}(X - X_s) = -(X - X_s) + (q_1 - q_{1s})\frac{\tau}{c_{A0}}. \quad (6)$$

$$\tau \frac{d}{dt}(T - T_s) = -(1+\beta)(T - T_s) + (q_1 - q_{1s})\,\tau J_1 \quad -(7).$$

Expanding $r_1(X, T)$ by Taylor's Series.

$$r(X, T) = r(X_s, T_s) + (X - X_s)\left(\frac{\partial r}{\partial X}\right)_s$$

$$+ (T - T_s)\left(\frac{\partial r}{\partial T}\right)_s + \frac{(X - X_s)^2}{2!}\frac{\partial^2 r}{\partial X^2}$$

$$+ \frac{(X - X_s)}{1!}\frac{(T - T_s)}{1!}\frac{\partial^2 r}{\partial X \partial T} + \frac{(T - T_s)^2}{2!}\frac{\partial^2 r}{\partial T^2}$$

$$r(X, T) = r(X_s T_s) + (X - X_s)\left(\frac{\partial r}{\partial X}\right)_s + (T - T_s)\left(\frac{\partial r}{\partial T}\right)_s$$

$$r_1(X, T) - r_1(X_s T_s) = (X - X_s)\left(\frac{\partial r}{\partial X}\right)_s + (T - T_s)\left(\frac{\partial r}{\partial T}\right)_s$$

Substitute for $(r_1 - r_{1s})$

$$\tau \frac{d(X-X_s)}{dT} = -(X-X_s) + \left[(X-X_s)\left(\frac{\partial r}{\partial X}\right)_s + (T-T_s)\left(\frac{\partial r}{\partial T}\right)_s\right]\frac{\tau}{C_{A0}}$$

$$X - X_s = x$$

$$\tau \frac{dx}{dt} = -x + \left[x\left(\frac{\partial r}{\partial X}\right)_s + y\left(\frac{\partial r}{\partial T}\right)_s\right]\left(\frac{\tau}{C_{A0}}\right)$$

$$T - T_s = y$$

$$\tau \frac{d}{dt} y = -(1+\beta) y + \left[x \left(\frac{\partial r}{\partial x} \right)_s + y \left(\frac{\partial r}{\partial T} \right)_s \right] \tau J_1$$

$$L = 1 - \frac{\tau}{G_{A0}} \left(\frac{\partial r}{\partial x}\right)_s$$

$$M = 1 + \beta$$

$$N = J_1 \tau \left(\frac{\partial r}{\partial T}\right)_s$$

$$\tau \frac{dx}{dt} = -Lx + Ny / J_1 G_{A0} \qquad (9)$$

$$\tau \frac{dy}{dt} = -y(M-N) + J_1 G_{A0} (1-L)x \qquad (10)$$

$$\begin{bmatrix} -L & N/J_1 G_{A0} \\ \\ J_1 G_{A0}(1-L) & -(M-N) \end{bmatrix}$$

$$|A - \lambda I| = 0 \qquad \mathrm{Det}\begin{vmatrix} -L-\lambda & N/J_1 G_{A0} \\ \\ J_1 G_{A0}(1-L) & -(M-N)-\lambda \end{vmatrix} = 0$$

$$\tau \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -L & N/J_1 G_{m0} \\ J_1 G_m (1-L) & M-N \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Linear Stability Theory states that if coefficient matrix has -ve eigen values then the disturbance as measured by $\dot{x}$ and $\dot{y}$ will slowly decrease and ultimately become zero.

Characteristic Eqn

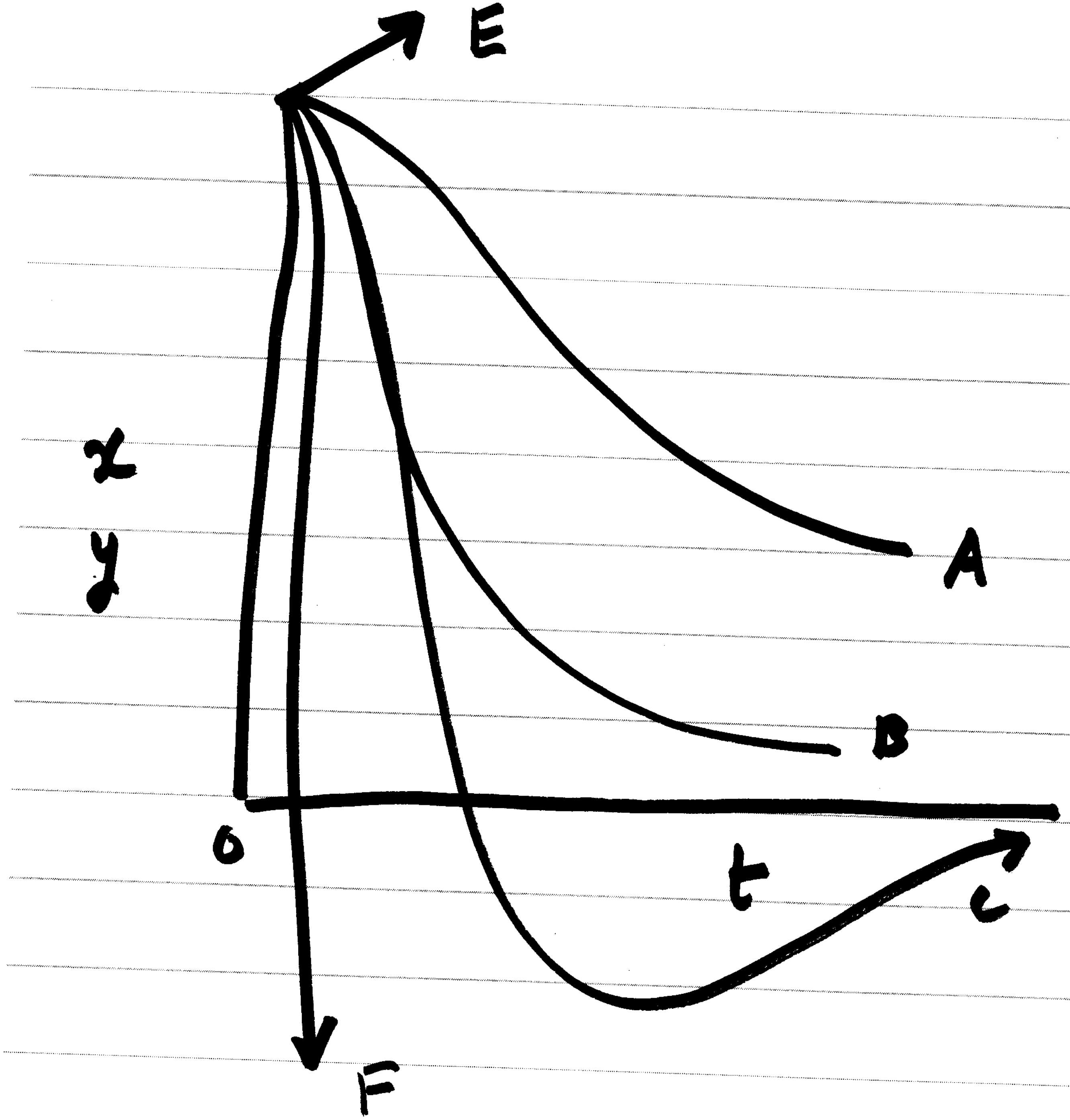
$$(-L-\lambda)(-M+N-\lambda) - \frac{N}{J_1 G_{A0}} \frac{J_1 G_{A0}(1-L)}{\cancel{(1-L)}} = 0$$

$$-L\cancel{N} + LN + L\lambda - \lambda N + \lambda M + \lambda^2 - N + L\cancel{N} = 0$$

$$\lambda^2 + \lambda(L+M-N) + (LM-N) = 0$$

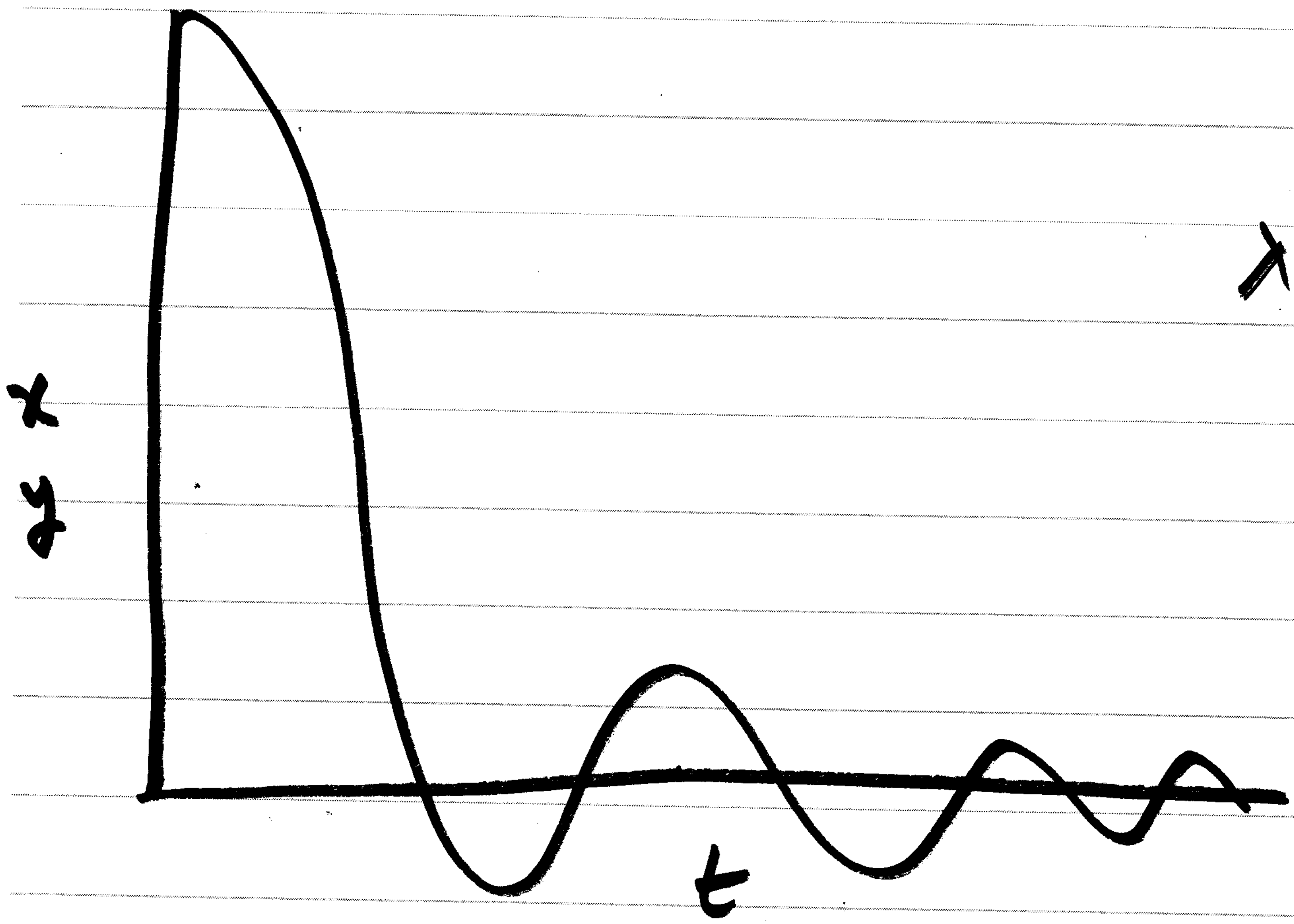
$$\lambda = \frac{-(L+M-N) \pm \sqrt{(L+M-N)^2 - 4(LM-N)}}{2}$$

$$\left.\begin{array}{l} L+M-N > 0 \\ LM-N > 0 \end{array}\right] \text{ for } \lambda \text{ -ve}$$

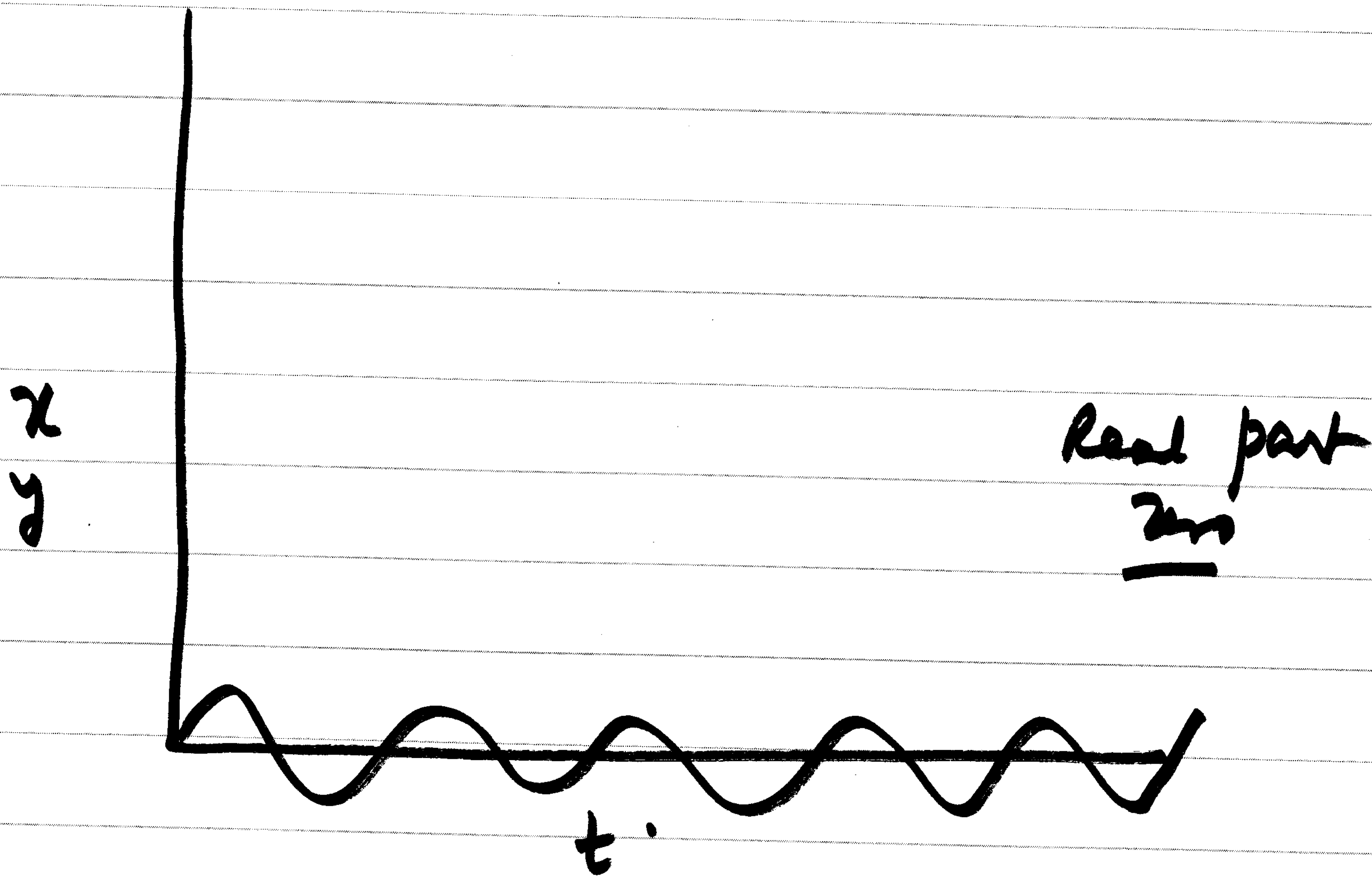


A, B, C
λ_1 and λ_2 are -ve

E and F
$\lambda_1 \lambda_2$ positive



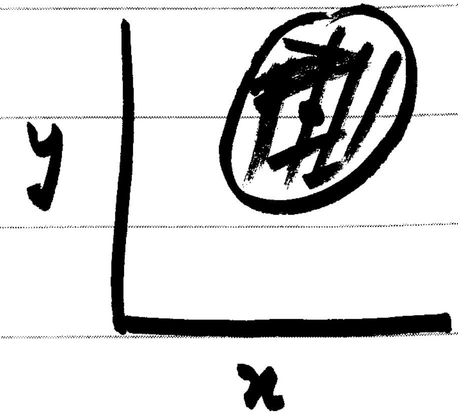
λ_1, λ_2 negative real


x
y
t
Real part

$$\tau \frac{dx}{dt} = -x + (g_1 - g_{1s}) \frac{\tau}{C_m}$$

$$\tau \frac{dy}{dt} = -(1+\beta) + (g_1 - g_{1s}) \underline{\tau J_1}$$

$$Q = \left[\frac{x}{X_s} , y/T_s \right] \begin{bmatrix} x/X_s \\ y/T_s \end{bmatrix}$$



$$\cancel{Q} = \boxed{\frac{x^2}{Q X_s^2} + \frac{y^2}{Q T_s^2} = 1}$$