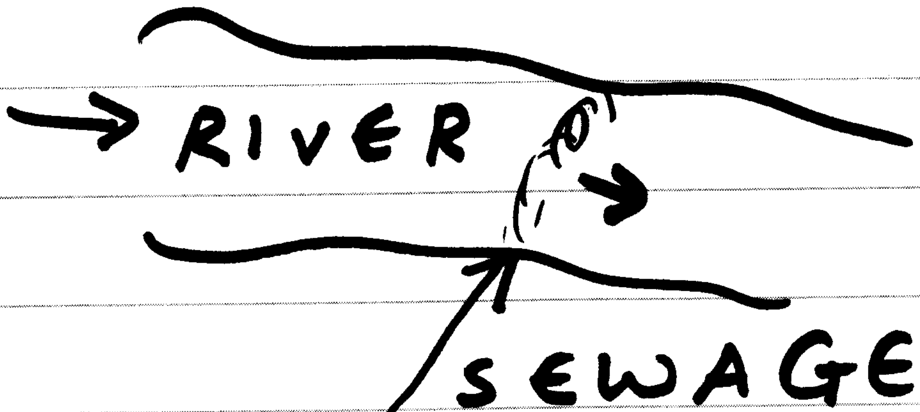
Lec - 10
(25-10-12

ENVIRONMENTAL.

REACTION ENGINEERING

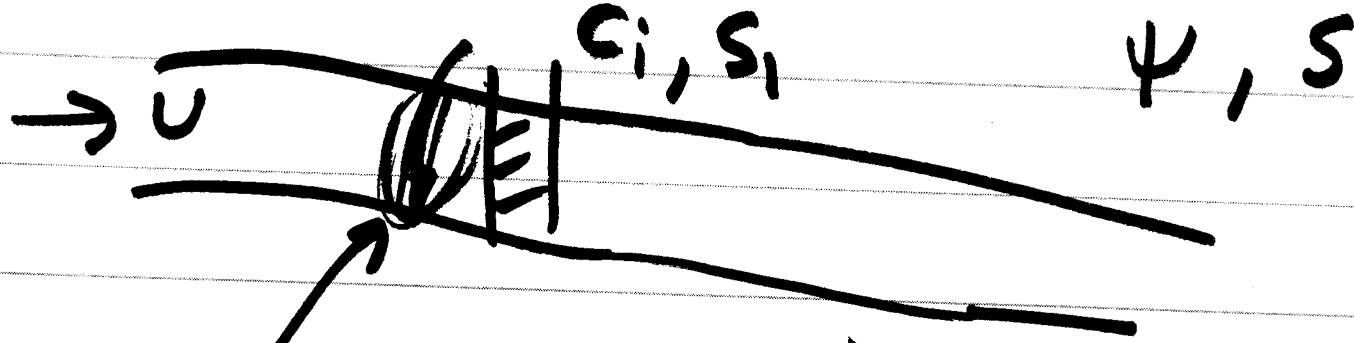
oxygen Sag Analysis



Steady State Analysis

$$I/P - O/P + Gen = \cancel{Acc}$$

$$1/p - 0/p + G = Acc$$



$$(U\cancel{A}c)_x - (U\cancel{A}c)_{x+\Delta x} - s_1 k_1 \cancel{A}\Delta x + k_3(c_s - c)\cancel{A}\Delta x$$

$$+ \overbrace{(\gamma_p - \gamma_{rp} - \gamma_s)}^{-\beta} \cancel{A}\Delta x = 0$$

$$-U\frac{dc}{dx} + k_3(c_s - c) - k_1 s_1 - \beta = 0$$

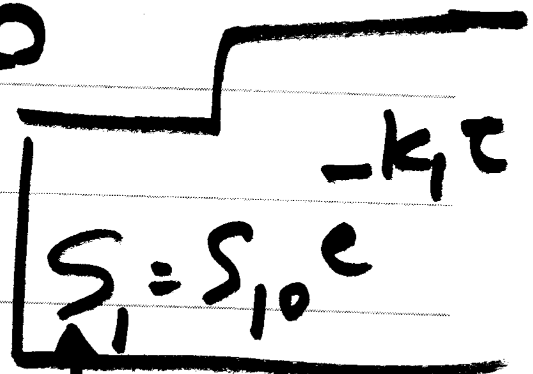
$$\beta = \gamma_{rp} + \gamma_s - \gamma_p$$

$$\tau = x/u$$

$$u\frac{dc}{dx} - k_3\,(c_s - c) + k_1 S_1 + \beta = 0$$

Let $\psi = c_s - c$ (oxygen deficit).

$$-u\frac{d\psi}{dx} - k_3\psi + k_1 S_1 + \beta = 0$$



$$S_1 = S_{10}\,e^{-k_1\tau}$$

Pollution

$$\boxed{u\frac{d\psi}{dx} + k_3\psi = k_1 S_1 + \beta \qquad \tau = 0 \quad \psi = \psi_i}$$

$$\frac{d\psi}{d\tau} + k_3 \psi = k_1 S_{10} e^{-k_1 \tau} + \beta$$

$\psi = C_s - C$

where $\tau = x/v$

Integrating Factor $e^{k_3 \tau}$.

$$\frac{d}{d\tau}\left(e^{k_3 \tau} \psi\right) = k_1 S_{10} e^{-k_1 \tau} \cdot e^{k_3 \tau} + \beta e^{k_3 \tau}$$

$$\boxed{\psi e^{k_3 \tau} = \frac{k_1 S_{10}}{(k_3 - k_1)} e^{-k_1 \tau} e^{k_3 \tau} + \frac{\beta}{k_3} e^{k_3 \tau} + \text{Constant}}$$

$\tau = 0 \quad \psi = \psi_i$

$$\psi_i = \frac{k_1 S_{10}}{k_3 - k_1} + \frac{\beta}{k_3} + \text{constant}$$

$$\text{Constant} = \psi_i - \frac{k_1 S_{10}}{k_3 - k_1} - \frac{\beta}{k_3}$$

Solution

$$\psi = \frac{k_1 S_{10} e^{-k_1 \tau}}{k_3 - k_1} + \frac{\beta}{k_3} + \left(\psi_i - \frac{k_1 S_{10}}{k_3 - k_1} - \frac{\beta}{k_3}\right) e^{-k_3 \tau}$$

$(c_s - c)$

$$\psi = \overbrace{\frac{k_1 S_{10}}{k_3-k_1}\left[e^{-k_1\tau} - e^{-k_3\tau}\right]}^{\text{BOD EXERTION}} + \underbrace{\frac{\beta}{k_3}\left(1-e^{-k_3\tau}\right)}_{\text{Net Load due to Photosyn Respiration Sediment respiration}}$$

$$+ \underbrace{\psi_i \, e^{-k_3\tau}}_{\text{Initial Deficit}}$$

Maximum Sag

$$\frac{d\psi}{d\tau} = 0$$

$$\frac{k_1 S_{10}}{(k_3 - k_1)} \left[-k_1 e^{-k_1 \tau} + k_3 e^{-k_3 \tau} \right]$$

$$+ \frac{\beta}{k_3} \left[k_3 e^{-k_3 \tau} \right]$$

$$- k_3 \psi_1 e^{-k_3 \tau} = 0$$

$$-k_1^2 s_{10} e^{-k_1 \tau} = k_3 \psi_i e^{-k_3 \tau} - \beta e^{-k_3 \tau} - \frac{k_1 k_3 s_{10} e^{-k_3 \tau}}{(k_3 - k_1)}$$

$$\frac{k_1^2 s_{10} e^{-k_1 \tau}}{(k_3 - k_1)} = e^{-k_3 \tau} \left[\beta + \frac{k_1 k_3 s_{10}}{k_3 - k_1} - k_3 \psi_i \right]$$

$$e^{(k_3 - k_1)\tau} = \frac{\beta + \frac{k_1 k_3 s_{10}}{k_3 - k_1} - k_3 \psi_i}{\left[k_1^2 s_{10} / (k_3 - k_1) \right]}$$

$$\tau_{max} = \frac{1}{(K_3 - K_1)} \ln\left[\frac{\beta(K_3 - k_1) + k_1 k_3 S_{10} - (k_3 - k_1) k_3 \psi_1}{k_1^2 S_{10}}\right]$$

Substituting τ_{max} in final soln

we get

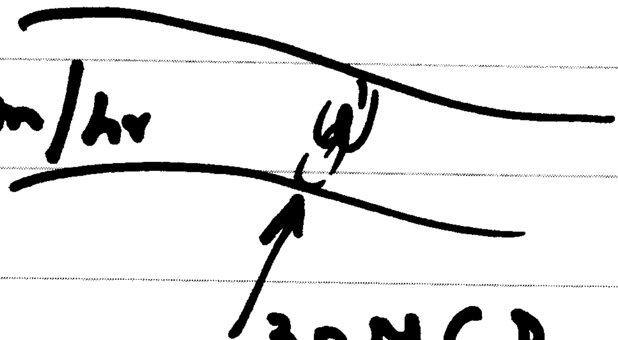
$$\psi_{max} = \frac{K_1 S_{10}}{K_3} e^{-k_1 \tau_{max}}$$

Q1: 5 DAY BOD: 1 mg/L

$10\ m^3/s$; 3 km/hr

DO: 90%

$C_s = 9.9\ \frac{mg}{L}$ at 15 C

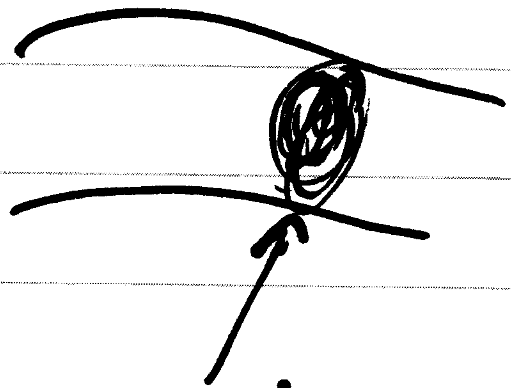


30 MGD

BOD 200 mg/L

BOD Removal Rate Constant: $K_1 = \frac{0.3}{d}$.

Reaeration Rate Constant: $0.7/d$
K_3



m^3/s

DO in Mixture: $\dfrac{(9)(10)(3600)\times 24}{10\times 3600\times 24+30\times 3800}$

$\boxed{1\,\text{Gal} = 3.8\,\text{Lit}}$

$= 7.95\ mg/L$

$(BOD)_5$ of Mixture =

$$\frac{30\times 3800\times 200+(1)(10)(3600)24}{10\times 3600\times 24+30\times 3800} = 27.7\ \frac{mg}{L}$$

$24.1\ mg/L$

$(BOD)_L$ = ultimate

$$\cancel{\frac{(S_1)_5}{(S_1)_L} = e^{-k_1 \tau}}$$

$(BOD)_5$ = 5 Day.

If rate constant is k_1 /d

then

$$\boxed{\frac{(BOD)_3}{(BOD)_L} = 1 - e^{-k_1 t}}$$

5 days

3 days

$$(BOD)_L = \frac{(BOD)_5}{(1 - e^{-k_1 \tau})}$$

$$(BOD)_L = \frac{(BOD)_5}{1-e^{-k\tau}}$$

$\psi_i = 9-7.95 = 1.05$

$= \cancel{21}\ mg/L$

$\beta = 0$; $K_3 = 0.7/d$

$K_1 = 0.3/d$

$$= \frac{\cancel{27.7}\ 24.1}{1-e^{-0.3\times 5}} = \cancel{35.6}\ 31.\ \frac{mg}{L}$$

$S_{10} = \cancel{35.6}\ 31\ mg/L$

$$\tau_{max} = \frac{1}{(K_3-K_1)}\left[\frac{\beta(K_3-K_1)+K_1K_3S_{10}-(K_3-K_1)K_2\psi_i}{K_1^2 S_{10}}\right]$$

Putting numbers: $\tau_{max} = \cancel{1.885}\ 2$ days

**

Distance at which max sag Occurs =

$$(3)\ \mathrm{km/hr}\ (1.\cancel{885}^{2}\ d) \times 24\ \frac{hr}{d}$$

$$= \cancel{135}^{144}\ \mathrm{km.}$$

$$\underline{\psi}_{max} = \frac{k_1 \zeta_0}{k_2} e^{-k_1 t_m} = \frac{(0.3)(35.6)}{0.7} e^{-.3 \times 1.885}$$

$$= \cancel{8.66}^{7.29}\ \mathrm{mg/L}$$

$$\psi_{max} = C_s - C \Rightarrow C = 9 - \cancel{8.66}^{7.29} = \cancel{0.34\ \frac{mg}{L}}\ 1.71\ \mathrm{mg/L}$$

$$(S)_{critical}: (35.6)\left[\exp(-0.3 \times \cancel{1.885}^{2}\right] = \cancel{25\ \frac{mg}{L}}\ 17\ \mathrm{mg/L}$$

General Treatment

$$(vAc)_x - (vAc)_{x+\Delta x} - k_1 S_1 A \Delta x - k_2 S_2 A \Delta x$$
$$+ k_3 (c_s - c) A \Delta x + (r_p - r_{rp} - r_s) A \Delta x$$
$$= 0$$

$$-v \frac{dc}{dx} + k_3 (c_s - c) - k_1 S_1 - k_2 S_2 - \beta$$

$$\psi = c_s - c$$

$$v \frac{d\psi}{dx} + k_3 (\psi) = k_1 S_1 + k_2 S_2 + \beta$$

Solution

$$\psi = \frac{k_1 S_{10} e^{-k_1 \tau}}{k_3 - k_1} + \frac{k_2 S_{20} e^{-k_2 \tau}}{k_3 - k_2} + \frac{\beta}{k_3} + \left[\psi_i - \frac{k_1 S_{10}}{k_3 - k_1} - \frac{\beta}{k_3} - \frac{k_2 S_{20}}{k_3 - k_2} \right] e^{-k_3 \tau}$$

$$\tau_{max} = \frac{1}{(k_3 - k_1)}\left[\frac{\beta(k_3 - k_1) + k_1 k_3 S_{10} - (k_3 - k_1) k_3 \psi_1}{k_1^2 S_{10}}\right]$$

$$+ \frac{1}{(k_3 - k_2)}\left[\frac{\beta(k_3 - k_2) + k_2 k_3 S_{20} - (k_3 - k_2) k_3 \psi_1}{k_2^2 S_{20}}\right]$$

$$\psi_{max} = \frac{k_1 S_{10}}{k_3} e^{-k_1 \tau_m} + \frac{k_2 S_{20}}{k_3} e^{-k_2 \tau_m}$$

WARMER CLIMES

$k_1 = 0.5/d.$

$C_s = 6\ mg/L$

$k_3 = 0.7/d.$

$S_{10} = 35.6\ mg/L$

$\cancel{E_i} = \cancel{7}.$

$\cancel{+ (10)(3600)(24)}$

$$\text{DO in mixture} = \frac{(5.4)\,10 \times 3600 \times 24}{10 \times 3600 \times 24 + 30 \times 3800}$$

$= 4.8\ mg/L$

$$\tau = \frac{1}{k_3 - k_1} \ln \left[\frac{\beta(k_3 - k_1) + k_1 k_3 S_{10} - (k_3 - k_1) k_3 \psi_1}{k_1^2 S_{10}} \right]$$

$$\frac{1}{(0.7 - 0.5)} \ln \left[\frac{0 + (0.5)(0.7)\,26.2 - (0.7 - 0.5)\,0.7\,(0.6)}{(0.5)^2 (26.2)} \right] = 1.6 \text{ days.}$$

$$\psi_i = 5.4 - 4.8 = 0.6 \text{ ms/L}$$

Distance $(3)(1.6)(24) = 115 \text{ KM} \simeq 1.6$ days

$$\psi_{max} = \frac{k_1}{k_3} S_{10} e^{-k_1 \tau_m} = \frac{(0.5)(26.2)}{0.7} e^{-0.5 \times 1.6} = 8.42 \text{ mg/L}$$

$\psi_m = \overset{1.2}{\cancel{3.5}}$ mg/L

$C = 5.4 - \underset{1.2}{\cancel{3.5}} = \overset{4.2}{\cancel{2.9}}$ mg/L

$S_{critical} = \cancel{S_{10}\,\exp\left[1 - 0.5 \times 3.5\right]}$

$S_{10}\,\exp\left[-0.5 \times 3.5\right]$

$= S_{10}\, e^{-1.75}$

$= 35.6\, e^{-1.75} = 6$ mg/L

General Case

$$\psi = \frac{k_1 S_{10} e^{-k_1 \tau}}{k_3 - k_1} + \frac{k_2 S_{20} e^{-k_2 \tau}}{k_3 - k_2}$$

$$+ \frac{\beta}{k_3} + \left[\psi_i - \frac{k_1 S_{10}}{k_3 - k_1} - \frac{\beta}{k_3} \right] e^{-k_3 \tau}$$

$$- \frac{k_2 S_{20}}{k_3 - k_2}$$

$$\tau_m = \frac{1}{(k_3 - k_1)}\left[\frac{\beta(k_3 - k_1) + k_1 k_3 S_{10} - (k_3 - k_1) k_3 \psi_1}{k_1^2 S_{10}}\right]$$

$$+ \frac{1}{(k_3 - k_2)}\left[\frac{\beta(k_3 - k_2) + k_2 k_3 S_{10} - (k_3 - k_2) k_3 \psi_i}{k_2^2 S_{20}}\right].$$