Introduction

Our understanding of mind and brain has come a long way over the millennia.

There was a time when people did not know that brain is the key organ responsible for our subjective experience. Greek philosopher Aristotle thought that the heart is the "seat of the soul" and the substrate for experience and selfhood. French philosopher Rene Descartes imagined that motor action is possible due to the action of "animal spirits" that rush through the nerves. The history of brain teaches us that though there was a considerable understanding of the brain's structure (anatomy) even half a millennium ago, when it came to brain function, all sorts of fantasies were paraded as knowledge for many centuries.

Around the middle of 18th century, with developments in physics and physiology, a physics-based understanding of brain function began to take shape. It became clear that nerve signals are not "animal spirits" but electric signals not very different from the currents that flow in an electrical circuit. Developments in microscope revealed the peculiar hairy morphology of neurons, and presented a vision of brain as a network of neurons. Progress in neurochemistry, neuropharmacology and neurophysiology unraveled how neurons converse among themselves using chemical signals. Breakthroughs in technology is offering us vast treasures of neuroscientific data spanning many scales from single molecules, to neurons, to networks to whole brain and behavior.

Understanding the brain as an organ is very different from understanding other organs of the body. The brain, first and foremost, is an information processing machine. Like any other organ in the body, the brain too is a mass of cells. But unlike any other organ in the body, brain is a network of cells, a network that clearly distinguishes itself in its sheer size, complexity and lability. An adult brain has about 100 billion neurons, each with about 1,000-10,000 connections. We thus have a staggering figure of about 10^{14}-10^{15} connections in the brain. Therefore, the brain network is perhaps more complex than the entire mobile network of the world, even if we assume that every one of the 7 billion denizens of the planet possess a mobile phone. Furthermore, this extremely complex cerebral network is quite labile, with neurons making and breaking connections at a time-scale that can be as short as a few tens of seconds.

Understanding brain function therefore means the ability to explain brain function in terms of the operations of this complex neural network. Thus the question "how does the brain see?" must be rephrased as "how do networks of neurons in the visual processing areas of the brain transduce the optical image that falls on the retina and process its many properties like form, color, motion etc?" And the answers to these questions are best clothed in the language of mathematics, which is the primary preoccupation of the science of computational neuroscience.

Models of brain can be classified broadly into two types: 1) biophysically realistic models and 2) abstract models. Biophysically realistic models are rooted in biophysics of neuron and brain, and aim to describe brain function in terms of electrical and chemical signaling of the brain. But these models can get extremely complex, computationally challenging, and often offer little insight into the essential information processing mechanisms that govern the function of a neural system. Therefore, modelers constant try to strike a balance between neurobiological realism with reliable and convenient abstraction. The present course also follows a course of development that starts from a biophysically realistic description, tending towards more abstract models by progressive and systematic simplification.

The chapters of this course material are organized as follows.

Chapter 1 – presents elements of neurobiology that form the necessary preparation for a student of computational neuroscience. The first part of the chapter describes the biology of a single neuron, and the basic neuronal signaling mechanisms, electrical and chemical. The latter part describes the overall organization of human nervous system.

Chapter 2 – this chapter presents the mathematical ingredients that will be used in the subsequent chapters.

Chapter 3 – presents the Hodgkin-Huxley model of action potential generation. The Hodgkin-Huxley model is one of the first significant mathematical models of neuron function. It describes how the dynamics of neuronal ion channels generates a sharp spike in neuronal membrane potential known as action potential.

Chapter 4 – Although action potential generation is a key event in neural signaling, there are other important mechanisms, like dendritic processing, axonal propagation and synaptic transmission, which are described in the present chapter.

Chapter 5 – this chapter begins the process of simplification of the more complex, biophysical neural models described so far. Starting from FitzHugh-Nagumo neuron model, coursing via a series of progressively simplified models, it ends in the definition of the McCulloch-Pitts neuron model.

Chapter 5 marks the end of Part I of this course material. Armed with the simplified models presented at the end of Chapter 5, we describe abstract models in the subsequent chapters.

Chapter 6 – this chapter describes network models based on McCulloch-Pitts neuron model. The first of these is the perceptron which has only two layers of McCulloch-Pitts neurons. The second is the multilayer perceptron which is a generalization of the perceptron to arbitrary number of layers.

Chapter 7 – the multilayer perceptron belongs to a class of networks known as feedforward networks. The present chapter describes a model known as Hopfield network which has extensive feedback loops. The model serves as an associative memory. Ideas related associative memory are applied to describe the function of hippocampus towards the end of the chapter.

Chapter 8 – The previous chapter presented the idea of Hebbian learning, a learning mechanism used to store patterns in Hopfield network. The present chapter starts with Hebbian learning and shows how weights trained by Hebbian learning can extract principal components from input data. The chapter ends with Linsker's model which uses Hebbian learning to explain evolution of response properties of neurons in the visual system.

Chapter 9 – This chapter introduces a variation of Hebbian learning known as competitive learning. The link between competitive learning and data clustering is explained. Next the chapter describes the Self-organizing Map (SOM), a neural network model based on competitive learning. The SOM model is applied to explain neural maps found in real brains.