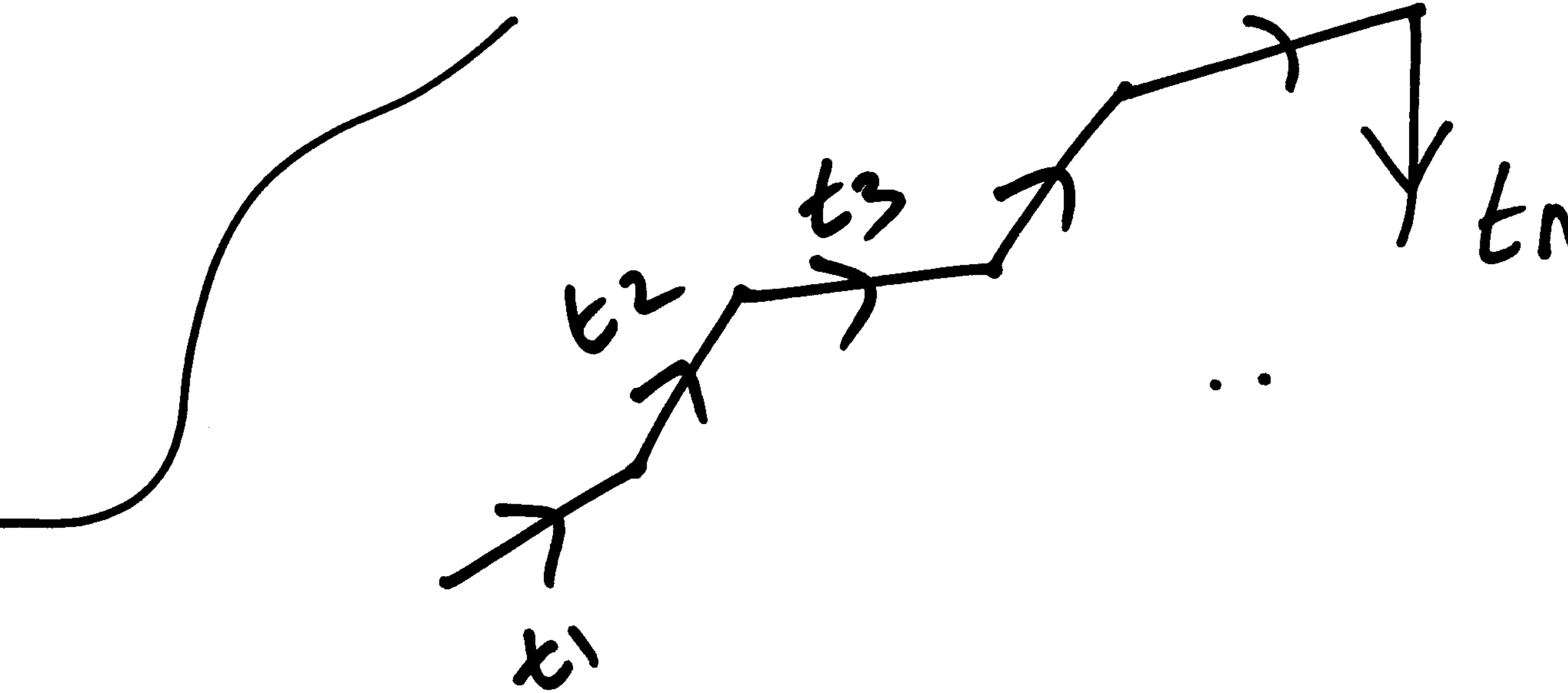
Worm-like chain model (WLC)



$$E_B = A\left(\sum_{i=1}^{N}\left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)\right)$$

$$A(1-\hat{t}_1 \cdot \hat{t}_2) + A(1-\hat{t}_2 \cdot \hat{t}_3)$$

$$+ \dots A(1-\hat{t}_{N-1} \cdot \hat{t}_N)$$

$$|\hat{t}| = 1$$

$$Z = \int d\vec{t}_1 \int dt_2 \int dt_3 \cdots \int dt_N \, e^{-\beta E_B}$$

$$\beta = \frac{1}{k_B T}$$

$$\int dt_1 = \int_0^{2\pi} \underline{d\phi} \int_0^{\pi} \underline{d\theta \sin\theta}\, e^{-\beta E_B}$$

$$\boxed{1 \left(d\phi \sin\theta\, d\theta \right)}$$

$$\int dt_1 = \int_0^{2\pi} d\phi_1 \int_{-1}^{+1} d\cos\theta_1$$

$$\int dt_2 = \int_0^{2\pi} d\phi_2 \int_{-1}^{1} d\cos\theta_2$$

$$\int dt_1 \int dt_2 \cdots\cdots \int dt_N \, e^{-\beta E}$$

$$\int d\phi_1 \int_{-1}^{1} d(\cos\theta_1) \int d\phi_2 \int d(\cos\theta_2) \cdots\cdots$$

$$\int d\phi_N \int d\cos\theta_N \, e^{-\beta E_B}$$

$$E_B = A \sum_i \left(1 - \hat{t}_i \cdot \hat{t}_{i+1}\right)$$
$$= A\left(\sum_i \left(1 - \cos\theta_i\right)\right)$$

$$\int_0^{2\pi} d\phi = 2\pi$$

$$e^{-\beta E_B} = e^{-\beta \sum_i \underbrace{A(1-\cos\theta_i)}_{}}$$

$$\overset{\alpha_i}{}$$

$$\cancel{= e^{-\beta E_1}} = e^{-\beta \sum_i \alpha_i}$$

$$= e^{-\beta(\alpha_1 + \alpha_2 + \alpha_3 \cdots)}$$

$$e^{-\beta\alpha_1} \cdot e^{-\beta\alpha_2} \cdots\cdots e^{-\beta\alpha_N}$$

$$Z = \left[\underbrace{\int_0^{2\pi} d\phi}_{2\pi} \int_{-1}^{1} d\cos\theta \; e^{-\beta A(1-\cos\theta)} \right]^N$$

$$(2\pi)^N e^{-\beta A N} \left[\frac{e^{+\beta A x}}{\beta A} \right]_{-1}^{1}$$

$$\uparrow$$

$$(2\pi)^N e^{-\beta A N} \int_{-1}^{1} dx \, e^{+\beta A x}$$

$$\uparrow$$

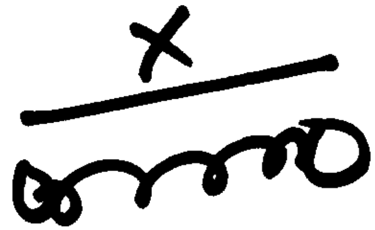
$$Z = (2\pi)^N e^{-\beta A N} \left[\frac{\sinh \beta A}{\beta A} \right]^N$$

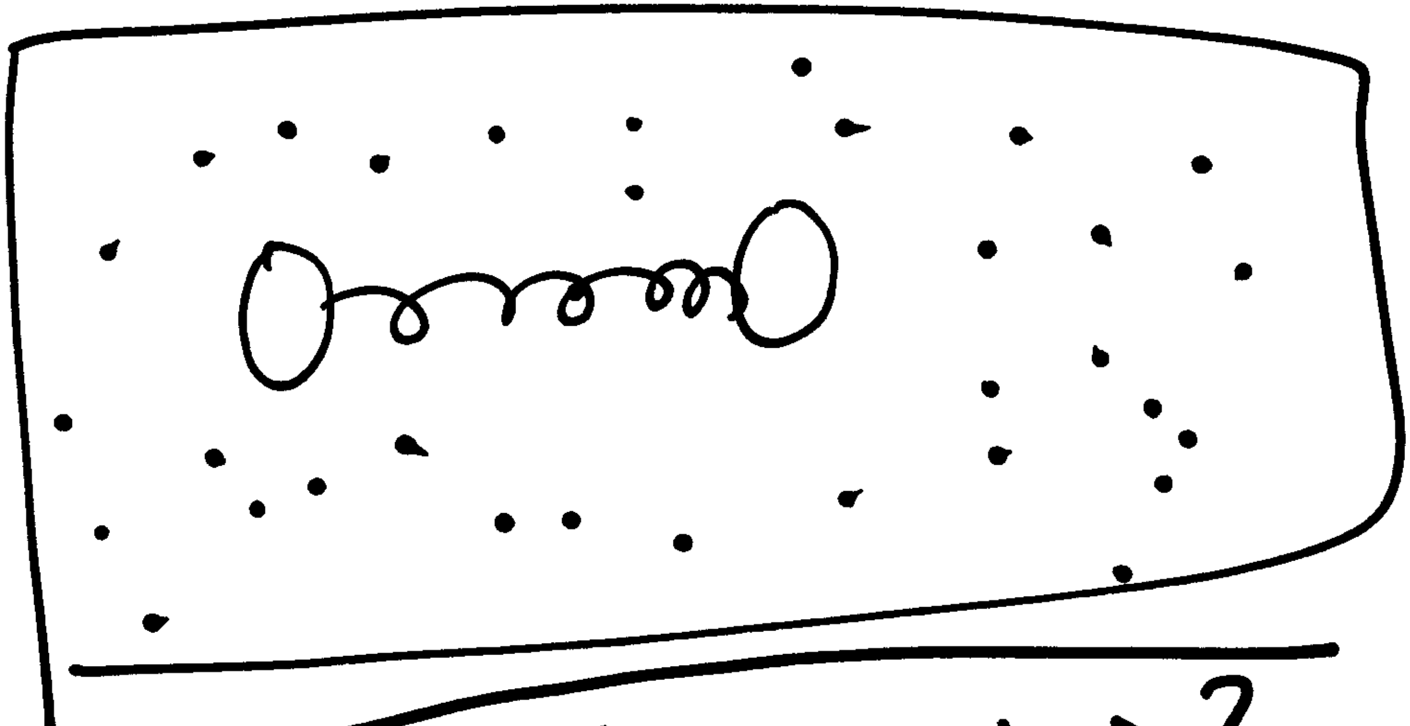
$$F = -k_B T \ln Z$$

N

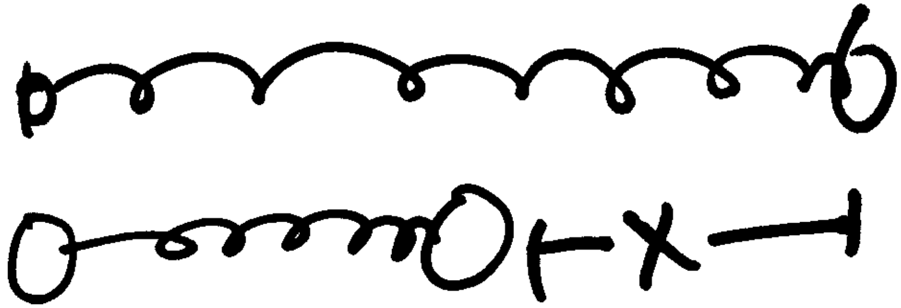
$\beta = \frac{1}{k_B T}$

A:





$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$



$$E = \frac{1}{2} k x^2$$

$$Z = \int_{-\infty}^{\infty} dx \, e^{-\frac{\beta}{2} k x^2}$$

$$\frac{\beta k}{2} = \tilde{k}$$

$$Z = \int_{-\infty}^{\infty} e^{-\beta \tilde{k} x^2} \, dx \qquad \sqrt{\frac{\pi}{\tilde{k}}}$$

$$P(x = x') = \frac{e^{-\beta \frac{k}{2} x'^2}}{\sqrt{\pi}} \sqrt{\frac{\beta k}{2}}$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \, P(x) \, dx$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 \frac{e^{-\frac{\beta k}{2} x^2}}{\sqrt{\frac{\pi}{\tilde{k}}}} \, dx$$

$$\sqrt{\frac{\tilde{k}}{\pi}}\int_{-\infty}^{\infty} x^2 e^{-\tilde{k}x^2}\,dx$$

$$\rangle = \frac{\sqrt{\tilde{k}}}{\sqrt{\pi}}\left(-\frac{\partial}{\partial \tilde{k}}\left[\underbrace{\int_{-\infty}^{\infty} e^{-\tilde{k}x^2}\,dx}_{\sqrt{\pi/\tilde{k}}}\right]\right)$$

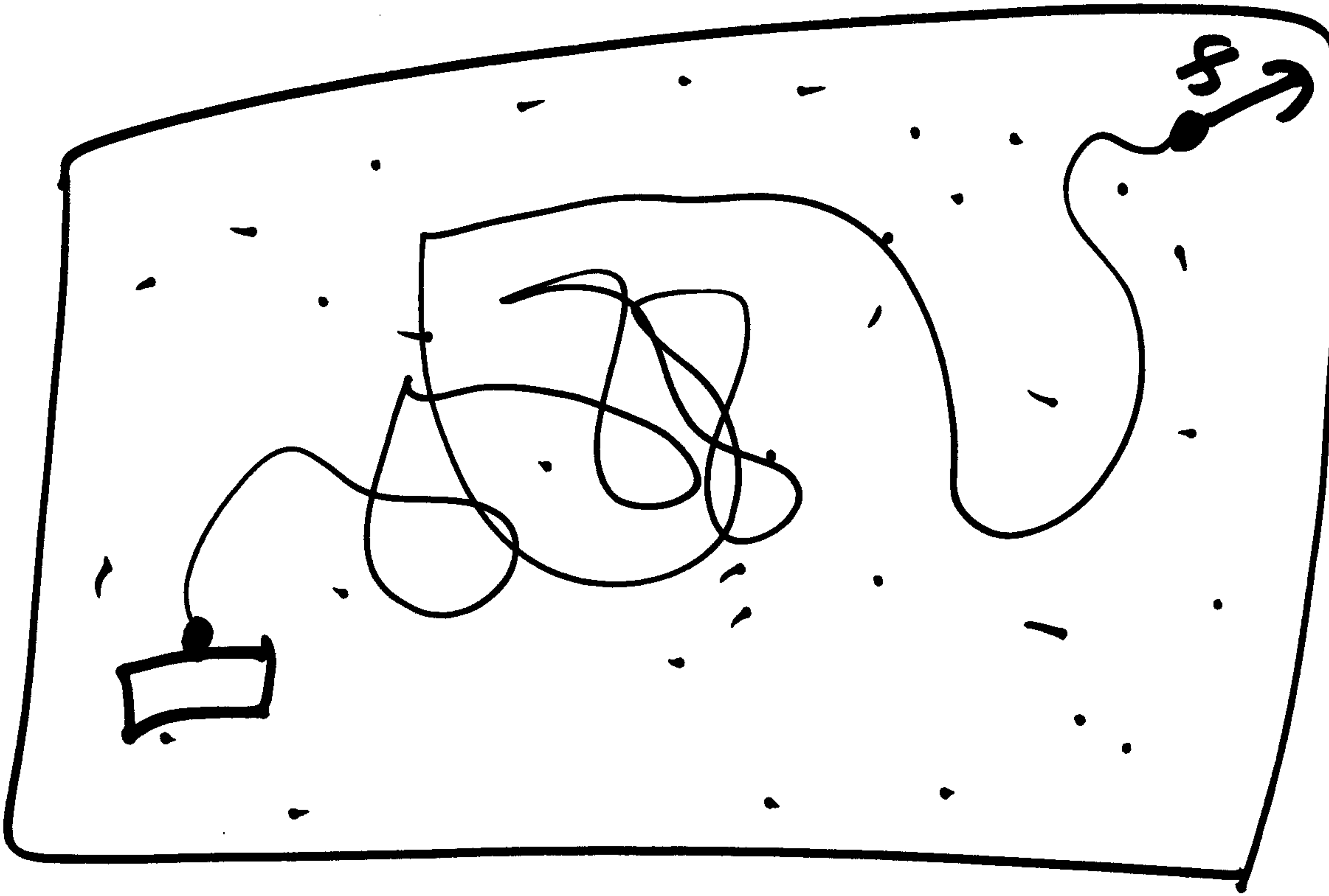
$$-\sqrt{\tilde{\kappa}}\,\frac{\partial}{\partial\tilde{\kappa}}\,\tilde{\kappa}^{-\frac{1}{2}} = \frac{\sqrt{\tilde{\kappa}}}{2}\,\tilde{\kappa}^{-3/2}$$

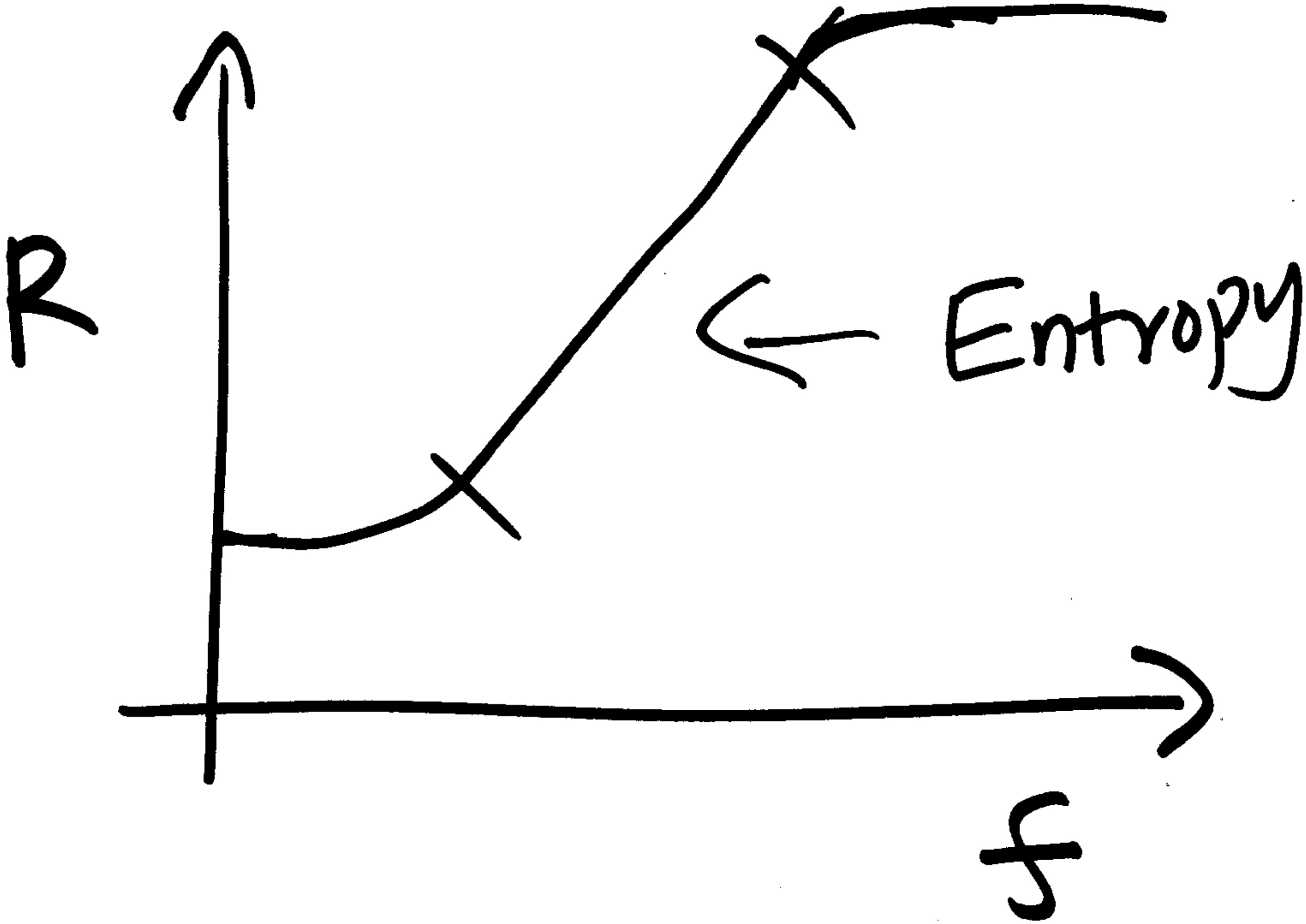
$$\frac{\sqrt{\tilde{\kappa}}}{\sqrt{\pi}}\;-\frac{\partial}{\partial\tilde{\kappa}}\sqrt{\pi/\tilde{\kappa}}$$

$$\langle x^2 \rangle = \frac{1}{2\tilde{K}} = \frac{2k_BT}{2K}$$

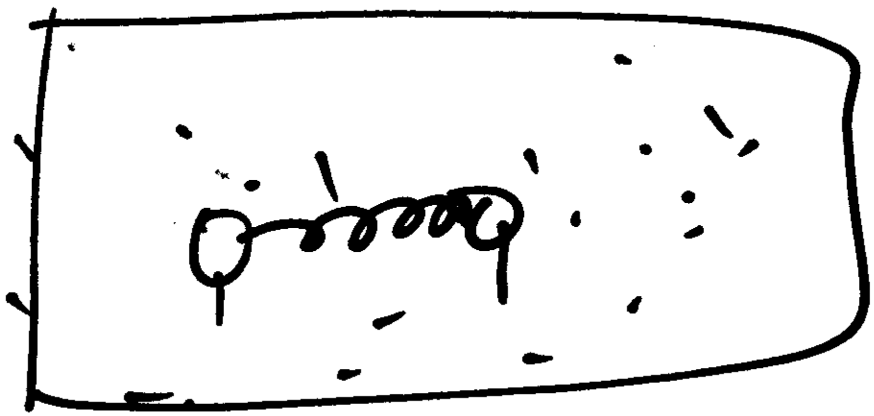
$$\boxed{\langle x^2 \rangle = \frac{k_BT}{K}} = \sigma$$

$$\langle x \rangle = 0$$

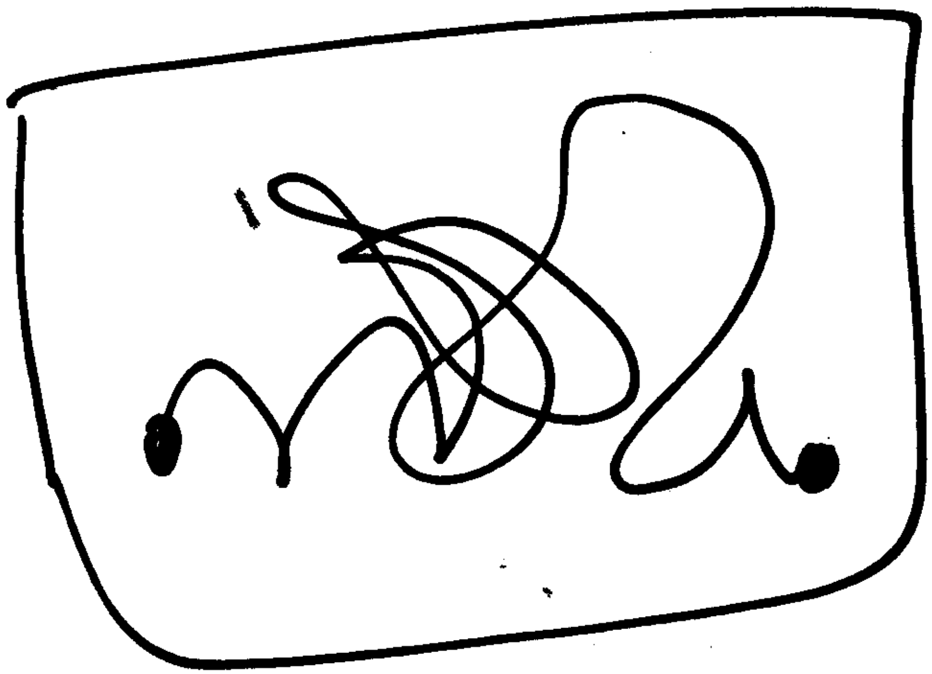




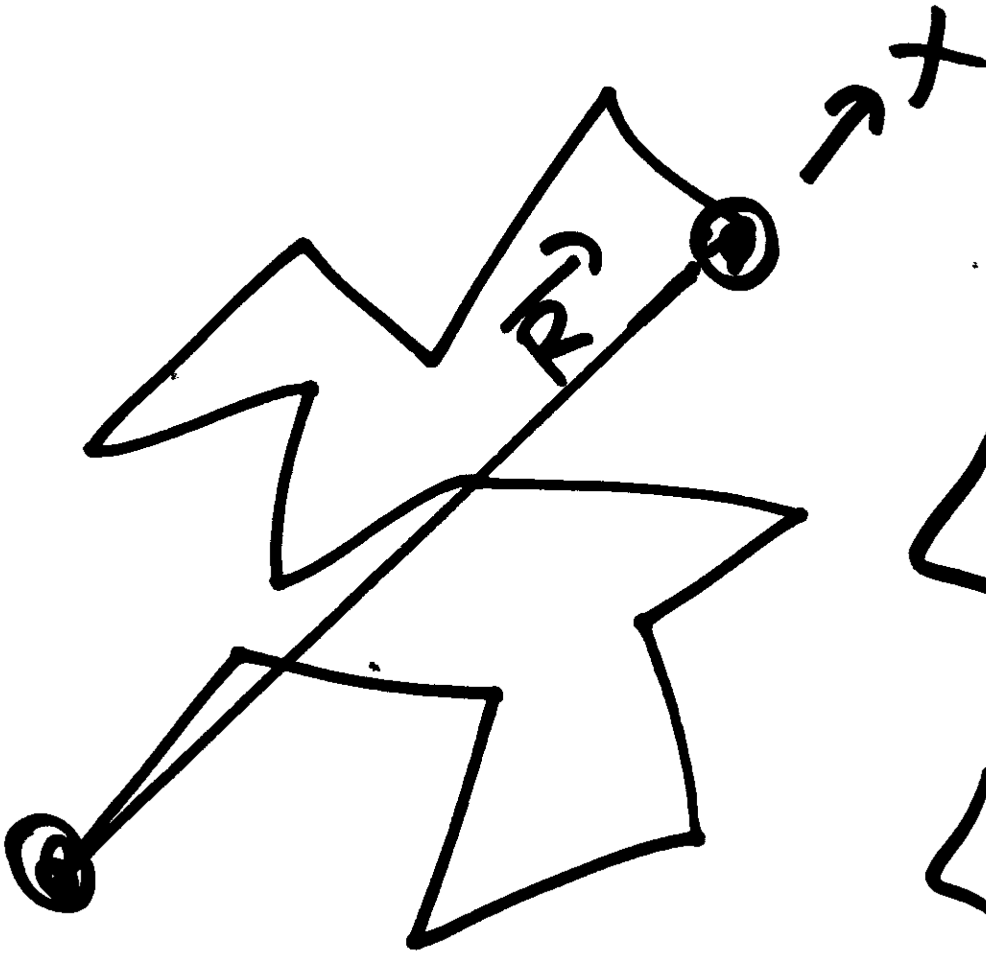
$$f = \underline{\underline{k}} R$$



$$\langle x^2 \rangle = \frac{k_B T}{K}$$



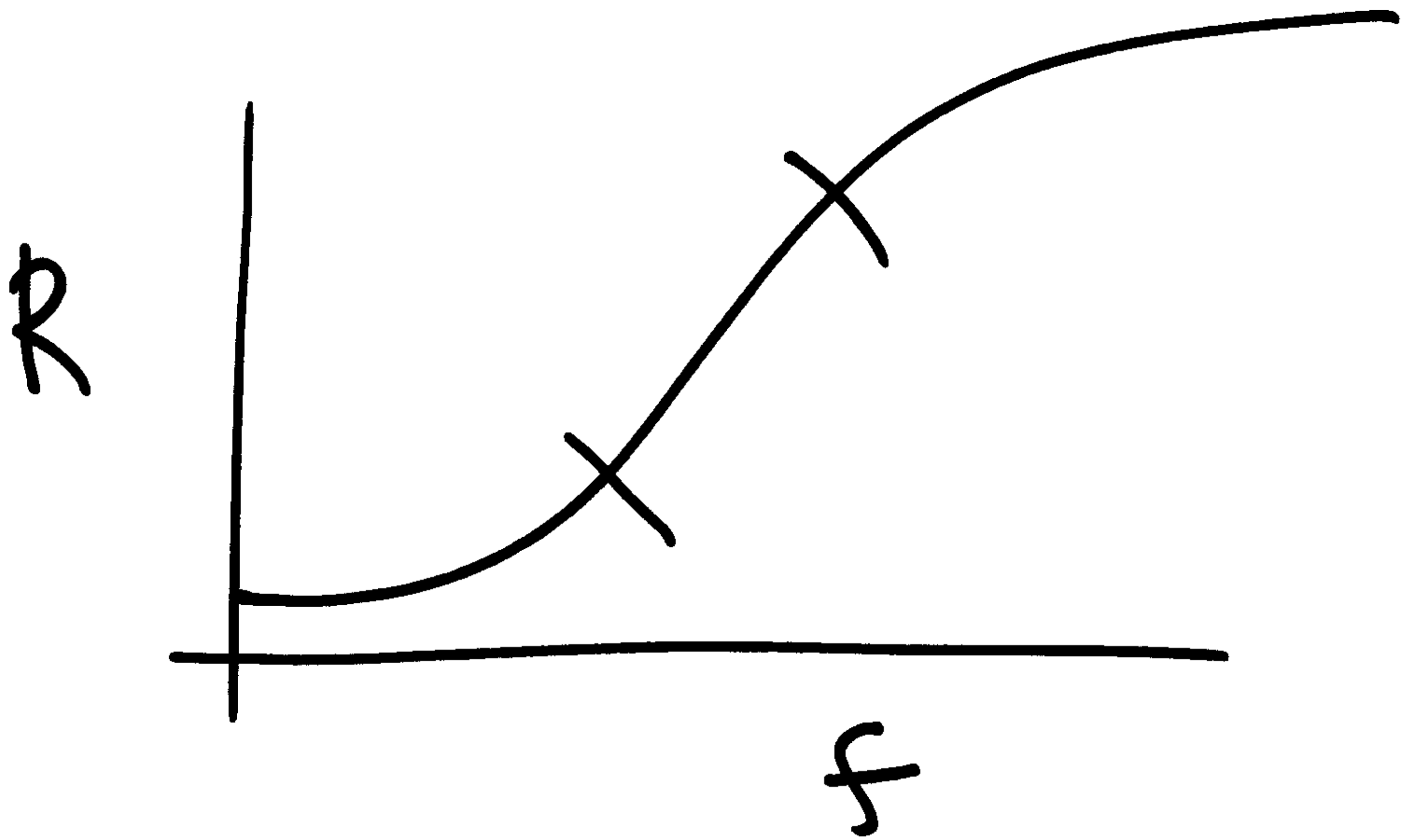
$$\langle R^2 \rangle =$$



$$\langle R_x^2 \rangle = \frac{b^2 N}{3}$$

$$\langle R^2 \rangle = \frac{k_B T}{K}$$

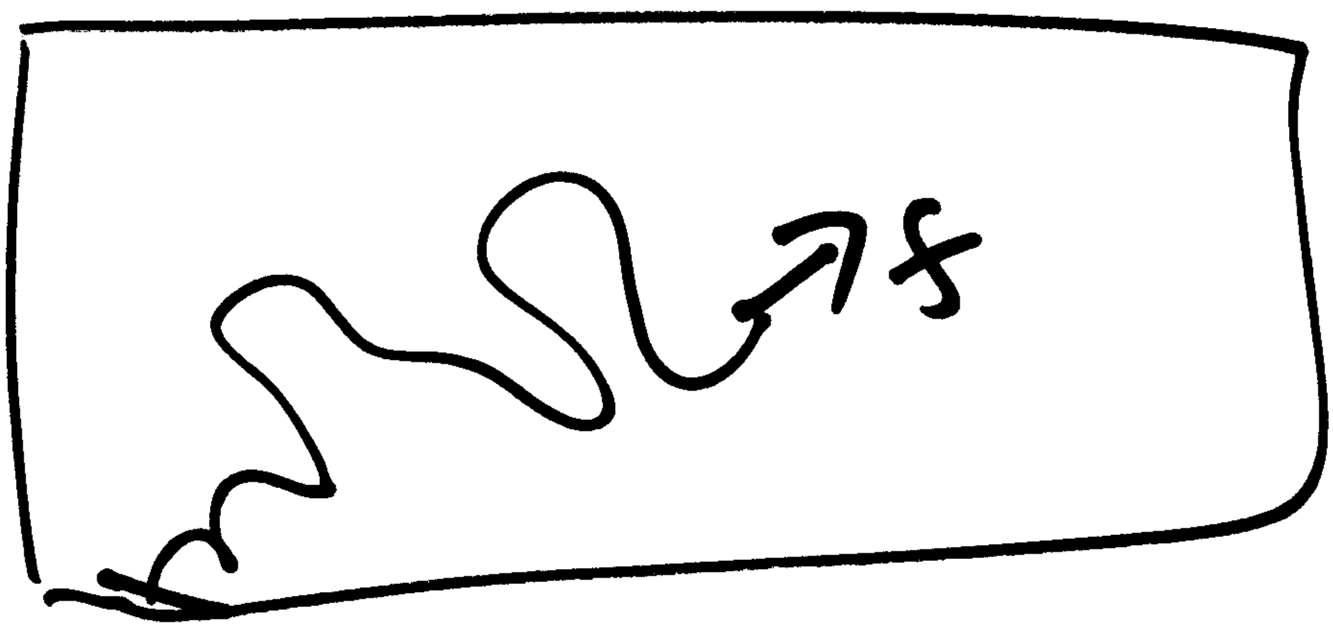
$$\boxed{K = \frac{3 k_B T}{b^2 N}}$$


R
f

$$G(P, T)$$

$$F(V, T)$$

$$G(f,T), \quad \frac{\partial G}{\partial f} = R$$



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.P	V
.T	S
f	d
M	N

$$G(f, T, N)$$

$$F(d, T, N)$$