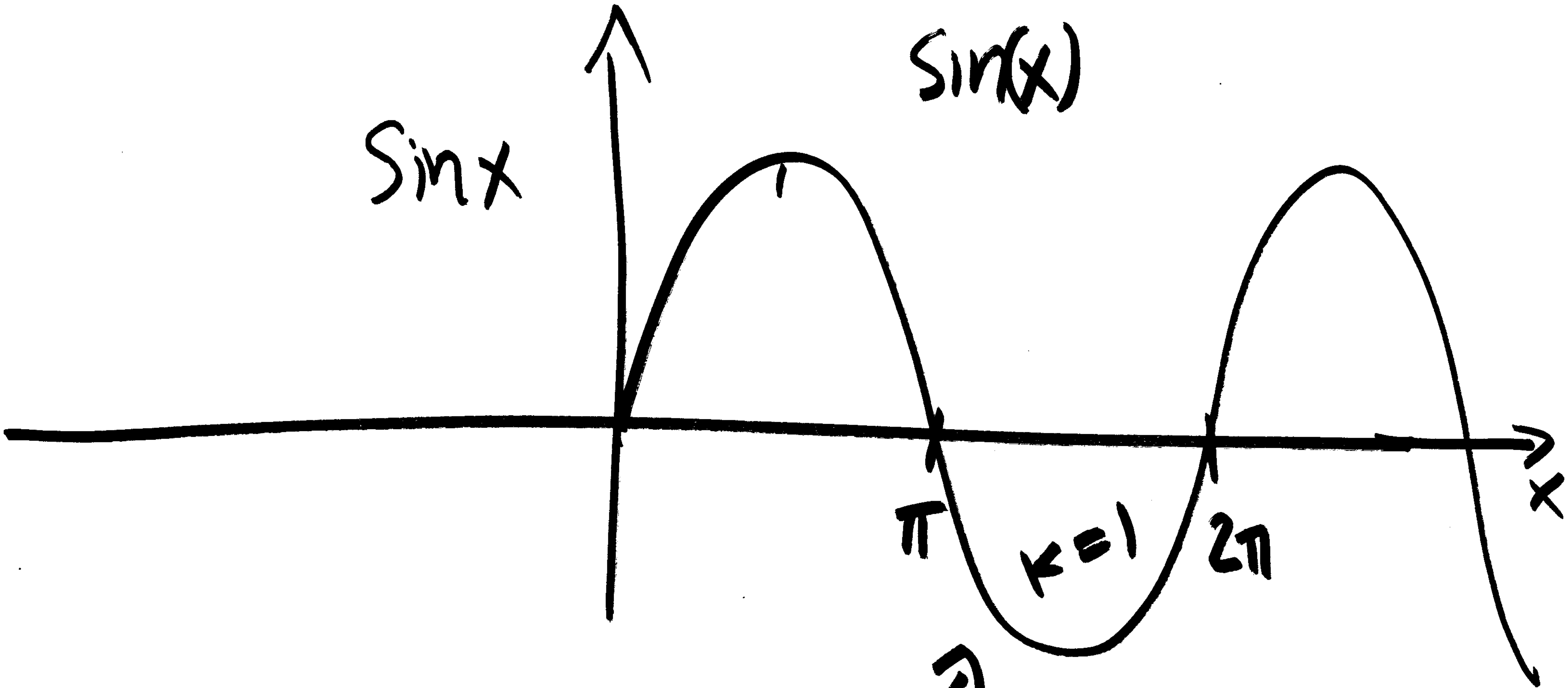


$\sin(\underline{\underline{k}}\,x)$

$k =$

$\sin(kx)$ $\quad\quad$ $\sin(wt)$

$\cos kx$ $\quad\quad$ $\cos(wt)$

e^{ikx} $\quad\quad$ e^{iwt}

$k \rightarrow$ spatial frequency

$w \rightarrow$ frequency

$$f(x) = \sin(kx)$$

$$f(x) = \cos(kx)$$

$$\boxed{f(x) = e^{ikx}}$$

$$c_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, e^{inx}$$

$$\downarrow$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\, e^{i2\pi kx}$$

$$g(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{ikx} \, dx$$

$$g(k)=\frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{\underline{f(x)}}\, e^{ikx}\, dx$$

$$e^{i3k} \qquad e^{i3x} =$$

$$\cos \underline{3x} + i\sin 3x$$

$$\cos 3x + i \sin 3x = e^{i3k} = f(x)$$

$$\downarrow$$

or

$$\left.\begin{array}{c}\text{a wave with frequency} \\ 3\end{array}\right\} g(k)$$

$$\downarrow$$

$$g(3)$$

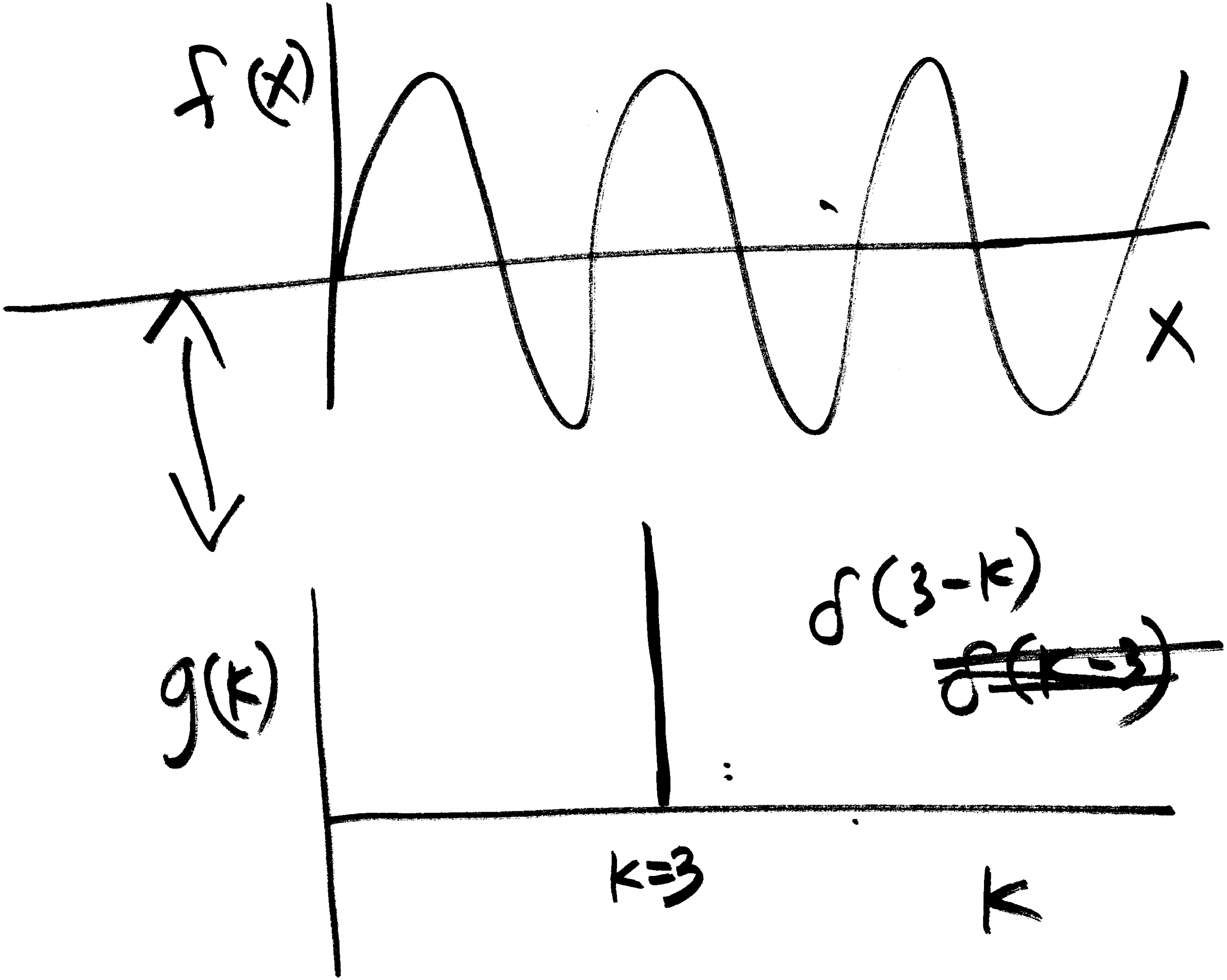
$$g(k) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x)\, e^{-ikx}\, dx$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i3x}\, e^{-ikx}\, dx$$

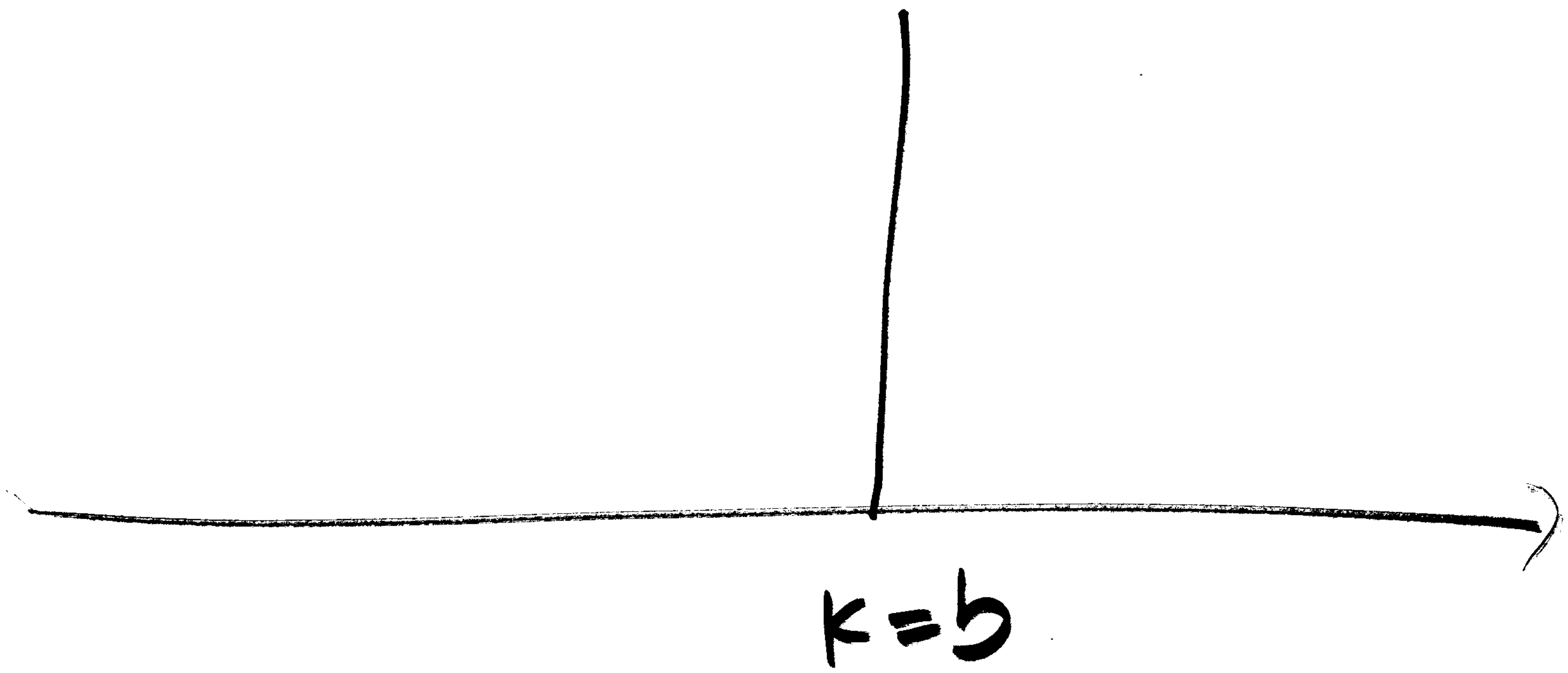
$$\delta(3-k) \longleftarrow$$ Dirac delta function

$$\uparrow$$

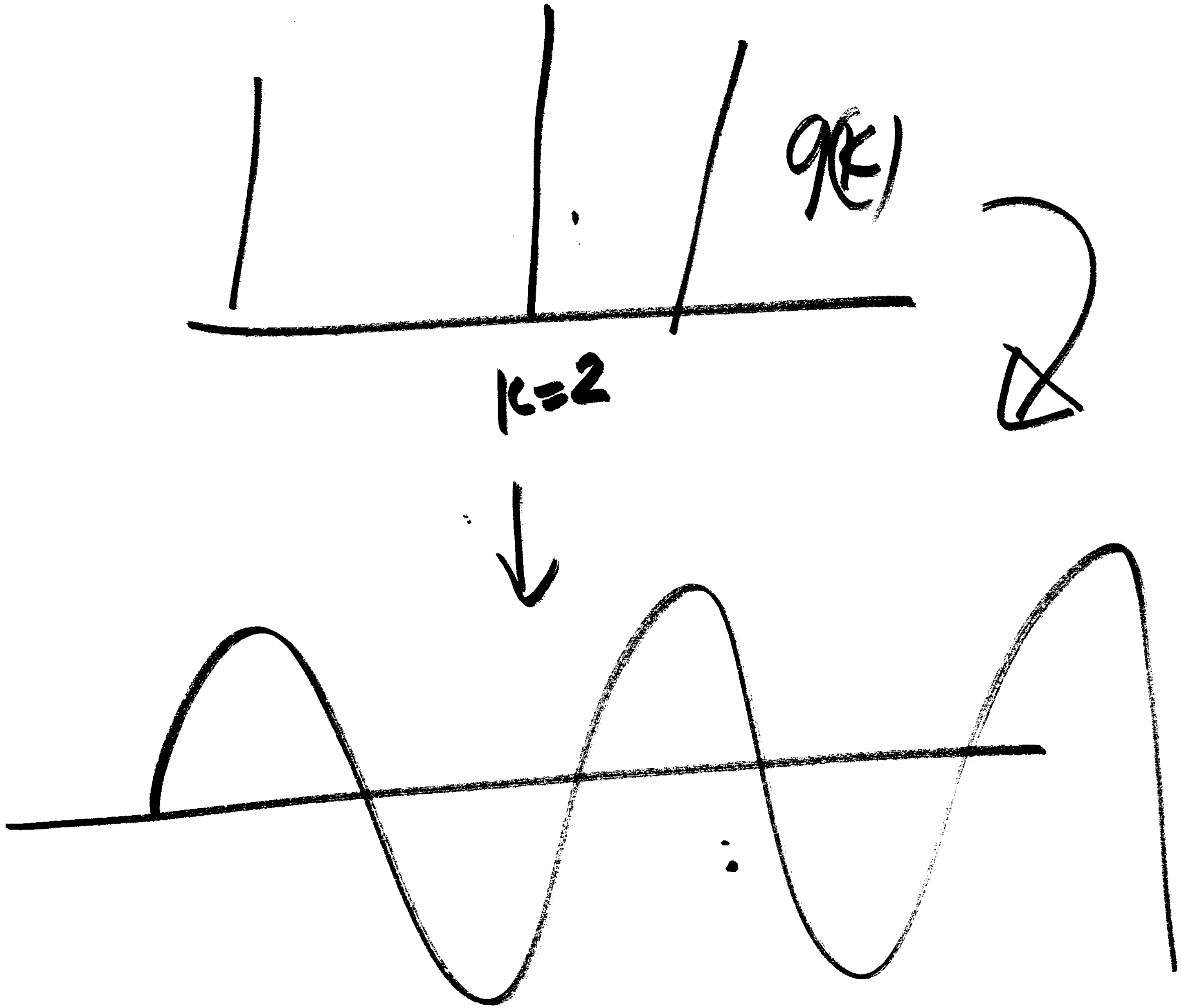
$$g(k) = \int_{-\infty}^{\infty} e^{ix(3-k)}\, dx$$

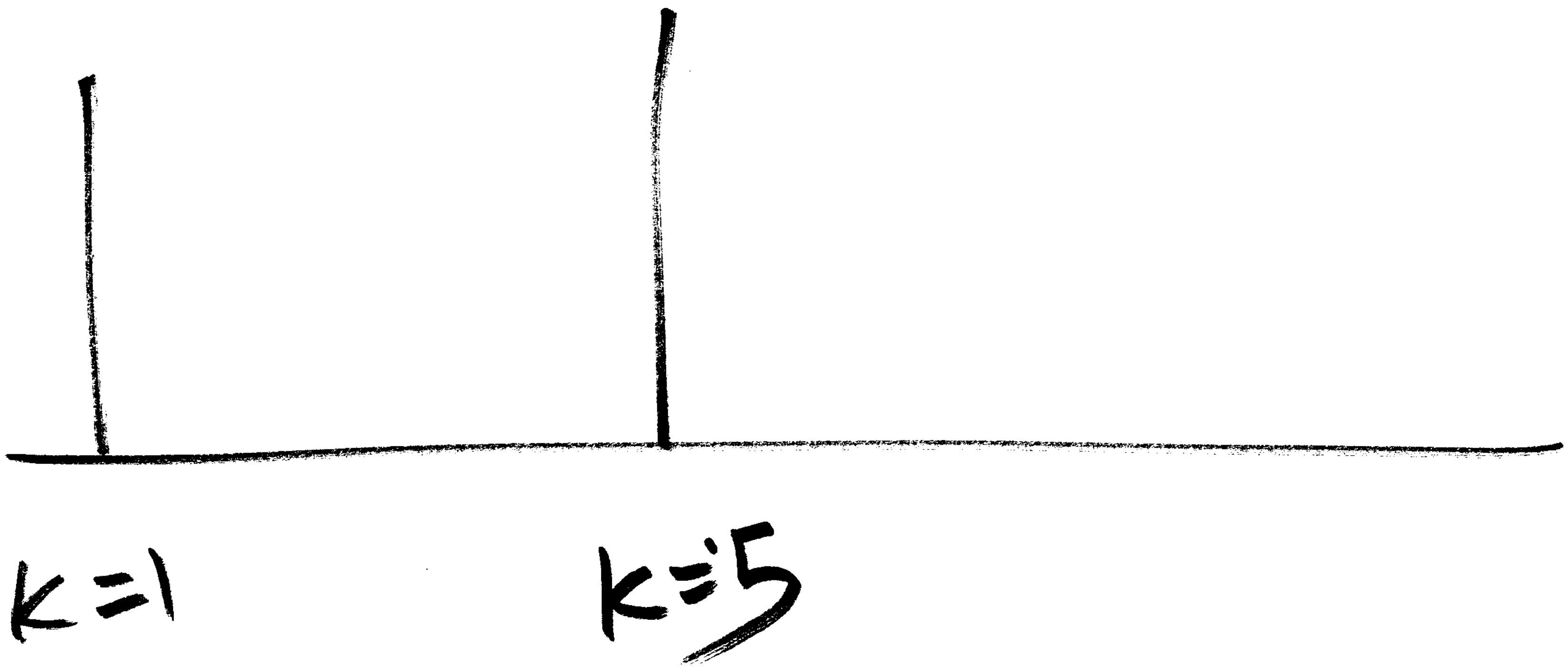


f(x)
x
g(k)
δ(3-k)
~~δ(k-3)~~
k=3
k



$$\delta(k-b)$$


g(k)
k=2



$g(k) \quad \delta(k-1) + \delta(k-5)$

$f(x) = ? = e^{ix} + e^{i5x}$

$$\int \delta(k-1)\, \cancel{dk}\, e^{ikx}\, dk$$

$$= e^{ix}$$

$$\int \delta(k-5)\, e^{ikx}\, dx = e^{i5x}$$

$$\int g(k)\,\delta(k-b)\,dk = g(b)$$

$$\int f(x)\,\delta(x-a)\,dx$$

$$= f(a)$$

$$\int \cancel{e^{ix}}\, e^{ikx}\,\delta(k-5)\,dk$$

$$= e^{i5x}$$

$$\delta(k-b) = 0 \quad \text{if } k \neq b$$

$$\delta(k-b) = \infty \quad \text{At } k = b$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\, \underline{e^{ikx}}\, dx = g(k)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\left[\cos kx + i\sin kx\right] dx$$

$$= g(k)$$

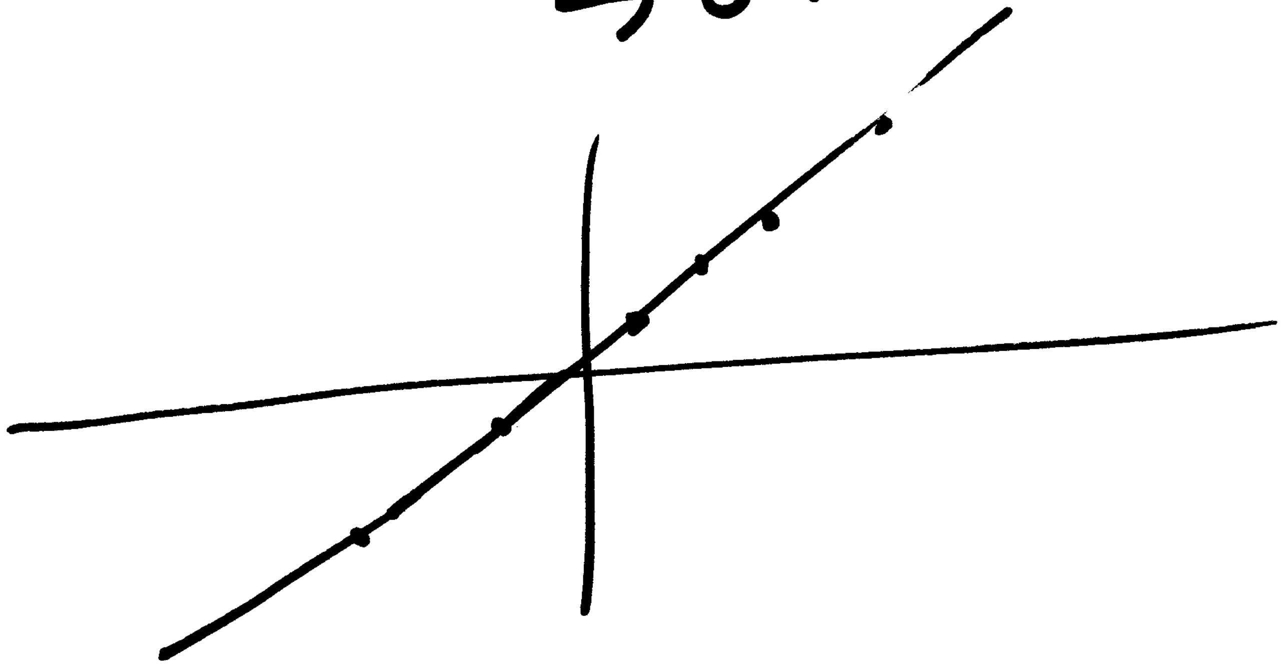
$$\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)\cos kx + \frac{i}{2\pi}\int_{-\infty}^{\infty} f(x)\sin kx \, dx$$

$$= g(k)$$

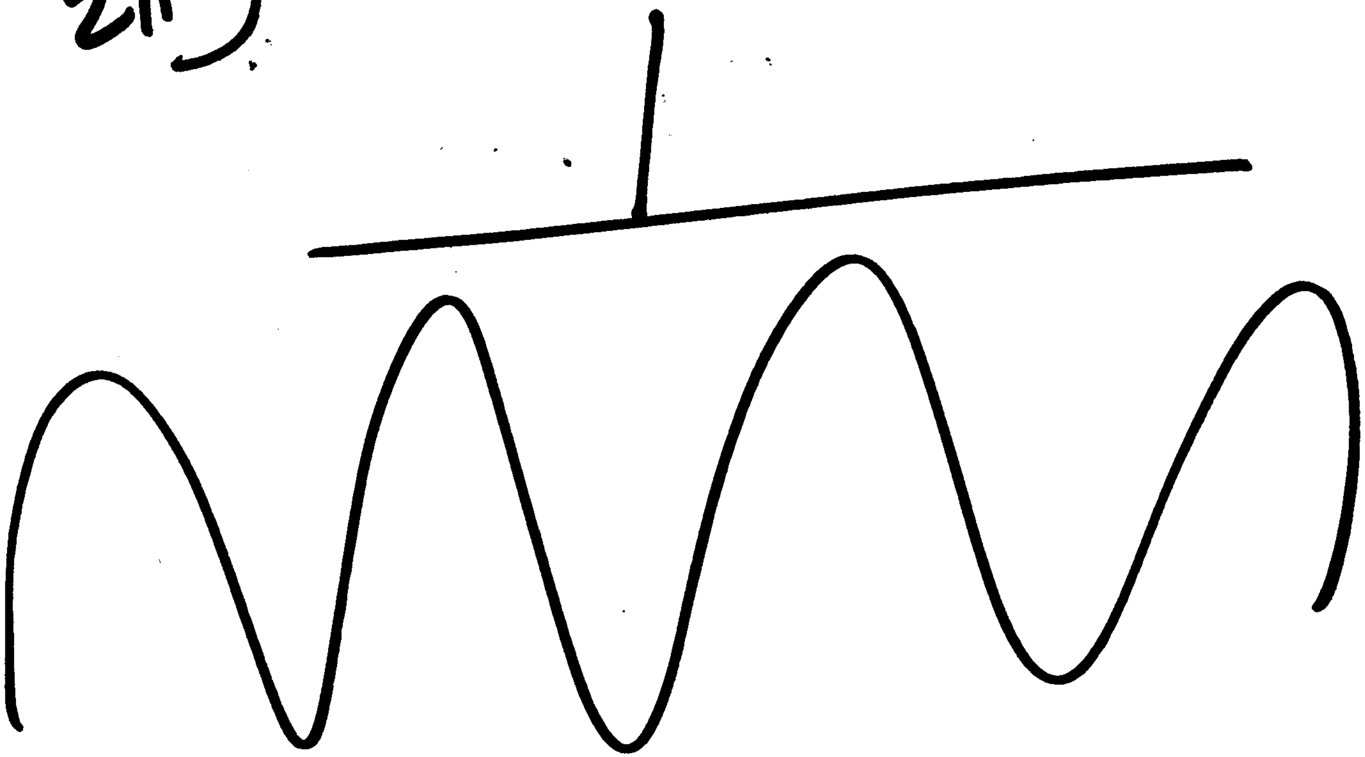
$$f(x) = \cancel{\text{odd}} - f(-x)$$

& if $f(x) = x$

$\Rightarrow$ odd function



$$g(k) = \frac{1}{2\pi}\int f(x)\, e^{ikx}\, dx = g(x)$$



Gaussian

$$f(x) = e^{-ax^2}$$

$$\downarrow$$

$$g(k) = e^{-bk^2}$$

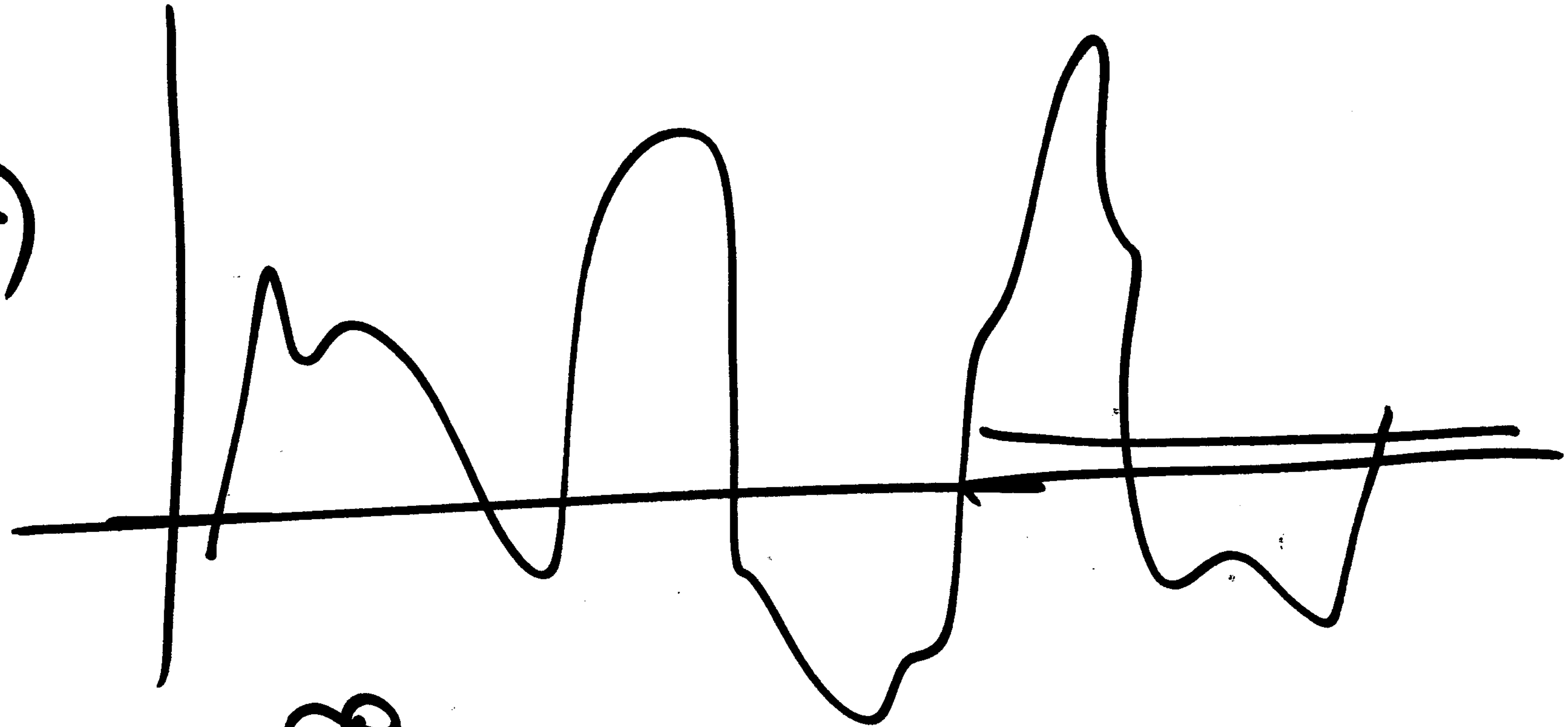
$$\boxed{f(x) = \frac{1}{x^2+a^2}}$$

FT

$$g(k) = \int_{-\infty}^{\infty} e^{ikx} \frac{1}{x^2+a^2}\, dx$$

IFT

$$\boxed{g(k) = e^{-ka}}$$



$$g(w) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)\, e^{-iwt}\, dt$$

$$\int_{-\infty}^{\infty} f(x)\, e^{ikx}$$

$$\sum_{n=-\infty}^{\infty} \cancel{f(x)}\, f(n)\, e^{inx}$$

$$g(z) = \sum_{n=-\infty}^{\infty} f(n)\, e^{-zn}$$

$$g(z) = \sum_{n=-\infty}^{\infty} f_n\, z^{n}$$

z – transform

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17/11/11

$$g(k) = \frac{1}{2\pi}\int_{-\infty}^{-\infty} f(x)\, e^{ikx}\, dx$$

$$g(k) = \sqrt{\frac{1}{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{ikx}\, dx$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\, e^{-i2\pi kx}\, dx$$