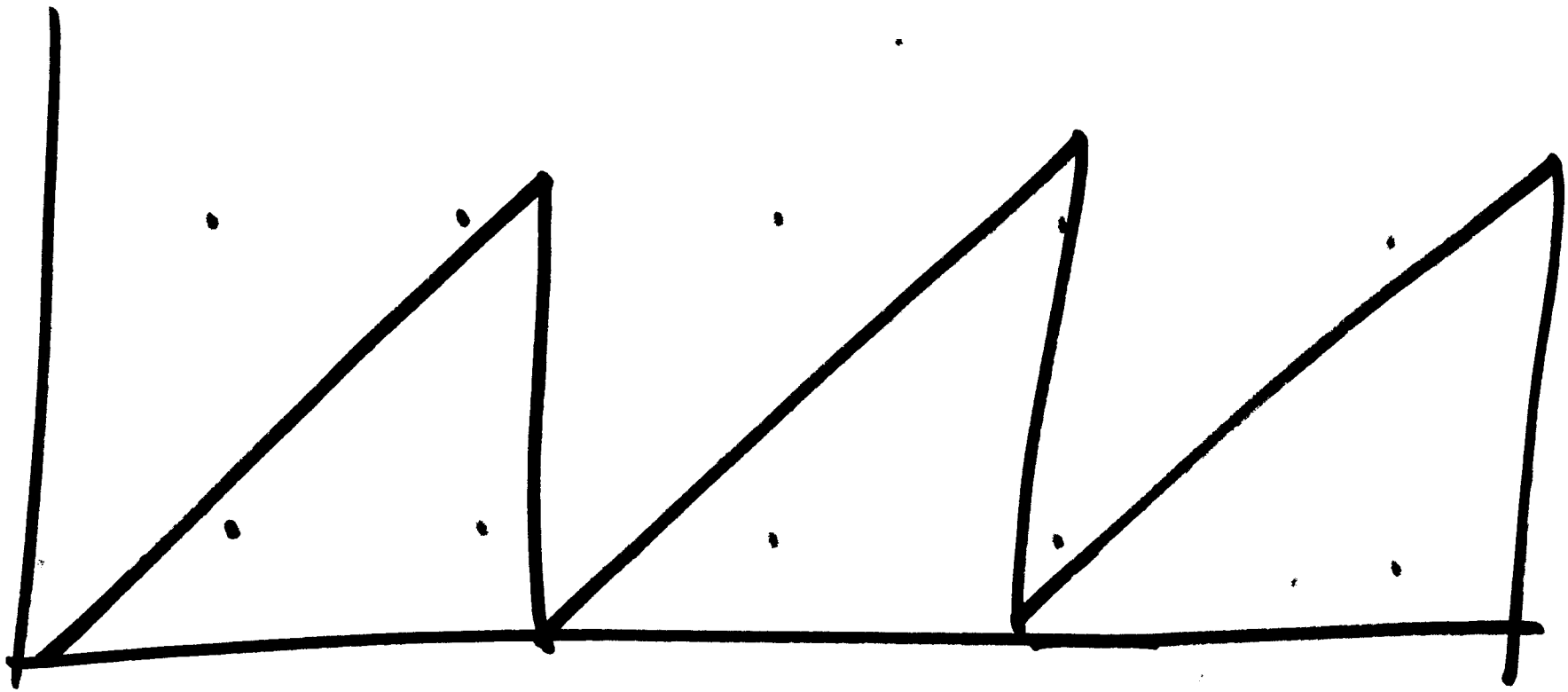
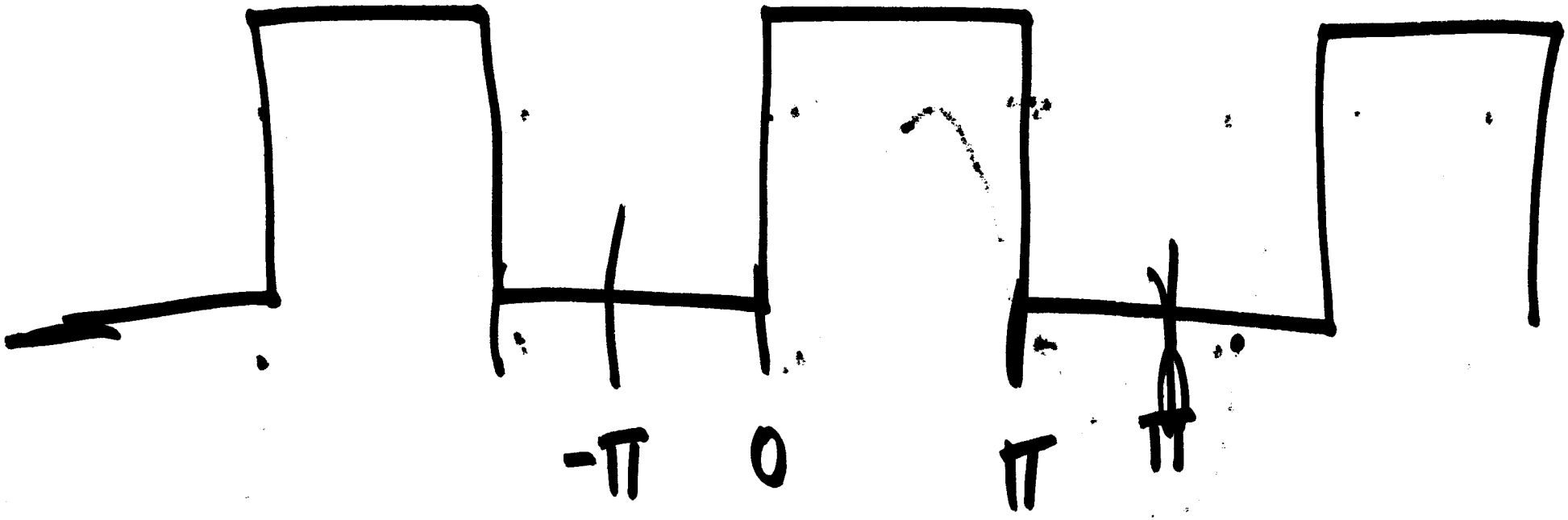
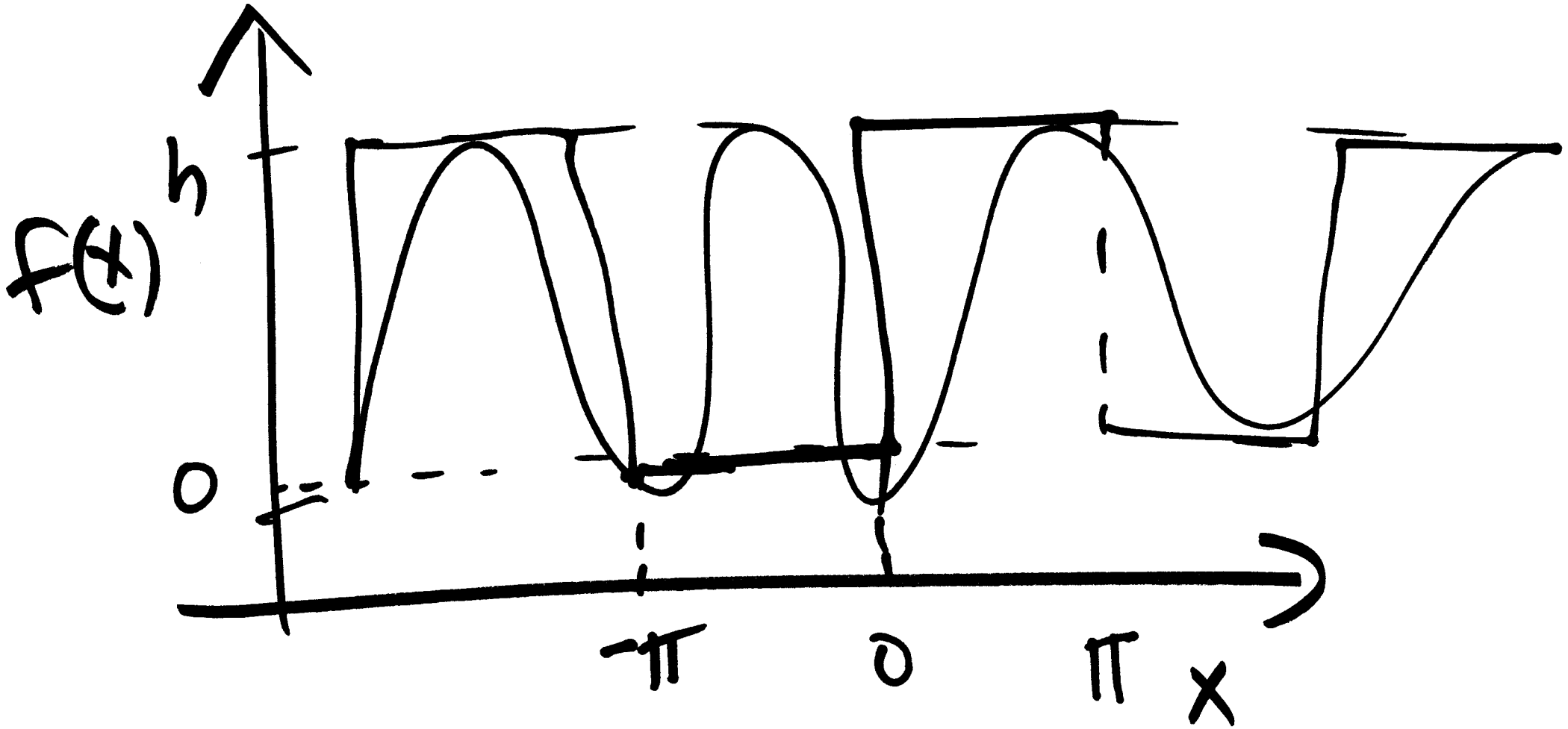


saw-tooth wave

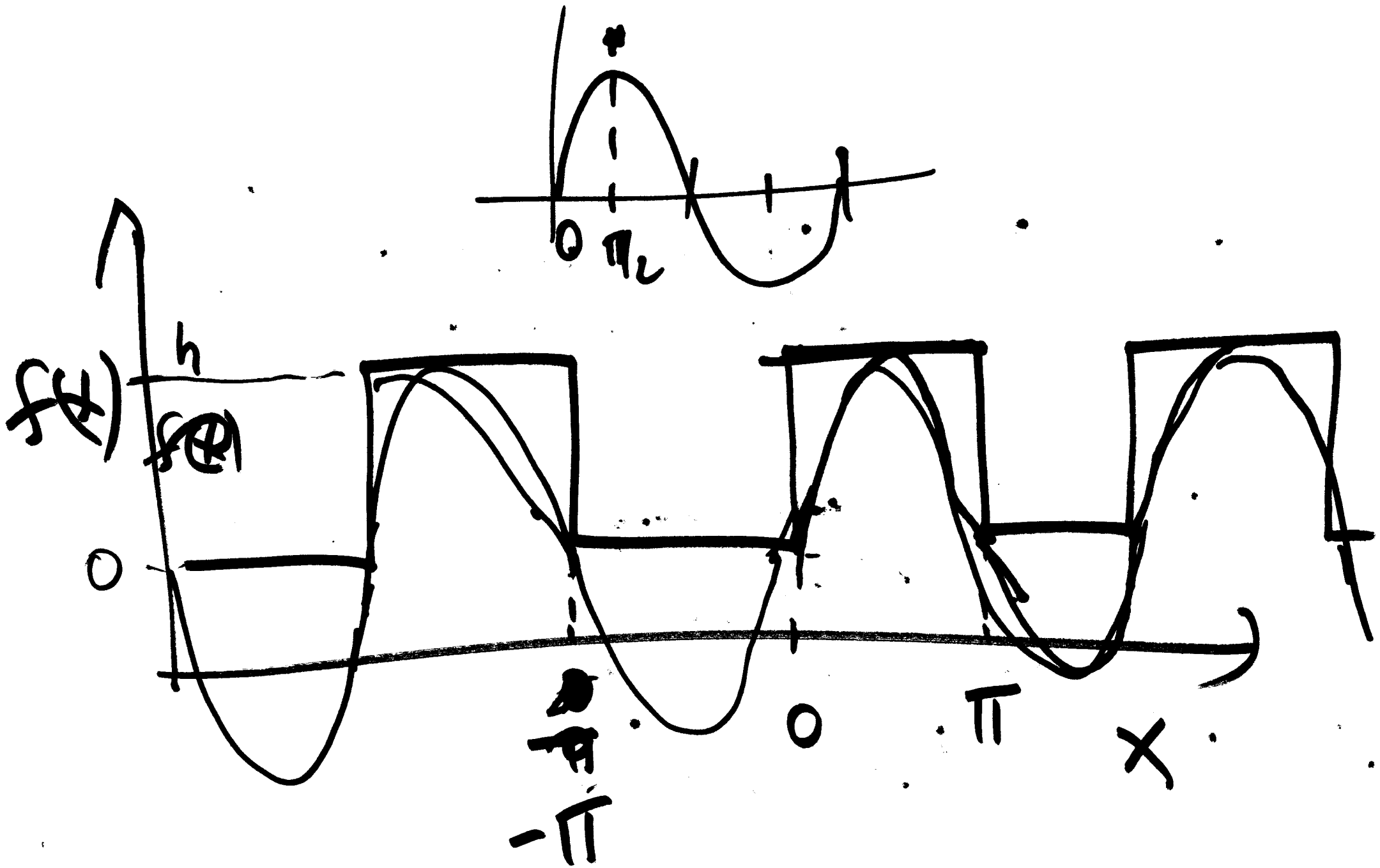
Square- wave



Prof. Ranjith. P (16/11/11)
Lec. 28

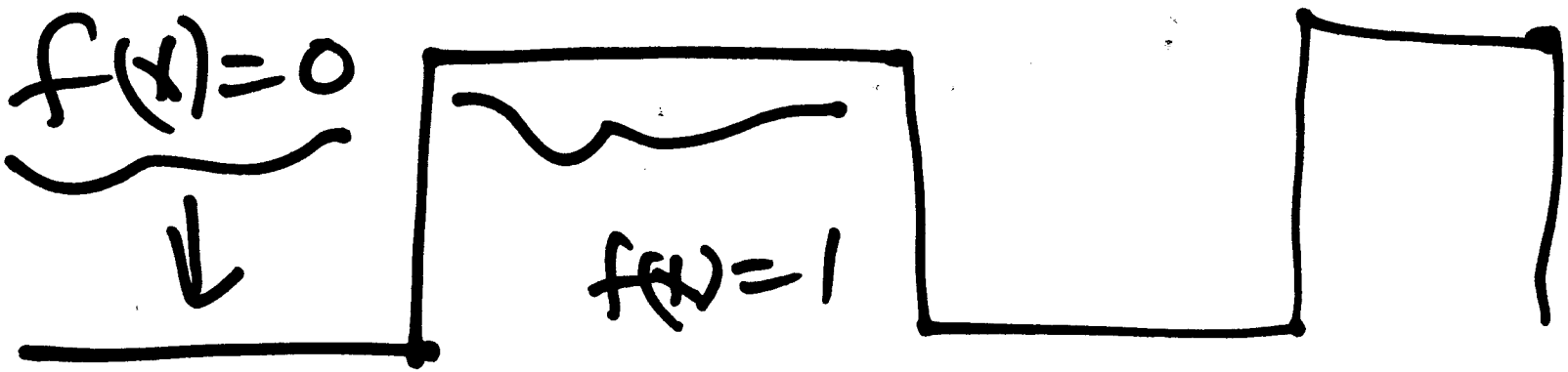


1


0
π/2
h
f(x)
0
-π
0
π
x

$$f(x) = \underline{a_0} + \sum_{n=1}^{\infty} \underline{a_n} \cos(nx)$$

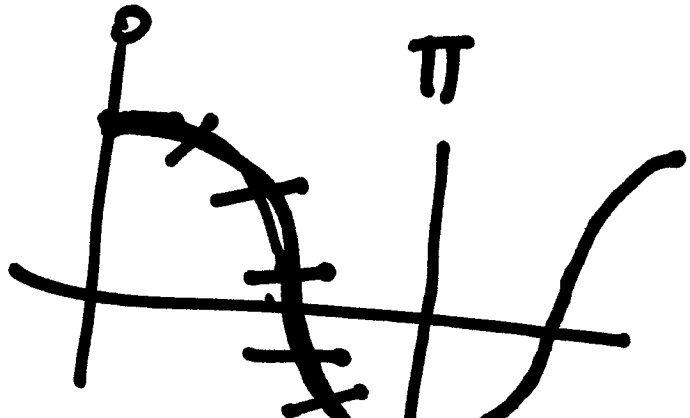
$$+ \sum_{n=1}^{\infty} \underline{b_n} \sin(nx)$$



$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$= \frac{1}{\pi}\left[\underbrace{\int_{-\pi}^{0} f(x)\cos(nx)\,dx}_{0} + \int_{0}^{\pi} f(x)\cos nx\,dx\right]$$

$$a_n = \frac{1}{\pi}h\int_{0}^{\pi}\cos nx\,dx = 0$$



$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\,\sin nx\,dx + \int_{0}^{\pi} f(x)\,\sin nx\,dx$$

$f(x) \to 0$

$$b_n = \frac{1}{\pi}\int_{0}^{\pi} f(x)\,\sin nx\,dx$$

$f(x) \to h$

$$b_n = \frac{b}{\pi} \int_0^{\pi} \sin nx \, dx$$

$$= \frac{b}{\pi} \left[-\frac{\cos nx}{n} \right]_0^{\pi}$$

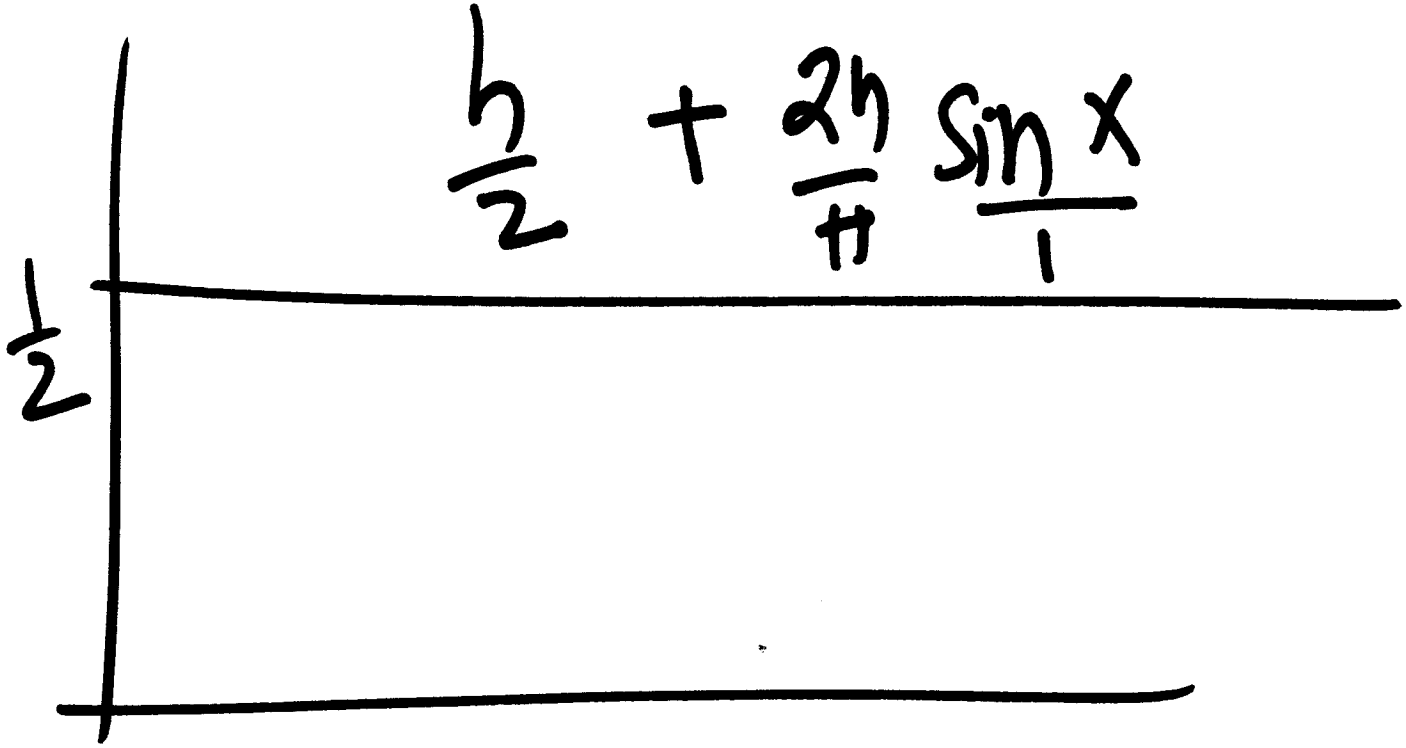
$$b_n = \frac{b}{\pi} \left[-\frac{\cos(n\pi)}{n} + 1 \right]$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \int_{0}^{\pi} f(x)\,dx + \cancel{\int_{-\pi}^{0} f(x)\,dx}$$

$$= \frac{h}{\pi}\int_{0}^{\pi} dx$$

$$= \frac{h}{\pi}\left[\pi\right] = h$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \underbrace{a_n \cos nx}_{0} + \sum_{n=1}^{\infty} b_n \sin nx$$


$\frac{1}{2} + \frac{2}{\pi} \sin x$
$\frac{1}{2}$

$$\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$\hat{i} \cdot \hat{j} = 0$$

$$\hat{i} \cdot \hat{i} = 1$$

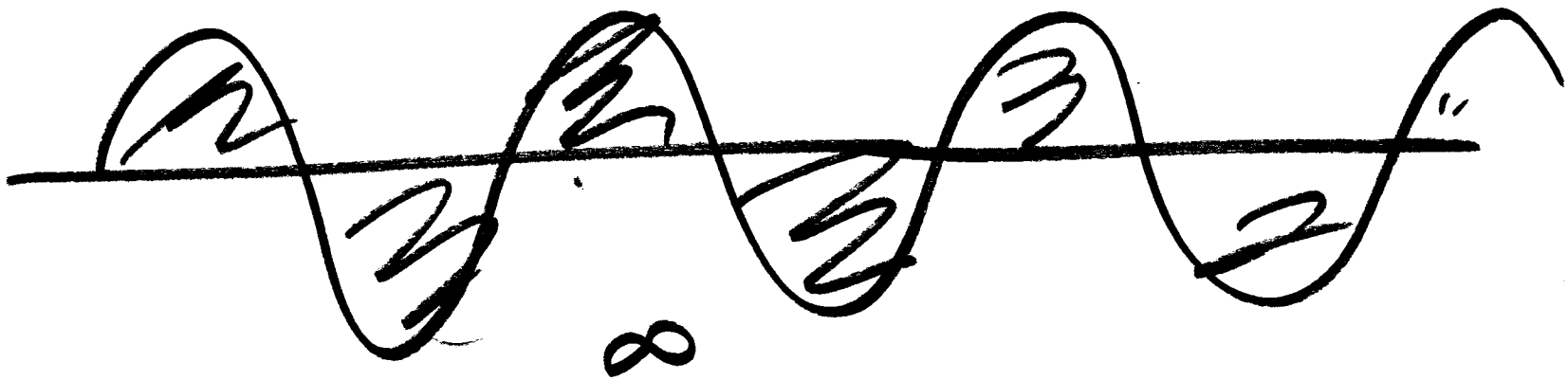
$$e^{inx} = \underline{\cos nx} + \underline{i \sin nx}$$

$$\int_{-\infty}^{\infty} e^{inx}\, e^{imx}\, dx = \int_{-\infty}^{\infty} e^{i(m+n)x}\, dx$$

$$= ?$$

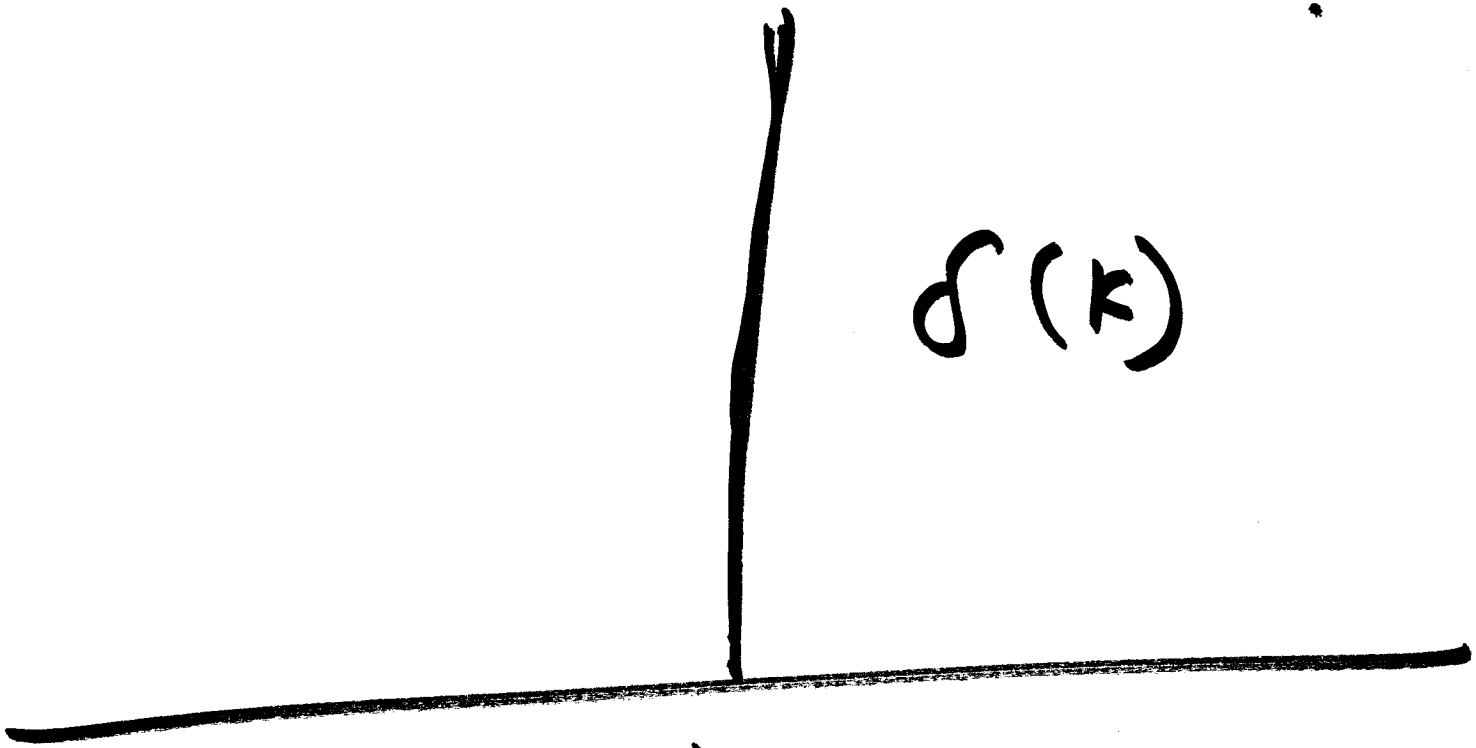
$$\int_{-\infty}^{\infty} e^{ikx} = 0 \quad \text{if} \quad k \neq 0$$

$$\cos kx + i \sin kx$$

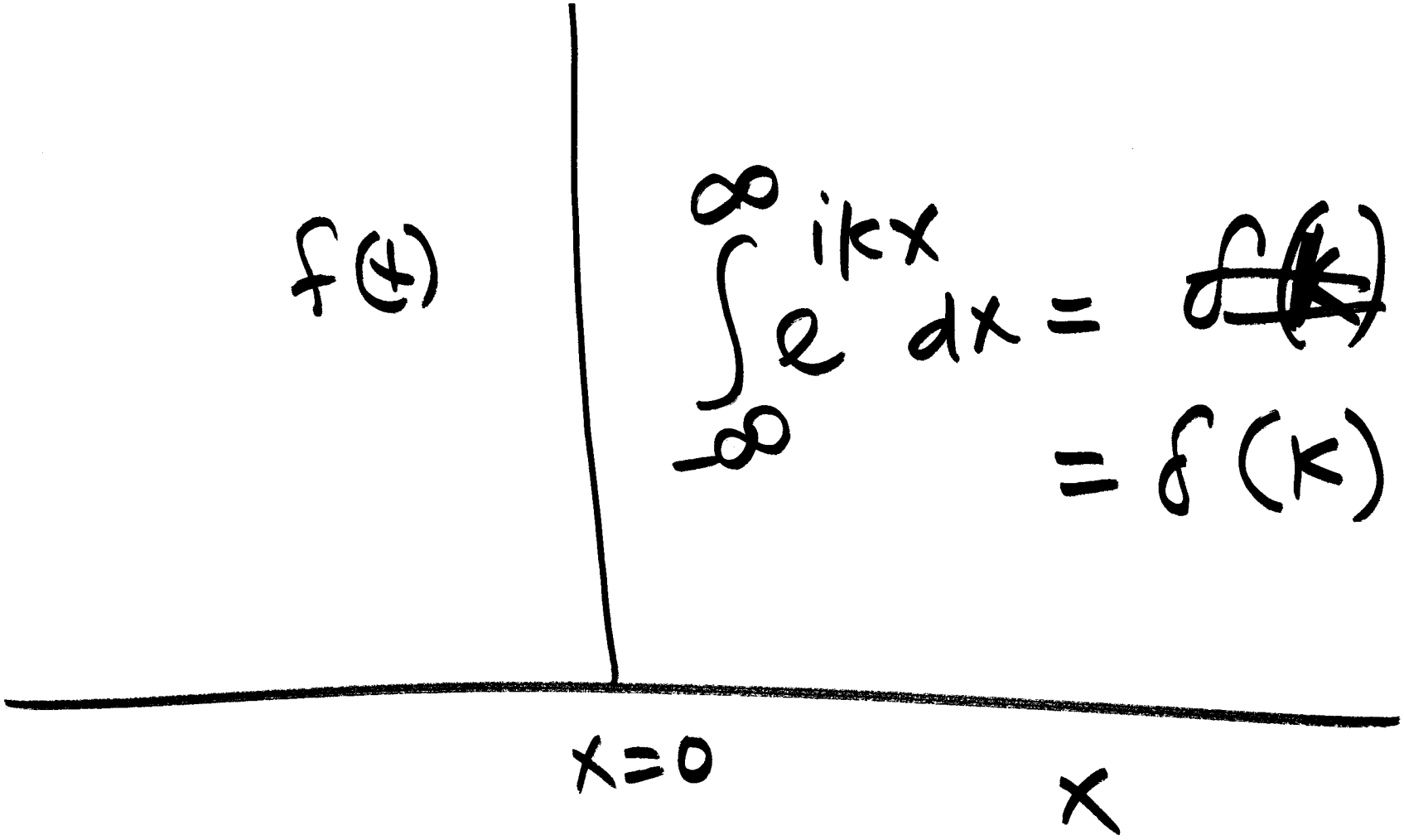


$$\text{If } k = 0, \int_{-\infty}^{\infty} e^{ikx} dx = \infty$$

Dirac delta function



$$\int_{-\infty}^{\infty} \delta(k)\, dk = 1$$



$$f(x) = \delta(x)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

or

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$$