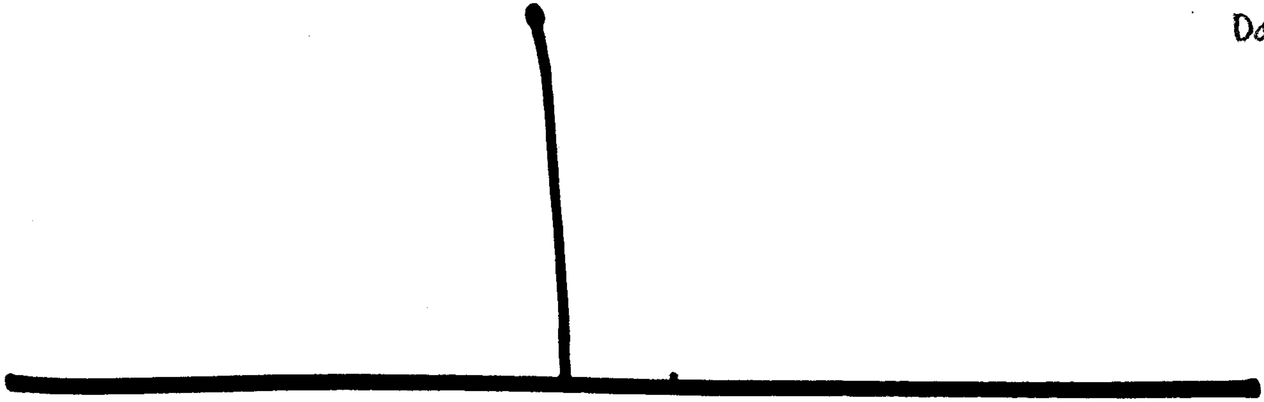
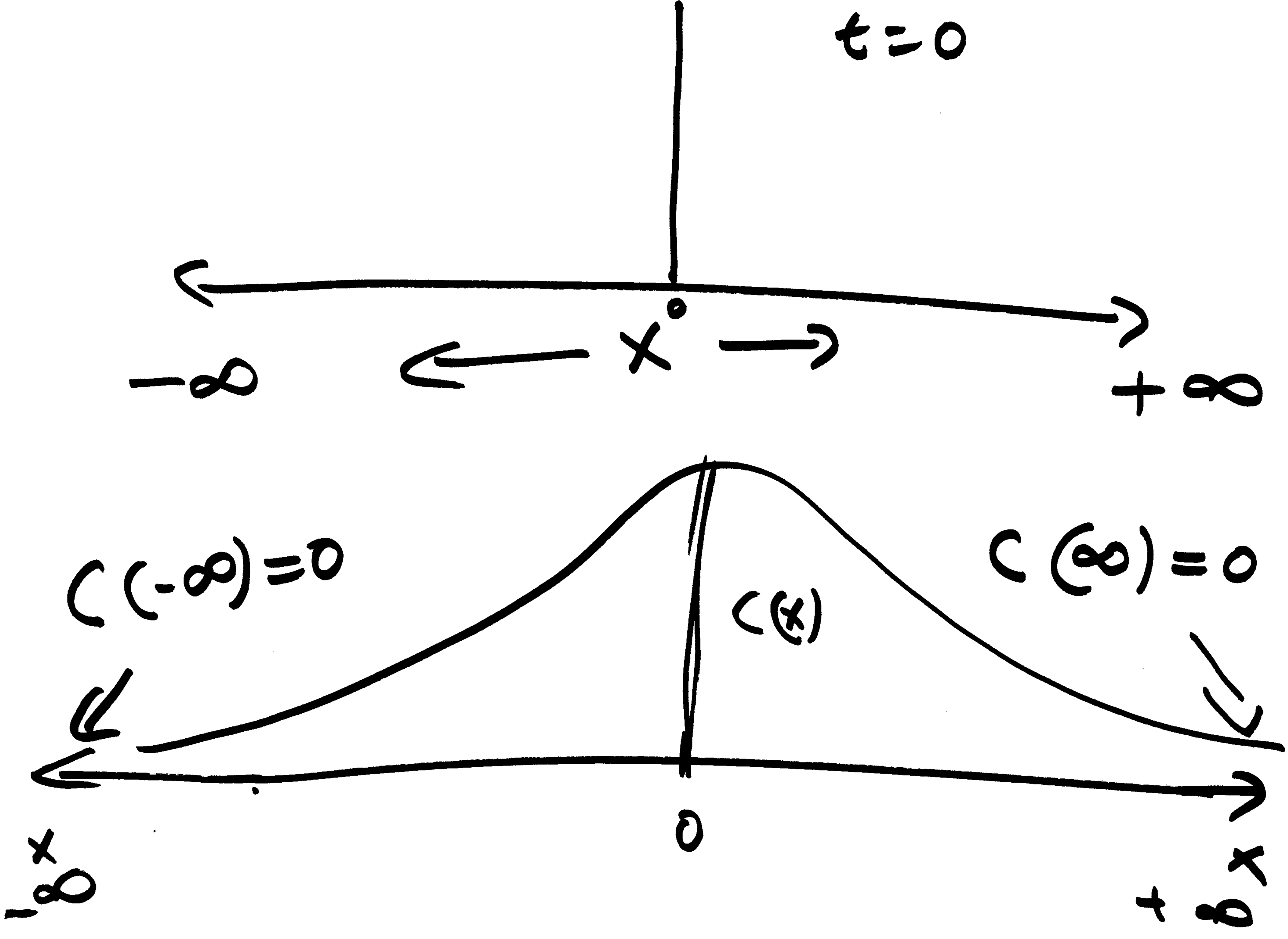


$x=0$

$$f(x) = \underbrace{\delta(x)}$$

Dirac's delta function

$$C(x) \propto \delta(x)$$

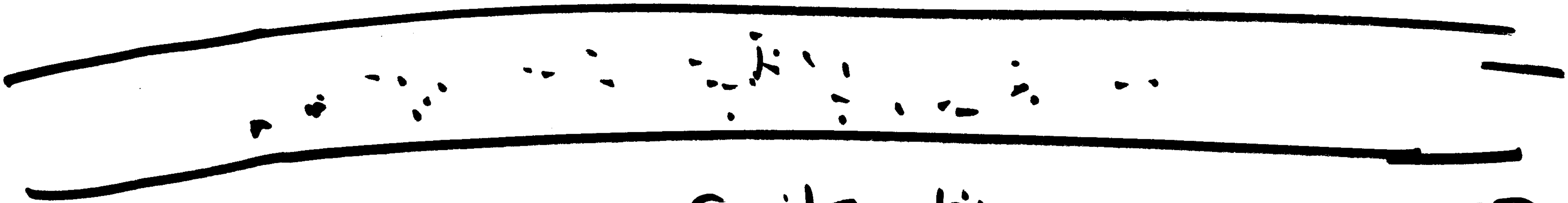


t=0
x°
−∞
+∞
C(−∞)=0
C(∞)=0
C(x)
0
x
−∞
x
+∞

$$c(x)$$

$$\left.\frac{\partial c}{\partial x}\right|_{x=-\infty} = 0 \qquad c(x=\infty) = 0 \qquad \left.\frac{\partial c}{\partial x}\right|_{x=\infty} = 0$$

$$c(x=-\infty) = 0$$



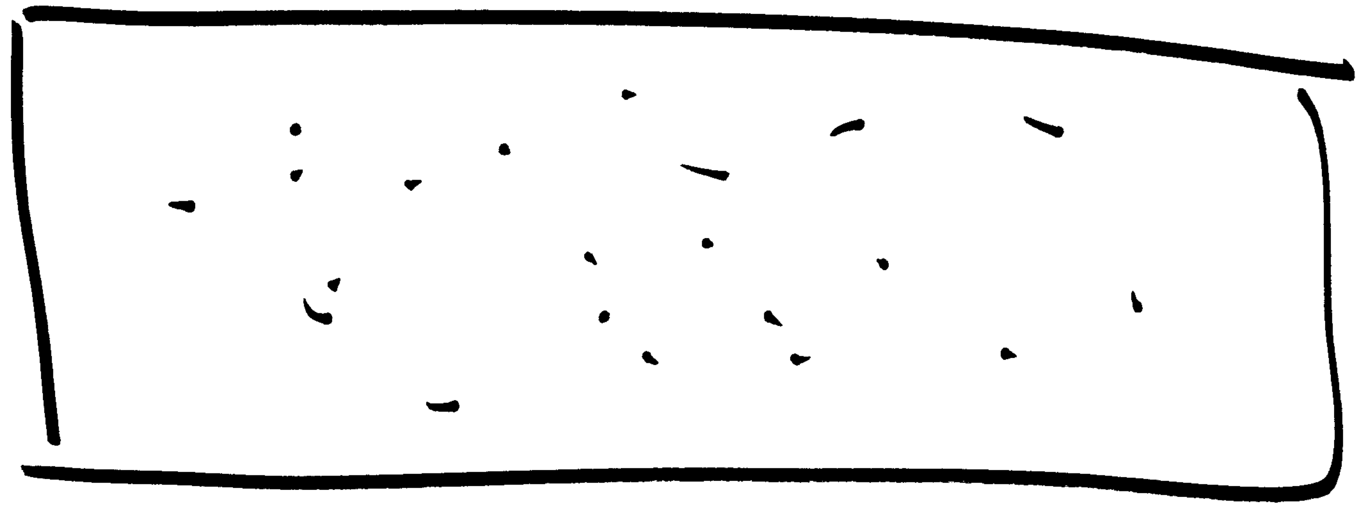
$-\infty$ finite time ∞

$$C(x)$$

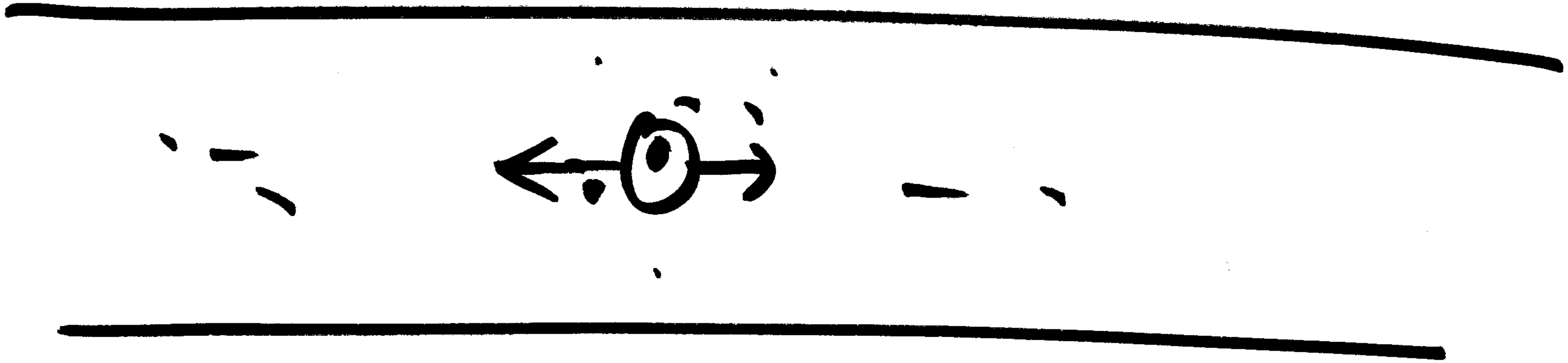
$$\int_{-\infty}^{\infty} C(x)\,dx = C_T$$

$$\boxed{\tilde{C}(x) = \frac{C(x)}{C_T}}$$

$$\int_{-\infty}^{\infty} \tilde{C}(x)\,dx = \frac{\int C(x)\,dx}{C_T} = 1$$



$$v_{rms} = \sqrt{\langle v^2 \rangle}$$



$$\sqrt{\langle x^2 \rangle} = x_{rms}$$

$$\langle x \rangle = ?$$

$$\langle x^2 \rangle = ?$$

$$\frac{1}{c_T}\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}\frac{1}{c_T}$$

$$\boxed{\frac{\partial \tilde{c}}{\partial t} = D\frac{\partial^2 \tilde{c}}{\partial x^2}}$$

$$\tilde{c} = \frac{c}{c_T}$$

$$\frac{\partial \tilde{C}}{\partial t} = D \frac{\partial^2 \tilde{C}}{\partial x^2}$$

$$\langle x^2 \rangle = ?$$

$$\frac{\partial}{\partial t} \int_{-\infty}^{\infty} x^2 \, \tilde{C} \, dx = D \int_{-\infty}^{\infty} x^2 \frac{\partial^2 C}{\partial x^2} dx$$

$$\frac{\partial}{\partial t}\underbrace{\int_{-\infty}^{\infty} x^2\, \tilde{C}(x)\, dx}_{\langle x^2 \rangle} = D\int_{-\infty}^{\infty} x^2 \frac{\partial \tilde{C}}{\partial x^2}\, dx$$

$$\frac{\partial}{\partial t}\langle x^2 \rangle = D\int_{-\infty}^{\infty} x^2 \frac{\partial \tilde{C}}{\partial x^2}\, dx$$

$u(x) \quad v(x)$

$$\frac{d}{dx}\left[u(x)\, v(x)\right]$$

$$\int \frac{d}{dx}\left[u\, v\right] dx = \int u \frac{dv}{dx} dx + \int v \frac{du}{dx} dx$$

$$\cancel{u \int dv}$$

$$\int_{-\infty}^{\infty} u \frac{dv}{dx}\,dx = \int_{-\infty}^{\infty} \frac{d}{dx}(uv)\,dx - \int_{-\infty}^{\infty} v \frac{du}{dx}\,dx$$

$$\frac{\partial}{\partial t}\langle x \rangle = D\int_{-\infty}^{\infty} \underbrace{x^2}_{u} \underbrace{\frac{\partial^2 \tilde{c}}{\partial x^2}}_{\frac{dv}{dx}} dx$$

$$\int u \frac{dv}{dx} = uv\Big|_{-\infty}^{\infty} - \int v \frac{du}{dx} dx$$

$$D\int_{-\infty}^{\infty} x^2 \frac{\partial^2 \tilde{c}}{\partial x^2}\, dx \qquad u = x^2$$

$$\frac{dv}{dx} = \frac{\partial^2 \tilde{c}}{\partial x^2}$$

$$\int_{-\infty}^{\infty} u \frac{dv}{dx}\, dx = uv\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} v \frac{du}{dx}\, dx$$

$$v = \frac{\partial \tilde{c}}{\partial x}$$

$$\frac{\partial}{\partial t}\langle x^2\rangle = -2D\int_{-\infty}^{\infty} \underset{\substack{\big\downarrow \\ u}}{x}\;\underset{\substack{\big\downarrow \\ \frac{dv}{dx}}}{\frac{\partial \tilde{c}}{\partial x}}\,dx$$

$$\frac{\partial}{\partial t}\langle x^2\rangle = 2D\underbrace{\int_{-\infty}^{\infty}\tilde{c}(x)\,dx}_{1}$$

$$\frac{\partial}{\partial t}\langle x^2\rangle = 2D$$

$$\langle x^2 \rangle = 2Dt$$

$$x_{rms} = \sqrt{\langle x^2 \rangle} = \sqrt{2Dt}$$

$$X_{rms} = \sqrt{2Dt}$$

$$D = 1 \frac{m^2}{s}$$

$$t = 10s$$

$$X_{rms} = \sqrt{2 \times 1 \times 10} = \sqrt{20}\ m$$

$$t = 100s$$

$$X_{rms} = \sqrt{2 \times 1 \times 100} = \sqrt{200}\ m$$

$t = 10s$, $X_{rms} = \sqrt{20} = 4.$___

$t = 100s$, $X_{rms} = \sqrt{200} \approx 15.$

x

$$x_{rms} = \sqrt{2Dt}$$

$$2Dt = \langle x^2 \rangle$$

$$t = \frac{\langle x^2 \rangle}{2D}$$

final

summary

$$\langle x^2 \rangle = 2Dt$$

$$\frac{\partial^2 \tilde{c}}{\partial t} = D \frac{\partial^2 \tilde{c}}{\partial x^2}$$