

Chapter 9

An Introduction to Proportional Navigation

Module 8: Lecture 22 PPN; TPN; GTPN; IPN

Keywords. Proportional Navigation, PPN, TPN, GTPN, IPN

Proportional navigation is, by far, the most important of the classical guidance laws. In this chapter we will give a brief description of the several variants of the PN law that has appeared in the literature.

To explain how the PN law works let us first define an engagement geometry in a 2-D plane. We shall use this engagement geometry or some variants of it in the subsequent lectures. The engagement geometry is two dimensional and is given in Figure 9.1. Point mass models are assumed for both the missile and the non-maneuvering target.

The missile lateral acceleration or latax is given by a_M . The missile and target

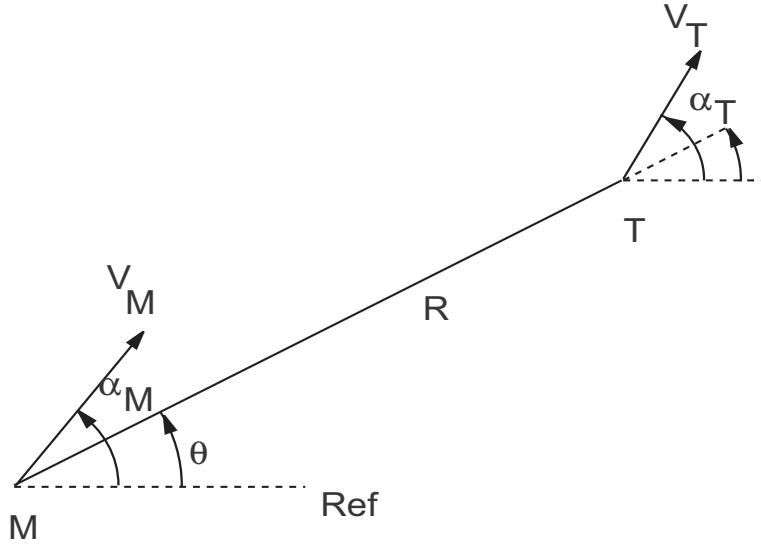


Figure 9.1: Missile-target engagement geometry

velocities are denoted by V_M and V_T , respectively. The equations of motion are,

$$V_r = \dot{R} = V_T \cos(\alpha_T - \theta) - V_M \cos(\alpha_M - \theta) \quad (9.1)$$

$$V_\theta = R\dot{\theta} = V_T \sin(\alpha_T - \theta) - V_M \sin(\alpha_M - \theta) \quad (9.2)$$

where, V_r is the rate of change of the LOS separation (i.e., the component of the relative velocity of the target with respect to the missile along the LOS) and V_θ is the angular velocity of the LOS (i.e., the component of this relative velocity perpendicular to the LOS).

Now the *Proportional Navigation (PN)* guidance law is defined as a law that generates a guidance command (or a latax) which ensures that the rate of rotation of the missile velocity vector is proportional to the rate of rotation of the LOS. That is,

$$\dot{\alpha}_M = N\dot{\theta} \quad (9.3)$$

where, N is the navigation constant. This was the essence of the PN guidance law when it was formulated in the early days of missile guidance research. This innocuous-looking guidance law has, over the years, spawned an enormous variety of guidance laws that have attempted to improve the performance of the basic PN law. These guidance laws are popularly known as PN-variants.

In fact, the PN law is no longer referred to in the form of (9.3). It is expressed in terms of the latax that is prescribed at different instants of time at various points on the trajectory. We shall examine the structure of some of these laws in this lecture.

9.1 Pure Proportional Navigation (PPN)

One of the first complete definition of the PN law is what is now known as the *Pure Proportional Navigation (PPN)* guidance law. This is also the most natural definition of the PN law. We know that,

$$\dot{\alpha}_M = \frac{a_M}{V_M} \quad (9.4)$$

So, from (9.3), we get,

$$a_M = NV_M \dot{\theta} \quad (9.5)$$

This is the *PPN* law. But there is one more point to be noted here. We know that (9.4) is valid only when the lateral acceleration a_M is perpendicular to the velocity V_M of the missile. So, according to the PPN law, the latax is given by (9.5) and is *applied perpendicular to the velocity vector of the missile*. If we ignore the *angle-of-attack* of the missile, then this direction of the latax is also the *natural direction of the lift force* which is generated by the airframe and the lifting surfaces (as explained earlier) whenever the missile maneuvers. Note that this lift force is responsible for generating the actual lateral acceleration or latax. However, the only problem here is that the angle-of-attack of a missile is *never* zero and for many highly maneuverable missiles it turns out to be quite high. This is where PPN departs from reality and its elegant results stop being fully applicable. In Figure 9.2(a), the PPN latax is shown.

9.2 True Proportional Navigation (TPN)

True proportional navigation is another variant of the proportional navigation guidance law. The logic behind it is as follows: The velocity of interest is actually the closing velocity and not the missile velocity itself, because it is the closing velocity which ultimately drives the LOS separation to zero. Moreover, it is also the LOS rate which we are trying to drive to zero. So, why not apply the missile latax perpendicular to the LOS and also make it proportional to the closing velocity? It isn't a very convincing argument, but the guidance system designers who were desperate to explore ways

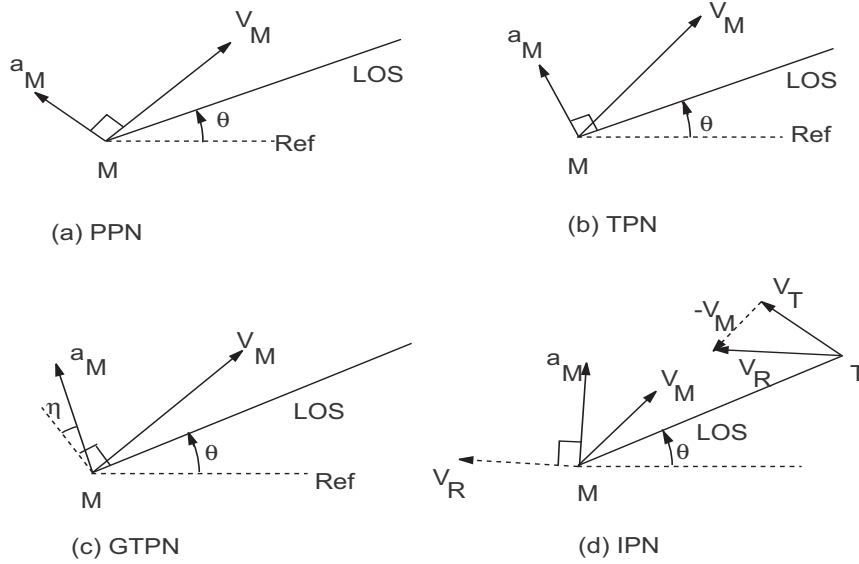


Figure 9.2: (a) PPN Latax (b) TPN Latax (c) GTPN latax (d) IPN Latax

of improving the performance of PN law found this convincing enough to try it out. Another reason was that V_M is not directly available unless the missile carries an inertial navigation unit, but V_c is easily available from the doppler data of the seeker. Of course, there were some other arguments regarding the invariance of the latax history with respect to the term $\frac{NV_M}{V_c} (= N')$ which clinched the argument in favour of true proportional navigation, but we will not go into this at the moment. So the TPN law was born with the following form,

$$a_M = N' V_c \dot{\theta} = -N' V_R \dot{\theta} \quad (9.6)$$

where, N' is called the *effective navigation ratio* to distinguish it from the navigation constant defined earlier, and V_c is the *closing velocity*. The word ratio is used to convey the fact that N' can be interpreted as the navigation constant N multiplied by the ratio V_M/V_c . But the main difference between the PPN and the TPN was that here the latax was applied *perpendicular to the LOS* and not to the missile velocity as in PPN (see Figure 9.2(b)). The main problem was that of implementation since this direction of the TPN latax is not a natural direction of the lifting force generated by the missile airframe which is ultimately responsible for the latax. But people came up with the idea of using thrusters which could be fired either in the forward or aft direction to impart

an additional longitudinal acceleration or deceleration. This acceleration coupled with the latax generated by the aerodynamic forces produced the desired latax in the desired direction. But this was not a very practical solution since it was a burden to provide for these extra thrusters. The next argument in favour of TPN was that if we want to use missiles for exo-atmospheric interception then we will have to use thrusters to generate a latax, because aerodynamic forces are non-existent at very high altitudes. So it is only a very little extra effort to deflect the force due to these thrusters in the desired direction. This was quite a convincing argument and quite a lot of work was done in the area of TPN mainly because TPN was found to be more analytically tractable than PPN.

Unfortunately, no matter what was done to make TPN perform better, its performance remained below that of PPN. Actually, this particular fact was not initially realized, because the initial analysis on TPN was done on a linearized geometry and because of the linearization much of the actual performance results were not very reliable. Later, when Guelman (1971, 1972, 1976) carried out a nonlinear analysis of PPN and TPN in a series of classic papers in the seventies, it was proved beyond any conceivable doubt that the performance of TPN was far worse than that of PPN.

9.3 Generalized TPN (GTPN)

But the TPN enthusiasts did not give up easily. They generalized the TPN law by saying that if we have the freedom of choosing our lateral acceleration direction, then why not make that a part of the guidance law and define the latax direction as being deviated by some angle from the normal to the LOS? This is shown in Figure 9.2(c). The idea was to increase the capturability performance of the guidance law further and (hopefully!) make it comparable to the PPN law. Well, it did improve matters a little bit, but not much. As an idea, it was indeed an excellent one, but when it came to performance evaluation, it did not measure up to the PPN performance. In fact some recent work in this area has shown that a realistic implementation of TPN or GTPN, that takes into account the fact that the closing velocity term is actually a time-varying one and not constant as assumed by earlier researchers, deteriorated the performance even further.

9.4 Ideal Proportional Navigation (IPN)

A further significant development in this direction occurred when another variant of the PN law called the *Ideal Proportional Navigation* was proposed. Here, the commanded latax was applied perpendicular to the *relative velocity between the missile and the target*. It is easy to see that the arguments in favour of this are similar to the arguments in favour of TPN, given above. But, the performance was significantly different. It was found that the capturability of this law was comparable to that of PPN and much better than that of TPN or its many generalizations. Only problem was that this law was as difficult to implement as TPN or any of its generalizations.

9.5 Concluding Remarks

There were quite a few other variations of the PN law which gave varying performance under various conditions. Their importance arises more from an academic point of view rather than from any real utility in the area of missile guidance. But both TPN and PPN are important – TPN, because of the vast literature that it has generated and consequently gave rise to many fundamental results in capturability which have a wider application than just missile guidance, and PPN, because of its natural application to guidance of missiles.

An important fact is that neither PPN nor TPN are the proper model for the actual performance of the PN guidance law. PPN makes the assumption that the angle-of-attack is zero, which is seldom the case. So the actual PN law is something that lies between the PPN and TPN. In fact, an analytical solution of a variant of PPN law in which the latax is applied at an angle (equal to the angle-of-attack) deviated from the normal to the velocity vector of the missile then this will give the correct performance analysis of the PN law. This is still an open and challenging problem of both academic as well as practical interest.

The earliest reference to the many variants of the PN law, but in a linearized setting, can be found in Murtaugh and Criel (1966). This paper also gives a fairly exhaustive analysis, but in a linearized framework.

Guelman's papers (1971, 1972) were perhaps the first significant results on PPN in

a nonlinear setting.

Guelman's (1976) was again the first significant result on TPN in a nonlinear setting. These results were later extended to a realistic framework with time-varying closing velocity. The generalization to TPN (GTPN) was first proposed by Yang et al. (1989). Ideal PN (IPN) was proposed and analysed by Yuan and Chern (1992).

References

1. M. Guelman: A qualitative study of proportional navigation. *IEEE Transactions on Aerospace and Electronic Systems*, AES-7, 4 (July 1971), 637-643.
2. M. Guelman: Proportional navigation with a maneuvering target. *IEEE Transactions on Aerospace and Electronic Systems*, AES-8, 3 (May 1972), 364-371.
3. M. Guelman: The closed-form solution of true proportional navigation. *IEEE Transactions on Aerospace and Electronic Systems*, AES-12, 4 (July 1976), 472-482.
4. A.S. Murtaugh, and H.E. Criel: Fundamentals of proportional navigation. *IEEE Spectrum*, 3, 12 (Dec. 1966), 75-85.
5. C.D. Yang, F.B. Hsiao, and F.B. Yeh: Generalised guidance law for homing missiles. *IEEE Transactions on Aerospace and Electronic Systems*, AES-25, 2 (Mar. 1989), 197-211.
6. P.J. Yuan and J.S. Chern: Ideal proportional navigation. *Journal of Guidance, Control and Dynamics*, 15, 5 (November-December 1992), 1161-1165.