Chapter 8

The Line of Sight Guidance Law

Module 7: Lecture 19
Introduction; LOS Guidance

Keywords. Beam Rider Guidance; CLOS Guidance; LOS Guidance

As mentioned in the previous chapter, the Line-of-sight (LOS) guidance law functions on the philosophy that if the missile remains on the line joining the launch platform and the target then it eventually must hit the target. By its very nature LOS guidance is a three-point guidance. Basically, it may be implemented in two ways:

- *Beam Rider (BR)*: Here the missile senses its own deviation from the LOS (defined by an electro-optical beam focussed continuously on the target) and uses this information to generate guidance commands. This can be thought of as similar to semi-active homing but not precisely so since the missile does not use the reflected energy from the target, but rather it uses the beam from the launch platform to the target itself.

- *Command-to-Line-of-Sight (CLOS)*: Here the radar at the launch platform tracks both the missile and the target. The deviation is sensed by the tracking radar and the guidance command is computed and sent to the missile through an uplink. This is

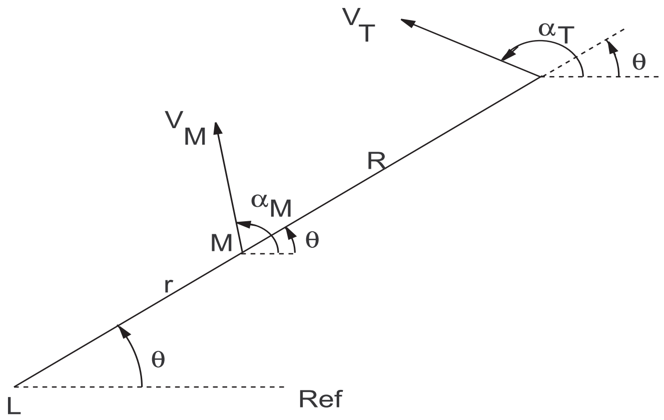

Figure 8.1: *Engagement geometry for LOS guidance*

similar to *command guidance*.

Essentially BR and CLOS are the two different mechanizations of the basic LOS guidance philosophy. In this chapter we will first study the basic LOS guidance law and then discuss its two mechanizations.

8.1 LOS Guidance

In Figure 8.1 we show the engagement geometry of the LOS guidance law. As in earlier cases we restrict our attention to a non-maneuvering target. We assume that V_T, α_T, and V_M are constants. The distance from the launch platform to the missile is LM=r and the distance from the missile to the target is MT = R. In the ideal case, the missile M should always be on the LOS (given by LT) between the launch platform and the target. Note that in LOS guidance the LOS refers to the LOS between points L and T, unlike in pursuit guidance analysis where the LOS was between the missile and the target. We have the following equations of motion,

$$V_r = \dot{r} = V_M \cos(\alpha_M - \theta) \tag{8.1}$$

$$V_R = \dot{R} = V_T \cos(\alpha_T - \theta) - V_M \cos(\alpha_M - \theta) \tag{8.2}$$

$$V_{\theta r} = r\dot{\theta} = V_M \sin(\alpha_M - \theta) \tag{8.3}$$

$$V_{\theta R} = R\dot{\theta} = V_T \sin(\alpha_T - \theta) - V_M \sin(\alpha_M - \theta) \tag{8.4}$$

Note that in this case V_r is the rate of change of the LOS separation between L and M, and V_R is the rate of change of the LOS separation between M and T. Similarly, $V_{\theta r}$ is the angular rate of the LOS between L and M, and $V_{\theta R}$ is the angular rate of the LOS between M and T.

Now, from (8.3) and (8.4) we can write,

$$\dot{\theta} = \frac{V_M \sin(\alpha_M - \theta)}{r} = \frac{V_T \sin(\alpha_T - \theta) - V_M \sin(\alpha_M - \theta)}{R} = \frac{V_T \sin(\alpha_T - \theta)}{R + r} \tag{8.5}$$

$$\Rightarrow \quad (R + r)V_M \sin(\alpha_M - \theta) = rV_T \sin(\alpha_T - \theta) \tag{8.6}$$

Differentiating (8.6) with respect to time, using the fact that $\dot{\alpha_M} = a_M/V_M$, and substituting (8.1)-(8.4) appropriately, we obtain the following sequence of equations,

$$\begin{aligned}
& (\dot{R} + \dot{r})V_M \sin(\alpha_M - \theta) + (R + r)V_M \cos(\alpha_M - \theta)(\dot{\alpha_M} - \dot{\theta}) \\
& \quad = \dot{r}V_T \sin(\alpha_T - \theta) + rV_T \cos(\alpha_T - \theta)(-\dot{\theta}) \\
\Rightarrow \quad & V_T \cos(\alpha_T - \theta)V_M \sin(\alpha_M - \theta) + (R + r)a_M \cos(\alpha_M - \theta) - V_T \sin(\alpha_T - \theta)V_M \cos(\alpha_M - \theta) \\
& \quad = V_T \sin(\alpha_T - \theta)V_M \cos(\alpha_M - \theta) - V_T \cos(\alpha_T - \theta)V_M \sin(\alpha_M - \theta) \\
\Rightarrow \quad & (R + r)a_M \cos(\alpha_M - \theta) = 2V_T V_M \sin(\alpha_T - \alpha_M) \\
\Rightarrow \quad & a_M = \frac{2V_T V_M \sin(\alpha_T - \alpha_M)}{(R + r)\cos(\alpha_M - \theta)} \qquad (8.7)
\end{aligned}$$

This gives an expression for the missile acceleration a_M required in an ideal implementation of the LOS guidance law in order to keep the missile on the LOS between the launch platform L and the target T. However, the expression for a_M given in (8.7) is a function of several time-varying quantities: R, r, α_M, and θ. Of these, it is possible to express θ and $(R + r)$ as functions of time as follows:

Consider Figure 8.1. From the geometry, since the target moves in a straight line, we can see that,

$$\tan\theta = \frac{(R_0 + r_0)\sin\theta_0 + V_T t \sin\alpha_T}{(R_0 + r_0)\cos\theta_0 + V_T t \cos\alpha_T}$$

$$\theta = \tan^{-1}\frac{(R_0 + r_0)\sin\theta_0 + V_T t \sin\alpha_T}{(R_0 + r_0)\cos\theta_0 + V_T t \cos\alpha_T} \tag{8.8}$$

$$R + r = \sqrt{(R_0 + r_0)^2 + (V_T t)^2 + 2(R_0 + r_0)V_T t \cos(\theta_0 - \alpha_T)} \tag{8.9}$$

Where, the subscript '0' denotes the initial values which are known. This gives an expression for the LOS angle θ and $R + r$ as functions of time.

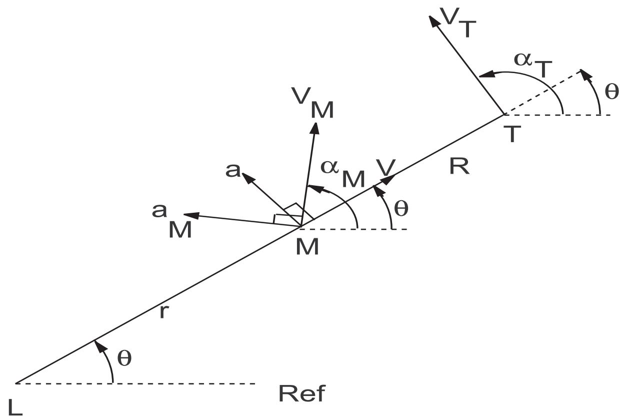

Figure 8.2: *Another engagement geometry for LOS guidance*

Now, from (8.6), we have,

$$(R + r)V_M \sin(\alpha_M - \theta) = rV_T \sin(\alpha_T - \theta)$$
$$\Rightarrow \sin(\alpha_M - \theta) = \frac{rV_T \sin(\alpha_T - \theta)}{(R + r)V_M}$$

So, α_M can be finally expressed as a function of t and the distance r of the missile from the launch platform. These expressions can be used to generate the missile latax for simulating an LOS guided missile target engagement for a non-maneuvering target.

An alternative way to obtain the missile acceleration, that is usually given in most books is also given below.

Consider the Figure 8.2. In this figure we consider the missile M to be a body moving along a rotating line (actually the LOS between the launch platform L and the target T that rotates at rate $\dot{\theta}$) with speed V. Then the acceleration experienced by M normal to the rotating line is denoted by a and is given by,

$$a = 2V\dot{\theta} + r\ddot{\theta} = \frac{d}{dt}(r\dot{\theta}) + \dot{r}\dot{\theta} \tag{8.10}$$

Note that the expression for a is obtained by adding the Coriolis acceleration $\dot{r}\dot{\theta}$ to the rate of change of the angular velocity obtained from $d/dt(r\dot{\theta})$.

Now, the actual missile velocity and accelerations are as as shown in Figure 8.2.

They are related to V and a as,

$$\begin{aligned} V &= V_M \cos(\alpha_M - \theta) \\ a &= a_M \cos(\alpha_M - \theta) \end{aligned} \tag{8.11}$$

Substituting in (8.10), we get

$$\begin{aligned} a_M \cos(\alpha_M - \theta) &= 2V_M \cos(\alpha_M - \theta)\dot{\theta} + r\ddot{\theta} \\ \Rightarrow a_M &= 2V_M\dot{\theta} + \frac{r\ddot{\theta}}{\cos(\alpha_M - \theta)} \end{aligned} \tag{8.12}$$

From Figure 8.2,

$$\dot{\theta} = \frac{V_T \sin(\alpha_T - \theta)}{R + r} \tag{8.13}$$

which on rearrangement and differentiation yields,

$$\dot{\theta}\frac{d}{dt}(R + r) + (R + r)\ddot{\theta} = -\dot{\theta}V_T \cos(\alpha_T - \theta) \tag{8.14}$$

Substituting

$$\frac{d}{dt}(R + r) = V_T \cos(\alpha_T - \theta) \tag{8.15}$$

and (8.13), we get

$$\ddot{\theta} = -\frac{2{V_T}^2 \sin(\alpha_T - \theta)\cos(\alpha_T - \theta)}{(R + r)^2} \tag{8.16}$$

Substituting these expressions for $\dot{\theta}$ and $\ddot{\theta}$ in (8.12), we get

$$a_M = \frac{2V_M V_T \sin(\alpha_T - \theta)}{R + r} - \frac{2r{V_T}^2 \sin(\alpha_T - \theta)\cos(\alpha_T - \theta)}{\cos(\alpha_M - \theta)(R + r)^2} \tag{8.17}$$

Substituting from

$$\dot{\theta} = \frac{V_M \sin(\alpha_M - \theta)}{r} = \frac{V_T \sin(\alpha_T - \theta)}{R + r} \tag{8.18}$$

we get

$$\begin{aligned} a_M &= \frac{2V_M V_T \sin(\alpha_T - \theta)}{R + r} - \frac{2V_M \sin(\alpha_M - \theta)V_T \cos(\alpha_T - \theta)}{\cos(\alpha_M - \theta)(R + r)} \\ &\Rightarrow \frac{2V_M V_T}{(R + r)\cos(\alpha_M - \theta)}\left[\sin(\alpha_T - \theta)\cos(\alpha_M - \theta) - \sin(\alpha_M - \theta)\cos(\alpha_T - \theta)\right] \\ &\Rightarrow \frac{2V_M V_T \sin(\alpha_T - \alpha_M)}{(R + r)\cos(\alpha_M - \theta)} \end{aligned} \tag{8.19}$$

which is the same as (8.7) derived earlier.