

Ascent Phase and Trajectories

The following are accomplished in a powered ascent phase of a launch vehicle

- 1) Vertical climb from the earth at least as far as necessary to escape earth's Atmosphere is negligible.
- 2) Execute necessary turn during flight so that the desired flight path angle (near '0') at the burnout is achieved.
- 3) Accelerate the payload to desired velocity for orbital injection at desired location (related to ΔV).

The above objectives are met with an attempt to maximize the payload. It should be noted that small changes in the overall ascent profile can have significant effects on the final payload that can be delivered as well as design of ascent vehicle itself.

$$\text{Vehicle acceleration, } a = \frac{dV}{dt} = \frac{T}{M} - \frac{D}{M} - g_e \sin \theta$$

1. Losses in ascent phase of a launch vehicle

The loss mechanisms which degrade the ascent performance (ΔV losses) can be identified from the equations of motion for the vehicle. These are *thrust losses*, *drag losses*, *gravity losses* and *steering losses*. The actual vehicle trajectory is determined based on minimizing these losses in ΔV subjected to the other constraints (for *e.g.* structural stress limits, bending moments, aerodynamic heating, crew comfort, range safety requirements, mission abort procedure, launch site location...). Therefore, optimization is often specific to the particular vehicle and mission objectives. Arriving at optimized trajectory many times rely on trial and error approach and on the experience of the trajectory specialist.

Thrust loss:

Recall, $T = \dot{m}_e U_e + A_e (p_e - p_a)$ where, $p_e = p_a$ refers to optimum expansion at which T is maximum. Under-expansion $p_e > p_a$ implies additional force is remaining unused and over-expansion, $p_e < p_a$ can be noted as beginning of negative contribution to thrust generation. If an engine is designed for sea level operation, it is then much less efficient than a fully expanded engine for the high altitude portions of its flight. For example due to pressure effect the Space Shuttle Main Engine have 20% loss of thrust at sea level compared to vacuum. So from the point of propulsion it is efficient to operate the vehicle at higher altitude. As most of the flight time is spent at high altitude, designers prefer large area-ratio nozzles. But large area-ratio nozzles operating at sea level produce separated exhaust airflows, performance losses, and high nozzle structural loads.

Drag loss:

This aerodynamic force which acts (opposite to the direction of motion) to resist the motion of the vehicle passing through air (or any fluid) was discussed in the previous section. The drag force on the vehicle is given by

$$D = C_D \frac{1}{2} \rho V^2 A_f$$

A_f = frontal cross sectional area of vehicle

C_D = drag coefficient (depends on vehicle shape, speed and inclination)

Since density is much smaller at higher altitude (decreases exponentially with altitude) so based on density it is preferred to operate at higher altitude (or get to high altitude as early as possible). On the other hand since drag increases as V^2 , a slow ascent will

minimize the effect of squared velocity in regions of higher density. Hence a trade-off is usually made between minimizing drag and other mandatory mission requirements.

Gravity loss:

Gravity loss denotes reduction in vehicle acceleration due to gravitational force on the launch vehicle. The gravity loss is mainly governed by the amount of time spent in vertical flight in lower altitudes. To minimize gravity loss, the vehicle should finish vertical ascent and attain horizontal flight as soon as possible. This involves higher work in lifting unburnt fuel through the gravitational field than in lifting the payload, since the fuel mass is much higher than payload mass.

As we have the burn time (t_b) given by, $t_b = \frac{I_{sp}}{(T/M)}$

The two extreme limits are $T/M = \infty$ (not good when there is atmosphere) and $T/M = 1$ (expending lot of fuel without much benefit)

Steering loss:

This loss is incurred from vectoring of the rocket thrust. Thrust vectoring results in a force component normal to the flight direction therefore fails to add to the vehicle velocity. Therefore, any turning of the vehicle at is undesirable. If done, it should be done early, at low vehicle velocity (at larger velocity larger angle for thrust vectoring is required).

Note: in previous class we derived equation of motion with thrust vector in line with the velocity vector and vehicle axis. In general (as shown below) this need not be so.

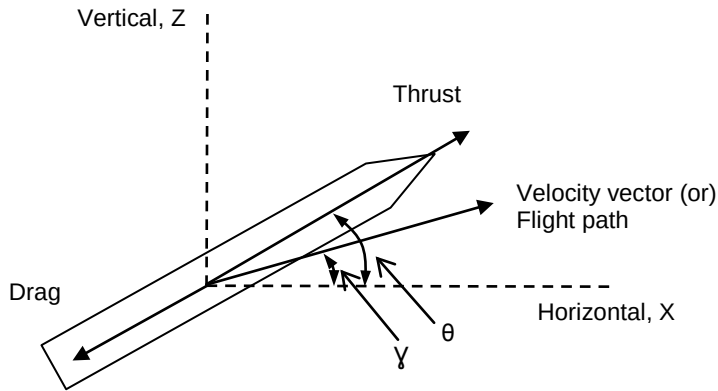


Figure 1 Aerodynamic forces acting on a rocket vehicle

An early pitch over to near-horizontal flight, followed by a long, shallow climb to altitude minimizes steering and gravity losses but dramatically increases drag and aggravates the problem by reducing the operating efficiency of the power plant. Similarly the steep vertical climb can minimize drag losses while obtaining maximum engine performance, at the price of expending considerable amount of fuel to go in a direction that is ultimately not desired.

2. Launch vehicle trajectories

Typical powered flight will begin with vertical lift off for a few hundred meters to clear the launch pad followed by a pitch maneuver. In general launch vehicle guidance system will execute maneuvers in certain plane (could be structural considerations/configuration as with Titan3 and Space shuttle) so a roll maneuver is performed prior to pitch program. Pitch maneuver is often specified in terms of pitch angle and angular rate as a function of time. There are special cases of pitch rate specification:

- 1) Constant turn-over rate.
- 2) Constant inclination to horizontal (i.e. turn-over rate = 0)
- 3) Pitch rate from gravity turn.

Most trajectories can be approximated by combinations of the segments of above special trajectories.

Note on gravity turn trajectory: Here the thrust vector is maintained parallel to the velocity vector throughout the flight path. With initial flight angle (θ) specified the vehicle will gradually turn toward horizontal under influence of gravity. With zero angle of attack acceleration (hence ΔV) is maximum (though, strictly true in absence of atmosphere). In case of ascent in presence of atmosphere a rocket using gravity turn would spend too much time in lower levels of atmosphere, where other factors (like drag) will act to offset the lack of steering loss. In general, gravity turns may comprise portions of an ascent profile but not complete (with exception to ascent from an airless planet)

The flight path of gravity turn trajectory is computed numerically. The differential equations of motion in terms of horizontal and vertical components of acceleration are (same as constant pitch angle case)

$$\frac{dV_z}{dt} = \frac{T \sin \theta}{M} - g \quad (1)$$

$$\frac{dV_x}{dt} = \frac{T \cos \theta}{M} \quad (2)$$

Here the pitch angle ' θ ' is not constant and equals to flight angle ' γ ' which also varies.

The flight angle ' γ ' is given by

$$\tan \gamma = \frac{V_z}{V_x}$$

Substituting for $\sin \gamma$ and $\cos \gamma$ in terms of velocity ratios

$$\frac{dV_z}{dt} = \frac{T}{M} \frac{V_z}{V} - g \quad (3)$$

$$\frac{dV_x}{dt} = \frac{T}{M} \frac{V_x}{V} \quad (4)$$

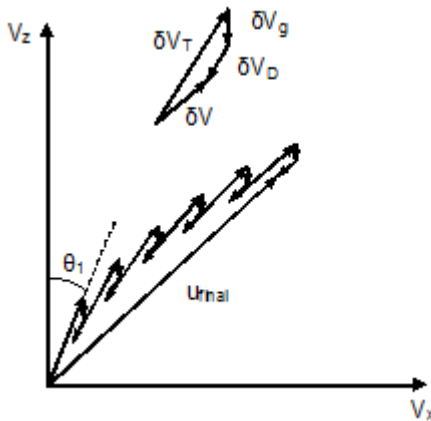
A manipulation is carried out after multiplying eqn. (3) with V_z and eqn. (4) by V_x and adding them leads to an expression dV / dt involving total velocity, $V = \sqrt{(V_x^2 + V_z^2)}$

$$\frac{dV}{dt} = \frac{T}{M} - g \sin \gamma \quad (5)$$

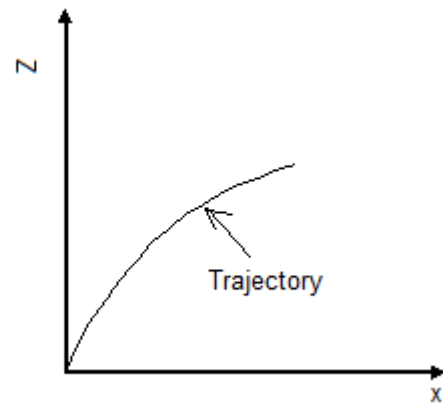
Again by multiplying eqn. (3) with V_x and eqn. (4) by V_z and subtracting leads to the following equation for $d\gamma / dt$

$$\frac{d\gamma}{dt} = -\frac{g}{V} \cos \gamma \quad (6)$$

The graphical representation of a typical gravity turn trajectory can be seen from Fig. 2. Here the initial pitch angle is θ_1 with respect to local gravity vector. The thrust and drag forces are always parallel and the gravity causes turning of the vehicle. The trajectory calculations are performed assuming constant values of D , M , g & θ over the time interval δt . The velocity change δV is given as a vector sum of three terms: δV_T , δV_g and δV_D (illustrated in Figure 2). Each of these terms is calculated from the velocity at the beginning of δt and the accuracy depends on the size of δt compared to burn time t_b . Having obtained velocity (V) as a function of time, the trajectory is obtained using $\delta y = V_z \delta t$ and $\delta x = V_x \delta t$ at each δt (shown qualitatively in the subfigure on the right side of Figure 2).



Velocity coordinates



Position coordinates

Figure 2 Approximate trajectory estimation of a vehicle in a gravity turn

Typical rocket vehicle acceleration with time and the dynamic pressure experienced by the vehicle is shown in Fig. 3. While acceleration increases due to reduced rocket mass, the dynamic pressure reaches a peak before dropping again. The peak dynamics pressure is an important design consideration (structural integrity of the rocket) in the trajectory design.

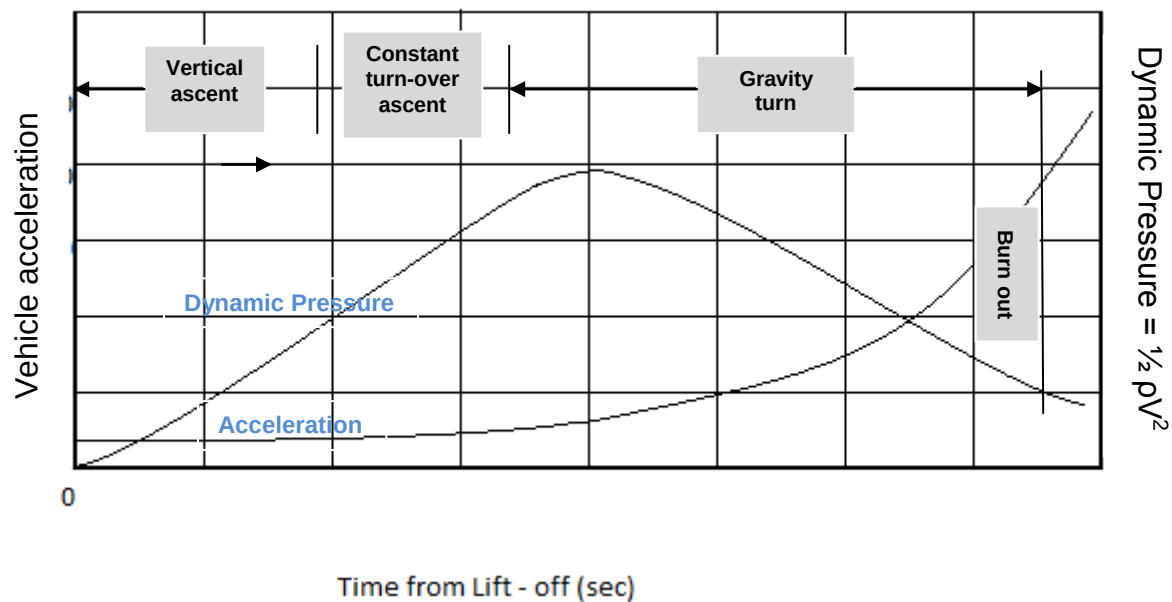


Figure 3 A typical rocket vehicle acceleration and dynamic pressure with time

Further reading:

1. Mechanics and Thermodynamics of Propulsion by Philip G Hill & Carl Peterson
Chapter # 10 (Section 10.3)