Module 8: Transonic Flow

Lecture 35: Transonic Flow

The flow pattern and the forces exerted on a moving body in this range differ considerably from those in the other speed ranges. Shock waves are an essential feature of transonic flow and their appearance on a moving body leads to a rapid increase in drag-coefficient with increasing Mach number – myth of the sonic barrier. In transonic flow, and also in hypersonic flow, it is often impossible to separate clearly the non-viscous flow from the effects of viscosity. The chief feature of the flow in this regime is the 'shock-wave boundary layer interaction'; an interplay between the strength and position of shock waves on a body and of boundary layer character, separation, etc.

If the fluid is assumed to be frictionless, the flow slips past solid boundaries. The flow is called transonic if both subsonic and supersonic regions are present in the flow field. If the free stream Mach number is increased continuously from zero, the transonic range begins when the highest local Mach number reaches unity and ends when the local lowest Mach number reaches unity.

Considering thin or slender bodies in the sense of small perturbation theory, and the characteristic body thickness ratio τ , the flow past such a body is transonic if the transonic parameter falls within the range

$$-1 \leq \frac{M_\infty^2 - 1}{\left[(\gamma + 1)\tau M_\infty^2\right]^{2/3}} \leq 1$$

Transonic flow past wedge sections

Consider flow past a simple wedge of vertex angle 2θ and length c followed by a straight section. As M_∞ increases from subsonic values, a shock wave system appears near the shoulder. The main nearly normal shock grows and moves downstream as M_∞ increases toward unity. At $M_\infty = 1$ it has moved to downstream infinity and a second shock wave has appeared at minus infinity. With further increase of M_∞ this bow wave approaches the wedge vertex, finally attaches and becomes straight. The flow regime then is fully supersonic. Thus for $M_\infty < 1$, there is a local supersonic region ahead of the main shoulder shock, while for $M_\infty > 1$ there is a local subsonic region behind the detached bow shock.

No discontinuous changes occur in the local Mach number distribution from the vertex to the shoulder in the whole range. The change over from subsonic to supersonic flow is smooth and continuous. The behaviour of the drag coefficient C_D is also smooth. C_D varies rapidly only if plotted as a function of Mach number which is an improper choice for this range. The proper independent variables is

$$\chi = \frac{1 - M_\infty^{\;2}}{\left[(\gamma+1)\tau M_\infty^{\;2}\right]^{2/3}}$$ and the dependent one is

$$\tilde{C}_D = \frac{C_D\left[(\gamma+1)M_\infty^{\;2}\right]^{1/3}}{\tau^{5/3}}$$

General features

1. Sonic velocity $(M = 1)$ occurs at the shoulder throughout the whole transonic range. Sonic velocity cannot be reached on any flat part of the body.
2. The local Mach number M becomes stationary as M_∞ passes through unity, i.e. $\left(dM/dM_\infty\right)_{M_\infty=1} = 0$.

   Consider M_∞ a little larger than unity. The detached bow wave will stand far ahead of the body and nearly normal to the flow. The Mach number ahead of the body is the Mach number M_2 downstream of a weak normal shock. It is related to the upstream Mach number $M_\infty \left(= M_1\right)$ by

   $$M_\infty - 1 = 1 - M_2$$

   As $M_\infty \to 1$ from supersonic values $M_2 \to 1$ from subsonic values. Hence, as far as local Mach number on the wedge is concerned, the conditions just below and just above $M_\infty = 1$ are identical. Consequently, the local Mach number M at a point on the wedge satisfies

   $$\left(\frac{dM}{dM_\infty}\right)_{M_\infty=1} = 0$$

Now
$$C_p = \frac{2}{\gamma M_\infty^{\;2}}\left[\left(\frac{2+(\gamma-1)M_\infty^{\;2}}{2+(\gamma-1)M^2}\right)^{\gamma/\gamma-1} - 1\right]$$

$$\Rightarrow \log\left(\frac{\gamma}{2}M_\infty{}^2 C_p + 1\right) = \frac{\gamma}{\gamma-1}\left\{\log[2+(\gamma-1)M_\infty{}^2 - \log[2+(\gamma-1)M^2\right\}$$

Differentiating wrt M_∞ and letting $M_\infty \to 1$ gives

$$\frac{\gamma C_p{}^* + \frac{\gamma}{2}\left(\frac{dC_p}{dM_\infty}\right)_{M_\infty=1}}{1+\frac{\gamma}{2}C_p{}^*} = \frac{2\gamma}{\gamma+1}, \quad C_p{}^* = (C_p)_{M_\infty=1}$$

or
$$\left(\frac{dC_p}{dM_\infty}\right)_{M_\infty=1} = \frac{4}{\gamma+1}\left(1-\frac{1}{2}C_p{}^*\right)$$

A point on the wedge where the pressure coefficient is C_p contributes $C_p\theta$ to the drag coefficient of the wedge. Hence, the slope of C_D vs. M_∞ curve at the sonic point is

$$\left(\frac{dC_D}{dM_\infty}\right)_{M_\infty=1} = \frac{4\theta}{\gamma+1} - \frac{2C_D{}^*}{\gamma+1}, \quad C_D{}^* = (C_D)_{M_\infty=1}$$

When the wedge angle is small, in the sense of small perturbation theory $\left(\frac{u}{U_\infty} << 1\right)$, it is permissible to neglect $\frac{1}{2}C_p{}^*$ compared to 1. Within the framework of small perturbation theory

$$\left(\frac{dC_p}{dM_\infty}\right)_{M_\infty=1} = \frac{4}{\gamma+1}, \quad \text{and} \quad \left(\frac{dC_D}{dM_\infty}\right)_{M_\infty=1} = \frac{4\theta}{\gamma+1}$$

In terms of transonic similarity parameters

$$\tilde{C}_p = C_p \frac{\left[(\gamma+1)M_\infty{}^2\right]^{1/3}}{\theta^{2/3}}$$

$$\tilde{C}_p = C_D \frac{\left[(\gamma+1)M_\infty{}^2\right]^{1/3}}{\theta^{5/3}}, \quad \chi = \frac{M_\infty{}^2 - 1}{\left[(\gamma+1)M_\infty{}^2\theta\right]^{2/3}}$$

$$\Rightarrow \left(\frac{d\tilde{C}_p}{d\chi} \right)_{\chi=0} = \left(\frac{d\tilde{C}_D}{d\chi} \right)_{\chi=0} = 2$$

The excellent agreement between theory and experiment demonstrates the validity of the approximations made in the transonic small perturbation theory and in the derivation of the transonic similarity laws.

More important for theoretical applications are wings of finite span and wing-body combinations. Good use can be made of the similarity rules and of slender body theory. The similarity rules are used to correlate experimental data with each other and with the few theoretically known results. Slender body theory can be used to obtain the lift and the drag due to lift, of low aspect ratio configurations, right through the transonic regime.