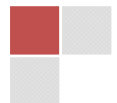


Module 2:One-dimensional gas dynamics

Lecture 8: One-dimensional gas dynamics (Contd.)



Pressure and particle velocity in a sound wave

The pressure disturbance accompanying the density wave for a perfect gas is

$$\frac{p}{p_1} = 1 + \gamma \bar{s} \Rightarrow \frac{p - p_1}{p_1} = \gamma \bar{s}$$

$\Rightarrow$ Pressure wave has the same shape as the density wave, differing by a constant factor γ .

As the wave progresses through the fluid, the pressure disturbance sets the fluid in motion, giving it a velocity u which is called the particle velocity and, in general, is much smaller than the wave speed a_1 .

A simple wave $\bar{s} = F(x - a_1 t)$, propagating to the right, produces a velocity disturbance

$$u = a_1 F'(x - a_1 t) = a_1 \bar{s}'$$

In a left ward propagating wave $u = a_1 G'(x + a_1 t) = -a_1 \bar{s}'$. The various parts of the wave are called condensation and rarefaction depending on whether the density is higher or lower than the undisturbed density ρ_1 .

The effect that the wave produces on the fluid depends on the gradient of this density and pressure distribution and on the direction of motion of the wave. Thus the portion of the wave that increases the density as it passes is called a compression and that which decreases the density is called an expansion. The corresponding distributions of particle velocity are

$$u = \pm a_1 \bar{s}'$$

for left and right propagating waves respectively. It may be seen that a compression accelerates the fluid in the direction of wave motion, whereas an expansion decelerates it. The non-simple wave is a superposition of two simple waves and the relation between particle velocity and

density is $\frac{u}{a_1 \bar{s}'} = \frac{F' - G'}{F' + G'}$.

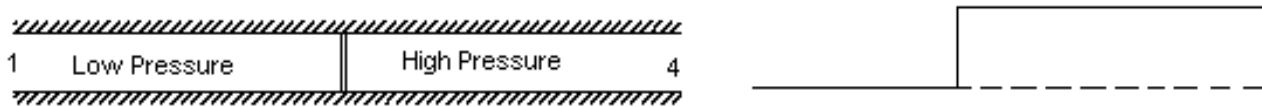
In the limit of vanishingly small disturbances the perturbation quantities may be written in differential form with u and $\bar{s}$ replaced by du and $d\rho/\rho_1$. The relations then become



$$\left. \begin{aligned} du &= \pm a_1 \frac{d\rho}{\rho_1} \\ \frac{dp}{p_1} &= \gamma \frac{d\rho}{\rho_1} \end{aligned} \right\} \Rightarrow dp = \frac{\gamma p_1}{\rho_1} d\rho = \pm \rho_1 a_1 du$$

'Linearized' Shock Tube

Shock tube is a simple device consisting of a tube that is divided by a membrane or diaphragm into two chambers in which pressures are different. A wave motion is set up when the diaphragm is suddenly removed or broken. If the pressure difference is very small, the wave motion may be approximated as isentropic and can be described by the acoustic equations. The shock tube in such a case is called acoustic or linearized.



At $t = 0$, immediately after the membrane is removed the wave has a shape of 'step' distribution. The particle velocity at this instant is 'zero' everywhere. The wave at $t = 0$ is described by

$$\bar{s}(x, 0) = F(x) + G(x) = \bar{s}_0(x) = \begin{cases} \bar{s}_4 & (x > 0) \\ 0 & x < 0 \end{cases}$$

$$u(x, 0) = a_1 F(x) - a_1 G(x) = 0$$

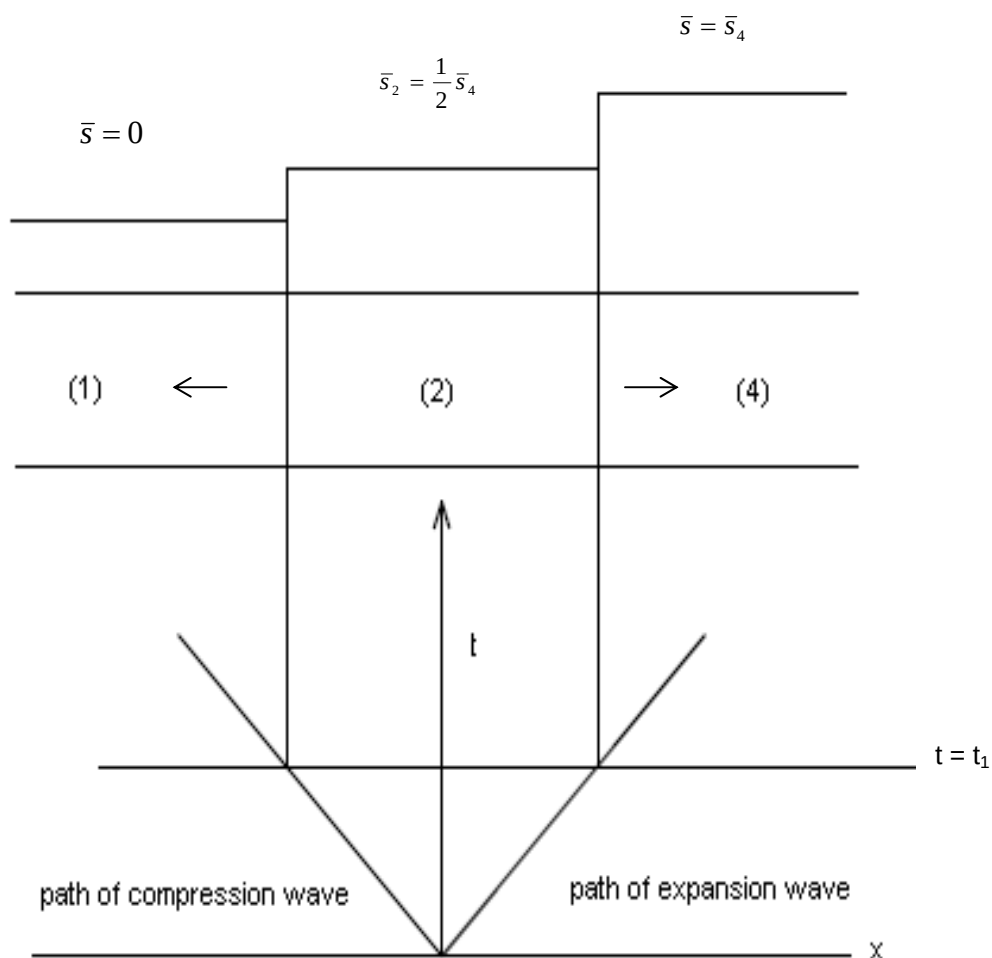
Simultaneous solution gives $F(x) = G(x) = \frac{1}{2} \bar{s}_0(x) = \begin{cases} \frac{1}{2} \bar{s}_4 & x > 0 \\ 0 & x < 0 \end{cases}$

Hence, the motion at any subsequent time is given by

$$\bar{s}(x, t) = \frac{1}{2} \bar{s}_0(x - a_1 t) + \frac{1}{2} \bar{s}_0(x + a_1 t) = \begin{cases} \bar{s}_4 & x > a_1 t \\ \frac{1}{2} \bar{s}_4 & -a_1 t < x < a_1 t \\ 0 & x < -a_1 t \end{cases}$$



$$u(x,t) = \frac{1}{2}a_1\bar{s}_0(x-a_1t) - \frac{1}{2}a_1\bar{s}_0(x+a_1t) = \begin{cases} 0 & x > a_1t \\ -\frac{1}{2}a_1\bar{s}_4 & -a_1t < x < a_1t \\ 0 & x < -a_1t \end{cases}$$



A compression wave is propagated to low pressure side and an expansion wave of equal strength to the high-pressure side.

