<u>Module: 1</u>
<u>Lecture: 1</u>

Aerospace Structural Dynamics

<u>Introduction to Aircraft Structural Dynamics</u>

Content

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- Force response of SDOF linear system
- Response due to support system
- Force response of SDOF due to arbitrary load in time scale
- Numerical solutions
- System with 2DOF – Free & Force response
- Co – ordinate Coupling
- Eigen value analysis / Modal Analysis of MDOF

(b) Continuous System

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    a. Vibration Analysis of String
    b. Axial Vibration of Bar
    c. Torsional Vibration of Rod
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- Approximate Method
    a) Work &Energy
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        - Rayleigh's Method
        - Rayleigh-Ritz Method
        - Galerkin Method

    d) Lagrange's Equation of Motion
        - Introduction to Finite Element Method

References

1) Theory of Vibration with application – W. T Thomson
2) Elements of Vibration Analysis – L. Meirovitch
3) Structural Dynamics: Theory & Computation – Mairo Paz
4) Dynamics of Structures – Anil Chopra

Introduction to Aircraft Structural Dynamics

History of vibration starts with the invention of the musical instrument like whistle and drum. Musical instruments are appreciated by Chinese, Japanese, Hindus and Egyptians as long ago as 4000 B.C.. The philosopher and mathematician Pythagoras (582-507 B.C.) is the first person to conduct scientific experiments on vibrating strings using simple apparatus called a monochord. Though the concept of pitch of sound was developed by the time of Pythagoras the relation between the pitch and frequency was not clear until the sixteenth century. The oldest book available on the subject of music is written by Aristoxenus, a pupil of Atistotle and the name of the book is 'Elements of Harmony' (320 B.C.). After this Euclid wrote 'Introduction of Harmony' in 300 B.C. In the year 20 B.C. famous roman architect Vitruvius wrote 'D Architectur Libri Decem' on the acoustic properties of theaters.

Understanding vibration involving acoustic effects like designing a musical instrument or an auditorium is followed by the invention of the worlds first seismograph in the year 132 A.D. Seismograph is an instrument to measure earthquakes. The famous historian and astronomer,

Figure 1: Replica of Zhang Heng's seismoscope

Zhang Heng, invented the wine-jar shaped instrument.

Afterwards in the seventeenth century, founder father of modern experimental science Galileo Galilei (1564-1642) discussed about vibrating bodies in his book 'Discourses Concerning Two New Sciences' published in 1638. In this book he has discussed about the relationship between frequency, length, tension and density of a vibrating stretched string. The book 'Harmonicorum Libre' (1636) authored by the father of acoustics Marin Mersenne (1588-1648), came two year before because of the prohibition imposed by the Inquisitor of Rome until 1638. This experimental study is further carried by Robert Hooke (1653-1703) to find the relation between the pitch and frequency of vibration of a string. Later Joseph Sauveur (1653-1716) coined the term "acoustics", John Wallis (1616-17030) observed the mode shape phenomenon.

The foundation stone of theoretical study of vibration is layed by Brook Taylor (1685-1731). Natural frequency of vibration derived by Taylor matches well with experimental values observed by Galileo Galilei and Mersenne. The derivation is further enhanced by Daniel Bernoulli (1700-17820), Jean D'Alembert (1717-1783) and Leonard Euler (1707-1783). Credit of studying first time the vibration of thin beams with different boundary conditions goes to the famous mathematicians Euler (1744) and Bernoulli (1751) and their modeling is known as Euler-Bernoulli thin beam theory. Charles Coulomb have studied and derived the equation of motion for the torsional vibration of the suspended cylinder first time in the year 1784. He found that the period of oscillation is independent of the angle of twist.

In case of theory related to the vibration of plate, it is first derived by the German scientist E.F.F. Chaldni (1756-1824). It is later corrected by G.R. Kirchhoff (1824-1887) by providing the proper boundary conditions in the year 1850. In relation to this the problem related to the vibrating membrane (sound emitted by drum) is first studied by Simon Poisson (1781-1840). In 1877 Lord Baron Rayleigh published his classic book on 'Theory of Sound'. The effort continues with the contributions of the famous scientists like Stodola, De Laval, Timoshenko, Mindlin and many other scientist.

From the history of development of theory of vibration we may start thinking about the inspiration behind the development. In a human body, three most prime interactions are through eye, ear and voice. Eye can differentiate between the different frequencies and wave lengths and make us see anything. Ear absorbs sounds of different frequencies and distinguish those properly from each other. In case of our voice we vibrate our voice-box in different frequencies and amplitude with proper co-ordination to communicate. If we look back to the activities we were discussing, all are time dependent phenomenon and also periodic in nature. These may be categorized as the type of dynamic behaviour where oscillation takes place about a certain equilibrium position and it may be defined as the vibratory motion or vibration. To understand the properties and characteristics related to vibration we need to model a physical system. A physical system in general is very complex in nature and difficult to analyse. It consists of various components. We need to identify the physical properties of the components. More precisely the properties governing the dynamic behaviour of the system must be modeled mathematically. Parameters are defined to represent physical properties of a system. A physical system is continuous with distributed parameters. There are two different ways of modeling a physical system a) lumped parameter system and b) continuous or distributed parameter system. Before we go further into the analysis of problems related to vibration, let us observe a few remarkable examples of vibration through the links provided below.

Vibration of components of ear to facilitate listening. Cochlea vibration (animation)
http://www.bbc.co.uk/science/humanbody/body/factfiles/hearing/hearing_animation.shtml
http://highered.mcgraw-hill.com/sites/0072495855/student_view0/chapter19/animation__effect_of_sound_waves_on_cochlear_structures__quiz_2_.html

Vibratory conveyor system
http://carriervibrating.com/videos/vibrating-conveyors-video
http://www.youtube.com/watch?v=MgHOXdXEdK0
http://www.key.net/files/videos/Marathon-vibratory-conveyor.wmv

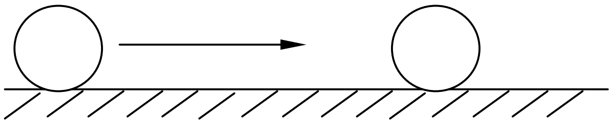
Electric tooth brush
http://www.explainthatstuff.com/electrictoothbrush.html

Disaster at Tacoma Narrows
http://www.youtube.com/watch?v=j-zczJXSxnw

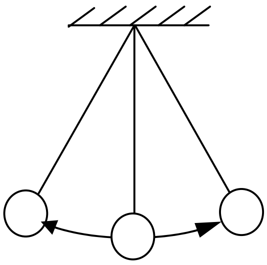
Flutter problems
http://www.youtube.com/watch?v=pEOmCkZyXzk
http://www.youtube.com/watch?v=qpJBvQXQC2M&NR=1&feature=fvwp
http://www.youtube.com/watch?v=DU7cOXgfqKE
http://www.youtube.com/watch?v=kQI3AWpTWhM&NR=1
http://www.youtube.com/watch?v=MEUGeEz6vd4

Aerospace Structural Dynamics

We assume the aircraft to be a rigid body for the study of aircraft stability and control



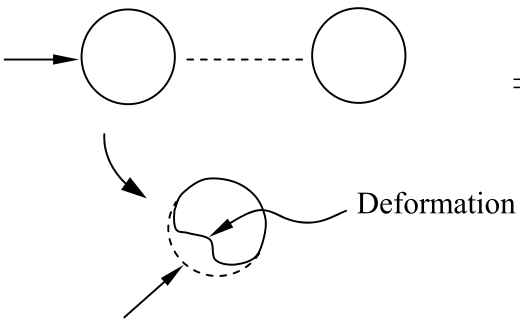
$\Rightarrow$ Translational Motion



$\Rightarrow$ Oscillatory Motion

In case of vibration $\rightarrow$ 3 types of motion

1. Rigid body motion
2. Elastic deformation
3. Periodic motion



$\Rightarrow$ Rigid body motion

- Based on this deformation, we design the ball
- In vibration there has to be oscillation as well $\Rightarrow$ Rigid body motion + elastic deformation + oscillatory motion $\rightarrow$ VIBRATION

Consequences of Vibration

- Passenger and Crew comfort (ideal zero vibration)
- Vibration gives rise to noise → Undesirable
- Change of structural integrity
- Reliability of vibration sensitive equipments

1) Discomfort depends on

a) Frequency
b) Amplitude
c) Duration and
d) Direction

> 4 Hz vertical vibration

Most Sensitive Range → 5 – 15 Hz

< 4 Hz Horizontal Vibration

Sl. No	Subjective Rating	Peak Velocity (m/sec) (RMS)	g at 16 Hz (RMS)
1	Comfortable	0.17	0.045
2	A little comfort	0.17 – 0.35	0.045 – 0.090
3	Fairly comfort	0.28 – 0.55	0.072 – 0.144
4	Uncomfortable	0.45 – 0.90	0.115 – 0.280
5	Very uncomfortable	0.70 – 1.45	0.18 – 0.36
6	Extremely uncomfortable	0.70 – 1.45	0.39

2) Structural Integrity

For the structure not to be in total failure (Some level of failure can be present)

→ Magnification of stresses σ_x or σ_y
→ Structural fatigue
→ Large deflection

3) Equipment Reliability

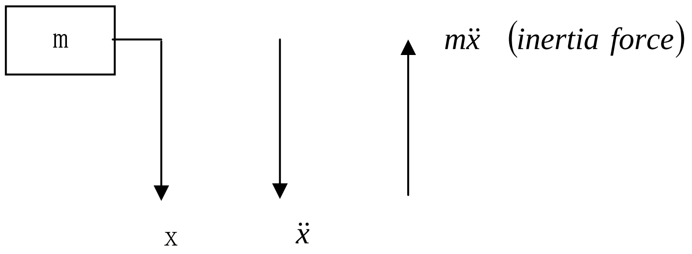
You cannot write the equation unless there is equilibrium in the dynamic system.

D'Alembert's Principle (1743)

In the case of static elastic deformation, the internal elastic forces, try to restore the system to the position of equilibrium.

In the case of pendulum, restoring force is force of gravity (Rigid body motion).

In the case of vibration, D'Alembert reasoned that the sum of the forces acting on a particle result in acceleration $\ddot{x}$ (in the same direction). The inertial force can be computed by multiplying by the mass of the body, $F = m\ddot{x}$. If the force so computed is applied to the body at its mass center in the direction opposite to its acceleration, the dynamic problem is reduced to one of statics. This is defined so-called D'Alembert principle.



A mechanical system in motion is always in the state of dynamic equilibrium under all forces including the inertia forces.

All forces including motion dependent forces i.e. elastic inertia and damping forces as well as external forces.
→ in case of vibration, mass is important.