Introduction to Aerospace Propulsion

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Lecture No - 22

In this lecture ...

- One dimensional compressible flows
- Stagnation properties
- Speed of sound and Mach number
- One-dimensional isentropic flow
- Variation of fluid velocity with flow area
- Isentropic flow through nozzles
  - Converging nozzles
  - Converging–diverging nozzles

One dimensional compressible flows

- Most of the analysis we considered so far neglected density variations.
- Flows that involve significant density variations: compressible flows.
- We shall consider one-dimensional compressible flows for an ideal gas with constant specific heats.
- Are frequently encountered in devices that involve the flow of gases at very high velocities.

Stagnation properties

- Enthalpy represents the total energy of a fluid in the absence of potential and kinetic energies.
- For high speed flows, though potential energy may be negligible, but not kinetic energy.
- Combination of enthalpy and KE is called stagnation enthalpy (or total enthalpy)

$$h_0 = h + V^2/2 \qquad \text{(kJ/kg)}$$

Stagnation enthalpy Static enthalpy Kinetic energy

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Stagnation properties

- Consider a steady flow through a duct (no shaft work, heat transfer etc).
- The steady flow energy equation for this is: $h_1 + V_1^2/2 = h_2 + V_2^2/2$

  or, $h_{01} = h_{02}$
- That is in the absence of any heat and work interactions, the stagnation enthalpy remains a constant during a steady flow process.

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Stagnation properties

- If the fluid were brought to rest at state2,

$$h_1 + V_1^2/2 = h_2 = h_{02}$$

- The stagnation enthalpy represents the enthalpy of a fluid when it is brought to rest adiabatically.
- During a stagnation process, the kinetic energy of a fluid is converted to enthalpy (internal energy flow energy), which results in an increase in the fluid temperature and pressure.

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Stagnation properties

- When the fluid is approximated as an ideal gas with constant specific heats,

$$c_p T_0 = c_p T + V^2/2$$

$$\text{or,} \quad T_0 = T + V^2/2c_p$$

- T_0 is called the stagnation temperature and represents the temperature an ideal gas attains when it is brought to rest adiabatically.
- The term $V^2/2c_p$ corresponds to the temperature rise during such a process and is called the dynamic temperature.

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Stagnation properties

- The pressure a fluid attains when brought to rest isentropically is called the stagnation pressure, P_0.
- For ideal gases, from isentropic relations,

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\gamma/(\gamma-1)}$$

Similarly, for density we have,

$$\frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(\gamma-1)}$$

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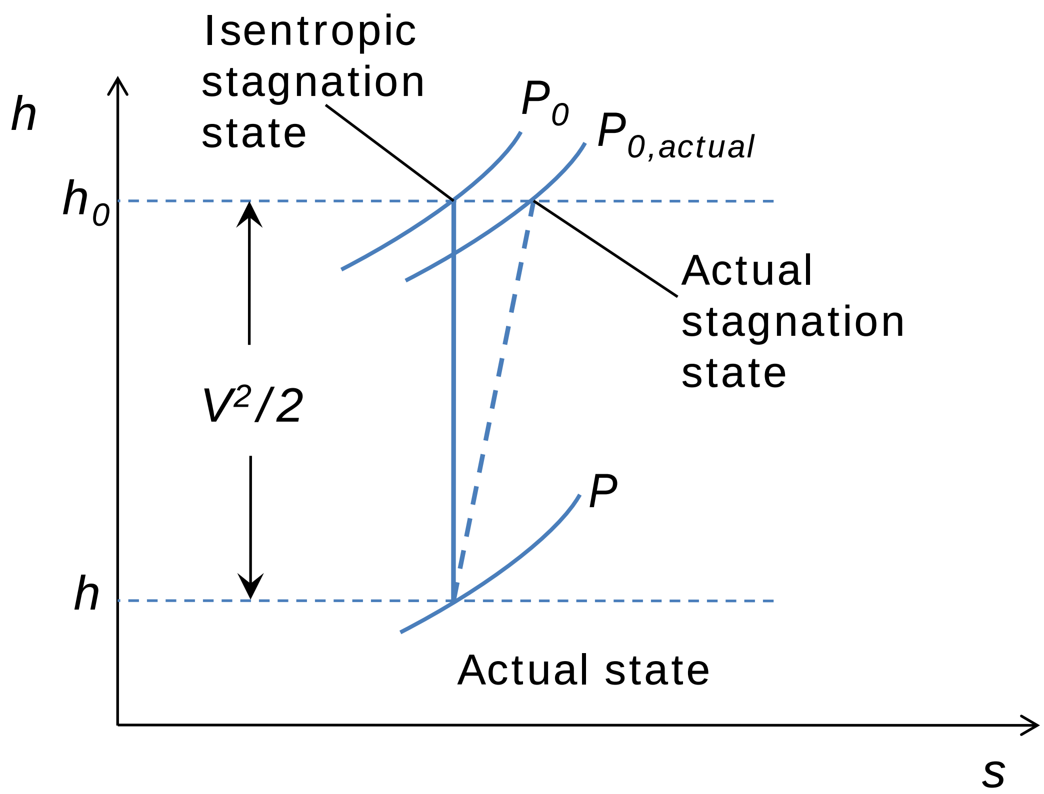
Stagnation properties



The actual state, actual stagnation state, and isentropic stagnation state of a fluid on an *h*-*s* diagram.

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Speed of sound and Mach number

- Speed of sound is the speed at which an infinitesimally small pressure wave travels through a medium.
- For an ideal gas, speed of sound, *c*, can be shown to be the following:

$$c = \sqrt{\gamma RT}$$

- Speed of sound is therefore a function of temperature.

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Speed of sound and Mach number

- Mach number is the ratio of actual velocity of the object/fluid to the speed of sound

$$\text{Mach number, } M = V/c$$

- Mach number is a function of the ambient temperature. So two objects moving at same speeds may have different Mach numbers depending upon the ambient temperature.
- $M=1$: Sonic flow, $M>1$: Supersonic flow; $M<1$ Subsonic flow; $M>>1$ Hypersonic flow; $M\approx1$: Transonic flow

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Variation of fluid velocity with flow area

- Consider mass balance for a steady flow process:

$$\dot{m} = \rho A V = \text{constant}$$

Differentiating and dividing the resultant equation by the mass flow rate,

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

Steady flow energy equation assuming W, Q, KE, PE ≅ o

$$h + \frac{V^2}{2} = 0 \quad \text{or,} \;\; dh + VdV = 0$$

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Variation of fluid velocity with flow area

Also, $Tds = dh - vdP$

For isentropic flows, $dh = vdP = dP/\rho$

Hence, $$\frac{dP}{\rho} + VdV = 0$$

Combining this and the earlier equations,

$$\frac{dA}{A} = \frac{dP}{\rho}\left(\frac{1}{V^2} - \frac{d\rho}{dP}\right)$$

Since it is known that, $\left(\partial\rho/\partial P\right)_s = 1/c^2$, rearranging,

$$\frac{dA}{A} = \frac{dP}{\rho V^2}\left(1 - M^2\right)$$

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Variation of fluid velocity with flow area

The above equation can also be written as:

$$\frac{dA}{A} = -\frac{dV}{V}\left(1 - M^2\right)$$

This equation governs the shape of a nozzle or a diffuser in subsonic or supersonic isentropic flow. Since *A* and *V* are positive quantities, it follows that,

For subsonic flows (M<1), $dA/dV < 0$
For supersonic flows (M>1), $dA/dV > 0$
For sonic flows (M=1), $dA/dV = 0$

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Variation of fluid velocity with flow area

- To accelerate a fluid, a converging nozzle is required at subsonic velocities and a diverging nozzle at supersonic velocities.
- The highest velocity that can be achieved in a converging nozzle is the sonic velocity.
- To accelerate to supersonic velocities, a diverging section after the converging section is required: Converging-diverging nozzle.

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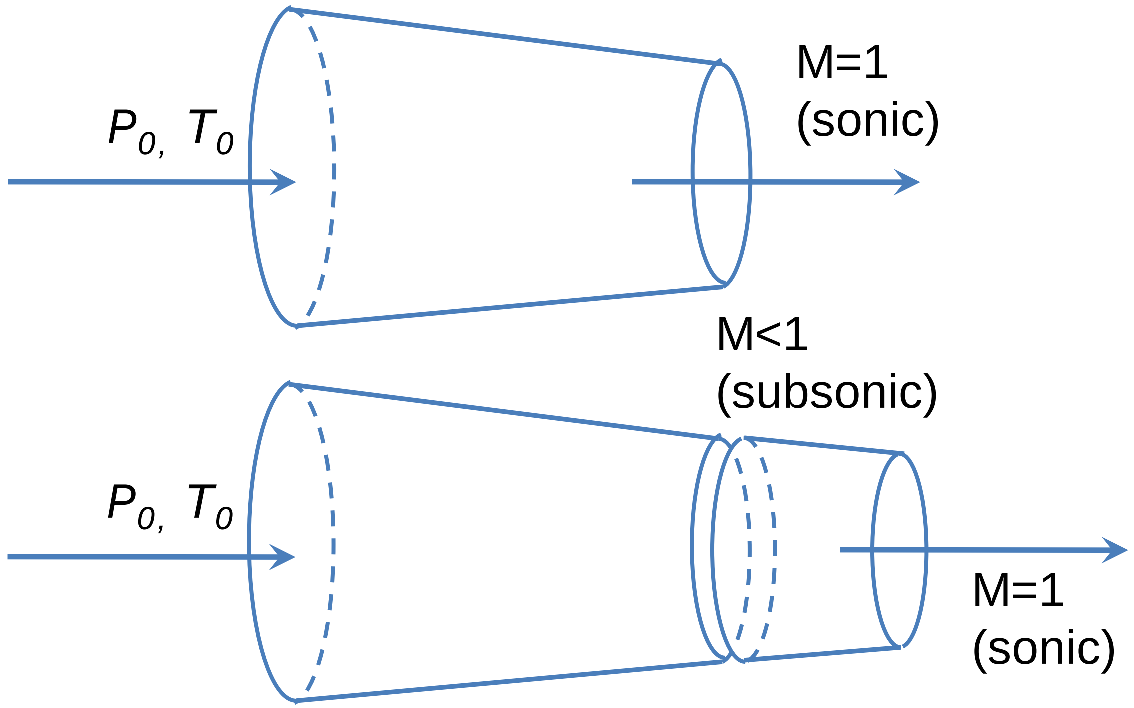
Variation of fluid velocity with flow area



Sonic velocity will occur at the exit of the converging extension, instead of the exit of the original nozzle, and the mass flow rate through the nozzle will decrease because of the reduced exit area.

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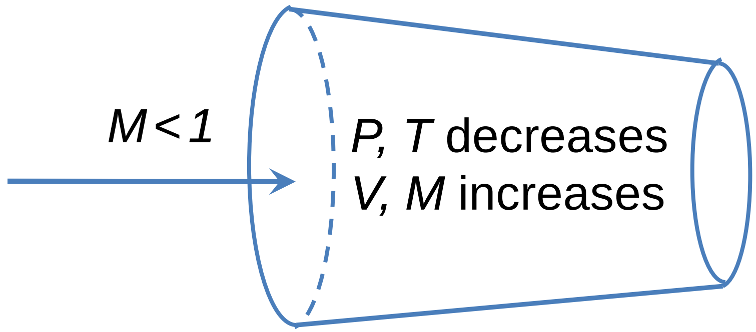
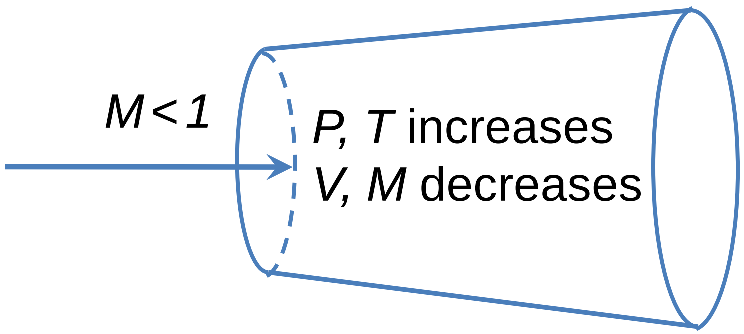
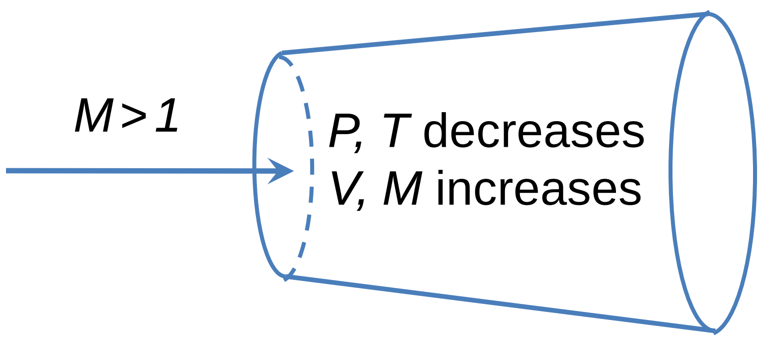
Variation of fluid velocity with flow area



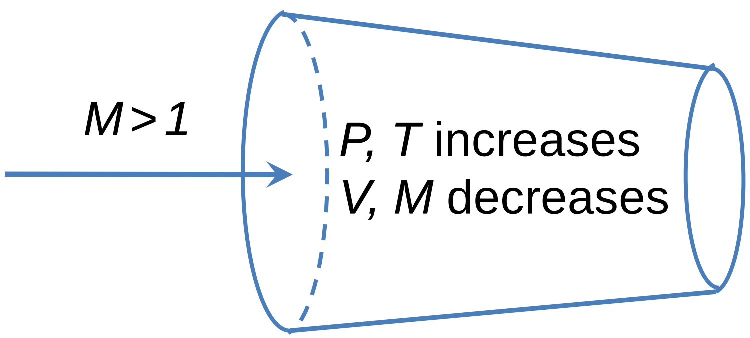
Subsonic nozzle



Subsonic diffuser



Supersonic nozzle



Supersonic diffuser

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Property relations for isentropic flow of ideal gases

- We know that:

$$T_0 = T + \frac{V^2}{2c_p} \quad \text{or,} \quad \frac{T_0}{\mathrm{T}} = 1 + \frac{V^2}{2c_p T}$$

$$\text{Since, } c_p = \frac{\gamma R}{\gamma - 1}, \quad c^2 = \gamma R T \text{ and } \mathrm{M} = \mathrm{V/c},$$

$$\frac{V^2}{2c_p T} = \frac{V^2}{2[\gamma R/(\gamma-1)]T} = \left(\frac{\gamma-1}{2}\right)\frac{V^2}{c^2} = \left(\frac{\gamma-1}{2}\right)M^2$$

Substituting in the above equation,

$$\frac{T_0}{\mathrm{T}} = 1 + \left(\frac{\gamma-1}{2}\right)M^2$$

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Property relations for isentropic flow of ideal gases

- Similarly for pressure and density:

$$\frac{P_0}{\mathrm{P}} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{1/(\gamma-1)}$$

- The above equations relate the stagnation properties with the corresponding static properties through the Mach number.

Property relations for isentropic flow of ideal gases

- Properties at a location where the Mach number is unity are called critical properties.
- Setting M=1 in the equations,

$$\frac{T^*}{T_0} = \left[\frac{2}{\gamma+1}\right]$$

$$\frac{P^*}{P_0} = \left[\frac{2}{\gamma+1}\right]^{\gamma/(\gamma-1)}$$

$$\frac{\rho^*}{\rho_0} = \left[\frac{2}{\gamma+1}\right]^{1/(\gamma-1)}$$

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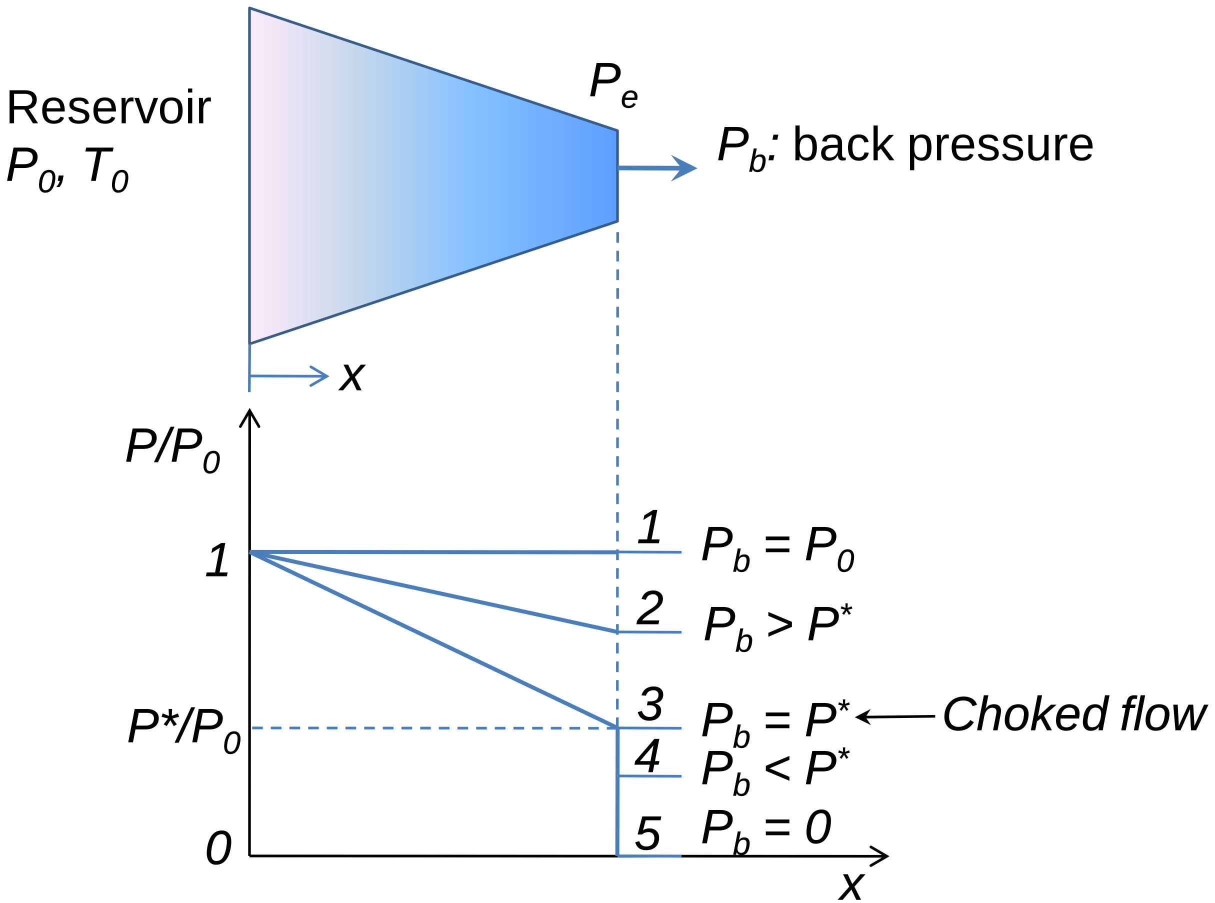
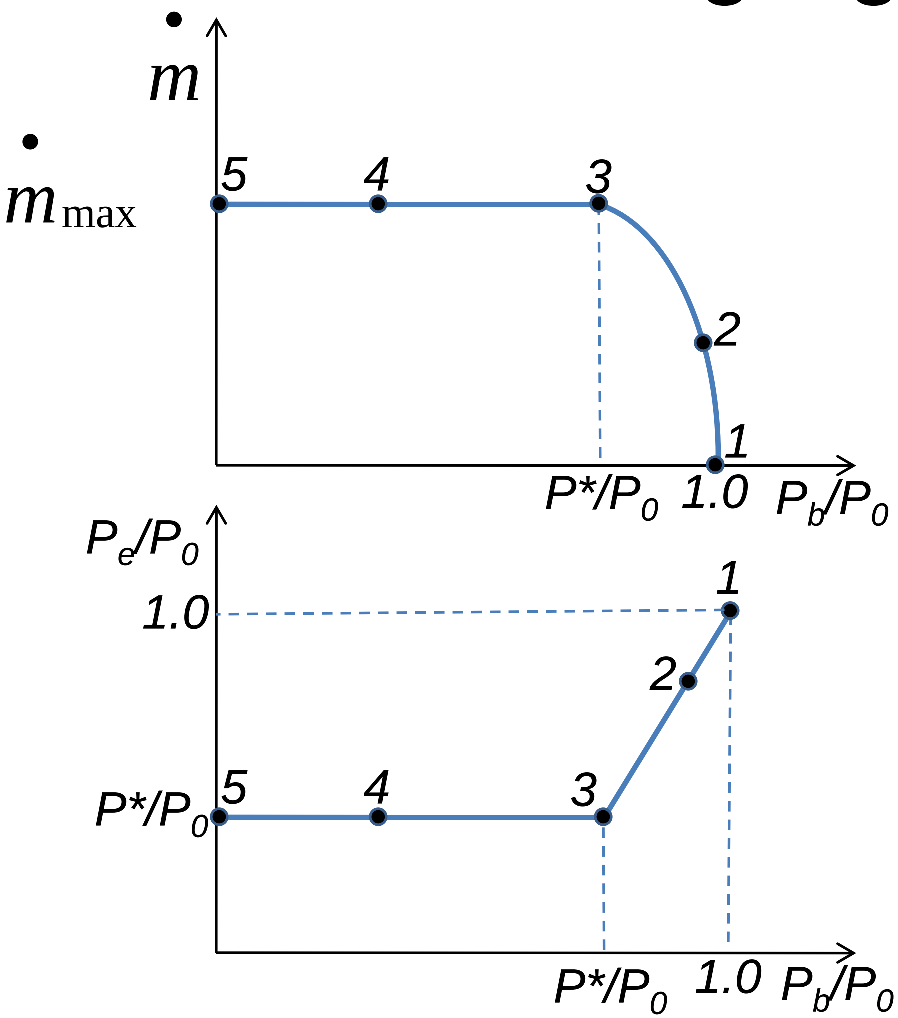
Isentropic flow through converging nozzles

- Converging nozzle in a subsonic flow will have decreasing area along the flow direction.
- We shall consider the effect of back pressure on the exit velocity, mass flow rate and pressure distribution along the nozzle.
- We assume flow enters the nozzle from a reservoir so that inlet velocity is zero.
- Stagnation temperature and pressure remains unchanged in the nozzle.

Isentropic flow through converging nozzles



Isentropic flow through converging nozzles



The effect of back pressure P_b *on the* mass flow rate and the exit pressure P_e.

Isentropic flow through converging nozzles

- From the above figure,

$$P_e = \begin{cases} P_b & \text{for } P_b \geq P^* \\ P^* & \text{for } P_b < P^* \end{cases}$$

- For all back pressures lower that the critical pressure, exit pressure = critical pressure, Mach number is unity and the mass flow rate is maximum (choked flow).
- A back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

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Isentropic flow through converging-diverging nozzles

- Maximum Mach number achievable in a converging nozzle is unity.
- For supersonic Mach numbers, a diverging section after the throat is required.
- However, a diverging section alone would not guarantee a supersonic flow.
- The Mach number at the exit of the converging-diverging nozzle depends upon the back pressure.

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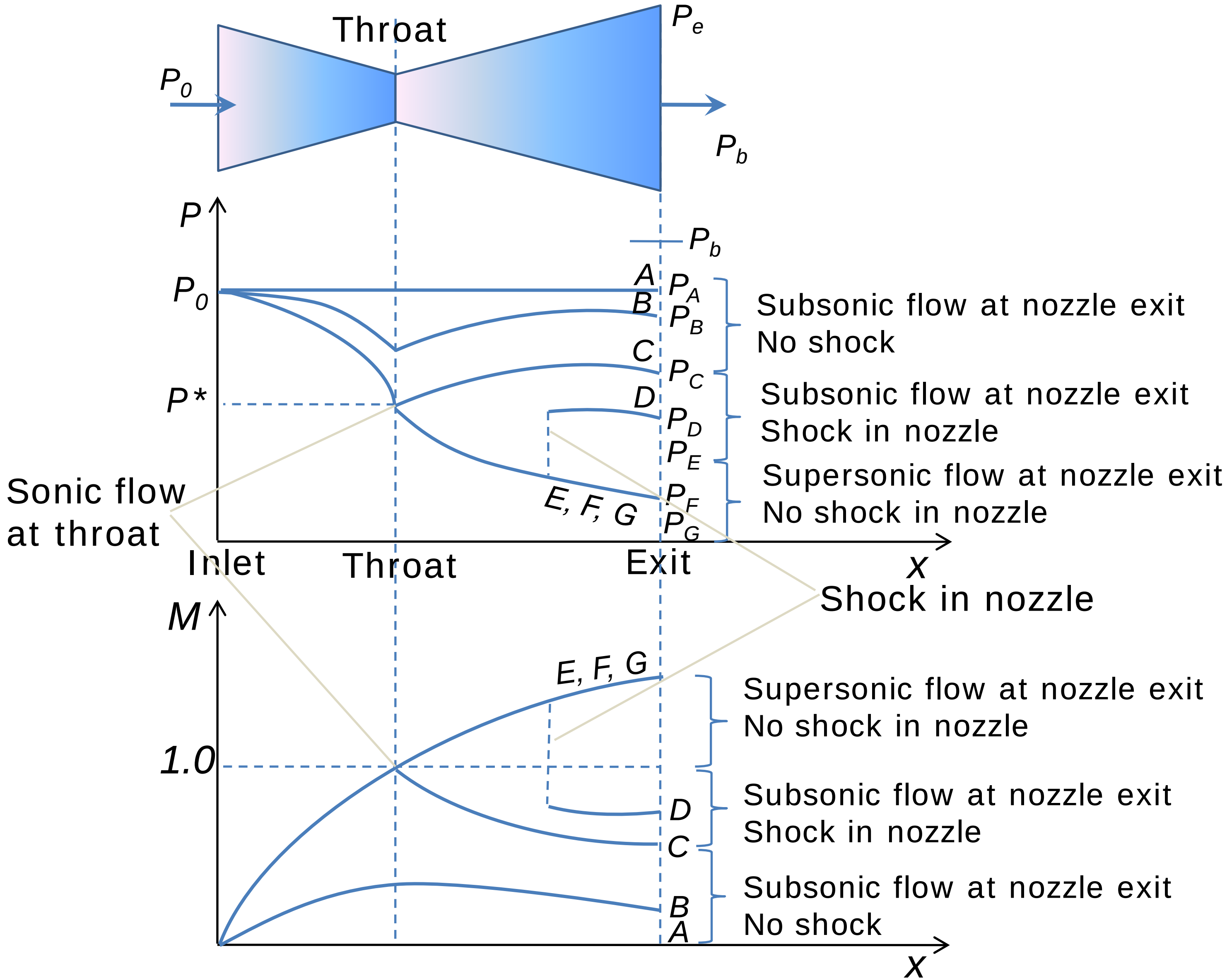

Throat
P_e
P_0
P_b
P
P_b
P_0
A
B
C
D
P_A
P_B
P_C
P_D
P_E
P_F
P_G
P^*
E, F, G
Subsonic flow at nozzle exit
No shock
Subsonic flow at nozzle exit
Shock in nozzle
Supersonic flow at nozzle exit
No shock in nozzle
Sonic flow
at throat
Inlet
Throat
Exit
x
Shock in nozzle
M
E, F, G
1.0
D
C
B
A
Supersonic flow at nozzle exit
No shock in nozzle
Subsonic flow at nozzle exit
Shock in nozzle
Subsonic flow at nozzle exit
No shock
x

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In the next lecture …

- Shock Waves and Expansion
  - Normal Shocks
  - Oblique Shocks
  - Prandtl–Meyer Expansion Waves
- Duct Flow with Heat Transfer and Negligible Friction (Rayleigh Flow)
  - Property Relations for Rayleigh Flow
- Duct flow with friction without heat transfer (Fanno flow)

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