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LectureNo• 16

## I $n$ the next lecture ...

- Solve problems from
- Entropy
- Carnot cycle
- Exergy
- Second law efficiency


## Problem 1

- A heat engine receives reversibly 420 $\mathrm{kJ} /$ cycle of heat from a source at $327^{\circ} \mathrm{C}$ and rejects heat reversibly to a sink at $27^{\circ} \mathrm{C}$. There are no other heat transfers. Consider three different rates of heat rejection (a) 210 kJ (b) 105 kJ and (c) 315 kJ . For each of these cases show which cycle is reversible, irreversible and impossible.


## Solution: Problem 1



## Solution: Problem 1

- From the Clausius inequality, we have

$$
\oint \frac{\delta Q}{T} \leq 0
$$

For heat engine 1,

$$
\oint \frac{\delta Q}{T}=\frac{Q_{H}}{T_{H}}-\frac{Q_{L}}{T_{L}}=\frac{420}{600}-\frac{210}{300}=0
$$

$\therefore$ Heat engine 1 operates on a reversible cycle.

## Solution: Problem 1

- We now look at heat engine 2.

For heat engine 2,
$\oint \frac{\delta Q}{T}=\frac{Q_{H}}{T_{H}}-\frac{Q_{L}}{T_{L}}=\frac{420}{600}-\frac{105}{300}=0.35$
Since $\oint \frac{\delta Q}{T}>0$, the cycle is impossible.

## Solution: Problem 1

- Heat engine 3

For heat engine 3,
$\oint \frac{\delta Q}{T}=\frac{Q_{H}}{T_{H}}-\frac{Q_{L}}{T_{L}}=\frac{420}{600}-\frac{315}{300}=-0.35$
Since $\oint \frac{\delta Q}{T}<0$, the cycle is irreversible and possible.

## Problem 2

- A block of iron weighing 100 kg and having a temperature of $100^{\circ} \mathrm{C}$ is immersed in 50 kg of water at a temperature of $20^{\circ} \mathrm{C}$. What will be the change in entropy of the combined system of iron and water? Specific heats of iron and water are 0.45 $\mathrm{kJ} / \mathrm{kg} \mathrm{K}$ and $4.18 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$, respectively.


## Problem 2



## Solution: Problem 2

- Let $T_{f}$ be the final temperature of the system after it reaches thermal equilibrium.
- From energy balance, we know that, $\left\{m \times c p \times\left(T-T_{f}\right)\right\}_{\text {iron }}=\left\{m \times c p \times\left(T_{f}-T\right)\right\}_{\text {water }}$

$$
100 \times 0.45 \times 10^{3}\left(373-T_{f}\right)=50 \times 4.18 \times 10^{3}\left(T_{f}-293\right)
$$

$$
\mathrm{T}_{\mathrm{f}}=307.3 \mathrm{~K}
$$

## Solution: Problem 2

$\Delta \mathrm{S}_{\text {total }}=\Delta \mathrm{S}_{\text {iron }}+\Delta \mathrm{S}_{\text {water }}$

- We know that for solids and liquids, $\mathrm{dV}=0$
$\Delta \mathrm{S}=\mathrm{m} \times \mathrm{c} \times \ln \mathrm{T}_{\mathrm{f}} / \mathrm{T}$
$\Delta S_{\text {iron }}=100 \times 0.45 \times 10^{3} \ln (307.3 / 373)$ $=-8.7189 \mathrm{~kJ} / \mathrm{K}$ (Why is this negative?)
$\Delta \mathrm{S}_{\text {water }}=50 \times 4.18 \times 10^{3} \ln (307.3 / 293)$ $=9.9592 \mathrm{~kJ} / \mathrm{K}$
$\Delta \mathrm{S}_{\text {total }}=-8.7189+9.9592=1.2403 \mathrm{~kJ} / \mathrm{K}$


## Problem 3

- An inventor claims to have developed a power cycle capable of delivering a net work output of 415 kJ for an energy input by heat transfer of 1000 kJ . The system undergoing the cycle receives heat from a source of 500 K and rejects heat to a sink of 300 K . Determine if this is a valid claim.


## Solution: Problem 3



## Solution: Problem 3

- We know that efficiency of the cycle

$$
\begin{aligned}
\eta_{\text {th }} & =W_{\text {net }} / Q_{H} \\
& =415 / 1000=0.415 \text { or } 41.5 \%
\end{aligned}
$$

- The maximum efficiency that any cycle can have while operating between $\mathrm{T}_{\mathrm{H}}=500 \mathrm{~K}$ and $T_{L}=300 \mathrm{~K}$ is given by the Carnot efficiency.

$$
\begin{aligned}
\eta_{\max } & =1-T_{L} / T_{H}=1-300 / 500 \\
& =0.40 \text { or } 40 \%
\end{aligned}
$$

Since $\eta_{\text {th }}>\eta_{\text {max }}$, the claim is not feasible.

## Problem 4

- A heat pump is to be used to heat a house during the winter. The house is to be maintained at $21^{\circ} \mathrm{C}$ at all times. The house is estimated to be losing heat at a rate of $120,000 \mathrm{~kJ} / \mathrm{h}$ when the outside temperature drops to $-7^{\circ} \mathrm{C}$. Determine the minimum power required to drive this heat pump.


## Solution: Problem 4



## Solution: Problem 4

- The heat pump must supply heat to the house at a rate of

$$
\begin{aligned}
\mathrm{Q}_{\mathrm{H}} & =120,000 \mathrm{~kJ} / \mathrm{h} \\
& =120,000 / 3600 \mathrm{~kJ} / \mathrm{s}=33.3 \mathrm{~kW}
\end{aligned}
$$

- The power required will be minimum when the heat pump operates on a reversible cycle.
- The COP for such a cycle is

$$
C O P_{H P}=\frac{1}{1-T_{L} / T_{H}}
$$

## Solution: Problem 4

- The COP for such a cycle is

$$
C O P_{H P}=\frac{1}{1-T_{L} / T_{H}}=\frac{1}{1-(-7+273) /(21+273)}=10.5
$$

- The minimum required power,

$$
\begin{aligned}
\mathrm{W}_{\text {net, in }} & =\mathrm{Q}_{\mathrm{H}} / \mathrm{COP}_{\mathrm{HP}}=33.3 / 10.5 \\
& =3.17 \mathrm{~kW}
\end{aligned}
$$

## Problem 5

- Air flows through an adiabatic compressor at $2 \mathrm{~kg} / \mathrm{s}$. The inlet conditions are 1 bar and 310 K and the exit conditions are 7 bar and 560 K. Determine the net rate of exergy transfer and the irreversibility. The ambient temperature can be taken as 298 K , the specific heat at constant pressure for air is $1.005 \mathrm{~kJ} / \mathrm{kgK}$ and the gas constant for air is $0.287 \mathrm{~kJ} / \mathrm{kgK}$.


## Solution: Problem 5

- Exergy change per unit mass is

$$
\begin{aligned}
\psi & =\left(h_{2}-h_{1}\right)-T_{0}\left(s_{2}-s_{1}\right)(\text { Assuming KE, PE } \cong 0) \\
& =c_{p}\left(T_{2}-T_{1}\right)-T_{0}\left(c_{p} \ln \frac{T_{2}}{T_{1}}-R \ln \frac{P_{2}}{P_{1}}\right) \\
& =1.005(560-310)-298\left(1.005 \ln \frac{560}{310}-0.287 \ln \frac{7}{1}\right) \\
& =240.58 \mathrm{~kJ} / \mathrm{kg}
\end{aligned}
$$

## Solution: Problem 5

$$
\Psi=\dot{m} \psi=2 \times 240.58 \mathrm{~kJ} / \mathrm{kg}=481.16 \mathrm{~kW}
$$

- Net rate of exergy change is 418.16 kW
- The actual work required is

$$
\begin{aligned}
W_{\text {act }} & =\dot{m}\left(h_{2}-h_{1}\right)=\dot{m} c_{p}\left(T_{2}-T_{1}\right) \\
& =2 \times 1.005 \times(560-310)=502.5 \mathrm{~kW}
\end{aligned}
$$

## Solution: Problem 5

- Therefore Irreversibility,

$$
\begin{aligned}
I & =W_{\text {act }} \text { Exergy } \\
& =502.5-481.2 \\
& =21.3 \mathrm{~kW}
\end{aligned}
$$

- Why is actual work higher than exergy?


## Problem 6

- A pipe carries a stream of a liquid with a mass flow rate of $5 \mathrm{~kg} / \mathrm{s}$. Because of poor insulation the liquid temperature increases from 250 K at the pipe inlet to 253 K at the exit. Neglecting pressure losses, calculate the irreversibility rate associated with the heat leakage. Take $\mathrm{T}_{0}$ as 293 K and specific heat for the liquid as $2.85 \mathrm{~kJ} / \mathrm{kg} \mathrm{K}$.


## Solution: Problem 6



## Problem 6

- Rate of heat transfer to the liquid:

$$
\begin{aligned}
\dot{Q} & =\dot{m} \times c \times\left(T_{2}-T_{1}\right) \\
& =5 \times 2.85 \times(253-250) \\
& =42.75 \mathrm{~kW}
\end{aligned}
$$

- Rate of entropy increase of the liquid:

$$
\begin{aligned}
\Delta \dot{S}_{\text {sys }} & =m \times c \times \ln \frac{T_{2}}{T_{1}} \\
& =5 \times 2.85 \times \ln \frac{253}{250}=0.17 \mathrm{~kW} / \mathrm{K}
\end{aligned}
$$

## Problem 6

- Rate of entropy decrease of the surroundings:

$$
\Delta \dot{S}_{\text {surr }}=\frac{Q}{T_{0}}=\frac{-42.75}{293}=-0.1459 k W / K
$$

- Hence, rate of entropy increase of the universe:

$$
\begin{aligned}
& \Delta \dot{S}_{\text {univ }}=\Delta \dot{S}_{\text {sys }}+\Delta \dot{S}_{\text {surr }} \\
& \quad=0.17-0.1459=0.0241 \mathrm{~kW} / \mathrm{K}
\end{aligned}
$$

## Problem 6

- Therefore Irreversibility, I

$$
\begin{aligned}
I & =T_{0} \Delta \dot{S}_{\text {univ }} \\
& =293 \times 0.0241=7.06 \mathrm{~kW}
\end{aligned}
$$

- The irreversibility associated with this process is 7.06 kW .


## Exercise Problem 1

- Air is compressed steadily by a 5 kW compressor from 100 kPa and $17^{\circ} \mathrm{C}$ to 600 kPa and $167^{\circ} \mathrm{C}$ at a rate of $1.6 \mathrm{~kg} / \mathrm{min}$. During this process, some heat transfer takes place between the compressor and the surrounding medium at $17^{\circ} \mathrm{C}$. Determine the rate of entropy change of air during this process.
- Ans: 0.0025 kW/K


## Exercise Problem 2

- An adiabatic vessel contains 3 kg of water at $25^{\circ} \mathrm{C}$. By paddle wheel work transfer, the temperature of water is increased to $30^{\circ} \mathrm{C}$. If the specific heat of water is $4.18 \mathrm{~kJ} / \mathrm{kgK}$, find the entropy change of the universe.
- Ans: $0.139 \mathrm{~kJ} / \mathrm{K}$


## Exercise Problem 3

- An inventor claims to have developed an engine that takes in 105 MJ at a temperature of 400 K , rejects 42 MJ at a temperature of 200 K and delivers 15 kWh of mechanical work. Is this a feasible engine?
- Ans: No as $\eta_{\text {th }}>\eta_{\text {rev }}$


## Exercise Problem 4

- Air enters a nozzle steadily at 300 kPa and $87^{\circ} \mathrm{C}$ with a velocity of $50 \mathrm{~m} / \mathrm{s}$ and exits at 95 kPa and $300 \mathrm{~m} / \mathrm{s}$. The heat loss from the nozzle to the surrounding medium at $17^{\circ} \mathrm{C}$ is estimated to be $4 \mathrm{~kJ} / \mathrm{kg}$. Determine (a) the exit temperature and (b) the exergy destroyed during this process.
- Ans: (a) $39.5^{\circ} \mathrm{C}$, (b) $58.4 \mathrm{~kJ} / \mathrm{kg}$


## Exercise Problem 5

- An iron block of unknown mass at $85^{\circ} \mathrm{C}$ is dropped into an insulated tank that contains 100 L of water at $20^{\circ} \mathrm{C}$. At the same time, a paddle wheel driven by a $200-\mathrm{W}$ motor is activated to stir the water. It is observed that thermal equilibrium is established after 20 min with a final temperature of $24^{\circ} \mathrm{C}$. Assuming the surroundings to be at $20^{\circ} \mathrm{C}$, determine (a) the mass of the iron block and (b) the exergy destroyed during this process.
- Ans: (a) 52.0 kg , (b) 375 kJ


## Exercise Problem 6

- An adiabatic turbine receives gas ( $c_{p}=1.09$ $\mathrm{kJ} / \mathrm{kg} \mathrm{K}$ and $\mathrm{c}_{\mathrm{v}}=0.838 \mathrm{~kJ} / \mathrm{kgK}$ ) at 7 bar and $1000^{\circ} \mathrm{C}$ and discharges at 1.5 bar and $665^{\circ} \mathrm{C}$. Determine the second law efficiency of the turbine assuming $\mathrm{T}_{0}=298 \mathrm{~K}$.
- Ans: 0.879


## In the next lecture ...

- Gas power cycles
- The Carnot cycle and its significance
- Air-standard assumptions
- An overview of reciprocating engines
- Otto cycle: the ideal cycle for sparkignition engines
- Diesel cycle: the ideal cycle for compression-ignition engines
- Dual cycles

