Wind Energy: Overview

Learning objectives:

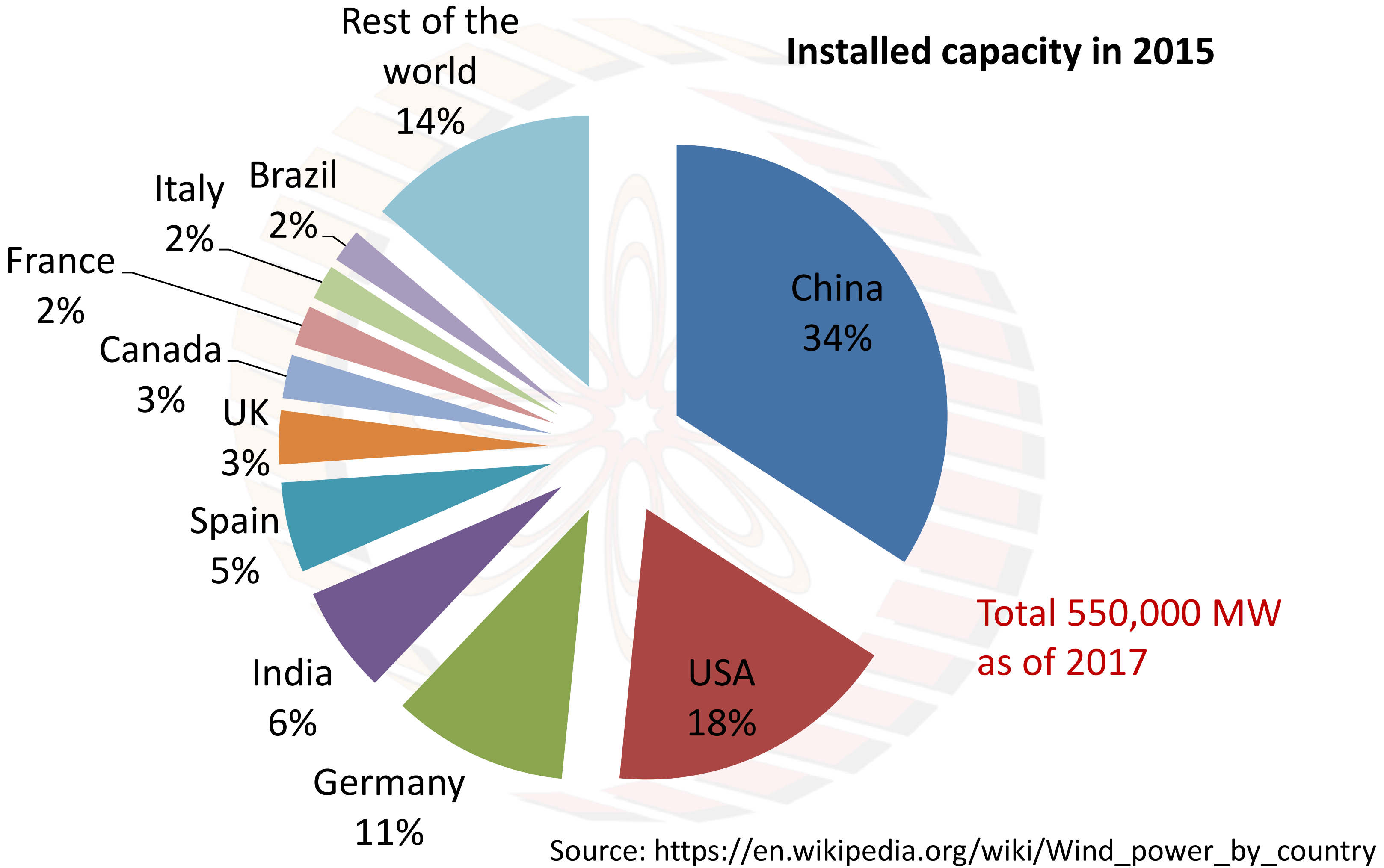
1) To understand the pattern of usage of wind energy internationally
2) To understand the pattern of usage of wind energy in India
3) To become aware of geographical issues associated with wind energy
4) To become aware of different types of windmills

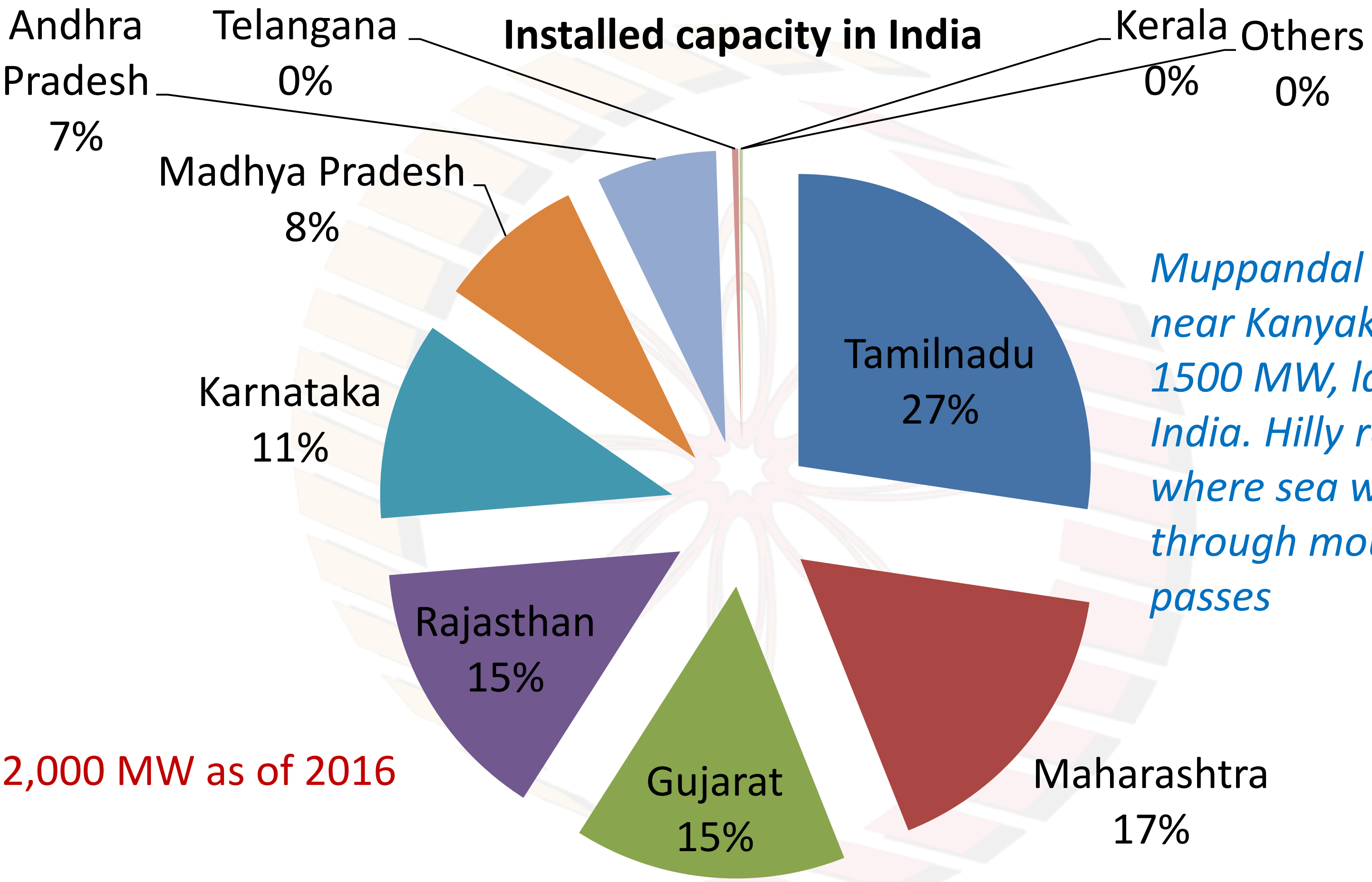
Historical usage of windmills

1)Grinding grains
2)Pumping water
3)Generating electricity

Requirements

1)At least 16 km/h winds
2)Low likelihood of bursts of wind
3)Access to transmission capacity

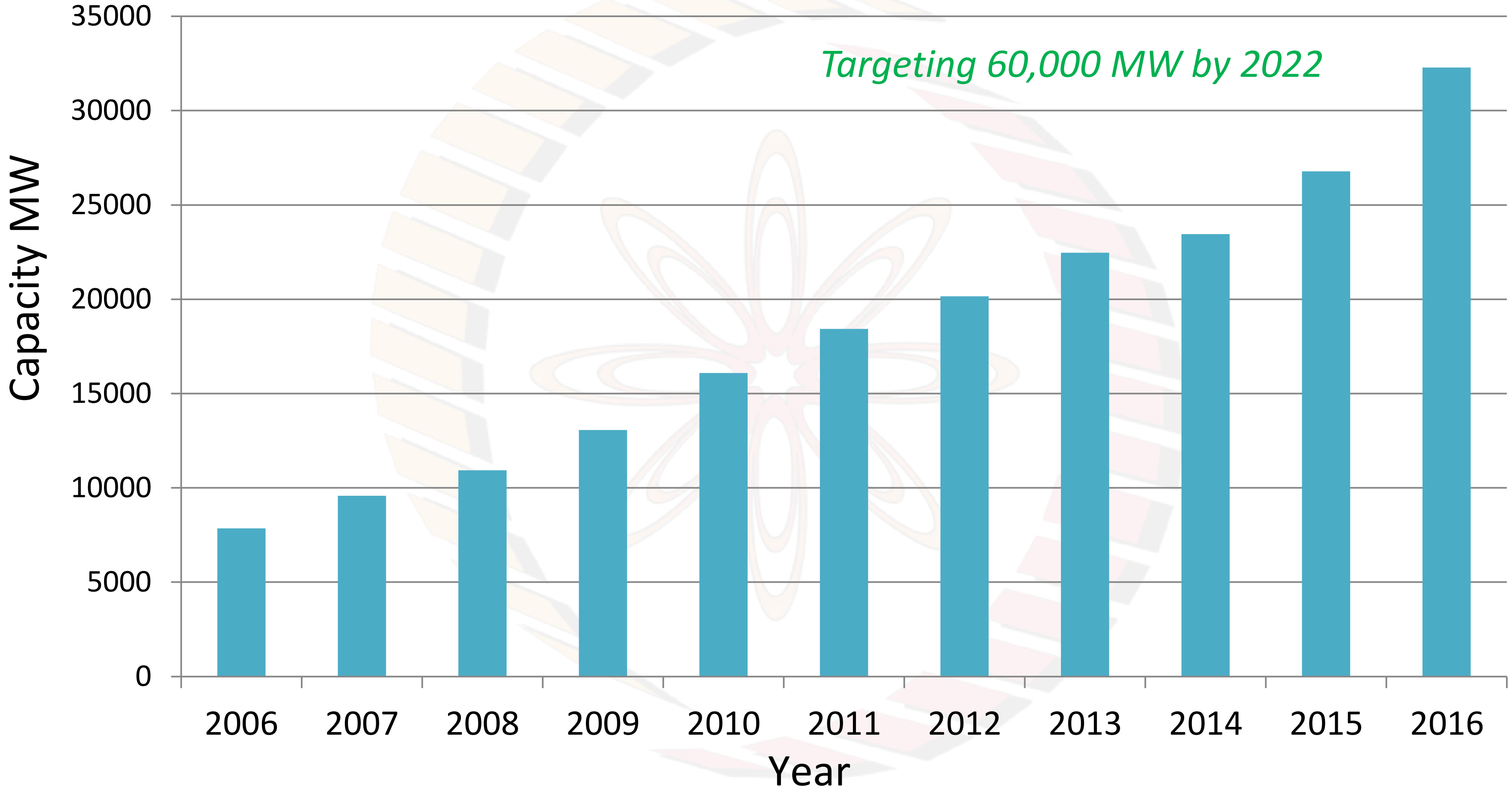

Installed capacity in 2015
Rest of the world 14%
Brazil 2%
Italy 2%
France 2%
Canada 3%
UK 3%
Spain 5%
India 6%
Germany 11%
USA 18%
China 34%
Total 550,000 MW as of 2017
Source: https://en.wikipedia.org/wiki/Wind_power_by_country



Muppandal windfarm, near Kanyakumari, 1500 MW, largest in India. Hilly region where sea winds go through mountain passes

Total 32,000 MW as of 2016

Source: https://en.wikipedia.org/wiki/Wind_power_in_India



Source: https://en.wikipedia.org/wiki/Wind_power_in_India

Types of windmills

1) Horizontal axis wind turbines
   a. Tall towers enable accessing stronger winds
   b. Blades capture wind energy throughout rotation

   a. Strong and huge towers required
   b. Complexity during construction
   c. Need to be turned to face the wind

Types of windmills

2) Vertical axis wind turbines

a. Generates power independent of wind direction
b. Low cost
c. Strong tower not needed since generator is on the ground

a. Low efficiency (only one blade works at a time)
b. May need wires to support
c. More turbulent flow near ground

Power generated:

Large wind turbine: 2-3 MW

Per year, at 25% capacity factor, it will generate:

$$2 \times 10^{6} \times 0.25 \times 3600 \times 24 \times 365 = 1.6 \times 10^{13}\ J$$

Therefore, 500 exa joules will require:

$$500 \times 10^{18} / 1.6 \times 10^{13} = 31 \times 10^{6}$$

31 Million wind turbines

Space requirement:

Rule of thumb is 7 times diameter of windmill

Approximately 500 m from other turbines

Each 2 MW turbine needs approximately 0.5 square km

Therefore 15.5 million square km needed to power the world!

1.5 times Size of China or USA

Conclusions:

1) Considerable interest in tapping wind energy both internationally as well as in India
2) Geographical locations play an important role in planning windmill installations
3) Various designs of wind mills considered historically

Wind Energy:
Energy considerations

Learning objectives:

1) To determine the relationship between wind speed and power
2) To understand typical performance characteristic and performance limits of windmills
3) To become aware of theoretical limits associated with capture of wind energy

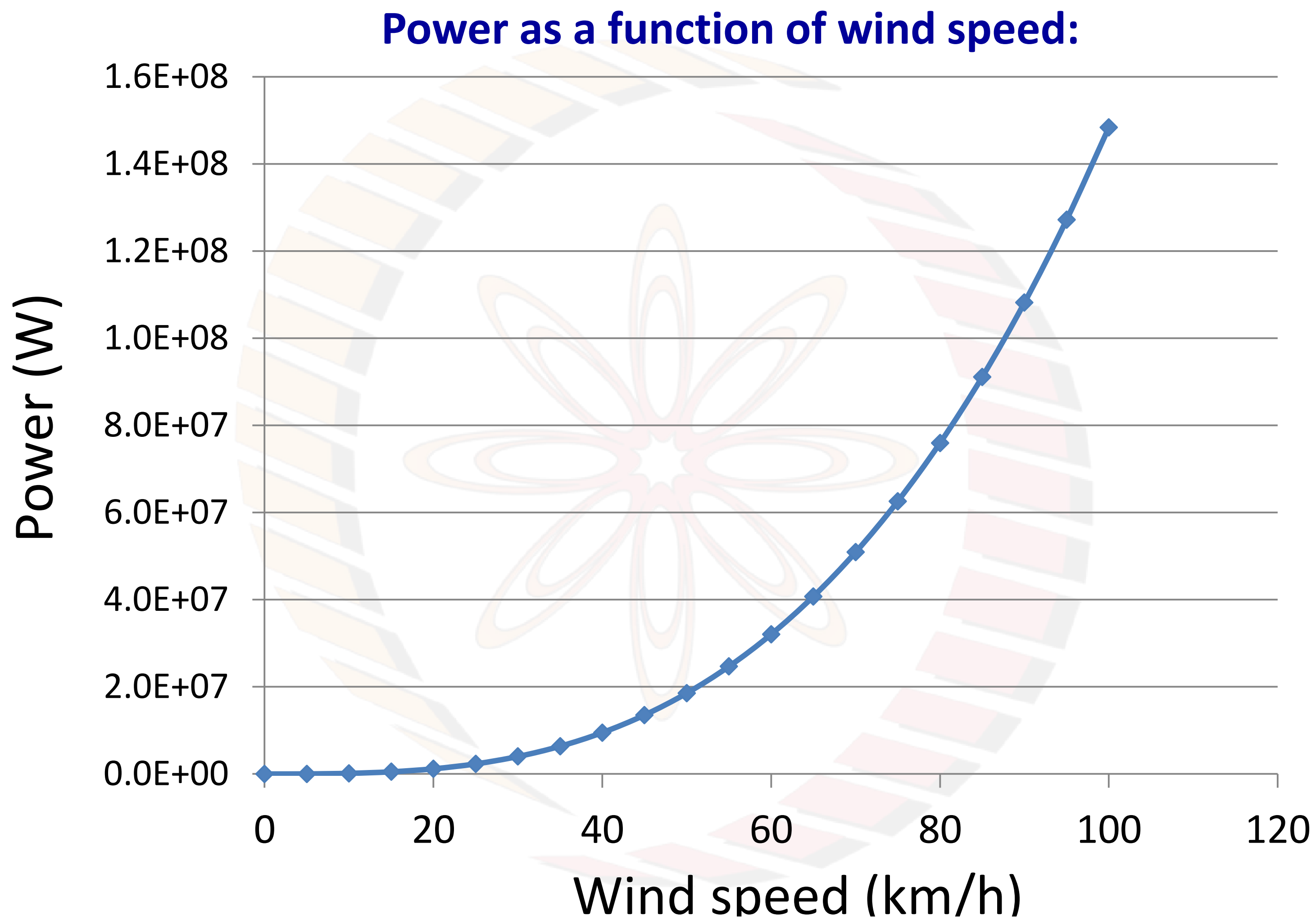
Energy calculations:

Energy calculations:

$$Kinetic\ Energy\ (KE) = \frac{1}{2}mv^2$$

$$KE = \frac{1}{2}\rho V v^2 = \frac{1}{2}\rho A l v^2$$

$$Power = \frac{dE}{dt} = \frac{1}{2}\rho A \frac{dl}{dt} v^2 = \frac{1}{2}\rho A v^3$$



Power as a function of wind speed:
1.6E+08
1.4E+08
1.2E+08
1.0E+08
8.0E+07
6.0E+07
4.0E+07
2.0E+07
0.0E+00
Power (W)
0
20
40
60
80
100
120
Wind speed (km/h)

Performance Characteristics:

Tip speed ratio: Ratio of rotational speed of blade to wind speed. Maximum of 10 for lift type blades

Cut in speed: Minimum wind speed at which the blades will turn. 10 km/h to 16 km/h

Rated speed: The wind speed at which the windmill generates its rated power. Usually it levels off in power beyond this speed. Around 40 km/h

Cut out speed: Usually at wind speeds above 70 km/h, the windmill is stopped to prevent damage

Theoretical Limit:

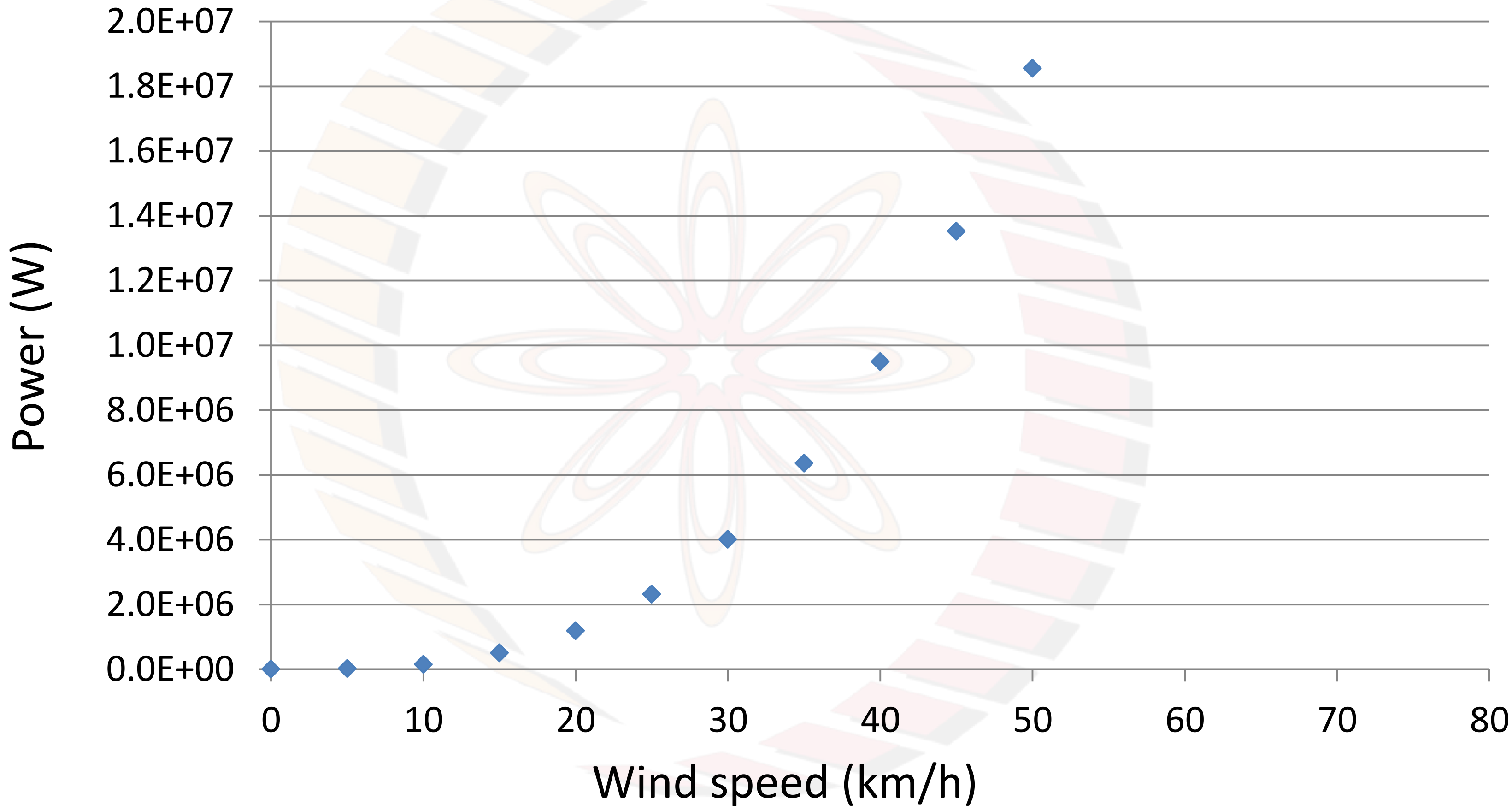
Betz law (1920)

- Wind fully stopped by windmill
- Wind unaffected by wind mill

$$\frac{16}{27} = 0.59$$

Practical efficiencies obtained: 10%-30% of energy originally available in wind

Power as a function of wind speed:



Blade types:

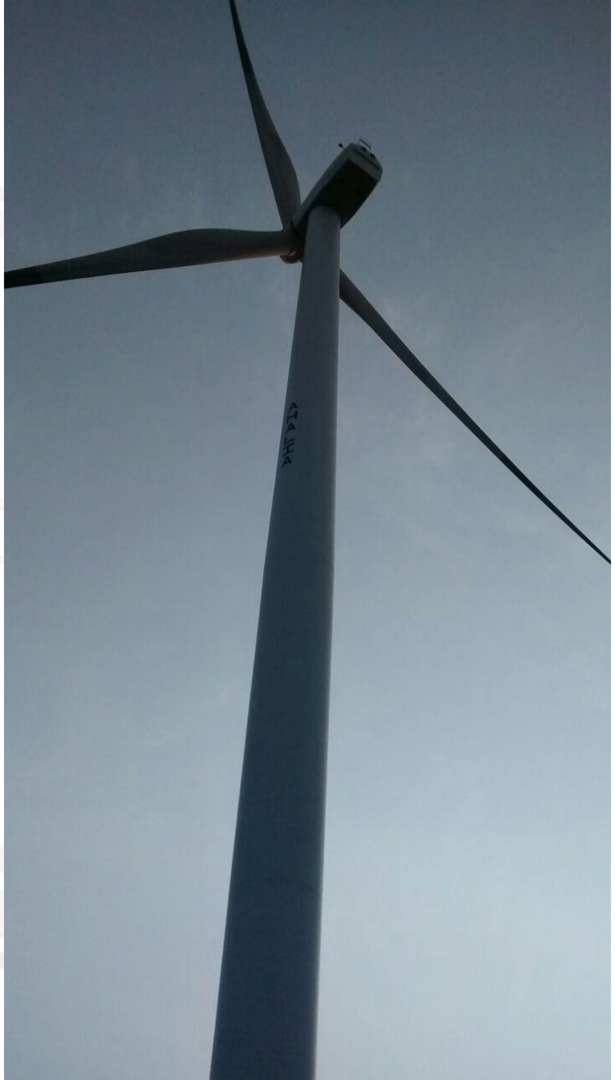
Drag type: Greater torque, lower rotational speed. Better suited for mechanical work

Lift type: Higher rotational speed. Better suited for power generation

Conclusions:

1) The power available in Wind is proportional to the third power of wind velocity
2) There are practical aspects that limit the range of wind velocities that can be effectively tapped
3) There is a theoretical limit to the extent to which energy available in the wind, can be captured

Wind Energy: Efficiency



Learning objectives:

1) To derive the Betz Limit
2) To understand its implications

Theoretical Limit:

Betz law (1920)

- Wind fully stopped by windmill
- Wind unaffected by wind mill

$$\frac{16}{27} = 0.59$$

Practical efficiencies obtained: 10%-30% of energy originally available in wind

Bernoulli's equation:

→ Conservation of energy – flow of fluids

$$\frac{1}{2}\rho V^2 + \rho g h + P = Constant$$

$\frac{1}{2} m v^2 = KE$

P

$\frac{1}{2} m v^2$ ✗

$\frac{1}{2} \rho v^2$ ✓

$$\frac{1}{2}\rho V^2 + P = Constant$$

$\frac{1}{2} \frac{m v^2}{Volume} = \frac{J}{m^3} = \frac{Nm}{m^3}$

$= \frac{N}{m^2}$ (Same as pressure)

Dynamic pressure + Static Pressure = Constant

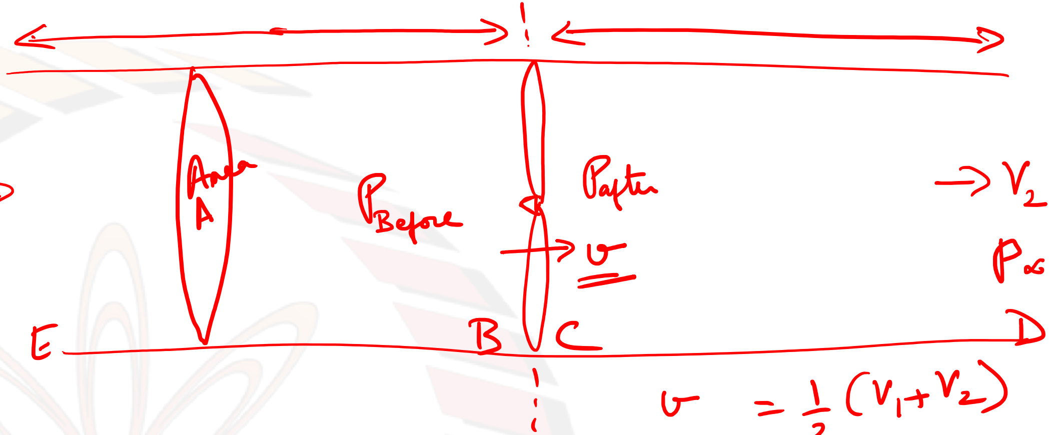
$$\frac{1}{2}\rho V^2 + P = \text{Constant}$$

$$\frac{1}{2}\rho V_1^2 + \cancel{P_\infty} = \frac{1}{2}\cancel{\rho v^2} + P_{Before}$$

$$\cancel{\frac{1}{2}\rho v^2} + P_{after} = \frac{1}{2}\rho V_2^2 + \cancel{P_\infty}$$

$$\frac{1}{2}\rho V_1^2 + P_{after} = \frac{1}{2}\rho V_2^2 + P_{Before}$$

$$P_{Before} - P_{after} = \frac{1}{2}\rho\left(V_1^2 - V_2^2\right)$$



$$v = \frac{1}{2}(V_1 + V_2)$$

Force = Pressure × Area $= A\left(P_{Before} - P_{after}\right) = \frac{1}{2}\rho A\left(V_1^2 - V_2^2\right)$

Change in momentum $= \rho A l \left(V_1 - V_2\right)$

Rate of change of momentum $= \dfrac{d}{dt}\left(\rho A l \left(V_1 - V_2\right)\right) = \rho A \dfrac{dl}{dt}\left(V_1 - V_2\right)$

= Force

Force $= \rho A v \left(V_1 - V_2\right)$

$$\frac{1}{2}\rho A\left(V_1^2 - V_2^2\right) = \rho A v\left(V_1 - V_2\right).$$

$$\frac{1}{2}\cancel{\rho A}\,\cancel{\left(V_1 - V_2\right)}\left(V_1 + V_2\right) = \cancel{\rho A}\,v\,\cancel{\left(V_1 - V_2\right)}$$

$$\frac{V_1 + V_2}{2} \text{ or } \boxed{\frac{1}{2}\left(V_1 + V_2\right) = v}$$

$$\frac{1}{2}\rho V_1{}^2 + P_\infty = \frac{1}{2}\rho v^2 + P_{Before}$$ ①

$$\frac{1}{2}\rho v^2 + P_{After} = \frac{1}{2}\rho V_2{}^2 + P_\infty$$ ②

$$P_{Before} - P_{After} = \frac{1}{2}\rho V_1{}^2 - \frac{1}{2}\rho V_2{}^2$$ ③

$$Force = A(P_{Before} - P_{After}) = \frac{1}{2}\rho A(V_1{}^2 - V_2{}^2)$$

$\rho A l V_1$ $\rho A l V_2$

$$Change\ in\ momentum = \boxed{\rho Al(V_1 - V_2)}$$

$v = \frac{dl}{dt}$

$$Force = Rate\ of\ change\ in\ momentum = \rho Av(V_1 - V_2)$$ ④

③ = ④

$$\therefore \rho A v(V_1 - V_2) = \frac{1}{2}\rho A({V_1}^2 - {V_2}^2)$$ 4 = 3

$$\boxed{\therefore v = \frac{1}{2}(V_1 + V_2)}$$

$$KE = \underline{\tfrac{1}{2}\rho A l \, V_1^2} - \tfrac{1}{2}\rho A l \, V_2^2$$

$$\text{Power} = \frac{d(KE)}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}\rho A l\left(V_1^2 - V_2^2\right)\right)$$

$$= \tfrac{1}{2}\rho A v \left(V_1^2 - V_2^2\right)$$

K E of wind far away from wind mill, approaching the wind mill

$$\tfrac{1}{2}\rho A l \, V_1^2 \qquad \therefore \text{Power} = \frac{d(KE)}{dt} = \tfrac{1}{2}\rho A \frac{dl}{dt} V_1^2 = \tfrac{1}{2}\rho A V_1^3$$

$$P_0 = \tfrac{1}{2}\rho A V_1^3$$

$$P = \tfrac{1}{2}\rho A v \left(V_1^2 - V_2^2\right) \qquad v = \tfrac{1}{2}\left(V_1 + V_2\right)$$

$$P = \frac{1}{2}\rho A v\left(V_1^2 - V_2^2\right)$$

$$= \frac{1}{4}\rho A\left(V_1 + V_2\right)\left(V_1^2 - V_2^2\right).$$

$$P = \frac{1}{4}\rho A\left(V_1^3 - V_1V_2^2 + V_2V_1^2 - V_2^3\right) \rightarrow 1$$

$$P_0 = \frac{1}{2}\rho A V_1^3 \rightarrow 2$$

$$v = \frac{1}{2}\left(V_1 + V_2\right)$$

$$\frac{P}{P_0}$$

$$\frac{P}{P_0} = \frac{1}{2}\left[1 - \frac{V_2^2}{V_1^2} + \frac{V_2}{V_1} - \frac{V_2^3}{V_1^3}\right] \rightarrow 3$$

$$\frac{V_2}{V_1} = \alpha$$

$$\frac{P}{P_0} = \frac{1}{2}\left[1 - \alpha^2 + \alpha - \alpha^3\right]$$

$$\frac{d\left(\frac{P}{P_0}\right)}{d\alpha} = \frac{1}{2}\left[-2\alpha + 1 - 3\alpha^2\right] = 0$$

$$-3\alpha^2 - 2\alpha + 1 = 0$$

$$\therefore \alpha = \frac{+2 \pm \sqrt{4+12}}{-6}$$

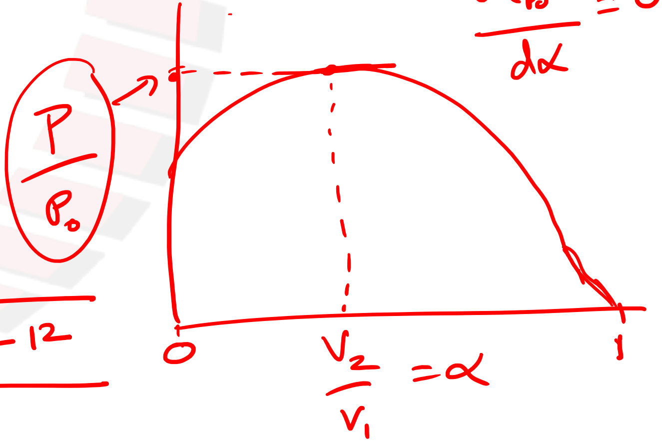
$$\therefore \alpha = \frac{2 \pm 4}{-6}$$

$$\alpha = -1, \frac{1}{3}$$

$$\frac{V_2}{V_1} = \alpha$$

$$\boxed{\frac{V_2}{V_1} = \frac{1}{3}}$$

$$\frac{d\left(\frac{P}{P_0}\right)}{d\alpha} = 0$$



$$Change\ in\ energy\ in\ wind = \frac{1}{2}\rho Al({V_1}^2 - {V_2}^2)$$

$$Power\ extracted\ from\ wind = P = \frac{dE}{dt} = \frac{1}{2}\rho Av({V_1}^2 - {V_2}^2)$$

$v = \frac{1}{2}[V_1 + V_2]$

$$\therefore P = \frac{1}{4}\rho A(V_1 + V_2)({V_1}^2 - {V_2}^2)$$

$$Kinetic\ Energy\ (KE)\ in\ incoming\ wind = \frac{1}{2}mv^2 = \frac{1}{2}\rho V{V_1}^2 = \frac{1}{2}\rho Al{V_1}^2$$

$$Powerin\ incomng\ wind = P_0 = \frac{dE}{dt} = \frac{1}{2}\rho A\frac{dl}{dt}{V_1}^2 = \frac{1}{2}\rho A{V_1}^3$$

$$\frac{P}{P_0} = \frac{\frac{1}{4}\rho A(V_1 + V_2)(V_1{}^2 - V_2{}^2)}{\frac{1}{2}\rho A V_1{}^3} = \frac{1}{2}\left[1 - \left(\frac{V_2}{V_1}\right)^2 + \frac{V_2}{V_1} - \left(\frac{V_2}{V_1}\right)^3\right]$$

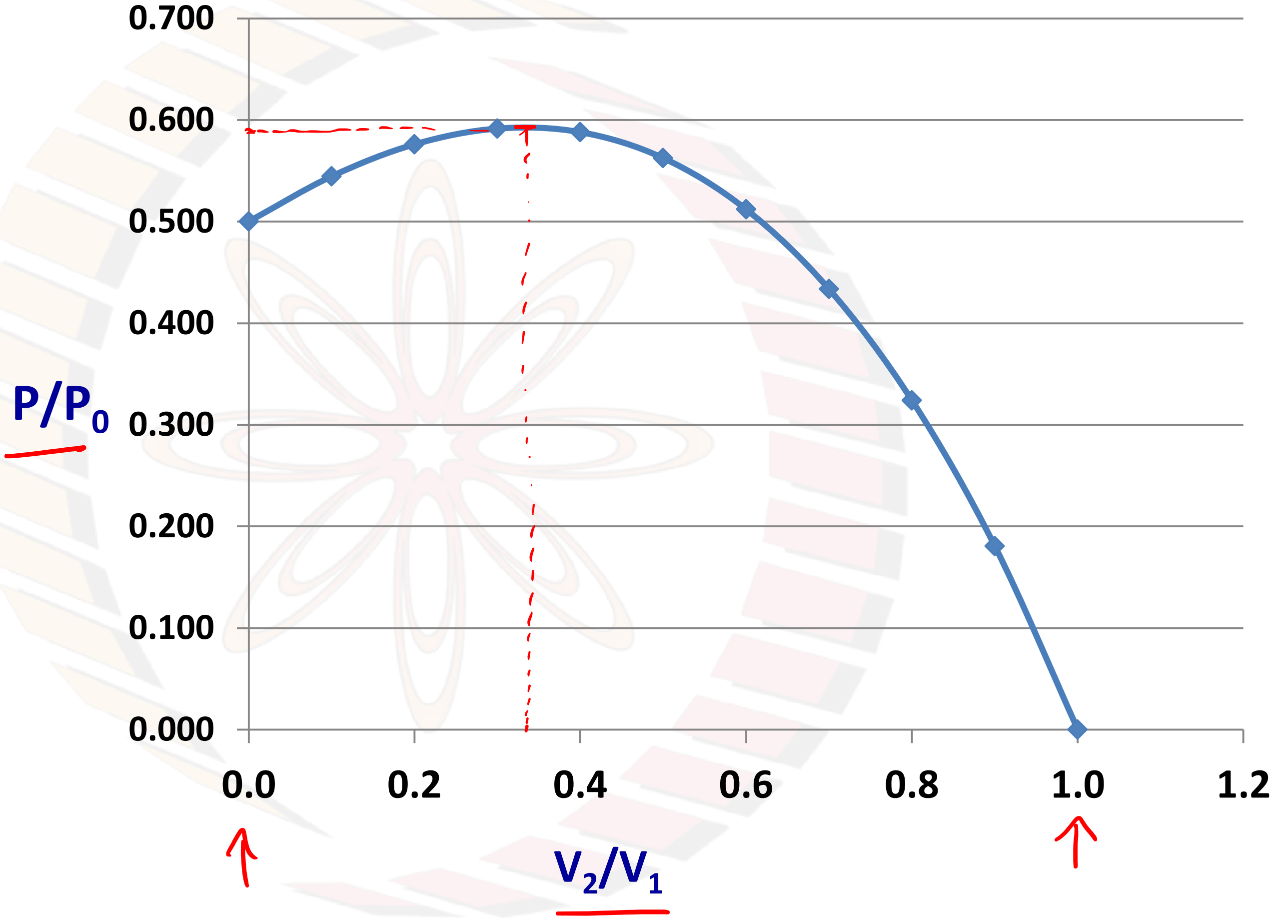
$$If\ we\ set\ \frac{V_2}{V_1} = \alpha$$

$$\frac{P}{P_0} = \frac{1}{2}[1 - \alpha^2 + \alpha - \alpha^3] \quad = \frac{1}{2}\left[1 - \frac{1}{9} + \frac{1}{3} - \frac{1}{27}\right]$$

$$= \frac{1}{2}\left[\frac{27 - 3 + 9 - 1}{27}\right]$$

$$= \frac{1}{2}\left[\frac{32}{27}\right] = \frac{16}{27} = \underline{0.593}$$

α	P/Po
0.0	0.500
0.1	0.545
0.2	0.576
0.3	0.592
0.4	0.588
0.5	0.563
0.6	0.512
0.7	0.434
0.8	0.324
0.9	0.181
1.0	0.000



Conclusions:

1) The Betz limit indicates that only about 59% of the energy available in the wind can actually be captured
2) Actual efficiencies will less than this limit

$$Change\ in\ energy\ in\ wind = \frac{1}{2}\rho Al({V_1}^2\ - {V_2}^2\ )$$

$$Power\ in\ wind = P = \frac{1}{2}\rho Av({V_1}^2\ - {V_2}^2\ )$$

$$If\ we\ set\ \frac{v}{V_1} = axial\ intereference\ factor\ (1-a) \qquad \therefore V_2 = V_1(1-2a)$$

$$P = \frac{1}{2}\rho A{V_1}^3\ (1-a)[(1-\ (1-2a)^2)$$

$$P = \frac{1}{2}\rho A{V_1}^3\ (1-a)(1-\ (1+4a^2-4a)$$

$$P = \frac{1}{2}\rho A{V_1}^3\ \ (4a^3-8a^2+4a) = 2\rho A{V_1}^3\ \ (a^3-2a^2+a)$$

$$P = 2\rho A {V_1}^3 \ (a^3 - 2a^2 + a)$$

$For\ maximum\ power\ to\ be\ tapped\ from\ wind\ energy$

$$\frac{dP}{da} = 0$$

$$\frac{dP}{da} = 3a^2 - 4a + 1 = 0$$

$$\therefore a = 1\ or\ a = \frac{1}{3}$$

$$at\ a = \frac{1}{3}, \qquad P = \frac{1}{2}\rho A {V_1}^3 \left(\frac{16}{27}\right) \approx 59\%\ of\ energy\ in\ the\ wind$$

Materials used in a windmill:

Drag Design:

Lift Design: