Chapter 1. "Probability Theory"

Eg. 1. In a class of 65 students.

Probability of atleast two students sharing a birthday.

Ans.

$$\underset{\substack{\downarrow \\ \text{Nobody sharing} \\ \text{a birthday}}}{p_0} + \underbrace{\underset{\substack{\downarrow \\ \text{two students} \\ \text{share a b'day}}}{p_2} + p_3 + \cdots + \underset{\substack{\downarrow \\ \text{All 65} \\ \text{students} \\ \text{sharing b'day}}}{p_{65}}}_{=\,p_A} = 1$$

$$p_0 + p_A = 1.$$

$$p_A = 1 - p_0 = 1 - \left[\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdots \frac{301}{365}\right]$$

$$= 1 - \frac{365 \cdot 364 \cdot 363 \cdots 301}{(365)^{65}}$$

$$= 1 - \frac{365!}{300!\,(365)^{65}}$$

Stirling approximation

$$365! = (365/e)^{365}$$

$$\simeq 1 - \frac{(365/e)^{365}}{(300/e)^{300}\,(365)^{65}}$$

$$> 70\,\%$$

Eg. 2. Four letters A, C, T, G represent monomers in a peptide chain

Chain is nine monomers long!

(A) A A A T C G A G T $\equiv$ Peptide chain

$\Rightarrow$ What is the probability of finding this sequence?

Permutation problem : Sequence is important !

Total size of sample space : $\underline{\underline{4^9}}$ [$\because$ Each monomer (total 9) is taken by one of the four (A, C, T, G)]

Probability of any one sequence $= 1/4^9 = \underline{\underline{4^{-9}}}$

(b) Probability of finding a sequence with $\left.\begin{array}{l}\text{four A's} \\ \text{two T's} \\ \text{two G's} \\ \text{one C}\end{array}\right\}$ Total nine monom.

Combination problem

A A A A T T G G C
A A A T G T G A C
⋮

Desired Probability $= \underbrace{\frac{1}{4^9}}_{\text{probability of 1 sequence}} \cdot \underbrace{\frac{9!}{4!\,2!\,2!\,1!}}_{\text{No. of such combinations}} \simeq 0.014$ (1.4 %)

Pb. 4. Fair sided die:  face value $= \mu \in [1, 2, 3, \ldots 6]$

How many times should I toss the die, such that the probability of getting atleast one '2' will become 2/3 !

$p(\mu) = 1/6$, $\mu \in [1, 2, 3, \ldots 6]$

$p(\mu = 2) = 1/6$

$p(\mu \neq 2) = 5/6$

Toss the die k. times:

Probability of $\mu \neq 2$: $(5/6)^k$ $\quad \because$ Each toss is independe

Probability of getting atleast one two

$$= 1 - (5/6)^k$$

$$= 2/3$$

Solve for k to get the required no. of tosses!

$$1 - (5/6)^k = 2/3$$

$k=1$: $\quad 1 - (5/6) = 1/6 < 2/3$

$k=2$: $\quad 1 - (5/6)^2 = 11/36 < 2/3$

$\vdots$

$k > 6$: $\quad 1 - (5/6)^k > 2/3$

Conclusion: I must toss atleast 7 times to get $P(\mu = 2) > 2/3$.

Pb: Fair sided die: $\mu \in [1, 2, 3, 4, 5, 6]$

Q. Probability of getting atleast two 5's in three tosses!

$x \in [1, 2, 3, 4, 6]$

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6} = \frac{5}{6^3} \equiv$ 5 5 x

x 5 5 ✓

5 x 5 ✓

$p(x) = 5/6$

$p(5) = 1/6$

Probability of two 5's $= 3 \cdot \frac{5}{6^3} = 15/6^3$

b) Atleast two 5's from three tosses of die.

$$P(\text{atleast two } 5\text{'s}) = P(\text{two } 5\text{'s}) + P(\text{three } 5\text{'s})$$

$$= 15/6^3 + 1/6^3$$

555

$$= 16/6^3$$

1. Probability concepts
   - Discrete random variables
   - Continuous random variable
   - Moments & their generators
   - Central limit theorem
   - Maximum Entropy principle

2. Postulates of S.M.
   - NVE – Micro
   - NVT – Canonical
   - NPT – Gibbs canonical
   - μVT – Grand canonical

3. Q.S.M.
   - Phonon gas in solids. (Low T. excitations)
   - Photon gas in cavity resonators "Black body radia
   - Electrons in methods/Fermi gas
   - Concept of Fermi surface.
   - BE/FD statistics.
   - Quantum $\longrightarrow$ Classical correspondence.

     High ρ, low T — Low ρ, High T.