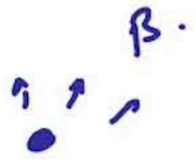
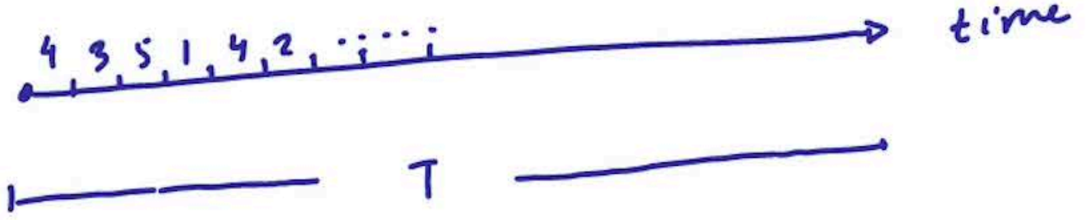
Poisson Distribution:

Example of radio-active decay. Co^{60}

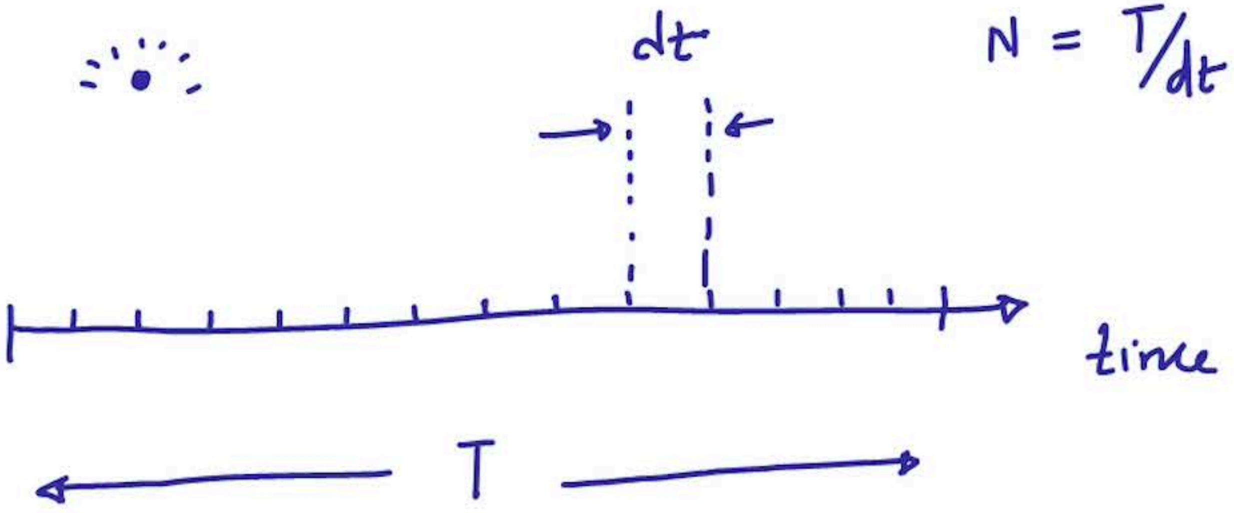




How many one minute intervals did you observe with m decays?

How many one minute intervals did you observe with n decays?

$N = T/dt$ intervals



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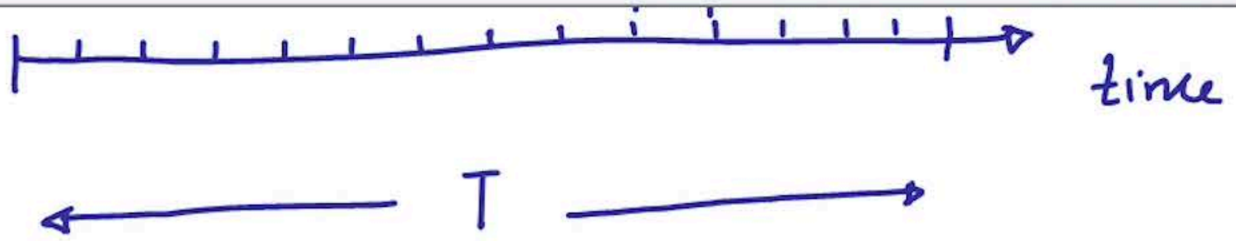


Lets try to reduce this dt further such that it becomes so small that either you get a event or no event !

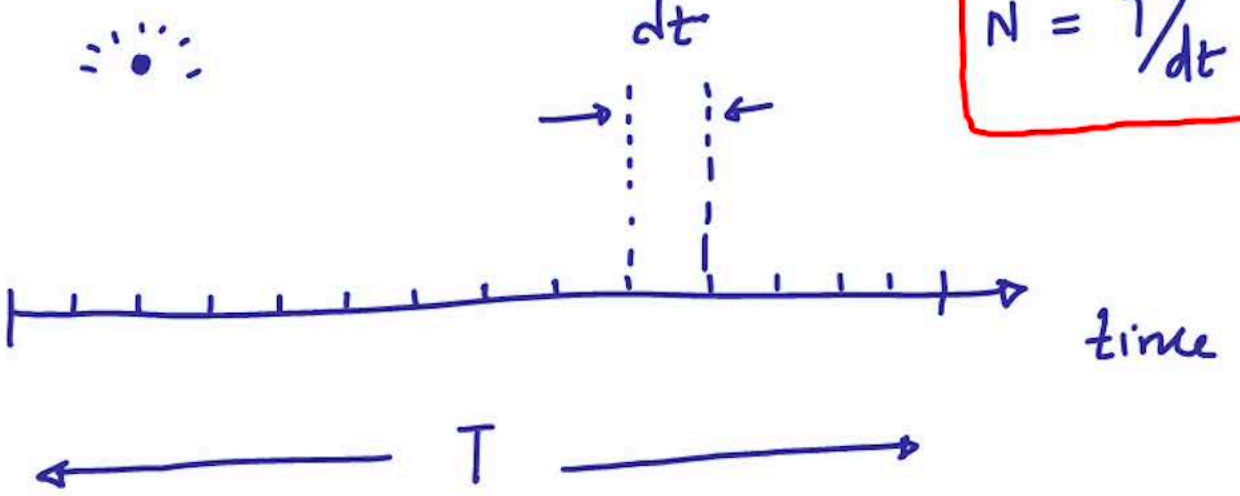
Probability of getting an event $= p \propto dt \Rightarrow p = \alpha\, dt$

α = Mean event rate

How many one minute intervals

did you observe with m decays?

$$\boxed{N = T/dt} \text{ intervals}$$



Lets try to reduce this dt further

such that it becomes so small

that either you get a event or no event!

$$\tilde{f}(k) = \left(e^{-ik} p + q\right)^N \quad \text{for } N \text{ intervals !}$$

$$= \left(e^{-ik}\alpha\, dt + 1 - \alpha\, dt\right)^N$$

$$= \left(1 + \alpha\, dt\left(e^{-ik} - 1\right)\right)^{T/dt} \qquad \because N = T/dt \gg 1$$

$$(1+x)^N = 1 + Nx + \overbrace{N(N-1)}^{\sim N^2}\frac{x^2}{2!} + \dots$$

$$= \sum_{j=0}^{\infty} \frac{(Nx)^j}{j!}$$

$$= e^{Nx}$$

$$\tilde{f}(k) = e^{T\alpha\left(e^{-ik}-1\right)} \quad \dots$$

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Characteristic f^n: $\quad \tilde{f}(k) = e^{T\alpha(e^{-ik}-1)}$ —— 1

$= \sum_{j=0}^{\infty} \frac{(Nx)^j}{j!}$

$= e^{Nx}$

To construct PDF: $\quad p_{\alpha T}(m) = \frac{1}{2\pi} \int_{k=-\infty}^{+\infty} e^{ikm} e^{\alpha T(e^{-ik}-1)} dk$

$$= \frac{1}{2\pi} \cdot e^{-\alpha T} \int_{k=-\infty}^{+\infty} e^{ikm} e^{\alpha T e^{-ik}} dk$$

$$= e^{-\alpha T} \cdot \frac{1}{2\pi} \int_{k=-\infty}^{+\infty} e^{ikm} \sum_{j=0}^{\infty} \frac{(\alpha T)^j e^{-ikj}}{j!} dk$$

1

1/1

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αT $\quad$ $m!$

in a duration of T with mean rate α"

$$\boxed{\tilde{p}(k) = e^{T\alpha(e^{-ik}-1)}} \longrightarrow \boxed{\ln \tilde{p}(k) = \alpha T (e^{-ik}-1)}$$

.... moments .... $\quad$ .... cumulants ....

$$\langle m \rangle_c = \langle m \rangle = \frac{\partial}{\partial(-ik)} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

$$\langle m^2 \rangle_c = \frac{\partial^2}{\partial(-ik)^2} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

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--- moments ----

$$\langle m \rangle_c = \langle m \rangle = \frac{\partial}{\partial(-ik)} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

$$\langle m^2 \rangle_c = \frac{\partial^2}{\partial(-ik)^2} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

$$\vdots$$

$$\langle m^n \rangle_c = \frac{\partial^n}{\partial(-ik)^n} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

$$\langle m \rangle = \bullet = \langle m \rangle_c = \alpha T$$

$$\langle m^2 \rangle = \bullet\,\bullet + (\bullet\,\bullet) = \langle m \rangle^2 + \langle m^2 \rangle_c$$
$$= (\alpha T)^2 .$$

$$\langle m^3 \rangle = \therefore + 3(:)\bullet + (\therefore)$$
$$= \langle m \rangle^3 + 3\langle m^2 \rangle_c \langle m \rangle + \langle m^3 \rangle_c$$
$$= (\alpha T)^3 + 3(\alpha T)^2 + (\alpha T)$$

$$\vdots$$

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$\partial(-ik)$ $k=0$

$\vdots$

$$\langle m^n \rangle_c = \frac{\partial^n}{\partial(-ik)^n} \ln \tilde{p}(k) \Big|_{k=0} = \alpha T$$

$$\langle m^3 \rangle = \therefore + 3 (\cdot) . + (\because)$$

$$= \langle m \rangle^3 + 3 \langle m^2 \rangle_c \langle m \rangle + \langle m^3 \rangle_c$$

$$= (\alpha T)^3 + 3(\alpha T)^2 + (\alpha T)$$

$\vdots$

Worked Example:

A radioactive source emitting α-particles.

1800 α-particles were observed to be emitted over 10 hours.

How many one minute intervals did you observe with

(a) no α-particles

(b) 5 α-particles

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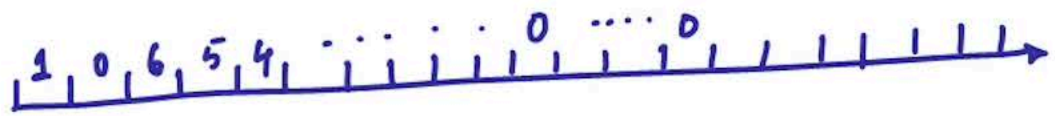
Solutions:



T = 10 hours 1800 events

Mean event rate, $\alpha = \frac{1800}{10 \times 60} = 3$ events/minute

$$p_T(n) = \frac{(\alpha T)^n e^{-\alpha T}}{n!} \Rightarrow p_{1\,min}(0) = \frac{(\alpha T)^0 e^{-3.1}}{0!} = e^{-3}$$

How many such one minute interval?

Ans. $p_{1\,min}(0) \times$ Total minutes $= e^{-3} \cdot 600$

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Mean event rate, $\alpha = \frac{\quad}{10 \times 60}$

$$p_T(m) = \frac{(\alpha T)^m e^{-\alpha T}}{m!} \Rightarrow p_{1\,min}(0) = \frac{(\alpha T)^0 e^{-3.1}}{0!} = e^{-3}$$

(a) How many such one minute interval?

Ans. $p_{1min}(0) \times \text{Total minutes} = e^{-3} \cdot 600 \text{ minutes} \approx 30 \text{ min}$

(b) $$p_{1min}(5) = \frac{(3.1)^5 \cdot e^{-3.1}}{5!} = \frac{3^5 e^{-3}}{5.4.3.2.1} \approx 60 \text{ min}$$

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3

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$$p_T(n) = \frac{(\alpha T)^n e^{-\alpha T}}{n!} \Rightarrow p_{1\,min}(0) = \frac{(\alpha\,1)\ e}{0!}$$

(a) How many such one minute interval?

Ans. $p_{1min}(0) \times \text{Total minutes} = e^{-3} \cdot 600 \text{ minutes} \approx 30 \text{ min}$

(b) $p_{1min}(5) = \dfrac{(3.1)^5 \cdot e^{-3.1}}{5!} = \dfrac{3^5\, e^{-3}}{5.4.3.2.1}$

How many such one minute intervals?

Ans. $p_{1min}(5) \times \text{Total minutes} = \dfrac{3^5\, e^{-3}}{5!} \times 600 \approx 60 \text{ min}$