Moments & fluctuations:

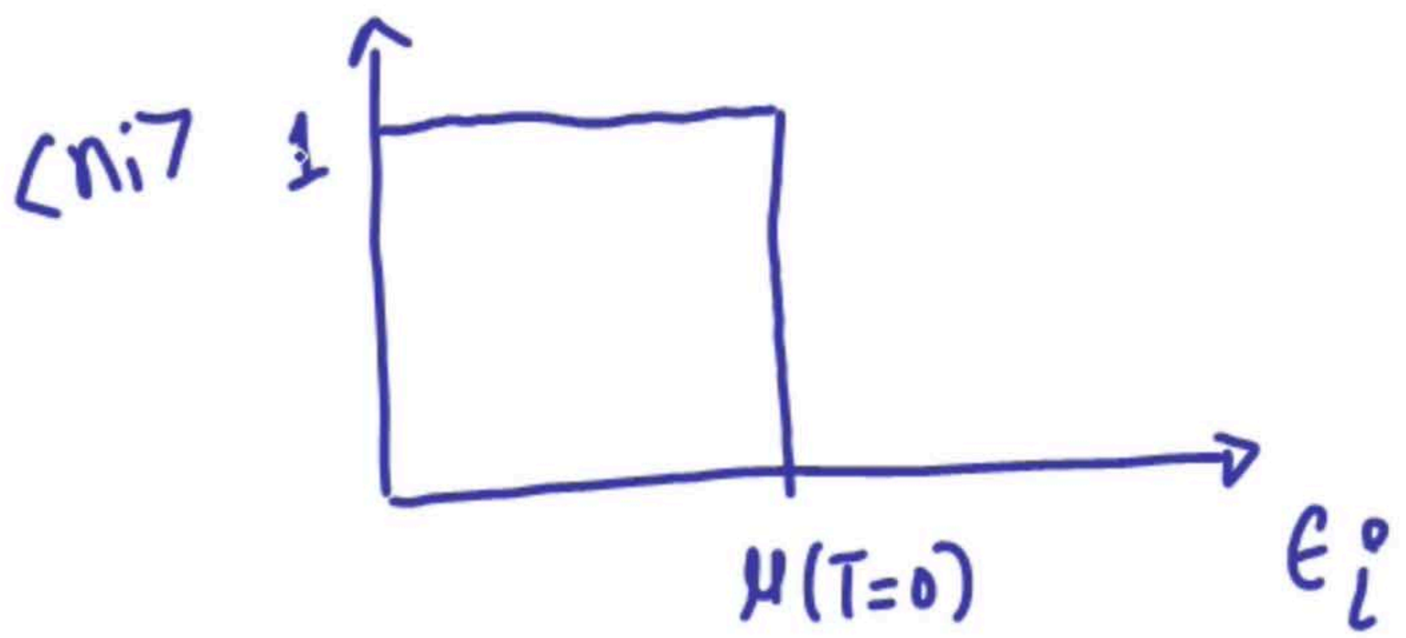
Fermions: $\left(\frac{1}{2}, \frac{3}{2} \ldots\right)$

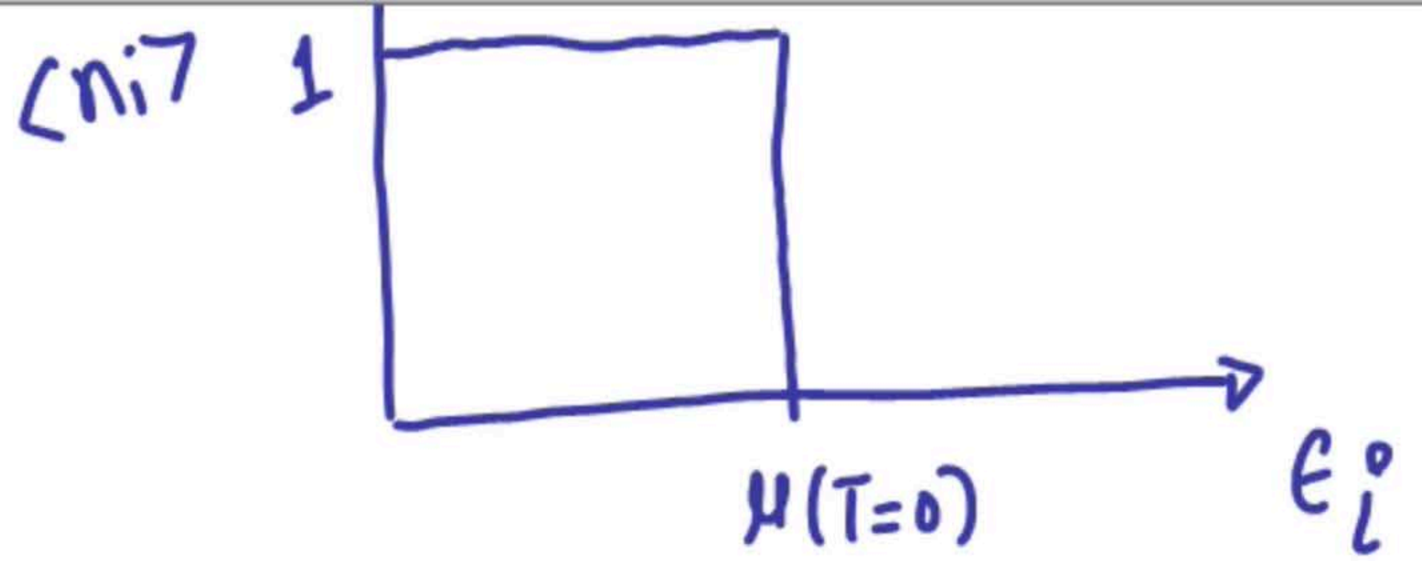
$n_i = 0, 1$

$$\langle n_i \rangle = \frac{1}{e^{\beta(\epsilon_i - \mu)} + 1}$$

at $\beta \to \infty \quad (T \to 0)$

$$\epsilon_j < \underbrace{\mu\,(T=0)}_{\text{Fermi energy}}$$





At $T \neq 0$: moments $\langle n_i n_j \rangle$ = Joint probability of finding a Fermion in level i & another Fermion in level j.

$i \neq j$

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$$\langle n_i n_j \rangle = \frac{\displaystyle\sum_{\{n_i\}} n_i\, n_j\, e^{-\beta \sum_{k=1}^{\infty} n_k (\epsilon_k - \mu)}}{\displaystyle\sum_{\{n_i\}} e^{-\beta \sum_{k=1}^{\infty} n_k (\epsilon_k - \mu)}} = \frac{\displaystyle\sum_{\{n_i\}} n_i n_j\, e^{-\beta \sum_{k=1}^{\infty} n_k (\epsilon_k - \mu)}}{Q(\mu, V, T)}$$

$$= \frac{1}{Q} \frac{\partial}{\partial(-\beta\epsilon_i)} \frac{\partial}{\partial(-\beta\epsilon_j)} Q.$$

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$$\sum_{\{n_i\}} e^{-\beta \sum_{k=1}^{\infty} n_k(\epsilon_k - \mu)} \qquad Q(\mu, V, T)$$

$$= \frac{1}{Q} \frac{\partial}{\partial(-\beta\epsilon_i)} \left(\frac{\partial}{\partial(-\beta\epsilon_j)} Q \right)$$

$$\langle n_i \rangle = \frac{1}{Q} \cdot \frac{\partial}{\partial(-\beta\epsilon_i)} Q$$

$$= \frac{1}{Q} \frac{\partial}{\partial(-\beta\epsilon_i)} \left(Q \langle n_j \rangle \right)$$

$$\langle n_i \rangle = \frac{1}{Q} \cdot \frac{\partial}{\partial(-\beta\epsilon_i)} Q$$

$$= \frac{1}{Q} \frac{\partial}{\partial(-\beta\epsilon_i)} \left(Q \langle n_j \rangle \right)$$

$$= \left(\frac{1}{Q} \frac{\partial}{\partial(-\beta\epsilon_i)} Q \right) \langle n_j \rangle + \left(\frac{1}{\cancel{Q}} \frac{\partial}{\partial(-\beta\epsilon_i)} \langle n_j \rangle \right) \cancel{Q}$$

$$= \langle n_i \rangle \langle n_j \rangle + \frac{\partial}{\partial(-\beta\epsilon_i)} \cancel{\frac{1}{1 + e^{-\beta(\epsilon_j - \mu)}}}$$

$$= \left(\frac{1}{Q} \frac{\partial Q}{\partial(-\beta\epsilon_i)} \right) \langle n_j \rangle + \left(\frac{1}{\cancel{Q}} \frac{\partial \langle n_j \rangle}{\partial(-\beta\epsilon_i)} \right) \cancel{Q}$$

$$= \langle n_i \rangle \langle n_j \rangle + \frac{\partial}{\partial(-\beta\epsilon_i)} \cancelto{0}{\frac{1}{1+e^{-\beta(\epsilon_j-\mu)}}}$$

$$\boxed{\langle n_i n_j \rangle = \langle n_i \rangle \langle n_j \rangle}$$

n_i & n_j are independent of each other.

→ Fluctuation of population number: $\langle n_i^2 \rangle_c = \langle n_i^2 \rangle - \langle n_i \rangle^2$

2.) Fluctuation of occupation number: $\langle n_j^2 \rangle_c = \langle n_j^2 \rangle - \langle n_j \rangle^2$

Recalling that: $\langle n_j \rangle = \dfrac{\sum_{\{n_i\}} n_j \, e^{-\beta \sum_{k=1}^{\infty} n_k (\epsilon_k - \mu)}}{Q}$

$$\langle n_j^2 \rangle_c = \frac{1}{Q} \cdot \frac{\partial^2 Q}{\partial(-\beta \epsilon_j)^2} - \frac{1}{Q^2} \left(\frac{\partial Q}{\partial(-\beta \epsilon_j)} \right)^2 :$$

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$$\langle n_j^2 \rangle_c = \frac{1}{Q} \cdot \frac{\partial^2 Q}{\partial(-\beta\epsilon_j)^2} - \frac{1}{Q^2}\left(\frac{\partial Q}{\partial(-\beta\epsilon_j)}\right)^2$$

$$= \frac{\partial}{\partial(-\beta\epsilon_j)} \cdot \frac{\partial}{\partial(-\beta\epsilon_j)} \ln Q$$

$$\frac{\partial}{\partial(-\beta\epsilon_j)} \ln Q = \frac{1}{Q} \cdot \frac{\partial Q}{\partial(-\beta\epsilon_j)}$$

$$= \langle n_j \rangle$$

$$= \frac{1}{e^{\beta(\epsilon_j - \mu)} + 1}$$

$$= \frac{\partial}{\partial(-\beta\epsilon_j)} \langle n_j \rangle$$

$$= \frac{e^{\beta}}{\left(e^{\beta(\epsilon_j - \mu)} + 1\right)^2}$$

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Lecture 35.mp4   Open with QuickTime Player

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$$\langle n_j^2\rangle_c \;\; \frac{\quad e^{\beta(\epsilon_j-\mu)}}{\left(e^{\beta(\epsilon_j-\mu)}+1\right)^2} \qquad e \qquad - \langle n_j\rangle$$

$$= e^{\beta(\epsilon_j-\mu)} \langle n_j\rangle^2$$

$$= \left(\frac{1}{\langle n_j\rangle} - 1\right)\langle n_j\rangle^2 = \langle n_j\rangle - \langle n_j\rangle^2$$

$$\langle n_j^2\rangle_c = \langle n_j\rangle - \langle n$$

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$$\langle n_j^2\rangle_c = \frac{\cdots}{\left(e^{\beta(\epsilon_j-\mu)}+1\right)^2} \qquad e \qquad = \langle n_j\rangle$$

$$= e^{\beta(\epsilon_j-\mu)}\langle n_j\rangle^2$$

$$= \left(\frac{1}{\langle n_j\rangle}-1\right)\langle n_j\rangle^2 = \langle n_j\rangle - \langle n_j\rangle^2$$

$$\langle n_j^2\rangle_c = \langle n_j\rangle - \langle n$$

$$= \frac{\partial}{\partial(-\beta\epsilon_j)}$$

$$\langle n_j^2 \rangle_c = \frac{e^{\beta(\epsilon_j - \mu)}}{\left(e^{\beta(\epsilon_j - \mu)} + 1\right)^2} \qquad e^{\beta(\epsilon_j - \mu)} = \frac{1}{\langle n_j \rangle} - 1$$

$$= e^{\beta(\epsilon_j - \mu)} \langle n_j \rangle^2$$

$$= \left(\frac{1}{\langle n_j \rangle} - 1\right) \langle n_j \rangle^2 = \langle n_j \rangle - \langle n_j \rangle^2$$

$$\boxed{\langle n_j^2 \rangle_c = \langle n_j \rangle - \langle n_j \rangle}$$

In terms of energies, $\langle n_j^2 \rangle_c = \dfrac{e^{\beta(\epsilon_j - \mu)}}{\left(e^{\beta(\epsilon_j-\mu)} + 1\right)^2}$

$$= \frac{e^{\beta(\epsilon_j - \mu)/2} \cdot e^{\beta(\epsilon_j - \mu)/2}}{\left(e^{\beta(\epsilon_j-\mu)} + 1\right)\left(e^{\beta(\epsilon_j-\mu)} + 1\right)}$$

$$\left(e^{\beta(\epsilon_j-\mu)}+1\right)\left(e^{\beta(\epsilon_j-\mu)}+1\right)$$

$$= \frac{2^2}{4\left(e^{\beta(\epsilon_j-\mu)/2}+e^{-\beta(\epsilon_j-\mu)/2}\right)^2}$$

$$= \frac{1}{4}\cdot\frac{1}{\cosh\left[\beta(\epsilon_j-\mu)/2\right]}^{2}.$$

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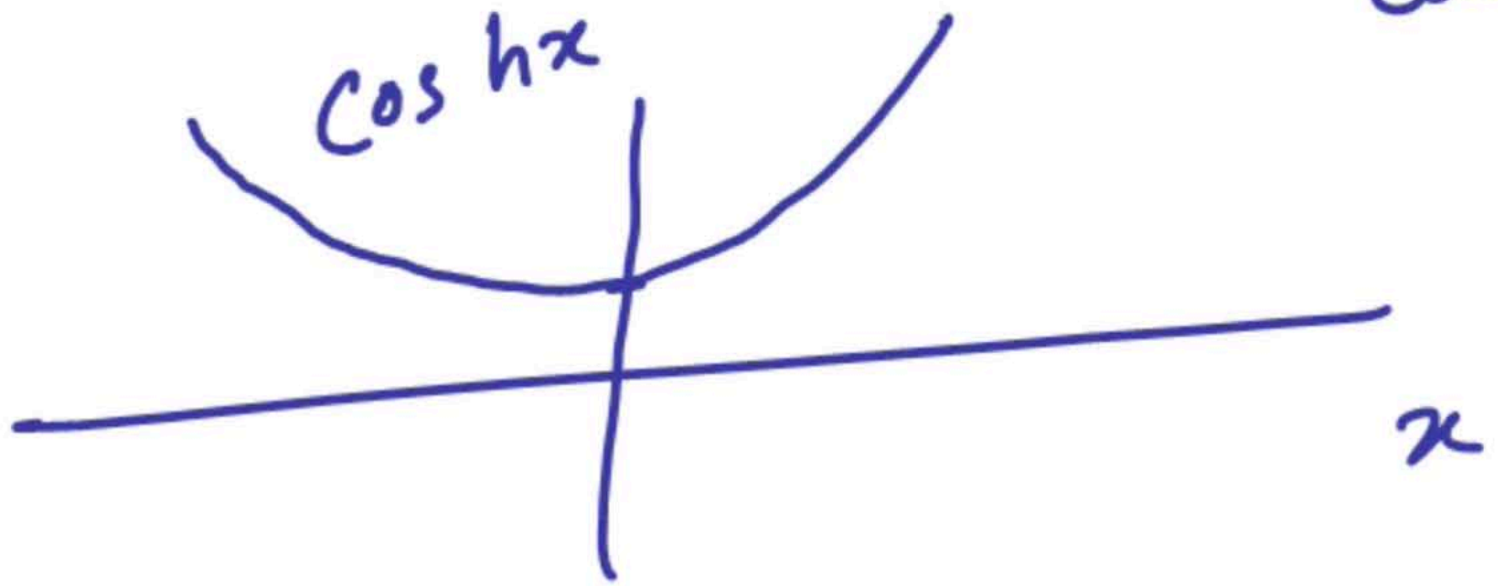
$$= \frac{1}{4} \cdot \frac{1}{\cosh\left[\beta(\epsilon_j - \mu)/2\right]}^{2}$$

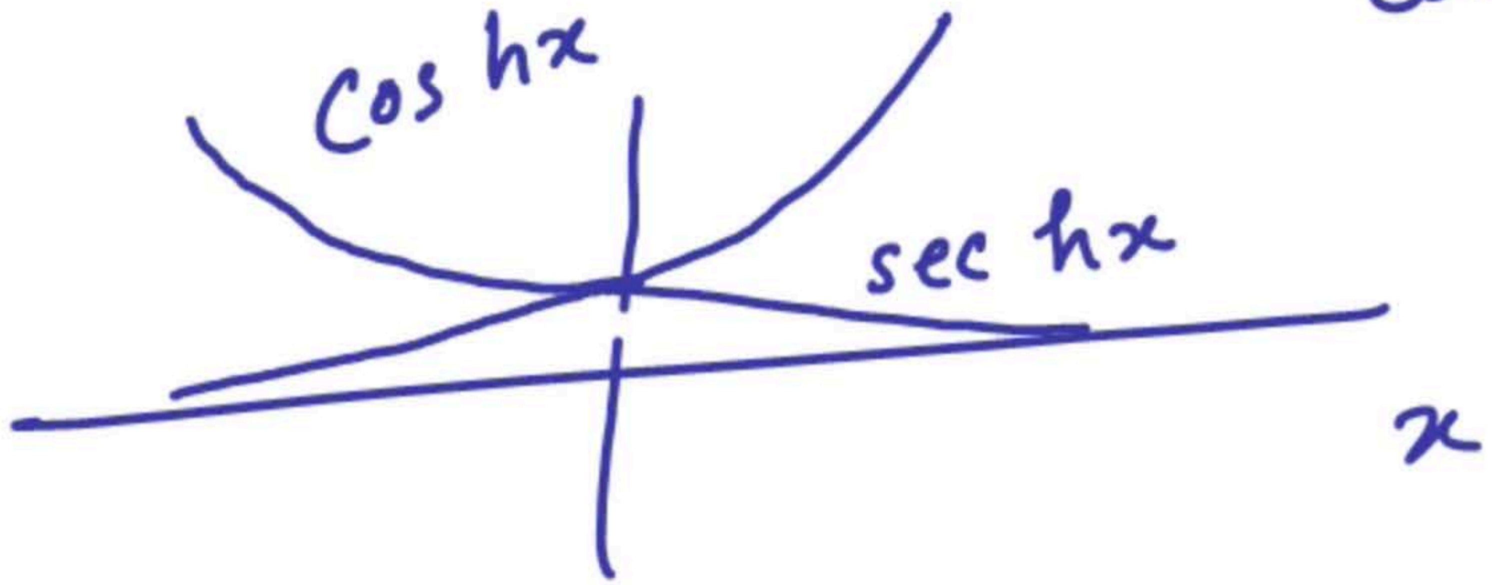
$$\langle n_j^2 \rangle_c = \frac{1}{4} \left(\operatorname{sech} \beta(\epsilon_j - \mu)/2 \right)^2$$

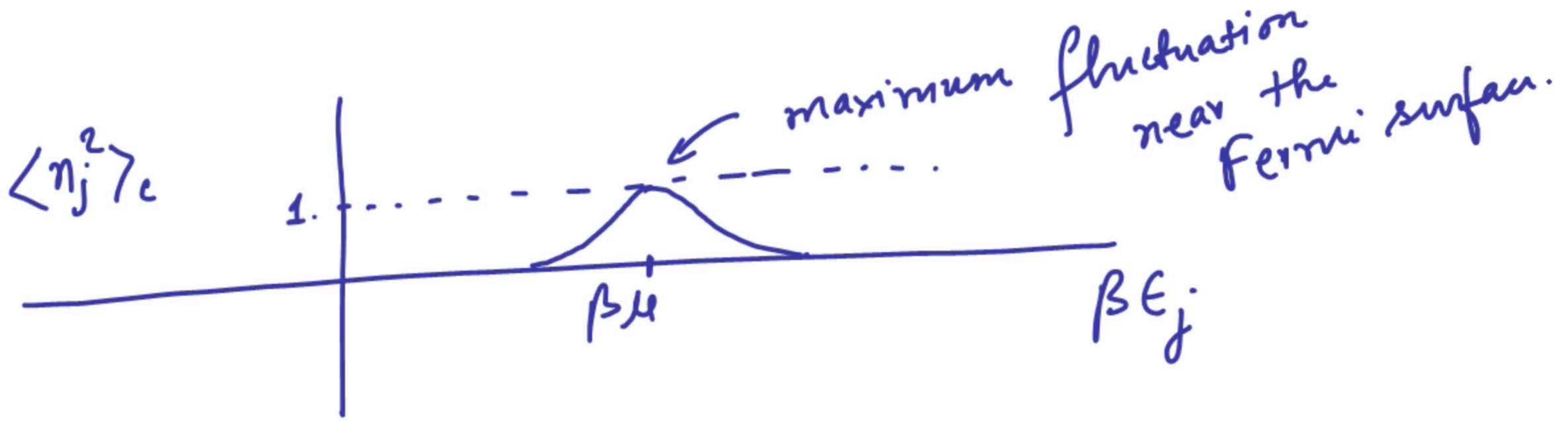
$$\cosh x = \frac{e^x + e^{-x}}{2}.$$



$$\langle n_j^2\rangle_c = \frac{1}{4}\left(\operatorname{sec\,h}\ \beta(\epsilon_j-\mu)/2\right)^2$$

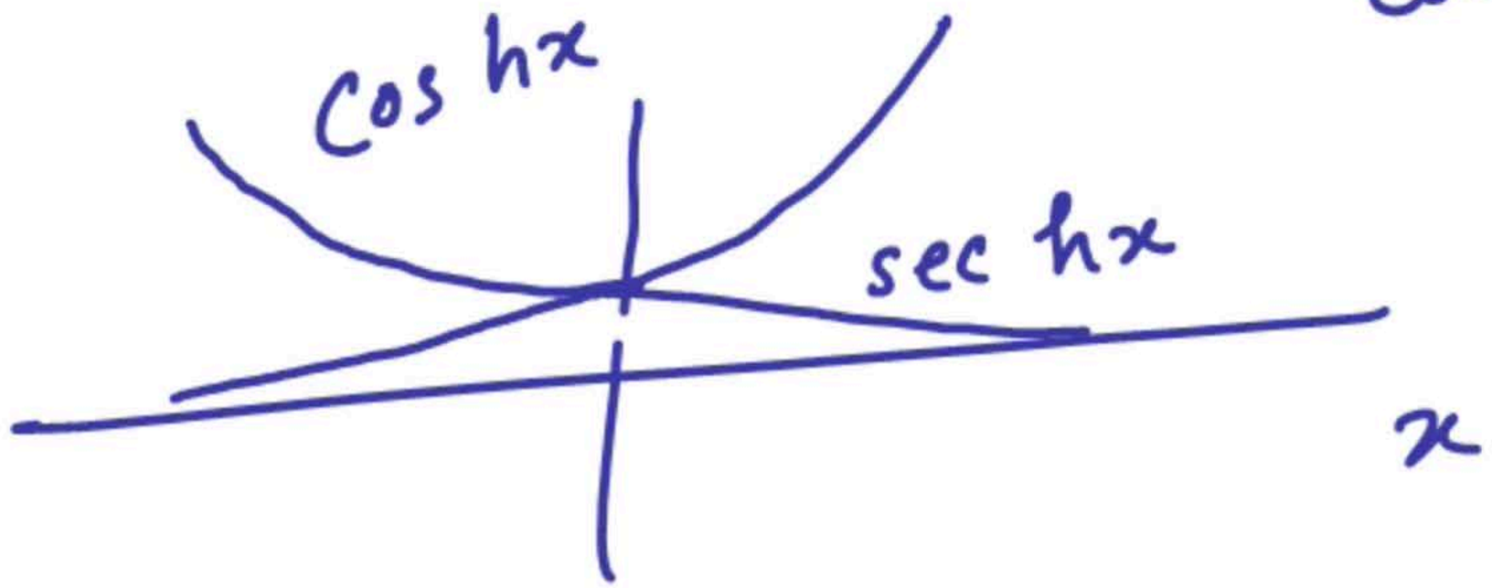
$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

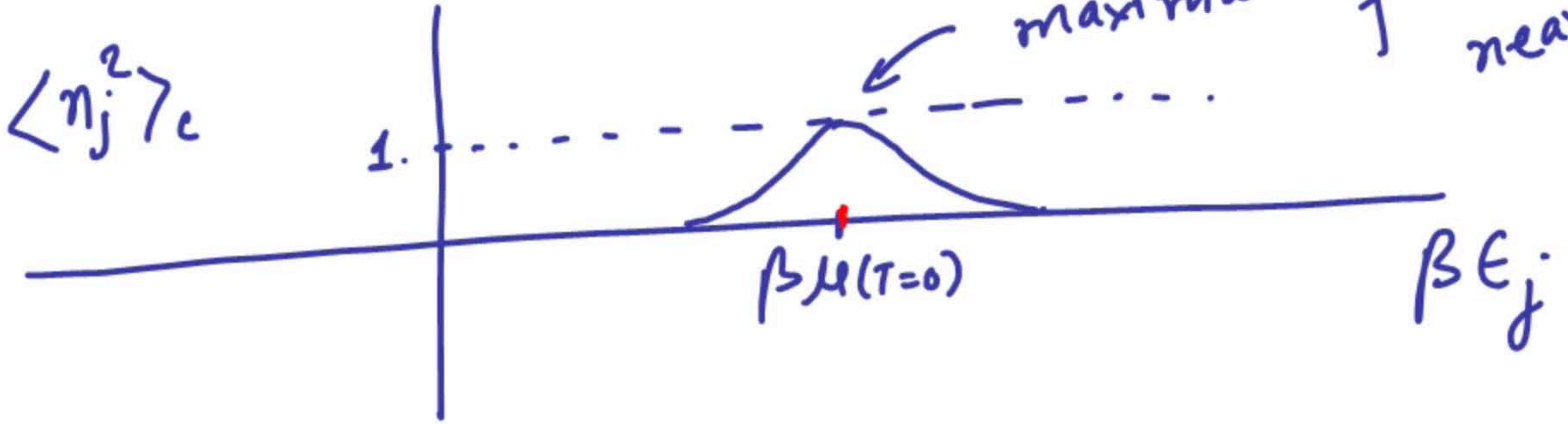




$$\langle n_j^2 \rangle_c = \frac{1}{4}\left(\mathrm{sec\,h}\ \beta(\epsilon_j - \mu)/2\right)^2$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$





maximum fluctuation near the Fermi surface.