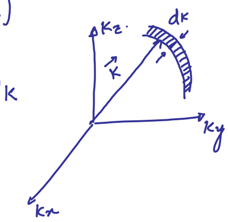
No. of modes in volume $d^3k = \dfrac{d^3k}{V_{min}} = \dfrac{4\pi k^2 dk}{(2\pi/L)^3}$



No. of modes in $d^3k = \dfrac{L^3}{2\pi^2} k^2 dk = g(k)\,dk$

$\omega = c\,k$ (c: acoustic speed)

$d\omega = c\,dk$

$$= \frac{L^3}{2\pi^2} \cdot \frac{\omega^2}{c^2} \cdot dk$$

$$= \frac{L^3}{2\pi^2} \cdot \frac{\omega^2}{c^2} \cdot \frac{d\omega}{c}$$

$$= \frac{L^3 \omega^2}{2\pi^2 c^3} d\omega$$

$$= g(\omega)\,d\omega$$

$$\int_{\omega=0}^{\omega_D = ck_D} g(\omega)\,d\omega = 3N$$

"Total no. of oscillators"

$$= \int_{\omega=0}^{\omega_D} \frac{L^3 \omega^2}{2\pi^2 c^3} d\omega$$

$$= \frac{L^3}{2\pi^2 c^3} \cdot \frac{\omega_D^3}{3} = 3N$$

$$\Rightarrow \frac{L^3}{2\pi^2 c^3} = \frac{9N}{\omega_D^3}$$

$$g(\omega) = \frac{L^3}{2\pi^2 c^3} \cdot \omega^2 = \frac{9N}{\omega_D^3} \cdot \omega^2$$

Debye's DOS.