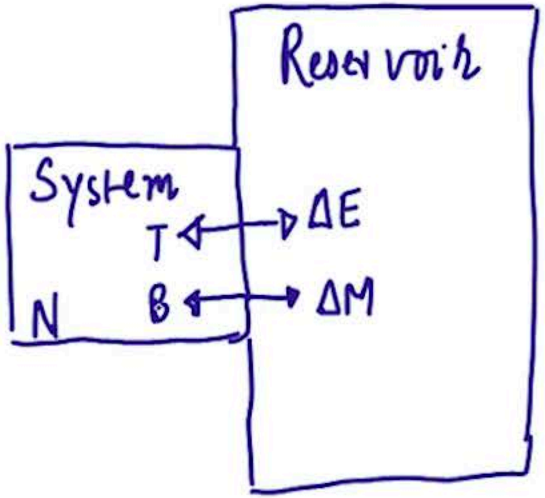
N spins in a uniform magnetic field. (N, B, T)



Generalized force	Extensive variable
P	ΔV
B	ΔM

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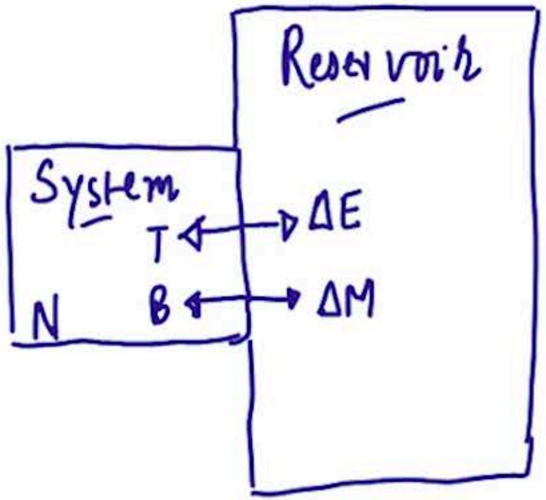

M_ν : Instantaneous magnetization

$$= \sum_{i=1}^{N} \mu_0 \sigma_i$$

Generalized force	Extensive variable
P	ΔV
B	ΔM

For $S = 1/2$ system,

$$\sigma_i = +1, -1$$

Energy scale : $\mathcal{H}(\mu) - BM_\mu = E_\mu$

Spins are non-interacting $\longrightarrow$ $\mathcal{H}(\mu) = 0$

Probability distribution fuction : $p(\mu) = p(\{\sigma_i\}) = \dfrac{e^{-\beta(-BM_\mu)}}{Z(N,B,T)}$

$$= \frac{e^{\beta B M_\mu}}{Z(N,B,T)} .$$

Probability distribution function: $\dfrac{\quad}{Z(N,B,T)}$

$$p(\mu) = \frac{e^{\beta B M_\mu}}{Z(N,B,T)}$$

Canonical partition function: $$Z(N,B,T) = \sum_{\mu=\{\sigma_i\}} e^{\mu_0 \beta B \sum_{j=1}^{N} \sigma_j}$$

$$= \sum_{\{\sigma_i\}} e^{\mu_0\beta B\sigma_1} \cdot e^{\mu_0\beta B\sigma_2} \cdots e^{\mu_0\beta B\sigma_N}.$$

Canonical partition function

$$\mu = \{\sigma_i\}$$

$$= \sum_{\{\sigma_i\}} e^{\mu_0 \beta B \sigma_1} \cdot e^{\mu_0 \beta B \sigma_2} \cdots e^{\mu_0 \beta B \sigma_N}$$

$$= \sum_{\sigma_1 = +1,-1} e^{\mu_0 \beta B \sigma_1} \sum_{\sigma_2 = \pm 1} e^{\mu_0 \beta B \sigma_2} \cdots \sum_{\sigma_N = \pm 1} e^{\mu_0 \beta B \sigma_N}$$

$$= \left(\sum_{\sigma_1 = \pm 1} e^{\mu_0 B \beta \sigma_1} \right)^N .$$

$$\sigma_1 = +1,-1 \qquad \sigma_2 = \pm 1 \qquad \sigma_N = \pm 1$$

$$= \left(\sum_{\sigma_1 = \pm 1} e^{\mu_0 B \beta \sigma_1} \right)^N$$

$$= \left(e^{\mu_0 \beta B} + e^{-\mu_0 \beta B} \right)^N$$

$$Z(N,B,T) = 2^N \cosh\left(\mu_0 \beta B\right)^N$$

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Average magnetization : $M = \langle M_\mu \rangle$

$$= \sum_{\mu} M_\mu \, p_\mu$$

$$= \frac{\displaystyle\sum_{\mu = \{\sigma_i\}} M_{\{\sigma_i\}} \, e^{\beta B M_{\{\sigma_i\}}}}{Z(N, B, T)}$$

$$Z(N,B,T) = 2^N \cosh(\mu_0 \beta B)^N \qquad (1)$$

Average magnetization : $M = \langle M_\mu \rangle$

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$$M = N\mu_0 \tanh(\mu_0 \beta B)$$

Magnetic susceptibility: $\left.\dfrac{\partial M}{\partial B}\right|_{B=0} = \chi_M(T)$

$$\chi_M(T) = N\mu_0 \operatorname{sech}^2(\mu_0\beta B)\,\mu_0\beta \Big|_{B=0}$$

$$= \frac{\mu_0^2 N}{\beta}$$

2/2

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