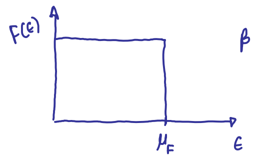
# Free electron Fermi gas :

At. Low T

$C_V = \text{Lattice vibrations} + \text{Free electrons}$

$\qquad \alpha T^3 \qquad\qquad \beta T$

For Fermions: $\langle n_j \rangle_{F.D} = \dfrac{1}{e^{\beta(\epsilon_j - \mu)} + 1} = F(\epsilon_j)$




$\beta = \infty \ (T = 0)$

$\mu\big|_{T=0} = \mu_F$ "Fermi Energy"

$= \epsilon_F$

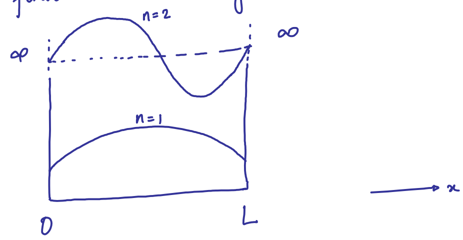
Total no. of Particles : $N = \sum_j \langle n_j \rangle$

Total Energy $E = \sum_j \epsilon_j \langle n_j \rangle$

$\Big\}$ All temperatures

$\left.\begin{matrix} N \gg 1 \\ V \gg 1 \end{matrix}\right\} n = N/V \rightsquigarrow$ finite "Thermodynamic limit"

Electron in a box :




$\psi(x) = \psi_0 \cdot \sin\left(\dfrac{n\pi x}{L}\right)$

$\vec{k} = (k_x, k_y, k_z)$

$= (n_x, n_y, n_z)\dfrac{\pi}{L}$

Exitations $n_x, n_y, n_z$ are $> 0$

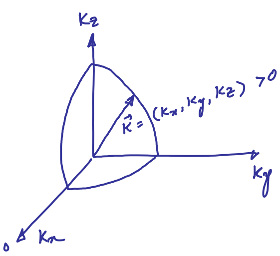
As the size $L \to \infty$ , the $k$'s become closed stacked.

$\sum_{\vec{k}} \xrightarrow[\text{as } N,V \to \infty]{} \int \underbrace{g(k)}_{dN/dk} dk$

To compute $g(k)$:

$dN$ modes in some $d^3k$

$= \dfrac{d^3k}{V_{min}} \cdot \underbrace{(2s+1)}_{s=1/2} \cdot \underbrace{\dfrac{1}{8}}_{n_{x,y,z} > 0}$



$dN$ modes in $d^3k$ $= \dfrac{4\pi k^2\, dk}{(\pi/L)^3} \, (2s+1) \cdot \dfrac{1}{8}$

$$= \frac{(2s+1) V k^2\, dk}{2\pi^2} = g(k)\, dk$$

$$\Rightarrow \quad g(k) = \frac{(2s+1)\, V\, k^2}{2\pi^2} \qquad \text{--- } (1)$$

Lets compute $N = \displaystyle\int_{k=0}^{k=k_F} \underbrace{g(k)}_{dN/dk}\, dk$ At $T = 0$

$$= \frac{(2s+1)}{2\pi^2} \cdot V \, \frac{k_F^3}{3}$$

$$k_F = \left( \frac{6\pi^2}{(2s+1)} \, \frac{N}{V} \right)^{1/3}$$

Compute Fermi energy: $\epsilon_F = \dfrac{\hbar^2 k_F^2}{2m} = \dfrac{\hbar^2}{2m} \left( \dfrac{6\pi^2}{(2s+1)} \, \dfrac{N}{V} \right)^{2/3}$

Compute Fermi temperature: $T_F \simeq \dfrac{\epsilon_F}{k_B} \simeq \dfrac{\hbar^2}{2mk_B} \left( \dfrac{6\pi^2}{2s+1} \cdot \dfrac{N}{V} \right)^{2/3}$

$$g(k)\, dk = \frac{(2s+1)\, Vk^2}{2\pi^2}\, dk = g(\epsilon)\, d\epsilon \qquad \text{--- } (2)$$

To get $g(\epsilon)$ "Density of states in terms of $\epsilon$"

$$g(\epsilon) = dN/d\epsilon$$

$$\int_0^{k_F} g(k)\, dk = \int_0^{\epsilon_F} g(\epsilon)\, d\epsilon$$

$$g(k)\, dk = \frac{(2s+1)\, Vk^2}{2\pi^2}\, dk$$

Using the fact that $\epsilon = \dfrac{\hbar^2 k^2}{2m} \Rightarrow k^2 = \dfrac{2m\epsilon}{\hbar^2}$

$$dk = \frac{1}{k} \cdot \frac{m\, d\epsilon}{\hbar^2}$$

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Take Cu for eg:

$n = \dfrac{N}{V} = 10^{28}/m^3$

$k_F \sim 10^9/m$

$\epsilon_F \sim 10^{-68} \cdot 10^{31} \cdot 10^{18}$ J $\sim 10^{-19}$ J $\sim$ eV

$T_F \sim 10^{-19}\, 10^{23}$ K $\sim 10^4$ K

At room temperature: $T = 300$ K

$$\frac{T}{T_F} << 1$$

Related to the fraction of excited electrons at $T$.

$$dk = \left(\frac{\hbar^2}{2m\epsilon}\right)^{1/2} \frac{m\, d\epsilon}{\hbar^2}$$

$$\therefore \quad g(k)\, dk = \frac{(2s+1)\, V}{2\pi^2} \cdot \left(\frac{2m\epsilon}{\hbar^2}\right) \left(\frac{\hbar^2}{2m\epsilon}\right)^{1/2} \frac{m}{\hbar^2}\, d\epsilon$$

$$= \underbrace{(2s+1)\, \frac{V m}{2\pi^2 \hbar^2} \left(\frac{2m}{\hbar^2}\right)^{1/2} \epsilon^{1/2}}_{g(\epsilon)}\, d\epsilon$$

$$g(\epsilon) = \frac{(2s+1)\, V m}{2\pi^2 \hbar^2} \left(\frac{2m}{\hbar^2}\right)^{1/2} \epsilon^{1/2} \quad \text{—} \; 3$$

Recalling $\epsilon_F = \dfrac{\hbar^2}{2m} \left(\dfrac{6\pi^2}{(2s+1)} \dfrac{N}{V}\right)^{2/3}$