

# Lecture 5: Intrinsic semiconductors-conductivity

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## 1 Carrier concentration in intrinsic Si

Consider intrinsic Si at room temperature. Intrinsic refers to pure Si, an ideal single crystal with impurity concentrations below ppb (parts per billion). There is an equilibrium concentration of electrons and holes at room temperature, due to thermal excitation. This is the intrinsic carrier concentration ( $n_i$ ) given by

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k_B T}\right) \quad (1)$$

$N_c$  and  $N_v$ , the effective density of states at the band edges, are dependent on temperature and the effective mass of the electron and holes respectively. For Si,  $m_e^*$  is  $1.08 m_e$  and  $m_h^*$  is  $0.60 m_e$ . From this the effective density of states can be calculated using

$$\begin{aligned} N_c &= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{\frac{3}{2}} \\ N_v &= 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{\frac{3}{2}} \end{aligned} \quad (2)$$

At room temperature the values of  $N_c$  and  $N_v$  for Si are  $2.81 \times 10^{25} \text{ m}^{-3}$  and  $1.16 \times 10^{25} \text{ m}^{-3}$ . These are the total number of available states at the band edges for the electrons and holes to occupy. Hence, the intrinsic carrier concentration at room temperature can be calculated using equation 1 with the band gap value of  $1.10 \text{ eV}$ . This gives  $n_i$  equal to  $10^{16} \text{ m}^{-3}$  or  $10^{10} \text{ cm}^{-3}$ . Thus, pure Si at room temperature has  $10^{10} \text{ cm}^{-3}$  electrons in the conduction band and an equal number of holes in the valence band. To place this number in context, the number of Si atoms per unit volume can be calculated from the density ( $2.3 \text{ g cm}^{-3}$ ) and the atomic weight ( $28 \text{ g mol}^{-1}$ ). This works out to be  $5 \times 10^{22} \text{ atoms cm}^{-3}$ . So there is approximately 1 electron/hole for every  $5 \times 10^{11}$  Si atoms!

This is a very low concentration compared to metals, where the electron concentration is comparable in magnitude to the atomic density and explains why semiconductors have a poor conductivity compared to metals. The conductivity is given by

$$\sigma_i = n_i e (\mu_e + \mu_h) = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2k_B T}\right) e (\mu_e + \mu_h) \quad (3)$$

For Si,  $\mu_e$  and  $\mu_h$  are  $1350 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$  and  $450 \text{ cm}^2 \text{V}^{-1} \text{s}^{-1}$  respectively. So, the room temperature conductivity is  $3 \times 10^{-6} \Omega^{-1} \text{cm}^{-1}$  or a resistivity of  $3.5 \times 10^5 \Omega \text{cm}$ . The room temperature conductivity of Cu is  $16 \times 10^{-7} \Omega \text{cm}$ , nearly 12 orders of magnitude lower than Si and the difference is directly related to the difference in concentration of the charge carriers.

## 2 Carrier concentration in intrinsic semiconductors

Pure Si is a very poor conductor. It is rarely used in the intrinsic form, except for specific optical sensor applications, at liquid  $\text{N}_2$  temperatures. Here it is usually lightly doped with Li to control the carrier concentration. For higher conductivity, using equation 3, we either need a material with a higher mobility or a higher intrinsic carrier concentration (i.e. lower band gap). Mobility and band gap values for 3 commonly used semiconductors are listed in table 1.

Based on mobilities, GaAs would be the material to choose for higher conductivity but GaAs also has a higher band gap so that the intrinsic carrier concentration would be low. The carrier concentration and conductivity for pure GaAs can be calculated in a manner similar to that done for Si. The effective mass values for GaAs are  $m_e^* = 0.067 m_e$  and  $m_h^* = 0.50 m_e$ . Using

Table 1: Carrier mobilities and band gaps for some common semiconductors

Material	$\mu_e$ ( $cm^2V^{-1}s^{-1}$ )	$\mu_h$ ( $cm^2V^{-1}s^{-1}$ )	$E_g$ (eV)
Ge	3900	1900	0.66
Si	1350	450	1.10
GaAs	8500	400	1.43

equation 2 the effective density of states can be calculated to be  $N_c = 4.3 \times 10^{23} m^{-3}$  and  $N_v = 8.85 \times 10^{23} m^{-3}$ . This gives an intrinsic carrier concentration of  $2.4 \times 10^6 cm^{-3}$  and hence a conductivity of  $3.4 \times 10^{-9} \Omega^{-1} cm^{-1}$ . A similar calculation can be done for Ge so that table 1 can be updated to give table 2 that contains the intrinsic carrier concentration and conductivity for the semiconductors listed in table 1.

It can be seen that while a higher mobility can lead to an increase in

Table 2: Carrier concentration and conductivity for the three semiconductors listed in table 1

Material	$E_g$ (eV)	$n_i$ ( $cm^{-3}$ )	$\sigma$ ( $\Omega^{-1} cm^{-1}$ )
Ge	0.66	$2.3 \times 10^{13}$	0.02
Si	1.10	$10^{10}$	$3 \times 10^{-6}$
GaAs	1.43	$2.4 \times 10^6$	$3.4 \times 10^{-9}$

conductivity the dominant term is the carrier concentration and consequently the band gap. Since the carrier concentration has an exponential dependence on  $E_g$  lowering  $E_g$  will have the greatest effect on conductivity. Based on table 2, Ge would be the semiconductor material of choice due to its high conductivity and low band gap. In fact, the first transistors were made using Ge. But later, Si has come to dominate the microelectronics industry. The reason for this is conductivity in Si can be increased by adding specific *impurities* to the material. This is also coupled to the fact that Si is the second most abundant material on earth (27%) while Ge is only present in the parts per million amount.

### 3 Fermi level in intrinsic semiconductors

Fermi level is defined as the energy level separating the filled states from the empty states at 0 K. It is also the highest filled energy level in a metal. This definition needs to be modified for a semiconductor since these have an energy gap between the filled states (valence band) and empty states (conduction band). The expression for the carrier concentration ( $n$  or  $p$ ) in

a semiconductor is given by

$$\boxed{\begin{aligned} n &= N_c \exp\left[-\frac{(E_c - E_F)}{k_B T}\right] \\ p &= N_v \exp\left[-\frac{(E_F - E_v)}{k_B T}\right] \end{aligned}} \quad (4)$$

In an intrinsic semiconductor  $n = p = n_i$ . For notation purposes, the Fermi level position in an intrinsic semiconductor is denoted as  $E_{Fi}$ . Hence, using equation 4 and rearranging, the Fermi energy can be written as

$$\boxed{E_{Fi} = E_v + \frac{1}{2}E_g - \frac{1}{2}k_B T \ln\left(\frac{N_c}{N_v}\right)} \quad (5)$$

Since, the effective density of states depend on the carrier effective mass, using equation 2, equation 5 can be rewritten as

$$\boxed{E_{Fi} = E_v + \frac{1}{2}E_g - \frac{3}{4}k_B T \ln\left(\frac{m_e^*}{m_h^*}\right)} \quad (6)$$

For Si, the intrinsic Fermi level is calculated to be 0.54 eV above the valence band edge ( $E_v$ ) at room temperature. Since the band gap is 1.10 eV this puts  $E_{Fi}$  very close to the center of the gap. If  $N_c = N_v$  ( $m_e^* = m_h^*$ ) then the intrinsic Fermi level will be at the center of the band gap *at all temperatures*. A schematic of the energy bands in an intrinsic semiconductor is shown in figure 1. This picture is true for any intrinsic semiconductor, with the Fermi level always close to the center of the gap.

In metals the Fermi level separates the filled and empty states. This is because metals have partially filled energy bands and the Fermi level lies within the band. This definition cannot be used for semiconductors. Another definition is that *Fermi level defines the average energy required to remove an electron from the material*. This is called the **work function**, denoted by  $\phi$ , and is the energy difference between the Fermi level and the vacuum level. This definition can also be used for semiconductors, with the realization that electrons in the CB and the holes in the VB are linked. So the Fermi level, *defines the average potential energy per electron for electrical work*. If an external electric field is applied the Fermi level in the material will no longer be uniform. This will be important when considering junctions formed between different semiconductors and between semiconductors and metals.

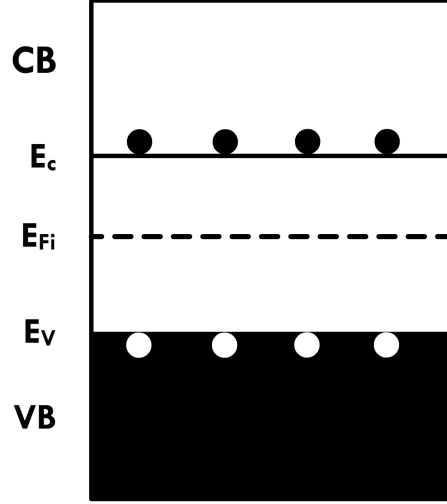


Figure 1: Band picture of an intrinsic semiconductor showing the VB and CB edge and location of the Fermi level ( $E_{Fi}$ ). The electron and hole concentrations are equal and the Fermi level is located close to the center of the band gap. Adapted from *Principles of Electronic Materials* - S.O. Kasap.

## 4 Temperature effect on $n_i$

The temperature effect on the carrier concentration can be understood by using equation 1. Temperature enters both the pre-exponential and the exponential term. Increasing temperature has two effects

1. Increases the effective density of states at the band edges ( $N_c$  and  $N_v$ ).
2. Decreases the exponential portion of equation 1, since  $T$  is in the denominator.

The exponential terms dominates, as can be seen by a semilog plot of  $n_i$  vs. inverse temperature, as shown in figure 2 for Si. The slope of the plot gives the band gap of the semiconductor. Changing the semiconductor material, changes the band gap and also the effective density of states. For different semiconductors the plot of  $n_i$  vs.  $T$  is similar with different slopes, corresponding to different  $E_g$  values. Intrinsic carrier concentrations for Ge, Si, and GaAs are compared in figure 3. The slopes correspond to their respective band gaps, given in table 1. The slope for GaAs is steepest, since it has the highest band gap.

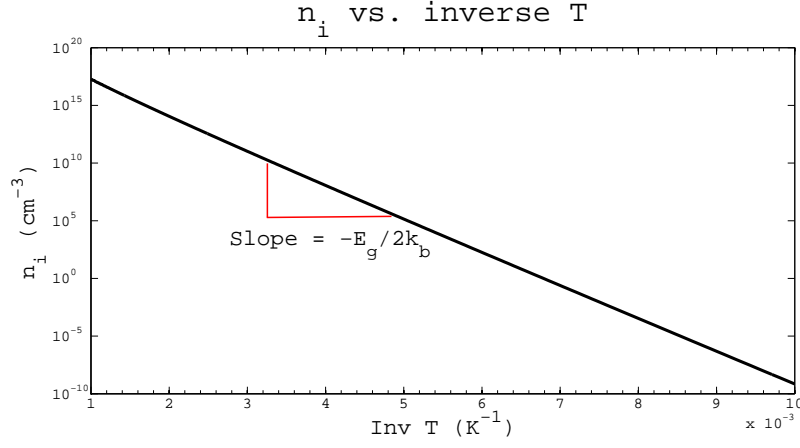


Figure 2:  $n_i$  vs. inverse T for Si in the range 100-1000 K. The plot is a straight line with slope given by  $-\frac{E_g}{2k_B}$ . The dominating temperature dependent term is the exponential one even though the effective density of states also increase with temperature. The plot was generated in MATLAB.

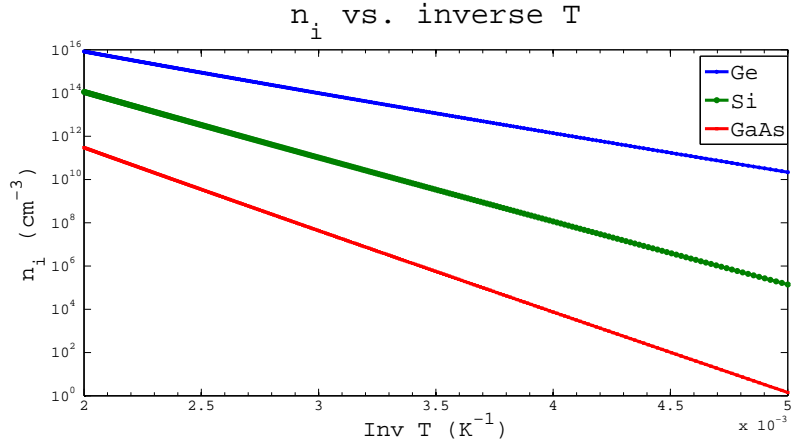


Figure 3:  $n_i$  vs. inverse T for Ge, Si, GaAs in the range 200-500 K. The plots are similar with different slopes given by their respective band gaps. GaAs has the lowest carrier concentration while Ge has the largest. The plot was generated in MATLAB.

Increase in concentration and/or mobility has the effect of increasing conductivity. While concentration increases with temperature mobility decreases with temperature. But because of the exponential term in equation 3, the dominating term is  $n_i$ . Consider two temperatures,  $T_1$  and  $T_2$  with intrinsic carrier concentrations,  $n_{i1}$  and  $n_{i2}$ . Using equations 1 and 2 the ratio of the carrier concentration is given by

$$\frac{n_{i1}}{n_{i2}} = \left(\frac{T_1}{T_2}\right)^{\frac{3}{2}} \exp\left[-\frac{E_g}{2k_B} \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \quad (7)$$

If  $T_1 > T_2$  then  $n_{i1} > n_{i2}$ .

For Si, when the temperature is doubled from room temperature (300 K) to 600 K, using equation 7, the change in carrier concentration is given by

$$\frac{n_i^{600K}}{n_i^{300K}} = 1.17 \times 10^5 \quad (8)$$

This leads to an approximately 5 orders of magnitude increase in conductivity. Problem is that almost all devices need to operate at or near room temperature, so that increasing temperature would not be a feasible option. So another method must be found to *increase and control* the conductivity in Si. This is achieved by **doping**.