Wave Propagation for communication

1) Bridge course $\rightarrow$ 1^{st} semester M. Tech /M.S. / Ph.D.

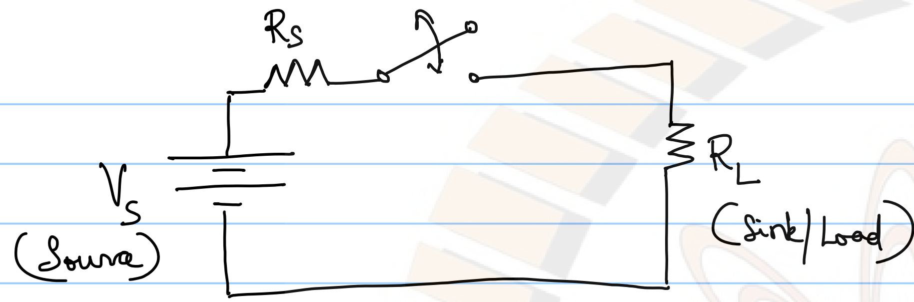
2) Recommended Text $\rightarrow$ R K Shevgaonkar
   Mathew N.O. Sadiku

3) Quiz - 1 (50 minutes) $\rightarrow$ 15 marks
   Quiz - 2 (") $\rightarrow$ 25 marks
   Final exam (180 minutes) $\rightarrow$ 50 marks
   Homeworks $\longrightarrow$ 10 marks

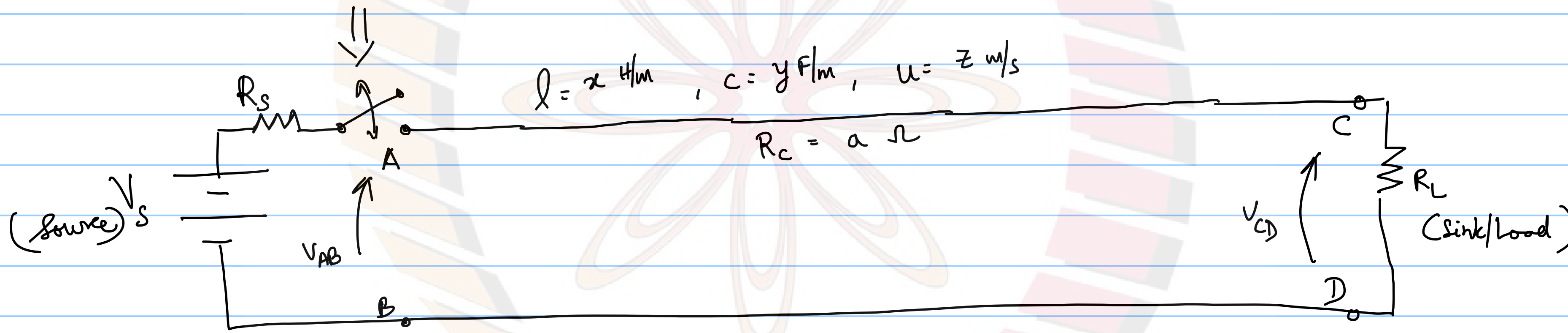
4) TA$_s$ $\longrightarrow$ 1) Ankit Arora
   2) Pratyush Pushkar
   3) Nirjhar Kumar

5) Transmission Lines & Waves

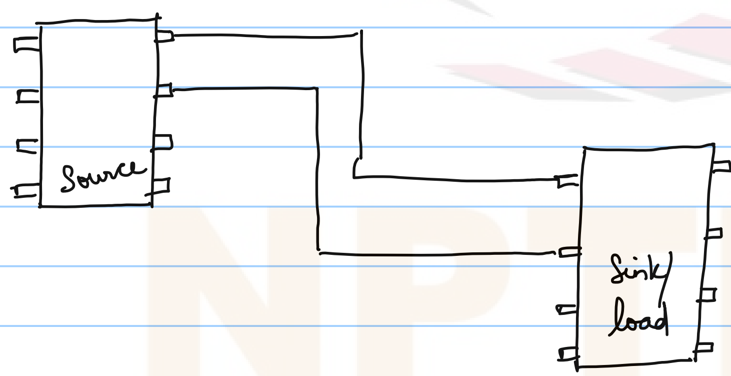
1)



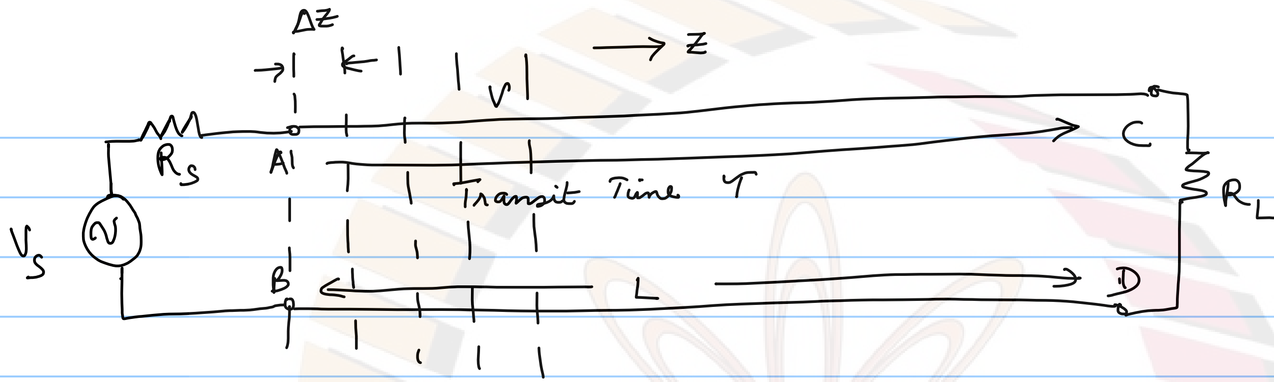
R_S
V_S
(Source)
R_L
(Sink/Load)



$l = x$ H/m , $c = y$ F/m , $u = z$ m/s
$R_c = a\ \Omega$
R_S
V_S
(Source)
A
B
C
D
V_{AB}
V_{CD}
R_L
(Sink/Load)



Source
Sink/load



Transit time $\tau = \dfrac{L}{v}$

(i) Frequency of source V_s is very low (f_s)

$$v = \downarrow f_s \lambda_s$$

$$\Rightarrow \lambda_s \uparrow$$

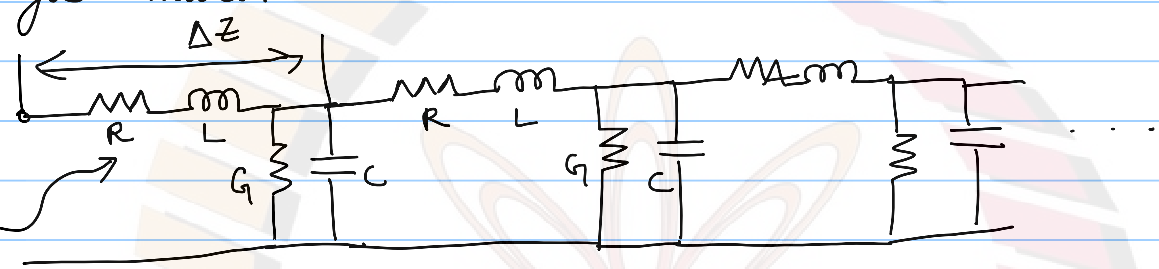
Low frequency circuit analysis could be used.

(ii) Frequency of source V_s to be high,

$$v = \uparrow f_s \lambda_s$$

$$\Rightarrow \lambda_s \downarrow$$

Phenomenological model :-



Distributed Resistance $\Rightarrow R = r\,\Delta Z$ ← m

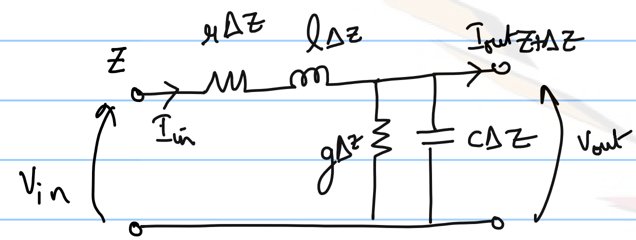
↳ Ω/m

, Distributed Inductance $\Rightarrow L = l\,\Delta Z$

↳ H/m

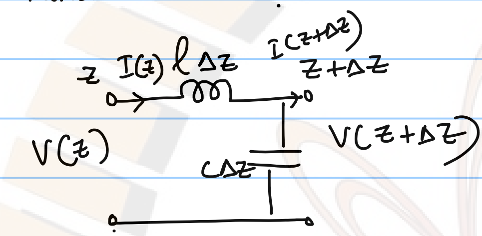
111^{rly} $G = g\,\Delta Z$, $C = c\,\Delta Z$

s/m ← ↳ F/m



Assume $r = 0$ & $g = 0$ } Lossless approximation

Lossless Transmission Line:-



KVL,

$$V(z) - (l\Delta z)\frac{dI}{dt} - V(z+\Delta z) = 0$$

$$V(z) - (l\Delta z)\frac{\partial I}{\partial t} - V(z+\Delta z) = 0$$

$$V(z+\Delta z) - V(z) = -l\Delta z \frac{\partial I}{\partial t}$$

$$\Rightarrow \frac{V(z+\Delta z) - V(z)}{\Delta z} = -l\frac{\partial I}{\partial t}$$

If $\Delta z \to 0$,

$$\boxed{\frac{\partial V}{\partial z} = -l\frac{\partial I}{\partial t}}$$

KCL,

$$I(z) - C\Delta z \frac{\partial V}{\partial t} - I(z+\Delta z) = 0$$

$$\Rightarrow \frac{I(z+\Delta z) - I(z)}{\Delta z} = -C \frac{\partial V}{\partial t}$$

$\Delta z \to 0$,

$$\boxed{\frac{\partial I}{\partial z} = -C \frac{\partial V}{\partial t}}$$

Telegrapher's equations.

1)

$$\frac{\partial V}{\partial z} = -l\frac{\partial I}{\partial t}$$

$$\frac{\partial I}{\partial z} = -c\frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial z^2} = -l\frac{\partial}{\partial z}\left(\frac{\partial I}{\partial t}\right)$$

$$= -l\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial z}\right)$$

$$= -l\frac{\partial}{\partial t}(-c)\frac{\partial V}{\partial t}$$

$$\boxed{\frac{\partial^2 V}{\partial z^2} = lc\,\frac{\partial^2 V}{\partial t^2}}$$

Dimensional analysis $\Rightarrow$ LHS units V/m^2

RHS units $\frac{\partial^2 V}{\partial t^2} \rightarrow V/s^2$

$\Rightarrow lc = \frac{1}{u^2}$

$u \rightarrow$ velocity (m/s)

$\hookrightarrow \dfrac{1}{\sqrt{lc}}$

$|||^{ly}$,

$$\boxed{\frac{\partial^2 I}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 I}{\partial t^2}} \qquad \boxed{\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2}}$$

wave equations

General Solution:-

forward   Backward

$$V(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-\left(t + \frac{z}{u}\right)$$

If $V(z,t) = f^+\left(t - \frac{z}{u}\right)$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left[f^+\left(t - \frac{z}{u}\right)\right]$$

Let $t - \frac{z}{u} = s$

$$\Rightarrow \quad \frac{\partial s}{\partial z} = \frac{-1}{u}$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}\left(f^{+}(s)\right)$$

$$= \frac{\partial}{\partial s} f^{+}(s) \quad \frac{\partial s}{\partial z}$$

$$= \frac{-1}{u} \quad \frac{\partial}{\partial s} f^{+}(s)$$

Now, $$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{-1}{u} \quad \frac{\partial}{\partial s} f^{+}(s)\right)$$

$$= \frac{-1}{u} \quad \frac{\partial}{\partial z}\left(\frac{\partial}{\partial s} f^{+}(s)\right)$$

$$= \frac{-1}{u} \quad \frac{\partial^2}{\partial s^2} f^{+}(s) \quad \frac{\partial s}{\partial z}$$

$$= \frac{1}{u^2} \quad \frac{\partial^2}{\partial s^2} f^{+}(s) = \frac{1}{u^2} \quad \frac{d^2}{ds^2} f^{+}(s)$$

RHS of voltage wave equation,

$$\frac{1}{u^2}\frac{\partial^2 V}{\partial t^2} = \frac{1}{u^2}\frac{\partial^2}{\partial t^2}\left(f^+(s)\right)$$

$$s = t - \frac{z}{u}$$

$$\frac{\partial s}{\partial t} = 1$$

$$RHS = \frac{1}{u^2}\frac{\partial^2}{\partial t^2} f^+(s)$$

$$= \frac{1}{u^2}\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t} f^+(s)\right)$$

$$= \frac{1}{u^2}\frac{\partial}{\partial t}\left(\frac{\partial f^+(s)}{\partial s}\cancel{\frac{\partial s}{\partial t}}^{1}\right)$$

$$= \frac{1}{u^2}\frac{\partial^2 f^+(s)}{\partial s^2}$$

$$= \frac{1}{u^2}\frac{d^2 f^+(s)}{ds^2}$$

$$V(z,t) = f^+\left(\overbrace{t - \frac{z}{u}}^{S}\right)$$

Let $u = 1\ m/s$ $\Rightarrow$ $S = 1 - \frac{0}{1}$

$= 1$

Let at $t = 1\ s$, $z = 0\ m$

Then at $t = 2s$, what is z?

$$t - \frac{z}{u} \Rightarrow 2 - \frac{z}{1} = S$$

$$= z = 2 - S = 2 - 1 = 1\ m$$

Given : $V(z,t) = f^+\left(t - \frac{z}{u}\right)$

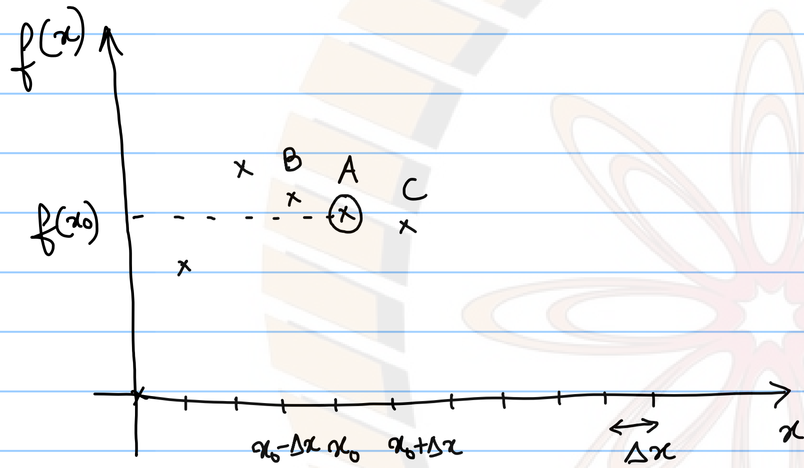
$$\frac{\partial V}{\partial z} = -l\frac{\partial I}{\partial t} \Rightarrow I(z,t) = \frac{1}{lu} f^+\left(t - \frac{z}{u}\right)$$

units of resistance

$$lu = l\frac{1}{\sqrt{lc}} = \sqrt{\frac{l}{c}}\ \Omega \quad \text{(Characteristic Resistance)}$$

$$= R_c$$

Introduction to Finite Differences :-



1) $$f(x_0+\Delta x) = f(x_0) + \frac{\Delta x}{1!} f'(x_0) + \frac{\Delta x^2}{2!} f''(x_0) + \frac{\Delta x^3}{3!} f'''(x_0) + \cdots \quad \text{— (I)}$$

$$\Rightarrow \frac{\Delta x}{1!} f'(x_0) = f(x_0+\Delta x) - f(x_0) - \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + \cdots$$

$\Delta x \rightarrow$ small

$$\Rightarrow f'(x_0) \cong \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$O(E) \cong \frac{\Delta x^2}{2!\,(\Delta x)} \cong \frac{\Delta x}{2}$$

Forward Differencing -

2) $f(x_0 - \Delta x) = f(x_0) - \frac{\Delta x}{1!} f'(x_0) + \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + \cdots$ —— 2

$$\Rightarrow \frac{\Delta x}{1!} f'(x_0) = f(x_0) - f(x_0 - \Delta x) + \frac{\Delta x^2}{2!} f''(x_0) - \frac{\Delta x^3}{3!} f'''(x_0) + \cdots$$

$$\Rightarrow f'(x_0) \cong \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$$

$$O(E) \cong \frac{\Delta x^2}{2!(\Delta x)} \cong \frac{\Delta x}{2}$$

Backward Differencing.

3) 1 − 2,

$$f(x_0 + \Delta x) - f(x_0 - \Delta x) = \frac{2\Delta x}{1!} f'(x_0) + \frac{2\Delta x^3}{3!} f'''(x_0) \cdots$$

$$\Rightarrow \frac{2\Delta x}{1!} f'(x_0) \cong f(x_0 + \Delta x) - f(x_0 - \Delta x)$$

$$\Rightarrow f'(x_0) \cong \frac{f(x_0 + \Delta x) - f(x_0 - \Delta x)}{2\Delta x} \rightarrow O(E) \cong \frac{2\Delta x^3}{3!(2\Delta x)} \cong \frac{\Delta x^2}{6}$$

Central Differencing

4) ① + ②,

$$f(x_0+\Delta x) + f(x_0-\Delta x) = 2f(x_0) + \frac{2\Delta x^2}{2!} f''(x_0) + \frac{2\Delta x^4}{4!} f''''(x_0) + \cdots$$

$$\Rightarrow \quad \frac{2\Delta x^2}{2!} f''(x_0) = f(x_0+\Delta x) + f(x_0-\Delta x) - 2f(x_0) - \frac{2\Delta x^4}{4!} f''''(x_0) - \cdots$$

$$\Rightarrow \quad f''(x_0) \cong \frac{f(x_0+\Delta x) + f(x_0-\Delta x) - 2f(x_0)}{\Delta x^2}$$

$$O(E) \cong \frac{2\Delta x^4}{4!\,\Delta x^2} \cong \frac{\Delta x^2}{12}$$

3) $$\frac{d^2V}{dx^2} = 0 \quad \text{(Laplace's equation)}$$

$$\Rightarrow \quad V(x_0+\Delta x) + V(x_0-\Delta x) - 2V(x_0) = 0$$

$$\Rightarrow \quad V(x_0) = \frac{V(x_0+\Delta x) + V(x_0-\Delta x)}{2}$$

4) If x & y are independent variables & $V(x,y)$,

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$\frac{\partial^2 V}{\partial x^2} \cong \frac{f(x_0+\Delta x, y_0) + f(x_0-\Delta x, y_0) - 2f(x_0,y_0)}{\Delta x^2} \quad \text{— (a)}$$

$$\frac{\partial^2 V}{\partial y^2} \cong \frac{f(x_0, y_0+\Delta y) + f(x_0, y_0-\Delta y) - 2f(x_0,y_0)}{\Delta y^2} \quad \text{— (b)}$$

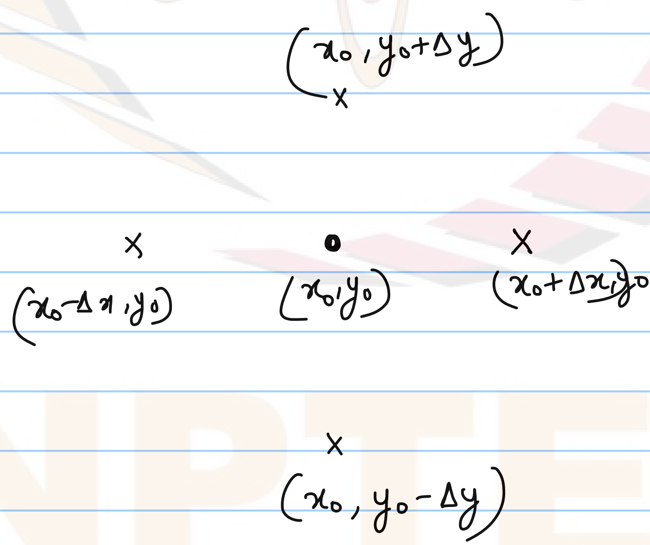
Let $\Delta x = \Delta y = h$,

$$(a) + (b) = 0$$

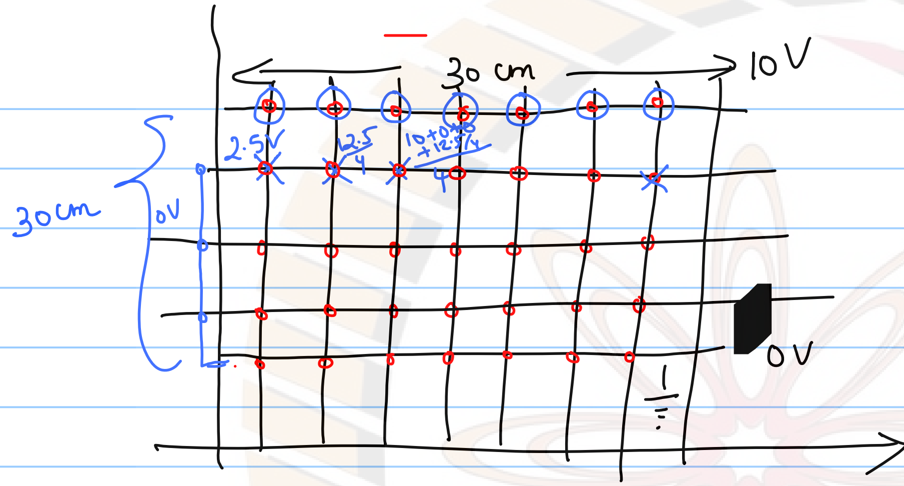
$$f(x_0+\Delta x, y_0) + f(x_0-\Delta x, y_0) - 2f(x_0,y_0) + f(x_0, y_0+\Delta y) + f(x_0, y_0-\Delta y) - 2f_{(x_0,y_0)} = 0$$

$$4\,V(x_0,y_0) = V(x_0+\Delta x, y_0) + V(x_0-\Delta x, y_0) + V(x_0, y_0+\Delta y) + V(x_0, y_0-\Delta y)$$

$$\Rightarrow \quad V(x_0,y_0) = \div 4$$

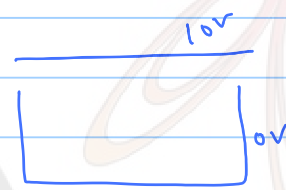


5)



$V(30, 30) = \text{Zeros}$

$V(\text{at all } x, 30) = 10$



$\frac{\partial V}{\partial x} = 0$; Neumann condition

$$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V}{\partial t^2}$$

Here, there are 2 independent variables.

$$\frac{V(z_0+\Delta z, t_0) + V(z_0-\Delta z, t_0) - 2V(z_0, t_0)}{\Delta z^2} = \frac{1}{\Delta t^2 u^2}\left\{\begin{array}{l} V(z_0, t_0+\Delta t) \\ + V(z_0, t_0-\Delta t) \\ - 2V(z_0, t_0) \end{array}\right\}$$

$$\Rightarrow V(z_0, t_0+\Delta t) = -V(z_0, t_0-\Delta t) + 2V(z_0, t_0) + \frac{u^2\Delta t^2}{\Delta z^2}\left\{\begin{array}{l} V(z_0+\Delta z, t_0) + \\ V(z_0-\Delta z, t_0) \\ - 2V(z_0, t_0) \end{array}\right\}$$

1) Let $V(z,t) = f^-\left(t + \frac{z}{u}\right)$

$$\frac{\partial V}{\partial z} = -l\frac{\partial I}{\partial t}$$

Let $s = t + \frac{z}{u}$, $f^-(s) \Rightarrow f^-$

$$LHS \Rightarrow \frac{\partial V}{\partial z} = \frac{dV}{ds}\frac{\partial s}{\partial z} = \frac{df^-}{ds}\left(\frac{1}{u}\right)$$

<u>RHS</u>

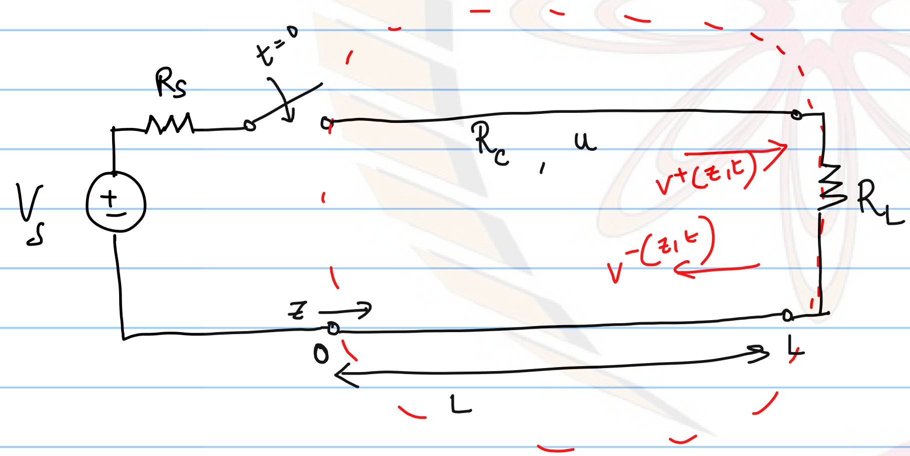
$$\frac{\partial I}{\partial t} = \frac{dI}{ds}\frac{\partial s}{\partial t} = \frac{dI}{ds}(1) = \frac{dI}{ds}$$

<u>LHS = RHS</u> $\Rightarrow \frac{df^-}{ds}\left(\frac{1}{u}\right) = -l\frac{dI}{ds}$

$$\Rightarrow I = \frac{-1}{lu}f^-(s) \Rightarrow \frac{-1}{lu}f^-\left(t + \frac{z}{u}\right)$$

$$I = \frac{-1}{R_c} f^-\left(t + \frac{z}{u}\right)$$

$$= \frac{-1}{R_c} V(z,t)$$



$$V(z,t) = V^+(z,t) + V^-(z,t)$$

$$I(z,t) = \frac{1}{R_c} V^+(z,t) - \frac{1}{R_c} V^-(z,t)$$

Reflection coefficient "Γ" $= \dfrac{V^-}{V^+}$

At $z = L$,

$$V(L,t) = R_L\left(I(L,t)\right)$$

$$V^+(L,t) + V^-(L,t) = R_L\left[\frac{1}{R_C}V^+(L,t) - \frac{1}{R_C}V^-(L,t)\right]$$

Rearranging this equation,

$$V^- = \left(\frac{R_L - R_C}{R_L + R_C}\right)V^+$$

Voltage Reflection coefficient at the load end,

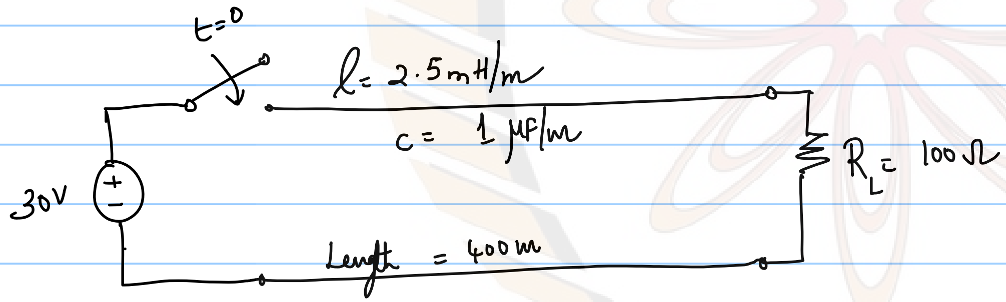
$$\boxed{\Gamma_{z=L} = \frac{R_L - R_C}{R_L + R_C}}$$

If $R_L \gg R_c$, $\Gamma_L \cong \frac{R_L}{R_L} \cong 1$

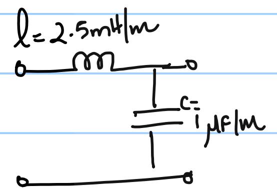
If $R_L \ll R_c$, $\Gamma_L \cong -1$

If $R_L = R_c$, $\Gamma_L = 0$

1)



Soln:-



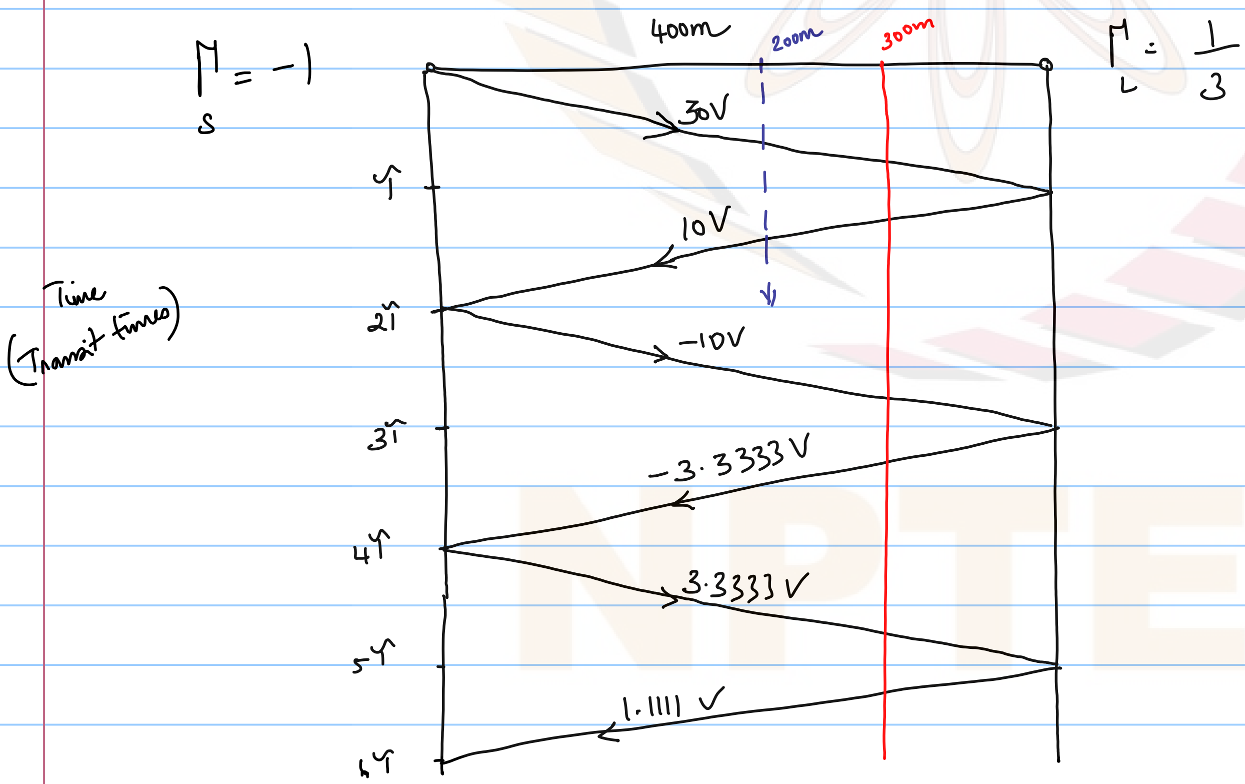
$$u = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{2.5\times10^{-3} \times 1\times10^{-6}}} = 2\times10^{4}\ m/s$$

$$\tau = \frac{L}{u} = \frac{400m}{2\times10^{4}\ m/s} = 2\times10^{-2}\ s$$

$$\Gamma_L = \frac{R_L - R_c}{R_L + R_c} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$\Gamma_S = \frac{R_S - R_c}{R_S + R_c} = \frac{0 - 50}{0 + 50} = -1$$

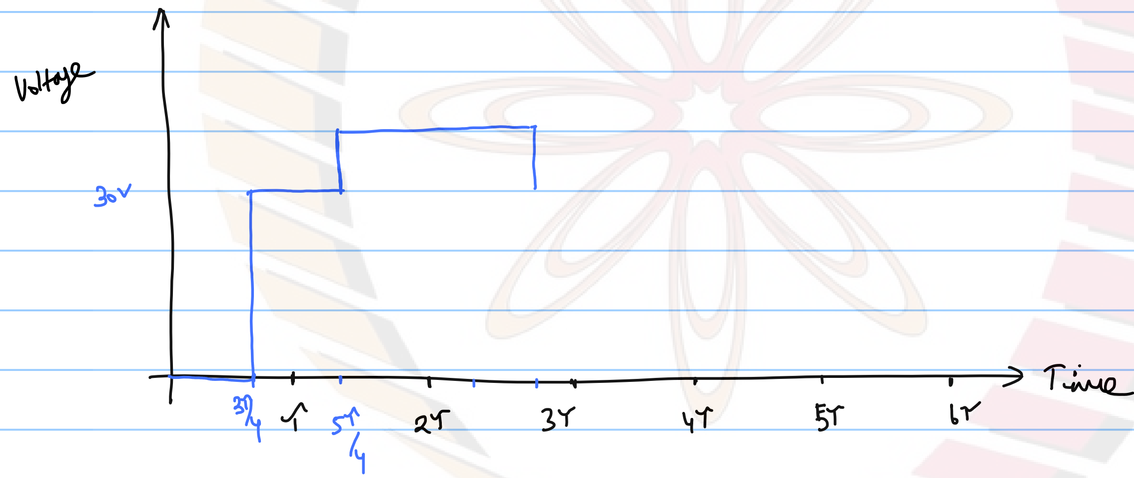
Bounce Diagram :-



At $z = 200\,m$, after 2 transit times,

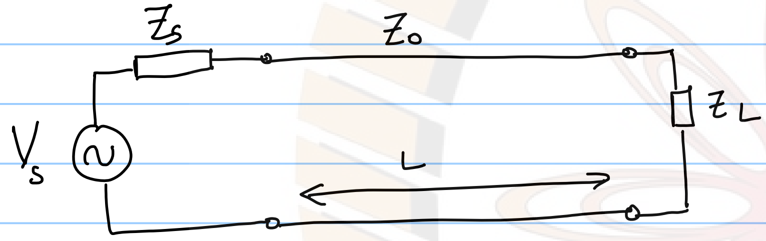
$V = 40V$

At, $z = 300m$, draw evolution of voltage,



AC signal in a lossless transmission line

1)



2)

$$\frac{\partial^2 V(z,t)}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 V(z,t)}{\partial t^2}$$

$$\frac{\partial^2 I(z,t)}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 I(z,t)}{\partial t^2}$$

3)

$$V(z,t) = Re\left\{ V(z)e^{j\omega t}\right\}$$

$$I(z,t) = Re\left\{ I(z)e^{j\omega t}\right\}$$

4) $$\frac{\partial^2 V}{\partial z^2} = \frac{1}{u^2}(j\omega)^2 \; V(z)\, e^{j\omega t}$$

$$= \left[\frac{1}{u^2}(j\omega)^2\right] V(z)$$

Let $\beta = \omega\sqrt{lc} \Rightarrow \frac{\partial^2 V}{\partial z^2} + \beta^2 V(z) = 0$

$\hookrightarrow$ radians/m (Phase constant) $\qquad \beta = \frac{2\pi}{\lambda}$

5) $$V(z) = \underbrace{V^+ e^{-j\beta z}}_{\text{Forward}} + \underbrace{V^- e^{+j\beta z}}_{\text{Backward}}$$

6) $$I(z) = \frac{1}{Z_0} V^+ e^{-j\beta z} - \frac{1}{Z_0} V^- e^{+j\beta z}$$

7)

$$V(z,t) = Re\left\{ |V^+| e^{-j\beta z} e^{j\omega t} \right\}$$

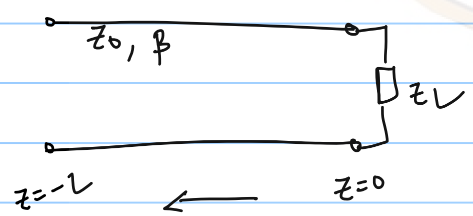
$$= |V^+| \cos(\omega t - \beta z)$$

$$= |V^+| \cos\left(\omega\left(t - \frac{\beta}{\omega} z\right)\right)$$

$$= |V^+| \cos\left(\omega\left(t - \frac{z}{u}\right)\right)$$

$u = \dfrac{\omega}{\beta} \longrightarrow$ Phase velocity

8)



$$V(z=0) = V_L = I(z=0)\, Z_L$$

$$V_0^+ + V_0^- = \frac{1}{Z_0}\left[V_0^+ - V_0^-\right] Z_L$$

$$\frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_L$$

$$= \Gamma_{z=0}$$

Let $Z_L = \infty$ (open circuit)

$$V(z) = V_0^+ \left(e^{-j\beta z} + \frac{V_0^-}{V_0^+} e^{+j\beta z} \right)$$

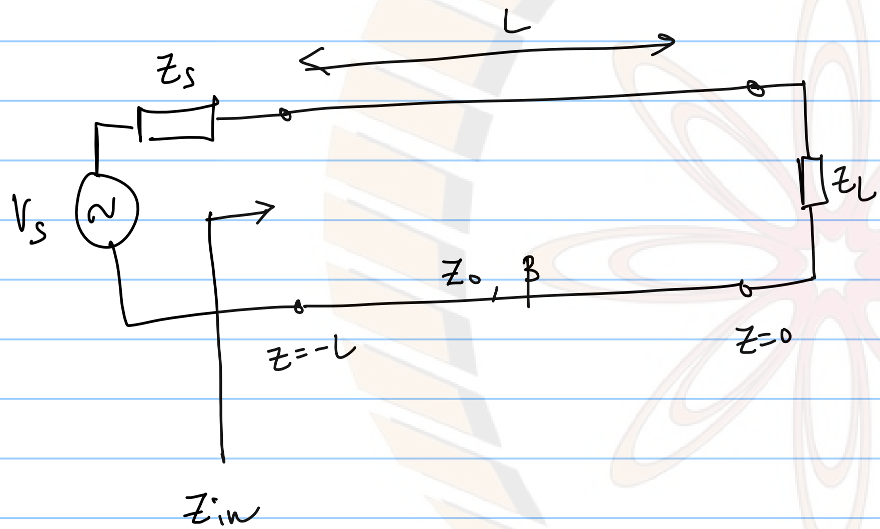
$$\Gamma_L \Big|_{z=0} = 1$$

$$= 2V_0^+ \cos(\beta z)$$

$$I(z) = \frac{V_0^+}{Z_0} \left(e^{-j\beta z} - \frac{V_0^-}{V_0^+} e^{+j\beta z} \right)$$

$$\Gamma_L \Big|_{z=0} = 1$$

$$= -j \frac{2V_0^+}{Z_0} \sin(\beta z)$$



$$V(-L) = V_0^+ \left(e^{+j\beta L} + \frac{V_0^-}{V_0^+} e^{-j\beta L} \right)$$

$$= V_0^+ \left(e^{+j\beta L} + \Gamma_L e^{-j\beta L} \right)$$

$$I(-L) = \frac{V_0^+}{Z_0} \left(e^{+j\beta L} - \Gamma_L e^{-j\beta L} \right)$$

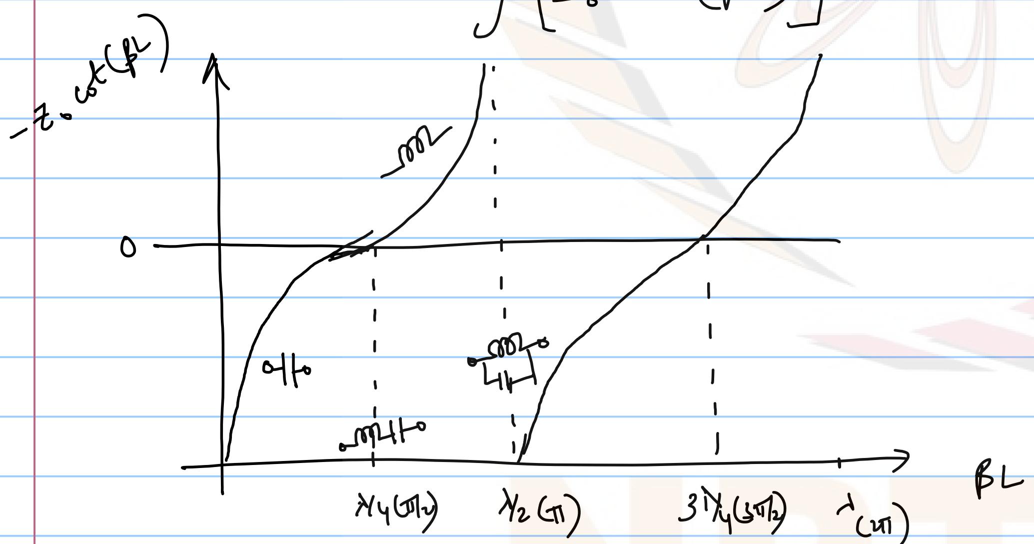
$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta L)}{Z_0 + jZ_L \tan(\beta L)} \right]$$

electrical length

Open ckt load,
$Z_L = \infty$,

$$Z_{in} = Z_0 \left(-j \cot(\beta L) \right) \ \Omega$$

$$= -j \left[Z_0 \cot(\beta L) \right]$$



(ii) SC, $Z_L = 0$

$$Z_{in} = j Z_0 \tan(\beta L)$$

(iii) $Z_L = Z_0$,

$$Z_{in} = Z_0 \quad //$$

1) $$\frac{\partial v}{\partial z} = -l\frac{\partial i}{\partial t}$$

$$\frac{v(z+1,t) - v(z,t)}{\Delta z} = -l\left(\frac{i(z,t+1) - i(z,t)}{\Delta t}\right)$$

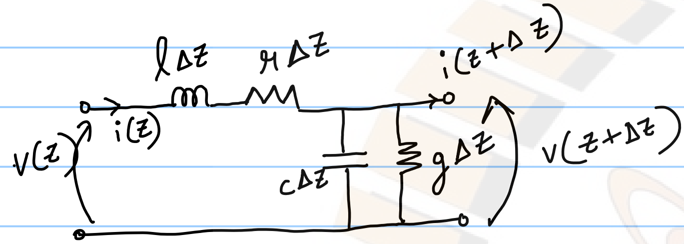
$$\Rightarrow \boxed{i(z,t+1) = i(z,t) - \frac{\Delta t}{l\,\Delta z}\left(v(z+1,t) - v(z,t)\right)}$$

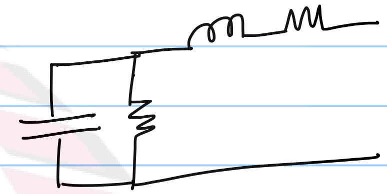
2) $$\frac{\partial i}{\partial z} = -c\frac{\partial v}{\partial t}$$

$$\frac{i(z,t) - i(z-1,t)}{\Delta z} = -c\,\frac{v(z,t+1) - v(z,t)}{\Delta t}$$

$$\Rightarrow \boxed{v(z,t+1) = v(z,t) - \frac{\Delta t}{c\,\Delta z}\left\{ i(z,t) - i(z-1,t)\right\}}$$

1)





KVL :-

$$v(z) - (r\Delta z)\, i(z) - l\Delta z \frac{\partial i}{\partial t} - v(z+\Delta z) = 0$$

Rearrange,

$$v(z+\Delta z) - v(z) = -r\Delta z\, i(z) - l\Delta z \frac{\partial i}{\partial t}$$

$$\Rightarrow \quad \frac{v(z+\Delta z) - v(z)}{\Delta z} = -r\, i(z) - l \frac{\partial i}{\partial t}$$

As $\Delta z \to 0$,

$$\Rightarrow \quad \boxed{\frac{\partial v}{\partial z} = -r\, i(z) - l \frac{\partial i}{\partial t}}$$

KCL :-

$$i(z) - (g\Delta z) - c\Delta z \frac{\partial v}{\partial t} - i(z+\Delta z) = 0$$

Rearrange & apply $\Delta z \to 0$,

$$\boxed{\frac{\partial i}{\partial z} = -gv - c\frac{\partial v}{\partial t}}$$

$$\frac{\partial v}{\partial z} = -ri - l\frac{\partial i}{\partial t}$$

$$\frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -r\, i(z) - l\, \frac{i(z,t+\Delta t) - i(z,t)}{\Delta t}$$

$$\left.\begin{aligned} \frac{\partial V}{\partial z} &= -(r+j\omega l)\, I \\ \frac{\partial I}{\partial z} &= -(g+j\omega c)\, V \end{aligned}\right]$$ Telegrapher's eqns in frequency domain

$$\boxed{\frac{\partial^2 V}{\partial z^2} = \underbrace{(r+j\omega l)}_{\text{(Series Impedance) } Z}\underbrace{(g+j\omega c)}_{\text{(Parallel admittance) } Y} V}$$ Wave eqn. in frequency domain

$$\frac{\partial^2 V}{\partial z^2} = \gamma^2 V$$

$\hookrightarrow \gamma = \sqrt{(r+j\omega l)(g+j\omega c)}$ $\longrightarrow$ Propagation constant $\rightarrow \alpha + j\beta$

$$\Rightarrow V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

Attenuation constant

Phase constant

$$\Rightarrow \quad V^+ e^{-\gamma z} = V^+ e^{-j\beta z}\left(e^{-\overset{\text{Positive}}{\alpha} z}\right)$$

$$V^+ e^{-\gamma z} = |V^+| \underbrace{e^{-\alpha z}}_{\text{attenuation}} \underbrace{e^{-j\beta z}}_{\text{Phase}}$$

Multiplying both sides with $e^{j\omega t}$,

phase $\rightarrow (\omega t - \beta z)$

$\alpha \rightarrow$ Nepers/m

If $\alpha = 1$ Np/m, the amplitude drops to $\frac{1}{e}$ over a distance of 1m.

Propagation length = The distance over which the amplitude drops to $\frac{1}{e}$.

"L" $= \frac{1}{\alpha}$

Input power = In
output " = Out

$$\text{Relative output power in dB} = 10 \log_{10}\left(\frac{\text{Out}}{\text{In}}\right)$$

Input voltage = $|V^+|$
Output " = $|V^+|e^{-\alpha z}$

$$\text{dB} = 10 \log_{10}\left(\left[\frac{|V^+|e^{-\alpha z}}{|V^+|}\right]^2\right)$$

$$= 20 \log_{10} \frac{\cancel{|V^+|}\, e^{-\alpha z}}{\cancel{|V^+|}}$$

If $\alpha = 1\ Np/m$, $z = 1\,m$,

$$\text{dB} = -8.68\ \text{dB}$$

$$\frac{V(z+\Delta z) - V(z)}{\Delta z}$$

$$V\left(z+\frac{\Delta z}{2}\right) \qquad = \qquad I(z)$$

$$\frac{u\Delta t}{\Delta z} \leq 1$$

$$\frac{u\Delta t}{\Delta z} = 1$$ (Magic time step criterion)

1) $$V = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$
$$I = I^+ e^{-\gamma z} + I^- e^{\gamma z}$$

2) $$\frac{d}{dz}\left(V^+ e^{-\gamma z} + V^- e^{\gamma z}\right) = -(r + j\omega l)\left\{ I^+ e^{-\gamma z} + I^- e^{\gamma z} \right\}$$

3) Separate forward & backward parts on LHS & RHS and equate them

forward:-
$$-\gamma V^+ e^{-\gamma z} = -(r + j\omega l)\, I^+ e^{-\gamma z}$$

$$\Rightarrow \quad \frac{V^+}{I^+} = \frac{r + j\omega l}{\gamma} = \frac{r + j\omega l}{\sqrt{(r + j\omega l)(g + j\omega c)}}$$

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \; \Omega$$

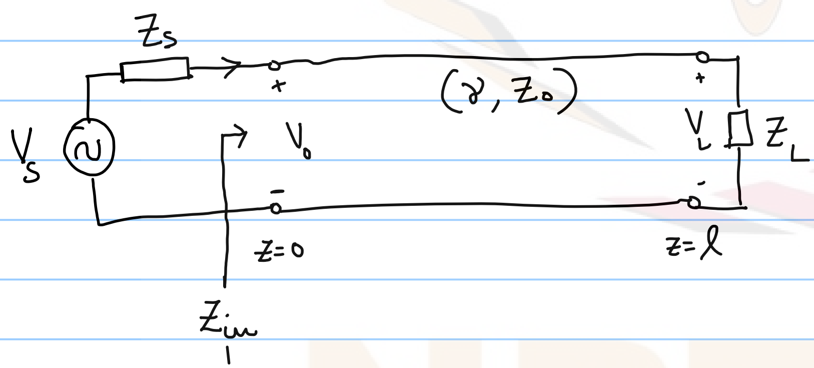
$$= \sqrt{\frac{\text{Series Impedance}}{\text{Parallel Admittance}}}$$

Equate the backward terms,

$$\gamma V^- e^{\gamma z} = -(r + j\omega l)\, I^- e^{\gamma z}$$

$$\Rightarrow \quad \frac{V^-}{I^-} = \frac{-(r+j\omega l)}{\gamma}$$

$$= -\sqrt{\frac{r+j\omega l}{g+j\omega c}}$$



In the transmission line section,

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad \text{—— (1)}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z} \quad \text{—— (2)}$$

$$V(z=0) = V_0$$
$$I(z=0) = I_0$$

Add (1) & (2), with $z = 0$ (source side)

$$V_0 = V_0^+ + V_0^-$$
$$I_0 Z_0 = V_0^+ - V_0^-$$

$$V_0 + I_0 Z_0 = 2V_0^+$$

$$\Rightarrow \quad V_0^+ = \frac{1}{2}\left(V_0 + I_0 Z_0\right)$$

① − ②,

$$V_0^- = \frac{1}{2}\left(V_0 - I_0 Z_0\right)$$

At the load side,

$$V_L = V(z=l)$$

$$I_L = I(z=l)$$

$$V_0^+ = \frac{1}{2}\left(V_L + Z_0 I_L\right)e^{\gamma l} \quad \text{——— ③}$$

$$V_0^- = \frac{1}{2}\left(V_L - Z_0 I_L\right)e^{-\gamma l} \quad \text{——— ④}$$

$$Z_{in} = \frac{V(z)}{I(z)} \text{ at input side}$$

$$= \frac{Z_0\left(V_0^+ + V_0^-\right)}{V_0^+ - V_0^-}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right]$$

Voltage Reflection Coefficient :-

$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma(z) = \frac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{2\gamma z}$$

$$\frac{V_{max}}{V_{min}} = \text{Voltage Standing Wave Ratio (VSWR)}$$

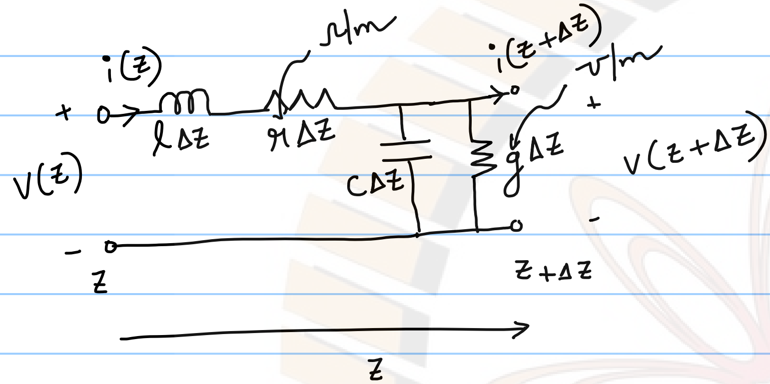
$$\frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$

If $\Gamma_L = 1$,

$$VSWR = \frac{1+1}{1-1} = \infty$$

$\Gamma_L = 0$,

$$VSWR = \frac{1+0}{1-0} = 1$$



KVL :-

$$v(z,t) - l\Delta z \left.\frac{\partial i}{\partial t}\right|_z - r\Delta z\, i(z,t) - v(z+\Delta z,t) = 0$$

$$\Rightarrow \quad v(z+\Delta z,t) - v(z,t) = -r\Delta z\, i(z,t) - l\Delta z \left.\frac{\partial i}{\partial t}\right|_z$$

$$\Rightarrow \quad \frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = -r\, i(z,t) - l\left.\frac{\partial i}{\partial t}\right|_z$$

Apply $\Delta z \to 0$

$$\Rightarrow \quad \boxed{\frac{\partial v}{\partial z} = -ri - l\frac{\partial i}{\partial t}} \quad \text{——} \quad (1)$$

KCL :-

$$i(z,t) - c\Delta z \left.\frac{\partial v}{\partial t}\right|_{z+\Delta z} - g\,\Delta z\, v(z+\Delta z, t) - i(z+\Delta z, t) = 0$$

$$\Rightarrow \frac{i(z+\Delta z, t) - i(z,t)}{\Delta z} = -g\,v(z+\Delta z, t) - c\left.\frac{\partial v}{\partial t}\right|_{z+\Delta z}$$

Apply $\Delta z \to 0$,

$$\Rightarrow \boxed{\frac{\partial i}{\partial z} = -gv - c\frac{\partial v}{\partial t}} \quad \text{——— (2)}$$

For periodic sinusoidal excitation,

$$\frac{\partial V}{\partial z} = -(r + j\omega l)\, I$$

$$\frac{\partial I}{\partial z} = -(g + j\omega c)\, V$$

$$\frac{\partial^2 V}{\partial z^2} = (r+j\omega l)(g+j\omega c)V$$

If $\gamma = \sqrt{(r+j\omega l)(g+j\omega c)}$, then,

$$\frac{\partial^2 V}{\partial z^2} = \gamma^2 V$$

complex

$\gamma \rightarrow$ Propagation constant (complex) /m

$\alpha + j\beta$

Attenuation Constant (Np/m)

Phase constant

$$V = V^+ e^{-\gamma z} + V^- e^{+\gamma z}$$

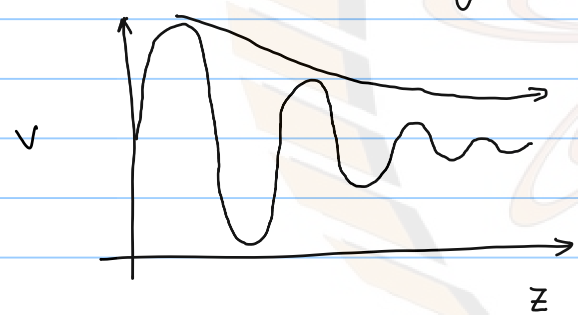
Forward wave, $V^+ e^{-\gamma z}$

$$\Rightarrow V^+ e^{-(\alpha + j\beta)z}$$

$$\Rightarrow \left[V^+ e^{-j\beta z}\right] e^{-\alpha z}$$

If $\alpha = 0$, $\longrightarrow$ Lossless transmission line

If $\alpha > 0$, $\longrightarrow$ Lossy transmission line



$$V^+ e^{-\gamma z} = |V^+| \underbrace{e^{-\alpha z}}_{\text{Amplitude}} \underbrace{e^{-j\beta z}}_{\text{phase}}$$

If $\alpha = 1$ Neper/m (Np/m), the amplitude of the wave drops to $\frac{1}{e}$ over a distance of 1m.

Propagation Length :-

The distance over which the amplitude drops to $\frac{1}{e}$ its original value

Propagation Length $L = \frac{1}{\alpha}$ (m)

If input power $= P_{in}$ (W)
output power $= P_{out}$ (W)

$$10 \log_{10}\left[\frac{P_{out}}{P_{in}}\right] \quad dB$$

$$\Rightarrow 10 \log_{10}\left[\frac{|V^+| e^{-\alpha z}}{|V^+|}\right]^2$$

$$\Rightarrow 20 \log_{10}\left[e^{-\alpha z}\right]$$

If $\alpha = 1\ Np/m$, If $z = 1m$,

$\Rightarrow \quad -8.68\ dB/m$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

Say, $Z_L = 50\,\Omega$ $\quad \Gamma = \frac{2-1}{2+1} = \frac{1}{3}$

$Z_0 = 25\,\Omega$

or

$Z_L = 100\,\Omega$ $\quad \Gamma = \frac{2-1}{2+1} = \frac{1}{3}$

$Z_0 = 50\,\Omega$

$\frac{Z_L}{Z_0} \longrightarrow$ Normalized Impedance

Lossless Transmission Line $\Rightarrow$ Z_0 is Real Ω

Let $Z_L = R + jX$

+ve (passive)

+ve or -ve

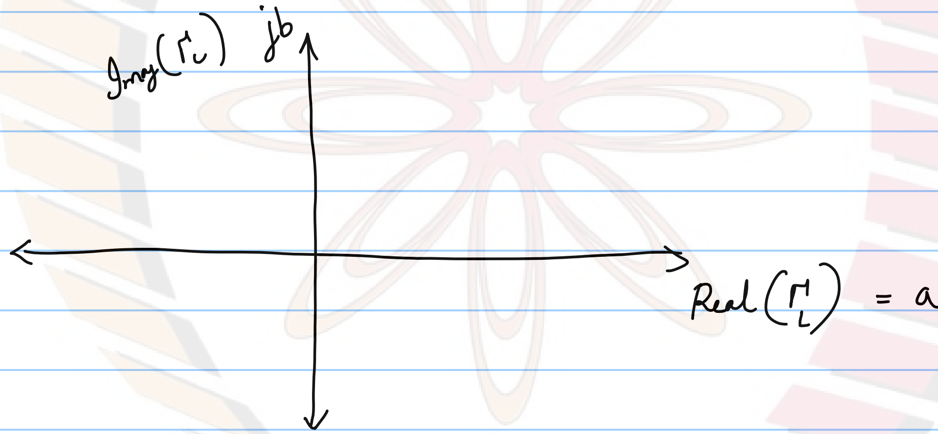
$$\frac{Z_L}{Z_0} = \frac{R}{Z_0} + \frac{jX}{Z_0}$$

$$\bar{Z}_L = r + jx$$

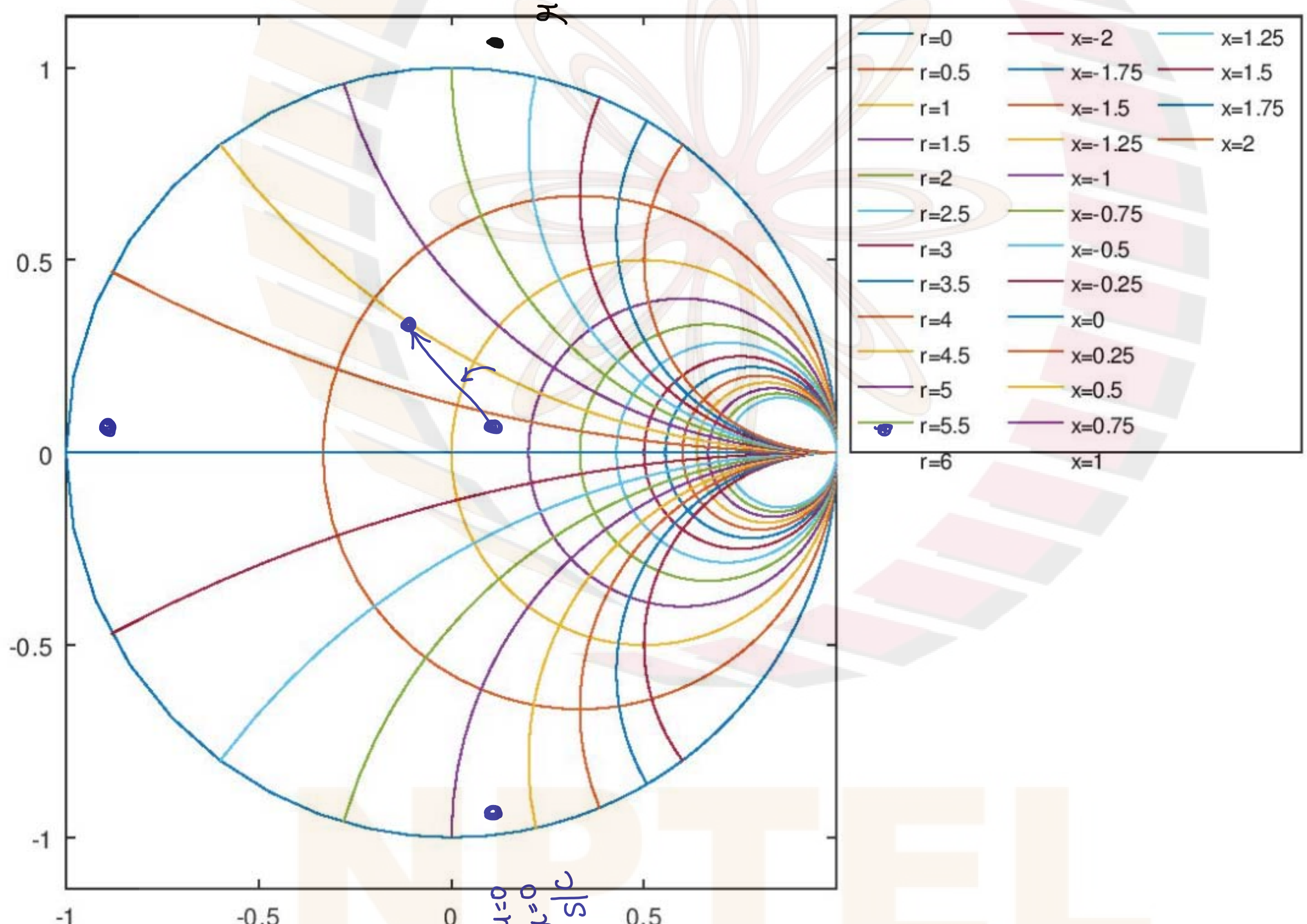
$$\Gamma_L = \frac{\bar{z}_L - 1}{\bar{z}_L + 1} = \frac{r + jx - 1}{r + jx + 1} = \frac{r - 1 + jx}{r + 1 + jx}$$

$$\Gamma_L \longrightarrow a + jb$$

$$\bar{z}_L = r + jx$$



$r=\infty$, $x=\infty$
o/c



$r=0$
$x=0$
s/c

$Y_0 \rightarrow$ Characteristic Admittance

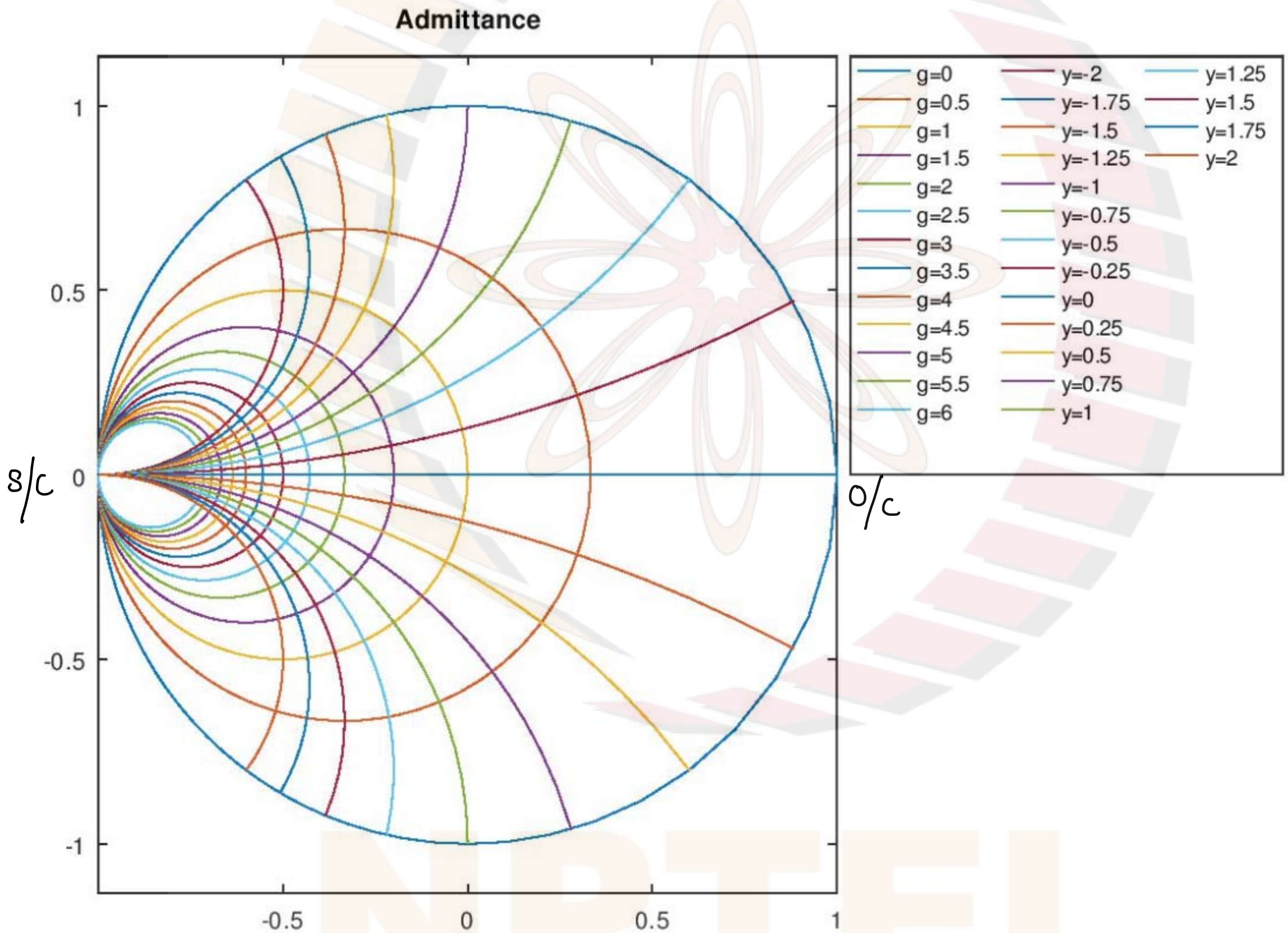
$Y_L \rightarrow$ Load admittance $\qquad g + jy$

$$\Gamma_L = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1} = \frac{\frac{Y_0}{Y_L} - 1}{\frac{Y_0}{Y_L} + 1} = \frac{Y_0 - Y_L}{Y_0 + Y_L}$$

$$= \frac{1 - \frac{Y_L}{Y_0}}{1 + \frac{Y_L}{Y_0}}$$

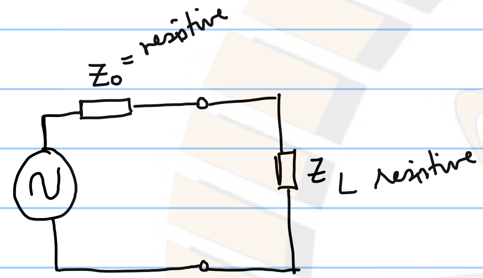
$$= \frac{1 - \bar{Y}_L}{1 + \bar{Y}_L}$$

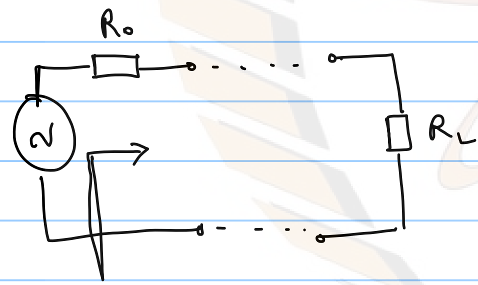
$$= \frac{-(\bar{Y}_L - 1)}{(\bar{Y}_L + 1)} = \frac{-(g + jy - 1)}{(g + jy + 1)}$$



Admittance
1
0.5
0
-0.5
-1
-0.5
0
0.5
1
g=0
g=0.5
g=1
g=1.5
g=2
g=2.5
g=3
g=3.5
g=4
g=4.5
g=5
g=5.5
g=6
y=-2
y=-1.75
y=-1.5
y=-1.25
y=-1
y=-0.75
y=-0.5
y=-0.25
y=0
y=0.25
y=0.5
y=0.75
y=1
y=1.25
y=1.5
y=1.75
y=2
S/C
O/C

Simsmith





$$I_{max} = \frac{V_{max}}{Z_0} \quad , \quad I_{min} = \frac{V_{min}}{Z_0}$$

$$\left| Z_{in} \right|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{VSWR}$$

$$\left| Z_{in} \right|_{max} = \frac{V_{max}}{I_{min}} = (VSWR)\, Z_0$$

1) For a transmission line,

$$r = 0.1 \ \Omega/m$$
$$l = 0.2 \ \mu H/m$$
$$c = 10 \ pF/m$$
$$g = 0.02 \ \mho/m$$

Soln:- Complex propagation constant γ :-

$$\gamma = \sqrt{(r+j\omega l)(g+j\omega c)} \qquad \left\{ \sqrt{Z_{\text{Series}} \, Y_{\text{parallel}}} \right\}$$

(i) At 1 MHz frequency,

$$= \sqrt{\left(0.1 + j\left(2\pi \times 1\times 10^{6}\right)\times\left(0.2\times 10^{-6}\right)\right)\left(0.02 + j\left(2\pi\times 1\times 10^{6}\right)\times\left(10\times 10^{-12}\right)\right)}$$

$$= \underset{\alpha}{0.117} + j\,\underset{\beta}{0.108} \quad /m$$

(ii) γ at 1 GHz

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega c)}$$

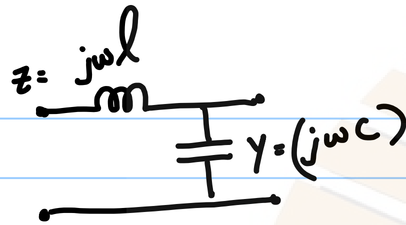
$$= \underset{\alpha}{1.4} + j\,\underset{\beta}{9} \quad /m$$

(iii) The ratio of propagation lengths

$$\frac{L_{1MHz}}{L_{1GHz}} = \frac{\left(\frac{1}{0.117}\right)}{\left(\frac{1}{1.4}\right)}$$

$$\simeq 11 \text{ or } 12$$

1)

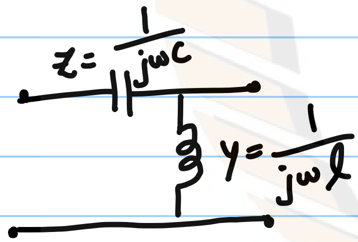


$$\gamma = \sqrt{zy} = j\omega\sqrt{lc}$$

$$\Rightarrow \beta = \omega\sqrt{lc}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{lc}}$$

Say:-



$$\gamma = \sqrt{zy} = \frac{1}{j\omega\sqrt{lc}} = \frac{-j}{\omega\sqrt{lc}}$$

$$\Rightarrow \beta = \frac{-1}{\omega\sqrt{lc}}$$

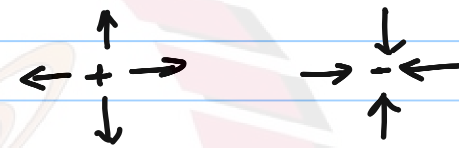
$$V_p = \frac{\omega}{\beta} = -\omega^2\sqrt{lc}$$

(Metamaterials)

Maxwell's equations, plane waves, interfaces

1)

$$\underline{\nabla} \cdot \underline{D}(t) = \rho(t)$$



$$\underline{\nabla} \cdot \underline{B}(t) = 0$$

$$\underline{\nabla} \times \underline{E}(t) = \frac{-\partial \underline{B}(t)}{\partial t}$$

$$\underline{\nabla} \times \underline{H}(t) = \underbrace{J(t)}_{\text{Conduction current density}} + \frac{\partial D(t)}{\partial t}$$

$$\underline{B}(t) = \underbrace{\mu(t)}_{(H/m)} * \underbrace{\underline{H}(t)}_{A/m}$$

$$\underline{D}(t) = \underbrace{\varepsilon(t)}_{(F/m)} * \underbrace{\underline{E}(t)}_{V/m}$$

a) Isotropic Media – Vacuum

b) Anisotropic " – Crystalline materials

c) Frequency Independent - Vacuum

d) Frequency dependent - Any material other than vacuum

Let $\rho = 0$, $J = 0$

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = -\mu \left[\underline{\nabla} \times \frac{\partial \underline{H}}{\partial t} \right]$$

$$= -\mu \left[\frac{\partial}{\partial t} (\underline{\nabla} \times \underline{H}) \right]$$

$$\underbrace{\nabla (\underbrace{\nabla \cdot E}_{0}) - \nabla^2 \underline{E}}_{\underline{\text{LHS}}} = -\nabla^2 \underline{E}$$

$\underline{\text{RHS}}$ $$= -\mu \frac{\partial}{\partial t} \left[\frac{\partial \underline{D}}{\partial t} \right] = -\mu \frac{\partial^2 \underline{D}}{\partial t^2}$$

$$= -\mu \varepsilon \frac{\partial^2 \underline{E}}{\partial t^2}$$

LHS = RHS $\Rightarrow$ $$\boxed{\nabla^2 \underline{E} = \mu \varepsilon \frac{\partial^2 \underline{E}}{\partial t^2}}$$

$1/m^2$ V/m $\frac{1}{s^2}$ V/m

$\Rightarrow$ $\frac{1}{\sqrt{\varepsilon \mu}}$ - units of m/s

$\Rightarrow$ $\varepsilon \rightarrow$ Vacuum ε_0 8.854×10^{-12}

$\mu \rightarrow$ Vacuum μ_0 $4\pi \times 10^{-7}$

Velocity in vacuum $\cong 3 \times 10^8$ m/s

$$\underline{\nabla} \times \underline{E}(t) = -\frac{\partial \underline{B}(t)}{\partial t}$$

$$\underline{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$$

Direction of propagation $+z$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cancel{\partial/\partial x}\ 0 & \cancel{\partial/\partial y}\ 0 & \partial/\partial z \\ E_x & \cancel{E_y}\ 0 & \cancel{E_z}\ 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}\left(0 - \frac{\partial}{\partial z}E_x\right) + \hat{k}(0-0)$$

$$= \hat{j}\,\frac{\partial E_x}{\partial z}$$

$$-\mu \frac{\partial \underline{H}}{\partial t} \qquad H_y$$

E_x H_y z direction propagation

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_x}{\partial t}$$

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 E_x}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

u is c

$$E_x(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-\left(t + \frac{z}{u}\right)$$

$$H_y(z,t) = \underbrace{\frac{1}{\mu_0 c}}_{\text{"}\Omega\text{"}} f^+\left(t - \frac{z}{u}\right)$$

$$\mu_0 c = \mu_0 \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \ \Omega = \eta_0 \ \Omega$$

$$\frac{\partial v}{\partial z} = -l \frac{\partial i}{\partial t}$$

$$\frac{\partial i}{\partial z} = -c \frac{\partial v}{\partial t}$$

$$\frac{\partial^2 v}{\partial z^2} = \frac{1}{u^2} \frac{\partial^2 v}{\partial t^2}$$

$$u = \frac{1}{\sqrt{lc}}$$

$$v(z,t) = f^+\left(t - \frac{z}{u}\right) + f^-\left(t + \frac{z}{u}\right)$$

Forward

$$i(z,t) = \underbrace{\frac{1}{lu}}_{\text{"}\Omega\text{"}} f^+\left(t - \frac{z}{u}\right)$$

$$lu = l \frac{1}{\sqrt{lc}} = \sqrt{\frac{l}{c}} \ \Omega$$ "R_c" or "Z_0"

In the case of vacuum,

$$\eta_0 \cong 377\ \Omega$$

$$\eta_0 = \frac{E_x^+}{H_y^+}$$

$$v(z,t) = v^+(z,t) + v^-(z,t)$$

$$i(z,t) = \frac{1}{Z_0} v^+(z,t) - \frac{1}{Z_0} v^-(z,t)$$

In the simulation, on the right edge, $E_x = 0$ (Perfect Electric Conductor)

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$Z_r = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}}$$

$$\mu = \mu_0 \mu_r$$ (for vacuum $\mu_r = 1$) for other media, $\mu_r \geq 1$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$Z_r = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \qquad \varepsilon_r > 1$$

$$= \frac{Z_0}{\sqrt{\varepsilon_r}}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$c_m = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}}$$

1) Velocity of the electromagnetic wave $c = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}}$

For vacuum, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

$\mu_0 = 4\pi \times 10^{-7}$ H/m

Velocity (vacuum) $C_0 \cong 3 \times 10^{8}$ m/s

2) Characteristic impedance $\eta = \sqrt{\frac{\mu}{\varepsilon}}$, for vacuum, $\eta_0 \cong 377\,\Omega$ or $120\pi\,\Omega$

3) Travel direction or propagation direction $\rightarrow z$

E-field $\longrightarrow E_x \hat{x}$

$$E(x,y,z,t) = E_x(z)\,\hat{x}\,e^{j\omega t}$$

$$\Rightarrow \quad \frac{\partial}{\partial x} = 0 \quad , \quad \frac{\partial}{\partial y} = 0$$

$$\Rightarrow \quad \frac{d^2 E_x(z)}{dz^2} = \underbrace{-\omega^2 \mu \varepsilon}_{\gamma^2} \; E_x(z)$$

$\gamma \rightarrow$ Propagation constant

$$\gamma = j\omega\sqrt{\mu \varepsilon} = j\beta$$

β ← phase constant

$$E_x = E_x^+ e^{-j\beta z} + E_x^- e^{+j\beta z}$$

$$E_x(t) = \left[E_x^+ e^{-j\beta z} + E_x^- e^{+j\beta z} \right] e^{j\omega t}$$

$$\nabla \times \underline{E} = -j\omega\mu \underline{H}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \cancel{\partial/\partial x} & \cancel{\partial/\partial y} & \partial/\partial z \\ E_x & \cancel{E_y} & \cancel{E_z} \end{vmatrix} = -j\omega\mu \underline{H}$$

$$-\hat{y}\left(-\frac{\partial E_x}{\partial z}\right) = -j\omega\mu \underline{H}$$

$$-j\beta E_x^+ e^{-j\beta z} + j\beta E_x^- e^{+j\beta z} = -j\omega\mu H_y$$

$$\Rightarrow \quad H_y = \frac{\beta}{\omega\mu} E_x^+ e^{-j\beta z} - \frac{\beta}{\omega\mu} E_x^- e^{+j\beta z}$$

$$\eta = \frac{E_x^+}{H_y^+}$$

E- field can be decomposed to E_x, E_y

H- field " " " " H_x, H_y

TEM waves

$$|\underline{H}| = \sqrt{H_x^2 + H_y^2}$$

$$= \sqrt{\left(\frac{E_y}{\eta}\right)^2 + \left(\frac{E_x}{\eta}\right)^2}$$

$$= \frac{|\underline{E}|}{\eta}$$

Polarization $\rightarrow$ The temporal evolution of E-field at a given location in space.

$+z \rightarrow$ Direction of travel

$$E_x = Re\left\{E_{x0}\, e^{j\omega t - j\beta z}\right\}$$

$$E_y = Re\left\{E_{y0}\, e^{j\omega t - j\beta z + j\phi}\right\}$$

$\phi \rightarrow +ve \rightarrow E_y$ lead E_x by ϕ

$\phi \rightarrow -ve \rightarrow E_y$ lags E_x by ϕ

$$\underline{E} = E_{x0}\cos(\omega t)\,\hat{x} + E_{y0}\cos(\omega t + \phi)\,\hat{y}$$

$$\Rightarrow \cos(\omega t) = \frac{E_x}{E_{x0}}$$

$$\sin(\omega t) = \sqrt{1 - \left(\frac{E_x}{E_{x0}}\right)^2}$$

$$\frac{E_y}{E_{y0}} = \cos(\omega t + \phi)$$

$$= \cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)$$

$$\frac{E_x}{E_{x0}}\cos\phi - \sqrt{1 - \frac{E_x^2}{E_{x0}^2}}\sin\phi$$

$$\cos^2\phi + \sin^2\phi = 1$$

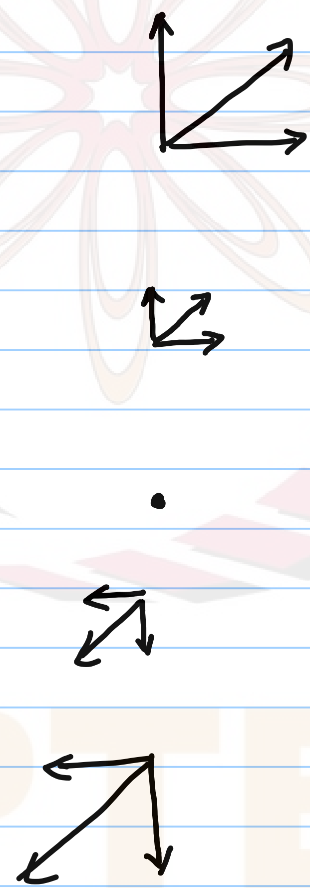
$$\left(\frac{E_x}{E_{x0}}\right)^2 - \frac{2E_xE_y\cos\phi}{E_{x0}E_{y0}} + \left(\frac{E_y}{E_{y0}}\right)^2 = \sin^2\phi$$

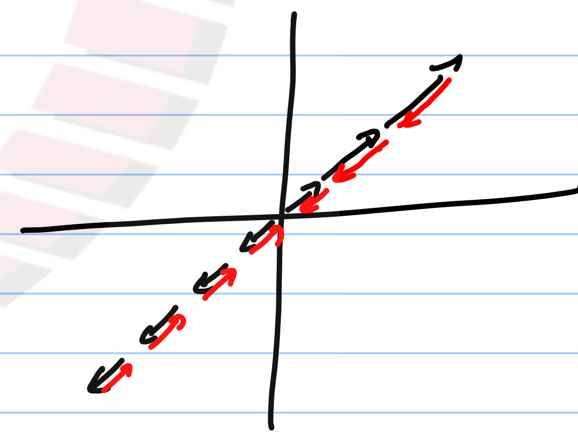
1) $E_x \neq E_y$, $\phi = 0$

$$\frac{E_x}{E_{x0}} = \frac{E_y}{E_{y0}}$$

$$\Rightarrow \quad E_y = \left(\frac{E_{y0}}{E_{x0}}\right) E_x \qquad (\text{Resembles } y = mx)$$

Linear polarization





Linear polarization

$$\phi = 0 \quad \text{or} \quad \pi \quad \text{or} \quad 2\pi \quad \text{or} \quad 3\pi \quad \cdots$$

$$\pm m\pi$$

Elliptical Polarization

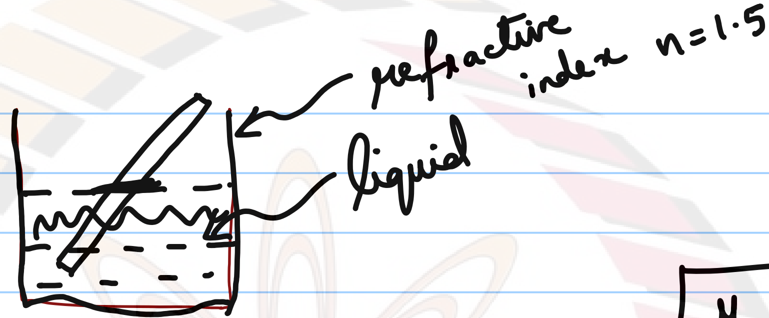
$$\phi \neq 0, \quad E_{x0} = E_{y0} \quad \text{or} \quad E_{x0} \neq E_{y0}$$

$$\phi = \pi/2, \quad E_{x0} \neq E_{y0}$$

Circular Polarization

$$\phi = \pm \text{ odd multiple of } \pi/2$$

$$E_{x0} = E_{y0}$$



$$\sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}$$

$$\sqrt{\mu_r\varepsilon_r} = 1.5$$

1) $$\nabla \times \underline{E} = -j\omega\mu_0\mu_r \underline{H}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega\varepsilon_0\varepsilon_r \underline{E}$$

2) If the medium has a conductivity σ,

$$\underline{J} = \sigma \underline{E}$$

$$\nabla \times \underline{H} = \sigma \underline{E} + j\omega\varepsilon_0\varepsilon_r \underline{E}$$

$$= (\sigma + j\omega\varepsilon_0\varepsilon_r)\underline{E}$$

$$= j\omega\varepsilon_0 \underbrace{\left(\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}\right)}_{\varepsilon_{rc}} \underline{E}$$

$$= j\omega\varepsilon_0\varepsilon_{rc}\underline{E}$$

$\uparrow$ Relative permittivity of conductive medium

{Complex Permittivity}

3) $\varepsilon_{rc} \longrightarrow \left(\varepsilon_r - \dfrac{j\sigma}{\omega\varepsilon_0}\right)$

$\varepsilon_{rc} \longrightarrow$ Complex / Frequency Dependent

$\underline{J} = \sigma \underline{E}$ is a characteristic of a conductor

$\varepsilon_0 \varepsilon_r \underline{E}$ is a characteristic of a dielectric

4) $\dfrac{\text{Conduction current density}}{\text{Displacement Current density}} \gg 1$

or $\dfrac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \gg 1$ $\longrightarrow$ Good conductors

If $\dfrac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \ll 1$ $\longrightarrow$ Good dielectric

If $\dfrac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \approx 1 \longrightarrow$ Neither good dielectric nor good conductor

5) Copper,

$$\varepsilon_r \approx 1$$

$$\sigma = 5.6 \times 10^7 \ \mho/m$$

$$f_T \text{ (Transition frequency) } \cong 10^{18} \text{ Hz}$$

Sea water, $\varepsilon_r \approx 80$

$$\sigma = 10^{-3} \ \mho/m$$

$$f_T \approx 225 \text{ kHz}$$

6) $$\nabla^2 \underline{E} = -\omega^2 \mu_0 \mu_r \varepsilon_0 \varepsilon_{rc} \underline{E}$$

$$\nabla^2 \underline{H} = -\omega^2 \mu_0 \mu_r \varepsilon_0 \varepsilon_{rc} \underline{H}$$

RHS $\rightarrow$ $j\omega \mu_0 \mu_r (\sigma + j\omega \varepsilon_0 \varepsilon_r) \underline{E}$

For majority of materials,

$$\mu_r = 1$$

7) For a plane wave, traveling in z- direction, x- polarized

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu_0 \varepsilon_0 \varepsilon_{rc} E_x$$

$$= \gamma^2 \underline{E_x}$$

$$\Rightarrow \quad \gamma = \sqrt{-\omega^2 \mu_0 \varepsilon_0 \varepsilon_{rc}}$$

$$= j\omega \sqrt{\mu_0 \varepsilon_0} \sqrt{\varepsilon_{rc}}$$

$$= j\omega \sqrt{\mu_0 \varepsilon_0} \left\{ \varepsilon_r - j \frac{\sigma}{\omega \varepsilon_0} \right\}^{1/2}$$

$$\alpha = Re\{\gamma\} = \omega \sqrt{\frac{\mu_0 \varepsilon_0 \varepsilon_r}{2}} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2 \varepsilon_r^2}} - 1 \right]^{1/2}$$

$$\beta = Im\{\gamma\} = \omega \sqrt{\frac{\mu_0 \varepsilon_0 \varepsilon_r}{2}} \left[\sqrt{1 + \frac{\sigma^2}{\omega^2 \varepsilon_0^2 \varepsilon_r^2}} + 1 \right]^{1/2}$$

Loss tangent

$$\tan\delta = \frac{\sigma}{\omega \varepsilon_0 \varepsilon_r}$$

$$\sigma \gg \omega \varepsilon_0 \varepsilon_r$$

$$\gamma \approx \sqrt{j\omega\mu_0\sigma}$$

$$\sqrt{j} = \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \cos\pi/4 + j\sin\pi/4$$

$$= \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$$

$$\gamma = \sqrt{\omega\mu_0\sigma}\left\{\frac{1+j}{\sqrt{2}}\right\}$$

$$\alpha = \beta = \sqrt{\frac{\omega\mu_0\sigma}{2}} \quad , \quad \omega\uparrow, \alpha\uparrow, \beta\uparrow$$

$$\sigma\uparrow, \alpha\uparrow, \beta\uparrow$$

Power density $\underline{P} = \underline{E} \times \underline{H}$

Net power $W = \oint_S (\underline{E} \times \underline{H}) \cdot d\hat{a}$

1) $\underline{E} \times \underline{H} = \underline{P}$ (power density)

$\underline{E} \to V/m$, $\underline{H} \to A/m$, $\underline{P} \to W/m^2$ (VA/m^2 if there is a phase difference between E & H)

2) Net outward power $= W = \oint_S (\underline{E} \times \underline{H}) \cdot d\hat{a}$

3) Instantaneous fields :-

$$\underline{E}(x,y,z,t) = Re\left\{ E_0(x,y,z)\, e^{j\omega t}\, e^{j\phi_e} \right\} \hat{e}$$

$$\underline{H}(x,y,z,t) = Re\left\{ H_0(x,y,z)\, e^{j\omega t}\, e^{j\phi_h} \right\} \hat{h}$$

E- field $\longrightarrow$ $E_0(x,y,z) \cos(\phi_e + \omega t)\, \hat{e}$

H- field $\longrightarrow$ $H_0(x,y,z) \cos(\phi_h + \omega t)\, \hat{h}$

Instantaneous power,

$$\underline{P} = \underline{E} \times \underline{H}$$

$$= E_0 H_0 \cos(\omega t + \phi_e) \cos(\omega t + \phi_h) \; (\hat{e} \times \hat{h})$$

$$= \frac{E_0 H_0}{2} \left\{ \cos(\phi_e - \phi_h) + \cos(2\omega t + \phi_e + \phi_h) \right\} (\hat{e} \times \hat{h})$$

Instantaneous power could be positive, negative or zero. Drawing conclusions is not easy.

Average power density :

$$\underline{P_{av}} = \frac{1}{T} \int_0^T \underline{P} \, dt$$

$$= \frac{1}{2} \, \mathrm{Re} \left\{ \left[E_0 e^{j\phi_e} e^{j\omega t} \hat{e} \right] \times \left[H_0 e^{-j\phi_h} e^{-j\omega t} \hat{h} \right] \right\}$$

$$\text{or} \left\{ \left(\frac{1}{2} E_0 H_0 \cos(\phi_e - \phi_h)\, \hat{e} \times \hat{h} \right) \right\}$$

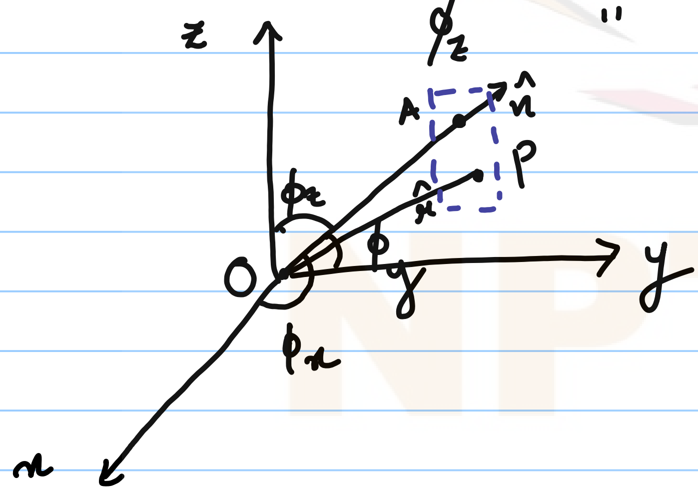
$$= \frac{1}{2} \, Re \left\{ \underline{E} \times \underline{H}^* \right\}$$

A plane wave travels in an arbitrary direction

$\hat{n} \rightarrow$ angles ϕ_x w.r.t x axis

ϕ_y " y axis

ϕ_z " z axis



$$\hat{n} = \cos\phi_x \, \hat{x} + \cos\phi_y \, \hat{y} + \cos\phi_z \, \hat{z}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\underline{k} = \underbrace{\beta}_{\frac{2\pi}{\lambda}} \hat{n} = \beta\cos\phi_x \, \hat{x} + \beta\cos\phi_y \, \hat{y} + \beta\cos\phi_z \, \hat{z}$$

$$\underline{E} \cdot \underline{k} = 0 \quad \text{or} \quad \underline{E} \cdot \hat{n} = 0.$$

for a z-directed plane EM wave,

$$\underline{k} = \beta\left(\cos\phi_x \, \hat{x} + \cos\phi_y \, \hat{y} + \cos\phi_z \, \hat{z}\right)$$

$$= \beta\left(\cos\frac{\pi}{2}\,\hat{x} + \cos\frac{\pi}{2}\,\hat{y} + \cos(0)\,\hat{z}\right)$$

$$= \beta \hat{z}$$

$$\underline{E} = \underline{E}_0 \, e^{-j\beta \hat{n}\cdot\underline{r}}$$

$$= \underline{E}_0 \, e^{-j\beta \hat{n}\cdot\underline{r}} \qquad \underline{k} = \underline{E}_0 \, e^{-j\underline{k}\cdot\underline{r}}$$

$$\underline{E} = \underline{E}_0 \, e^{-j\underline{k}\cdot\underline{r}}$$

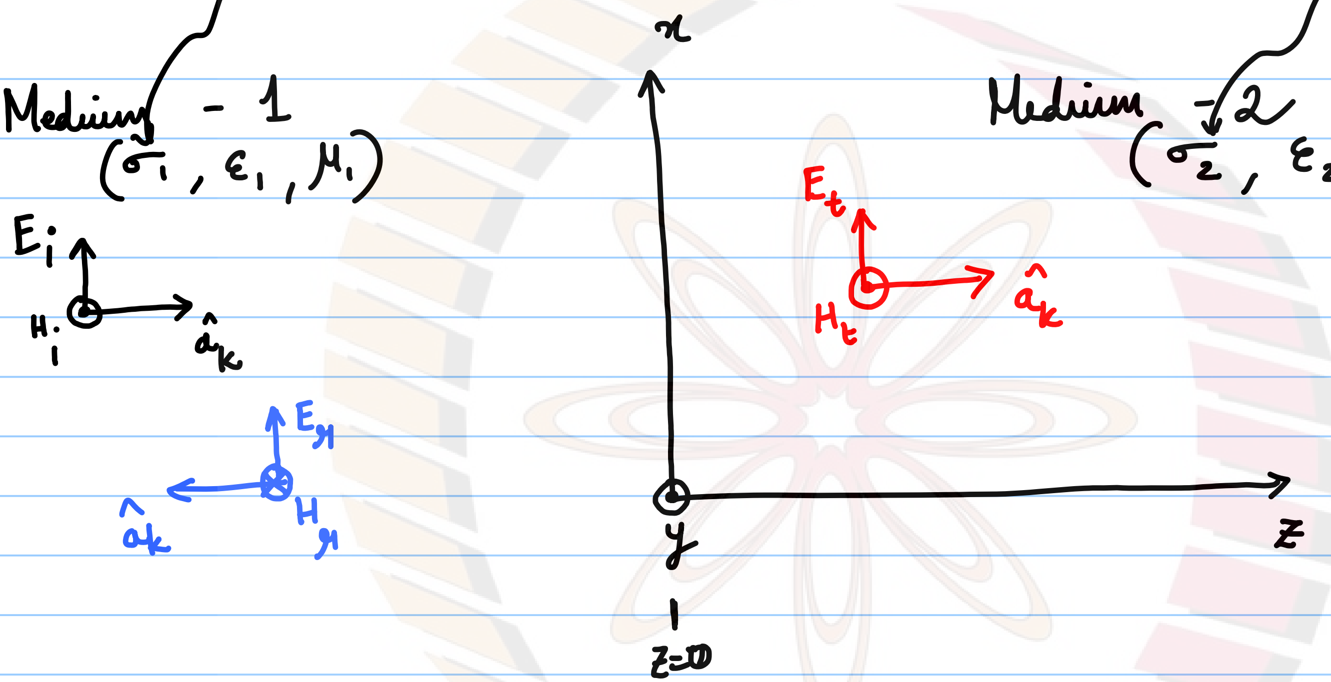
$$= \underline{E}_0 \, e^{-j\beta(x\cos\phi_x)} \, e^{-j\beta(y\cos\phi_y)} \, e^{-j\beta(z\cos\phi_z)}$$

$$\frac{\omega}{k} \rightarrow \text{phase velocity}$$

$$V_{pz} = \frac{\omega}{k_z} = \frac{\omega}{\beta\cos\phi_z} = \frac{V_0}{\cos\phi_z}$$

Group velocity $= \dfrac{d\omega}{dk}$ cannot be higher than c.

1) Medium - 1 $(\overset{0}{\sigma_1}, \varepsilon_1, \mu_1)$ Medium - 2 $(\overset{0}{\sigma_2}, \varepsilon_2, \mu_2)$



2) $$\left.\begin{aligned} E_i(z) &= E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x \\ H_i(z) &= H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y \\ &= \frac{E_{io}}{\eta_1}\, e^{-\gamma_1 z}\, \hat{a}_y \end{aligned}\right\} \text{Incident field}$$

3) $E_r(z) = E_{r0}\, e^{\gamma_1 z}\, \hat{a}_x$

$H_r(z) = H_{r0}\, e^{\gamma_1 z}\, (-\hat{a}_y)$

$= -\dfrac{E_{r0}}{\eta_1}\, e^{\gamma_1 z}\, \hat{a}_y$

Reflected field

4) $E_t(z) = E_{t0}\, e^{-\gamma_2 z}\, \hat{a}_x$

$H_t(z) = H_{t0}\, e^{-\gamma_2 z}\, \hat{a}_y$

5) Total fields in Medium 1 :-

$E_1 = E_i + E_r \quad , \quad H_1 = H_i + H_r$

In medium 2,

$E_2 = E_t \quad , \quad H_2 = H_t$

b) At $z = 0$,

$E_{tangential} \rightarrow$ Continuous

$H_{tangential} \rightarrow$ Continuous

$$E_{1_{tan}} = E_{2_{tan}} \quad \& \quad H_{1_{tan}} = H_{2_{tan}}$$

$$E_i(0) + E_r(0) = E_t(0) \quad \text{———} \quad (1)$$

$z=0$

$$H_i(0) + H_r(0) = H_t(0)$$

$$\frac{E_i(0)}{\eta_1} + \frac{E_r(0)}{\eta_1} = \frac{E_t(0)}{\eta_2} \quad \text{———} \quad (2)$$

$$\boxed{\frac{E_{ro}}{E_{io}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} = \Gamma$$

$$\boxed{\frac{E_{to}}{E_{io}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right)} = \tau$$

$$1 + \Gamma = \tau$$

1 Medium $\rightarrow$ $\sigma_1 = 0$, $\eta_1 \neq 0$

2 Medium $\rightarrow$ $\sigma_2 = \infty$

$\Gamma = -1$, $\tau = 0$

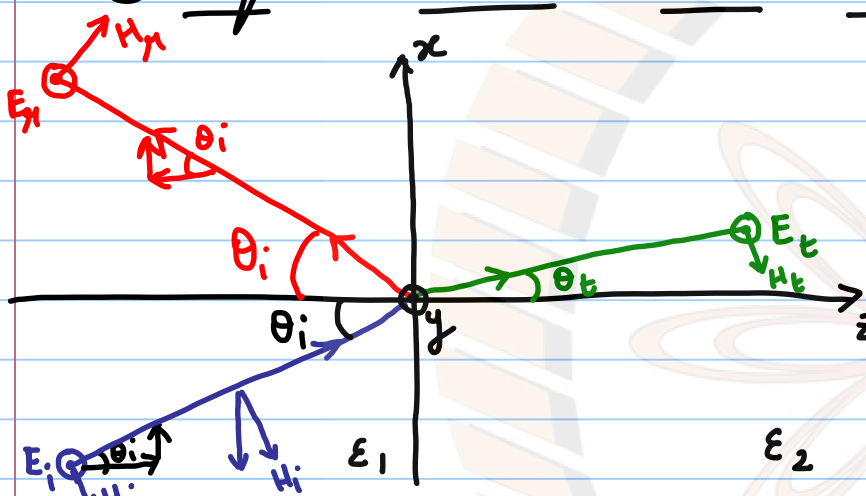
$$E_1 = E_i + E_r = \left(E_{io} e^{-\gamma_1 z} + E_{ro} e^{\gamma_1 z}\right)\hat{a}_x$$

$$= E_{io} e^{-j\beta_1 z} + E_{ro} e^{+j\beta_1 z}$$

$$= E_{io} e^{-j\beta z} + - E_{io} e^{+j\beta_1 z}$$

$$= -2j\, E_{io} \sin(\beta_1 z)\ \hat{a}_x$$

Oblique Incidence on Dielectric/Dielectric interfaces



Plane of incidence :- xz plane

E-field :- $\perp$r to plane of incidence

$\perp$r polarization

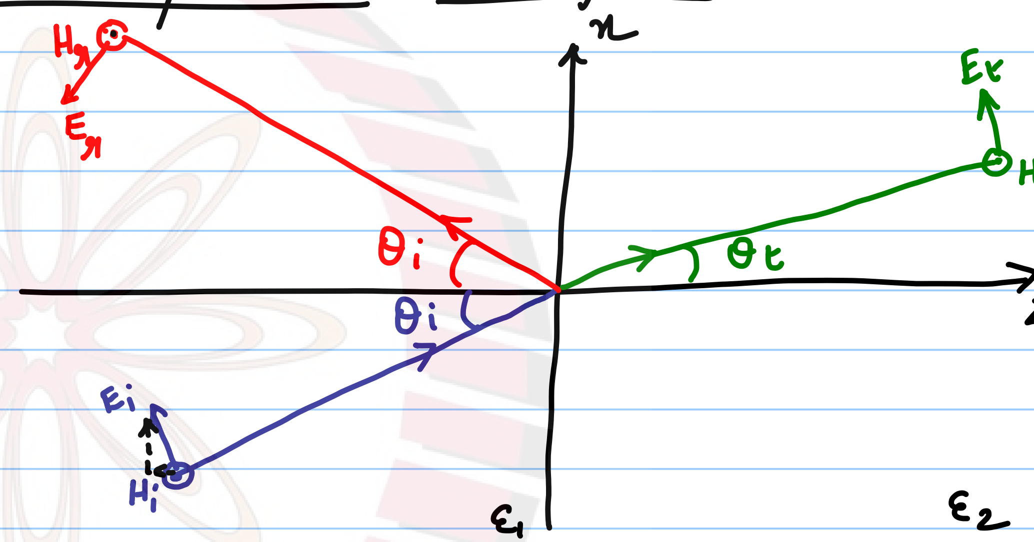
1) Incident wave :-

E-field :- $\underline{E_i} = \underline{E_{io}}\, e^{-j\underline{k_i}\cdot\underline{r}}$

$\underline{k_i} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$= \beta_1\hat{n}$

n_x, n_y, n_z



H-field :- $\perp$r to plane of incidence.

$\parallel$el polarization

1) Incident wave :-

E-field : $\underline{E_i} = \underline{E_{io}}\, e^{-j\underline{k_1}\cdot\underline{r}}$

$\underline{k_i} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$= \beta_1\hat{n}$

n_x, n_y, n_z

$$n = [\sin\theta_i \,,\, 0 \,,\, \cos\theta_i]$$

$$\Rightarrow \underline{k_i} = \beta_1 (\sin\theta_i \,\hat{x} + 0\hat{y} + \cos\theta_i \hat{z})$$

$$= \beta_1 \sin\theta_i \,\hat{x} + \beta_1 \cos\theta_i \,\hat{z}$$

Now, $\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$\Rightarrow \underline{E_i} = \underline{E_{io}}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$$

$$n = [\sin\theta_i \,,\, 0 \,,\, \cos\theta_i]$$

$$\Rightarrow k_i = \beta_1 \sin\theta_i \,\hat{x} + \beta_1 \cos\theta_i \,\hat{z}$$

$$\Rightarrow \underline{E_i} = \underline{E_{io}}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}$$

2) Reflected Wave :-

$$\underline{E_r} = \underline{E_{ro}}\, e^{-j\underline{k_r}\cdot\underline{r}}$$

$$\underline{k_r} = \beta\, \hat{n}$$

$$n = [\sin\theta_i \,,\, 0 \,,\, -\cos\theta_i]$$

$$\underline{k_r} = \beta(\sin\theta_i \,\hat{x} - \cos\theta_i \,\hat{z})$$

$$\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\underline{E_r} = \underline{E_{r0}}\, e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)}$$

3) Transmitted wave :-

E-field :- $\underline{E_t} = \underline{E_{t0}}\, e^{-j\underline{k_t}\cdot\underline{r}}$

$$\underline{E_t} = \underline{E_{t0}}\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

3) $|H_i| = \dfrac{|E_i|}{\eta_1}$

$|H_r| = \dfrac{|E_r|}{\eta_1}$

$|H_t| = \dfrac{|E_t|}{\eta_2}$

$$\underline{E_r} = \underline{E_{r0}}\, e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)}$$

$$\underline{E_t} = \underline{E_{t0}}\, e^{-j\beta_2 (x\sin\theta_t + z\cos\theta_t)}$$

The interface is at $z=0$
Apply boundary conditions

$$E_i + E_r = E_t$$

$$H_i \cos\theta_i - H_r \cos\theta_i = H_t \cos\theta_t$$

At $z=0$,

$$E_{io} + E_{ro} = E_{to} \quad \text{——— } (1)$$

$$\frac{E_{io}}{\eta_1}\cos\theta_i - \frac{E_{ro}}{\eta_1}\cos\theta_i = \frac{E_{to}}{\eta_2}\cos\theta_t \quad \text{——— } (2)$$

$$\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$$\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

The interface is at $z=0$,
Apply boundary conditions :

At $z=0$,

$$E_{io}\cos\theta_i - E_{ro}\cos\theta_i = E_{to}\cos\theta_t \quad \text{——— } (1)$$

$$H_{io} + H_{ro} = H_{to} \quad \text{——— } (2)$$

$$\frac{E_{io}}{\eta_1} + \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_1\cos\theta_i - \eta_2\cos\theta_t}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}$$

$$\tau_{\parallel} = \frac{E_{to}}{E_{io}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

For $\Gamma_{\perp} = 0$,

$$\eta_2 \cos\theta_i - \eta_1 \cos\theta_t = 0$$

Let $\theta_B = \theta_i$

$$\eta_2 \cos\theta_B - \eta_1 \cos\theta_t = 0$$

$$\Rightarrow \theta_{B_\perp} = \cos^{-1}\left\{\frac{\eta_1}{\eta_2}\cos\theta_t\right\}$$

Brewster's angle

For $\Gamma_{\parallel} = 0$,

$$\eta_1 \cos\theta_i - \eta_2 \cos\theta_t = 0$$

Let $\theta_B = \theta_i$

$$\eta_1 \cos\theta_B - \eta_2 \cos\theta_t = 0$$

$$\Rightarrow \theta_{B_{\parallel}} = \cos^{-1}\left\{\frac{\eta_2}{\eta_1}\cos\theta_t\right\}$$

Brewster's angle

Snell's law,

$$n_1 \sin\theta_i = n_2 \sin\theta_t$$

or $\sin\theta_t = \dfrac{n_1}{n_2}\sin\theta_i$

$$\beta_1 = \frac{2\pi}{\lambda_0/n_1} \quad , \quad \beta_2 = \frac{2\pi}{\lambda_0/n_2}$$

$$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2} \sin\theta_B$$

$$\cos\theta_t = \sqrt{1 - \frac{\beta_1^2}{\beta_2^2} \sin^2\theta_B}$$

$$\beta_1 = \omega\sqrt{\mu_1\varepsilon_1} \quad , \quad \beta_2 = \omega\sqrt{\mu_2\varepsilon_2}$$

$$\Rightarrow \eta_2 \cos\theta_B - \eta_1 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_B} = 0$$

$$\& \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \quad , \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

$$\theta_{B_\perp} = \tan^{-1}\left\{\sqrt{\frac{\mu_2}{\mu_1}}\sqrt{\frac{\mu_2\varepsilon_1-\mu_1\varepsilon_2}{\mu_2\varepsilon_2-\mu_1\varepsilon_1}}\right\}$$

For $\mu_1=\mu_2=\mu_0$,

$$\Rightarrow \theta_{B_\perp} = \tan^{-1}\left\{\sqrt{\frac{\mu_0}{\mu_0}\left(\frac{\varepsilon_1-\varepsilon_2}{\varepsilon_2-\varepsilon_1}\right)}\right\}$$

$$\theta_{B_{\parallel}} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{\frac{\mu_1\varepsilon_2-\mu_2\varepsilon_1}{\mu_2\varepsilon_2-\mu_1\varepsilon_1}}\right\}$$

for $\mu_1=\mu_2=\mu_0$,

$$\theta_{B_{\parallel}} = \tan^{-1}\left\{\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\sqrt{\frac{\mu_0\varepsilon_2-\mu_0\varepsilon_1}{\mu_0\varepsilon_2-\mu_0\varepsilon_1}}\right\}$$

Majority of the materials have $\mu_1=\mu_2=\mu_0$

$$n_1\sin\theta_i = n_2\sin\theta_t$$

or

$$\beta_1\sin\theta_i = \beta_2\sin\theta_t$$

$$\beta_1 = \frac{2\pi}{\lambda_0/n_1}$$

$$\beta_2 = \frac{2\pi}{\lambda_0/n_2}$$

$$\Rightarrow \sin\theta_t = \frac{\beta_1}{\beta_2}\sin\theta_i$$

If $\sin\theta_t = 1$, for a given material system (n_1 & n_2), at a specific angle of incidence θ_i, that angle of incidence is called Critical Angle.

θ_t at $\theta_i = \theta_c$ is $\pi/2$.

$$\frac{\beta_1}{\beta_2} \sin\theta_i \geqslant 1$$

$$\frac{\omega\sqrt{\mu_1 \varepsilon_1}}{\omega\sqrt{\mu_2 \varepsilon_2}} \sin\theta_i \geqslant 1$$

$$\Rightarrow \quad \sin\theta_i \geqslant \left(\frac{\sqrt{\mu_2 \varepsilon_2}}{\sqrt{\mu_1 \varepsilon_1}}\right)$$

$$\Rightarrow \quad \sin\theta_i \geqslant \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

$$\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} < 1$$

$$\sqrt{\varepsilon_2} < \sqrt{\varepsilon_1}$$

$$\Gamma_{\parallel} = \frac{E_{ro}}{E_{io}} = \frac{\eta_1 \cos\theta_i - \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$= \frac{\eta_1 \cos\theta_i - \eta_2 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}}{\eta_1 \cos\theta_i + \eta_2 \sqrt{1 - \frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i}}$$

$$= \frac{\eta_1 \cos\theta_i - j\eta_2 \sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}{\eta_1 \cos\theta_i + j\eta_2 \sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1}}$$

Let $\eta_1 \cos\theta_i = X$

Let $\eta_2 \sqrt{\frac{\beta_1^2}{\beta_2^2}\sin^2\theta_i - 1} = Y$

$$= \frac{X - jY}{X + jY}$$

$$|\Gamma_{\parallel}| = 1$$

Total Internal Reflection (TIR)

Also $|\Gamma_{\perp}| = 1$

$$E_i = E_{i_{\parallel}} + E_{i_{\perp}} e^{j\phi}$$

Let incident wave be linearly polarized,

$$\phi = 0 .$$

$E_{r_{\parallel}}$ $E_{r_{\perp}}$ with no phase.

Reflected wave → linearly polarized

Transmitted " → " "

2. <u>Circularly polarized :-</u>

$$|E_{i_{\parallel}}| = |E_{i_{\perp}}|$$

$$\phi = \pm \pi/2$$

Reflected → $\phi = \pm \pi/2$, $E_{r_{\parallel}} \neq E_{r_{\perp}}$, Elliptical

Transmitted → $\phi = \pm \pi/2$, $E_{t_{\parallel}} \neq E_{t_{\perp}}$, "

Perpendicular polarization (Simulation setup):-

1) H_x , E_y , H_z $\qquad$ xz plane $\longrightarrow$ propogation

k_x , k_z

2) $$\nabla \times \underline{H} = \varepsilon \frac{\partial E}{\partial t}$$

LHS

$$\nabla \times \underline{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & 0 & \partial/\partial z \\ H_x & 0 & H_z \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{k}(0)$$

$$= \hat{j}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)$$

RHS

$$\varepsilon \frac{\partial E}{\partial t} = \varepsilon \frac{\partial E_y}{\partial t} \hat{j}$$

$$\varepsilon \frac{\partial E_y}{\partial t} \cong \varepsilon \frac{E_y(x, z, t+\Delta t) - E_y(x, z, t)}{\Delta t}$$

LHS in difference form,

(i) $$\frac{\partial H_x}{\partial z} \cong \frac{H_x(x, z, t) - H_x(x, z-\Delta z, t)}{\Delta z}$$

(ii) $$\frac{\partial H_z}{\partial x} \cong \frac{H_z(x, z, t) - H_z(x-\Delta x, z, t)}{\Delta x}$$

LHS = RHS & Bringing unknown quantity to the left,

$$E_y(x, z, t+\Delta t) = E_y(x, z, t) + \frac{\Delta t}{\varepsilon \Delta}\Big[H_x(x, z, t) - H_x(x, z-\Delta z, t) - H_z(x, z, t) + H_z(x-\Delta x, z, t)\Big]$$

2) $$\nabla \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$= -\mu \frac{\partial \underline{H}}{\partial t}$$

LHS

$$\nabla \times \underline{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & 0 & \partial/\partial z \\ 0 & E_y & 0 \end{vmatrix}$$

$$= \hat{i}\left(-\frac{\partial E_y}{\partial z}\right) - \hat{j}(0) + \hat{k}\left(\frac{\partial E_y}{\partial x}\right)$$

RHS

$$-\mu \frac{\partial \underline{H}}{\partial t} = -\mu \frac{\partial H_x}{\partial t}\hat{i} - \mu \frac{\partial H_z}{\partial t}\hat{k}$$

LHS = RHS & Equating $\hat{i}$ components,

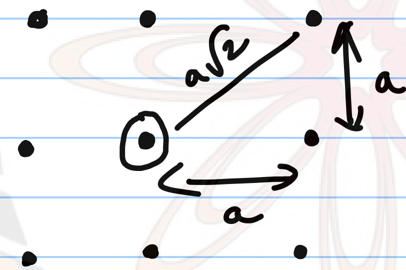
$$-\frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t}$$

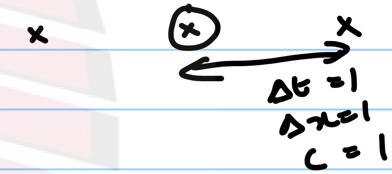
$$\Rightarrow \frac{\text{"}H_x(x,z,t+\Delta t)\text{"} - H_x(x,z,t)}{\Delta t} = \frac{1}{\mu \Delta}\left\{ E_y(x,z+\Delta z,t) - E_y(x,z,t) \right\}$$

LHS = RHS & Equating $\hat{k}$ components,

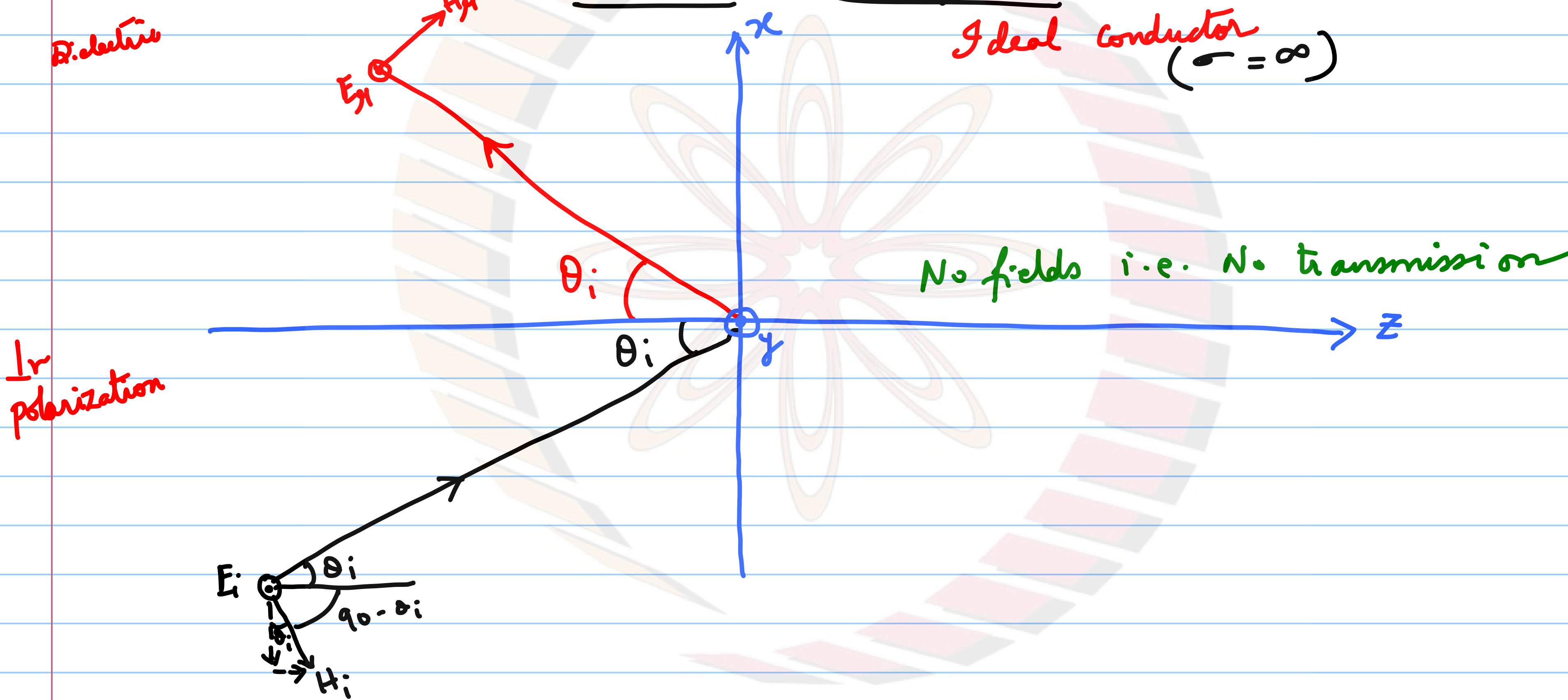
$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t}$$

$$H_z(x,z,t+\Delta t) = H_z(x,z,t) - \frac{1}{\mu\Delta}\left\{ \cdots \right\}$$





Dielectric - Ideal Conductor Interface



$$\underline{E}_i = E_{i_0} e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)} \hat{y}$$

$$\underline{H}_i = \frac{E_{io}}{\eta_1} e^{-j\beta_1 (x \sin\theta_i + z\cos\theta_i)} \left(-\cos\theta_i \, \hat{x} + \sin\theta_i \, \hat{z} \right)$$

$$\underline{E}_r = E_{ro} \, e^{-j\beta_1 (x\sin\theta_i - z\cos\theta_i)} \, \hat{y}$$

$$\underline{H}_r = \frac{E_{ro}}{\eta_1} e^{-j\beta_1 (x \sin\theta_i - z\cos\theta_i)} \left(\cos\theta_i \, \hat{x} + \sin\theta_i \, \hat{z} \right)$$

Apply boundary conditions at $z = 0$;

$$\underline{E}_{1\,tan} = \left(\underline{E}_i + \underline{E}_r \right)_{tan} = 0$$

$$\underline{H}_{1\,norm} = \left(\underline{H}_i + \underline{H}_r \right)_{norm} = 0$$

At $z=0$,

$$E_{io}\, e^{-j\beta_1(x\sin\theta_i)} + E_{ro}\, e^{-j\beta_1(x\sin\theta_i)} = 0$$

$$\Rightarrow \quad E_{ro}\, \cancel{e^{-j\beta_1 x\sin\theta_i}} = -E_{io}\, \cancel{e^{-j\beta_1 x\sin\theta_i}}$$

$$\Rightarrow \quad \boxed{E_{ro} = -E_{io}}$$

$$\Gamma = 1\angle 180^\circ \qquad \underline{\underline{(-1)}}$$

At $z=0$,

$$\frac{E_{io}}{\eta_1}\sin\theta_i\, e^{-j\beta_1 x\sin\theta_i} + \frac{E_{ro}}{\eta_1}\sin\theta_i\, e^{-j\beta_1 x\sin\theta_i} = 0$$

$$\Rightarrow \quad \frac{E_{ro}}{\cancel{\eta_1}}\cancel{\sin\theta_i}\, \cancel{e^{-j\beta_1 x\sin\theta_i}} = -\frac{E_{io}}{\cancel{\eta_1}}\cancel{\sin\theta_i}\, \cancel{e^{-j\beta_1 x\sin\theta_i}}$$

$$\Rightarrow \quad \boxed{E_{ro} = -E_{io}}$$

Total fields in Medium ① :-

$$\underline{E} = \underline{E_i} + \underline{E_r}$$

$$= E_{i0}\, e^{-j\beta_1 x \sin\theta_i} \left(e^{-j\beta_1 z \cos\theta_i} - e^{j\beta_1 z \cos\theta_i} \right) \hat{y}$$

$$= -2j\, E_{i0} \sin(\beta_1 z \cos\theta_i)\, e^{-j\beta_1 x \sin\theta_i}\, \hat{y}$$

|||ly,

$$\underline{H} = \underline{H_i} + \underline{H_r}$$

$$= -2 \frac{E_{i0}}{\eta_1} \cos\theta_i \left(\cos(\beta_1 z \cos\theta_i) \right) e^{-j\beta_1 x \sin\theta_i}\, \hat{x}$$

$$- 2j \frac{E_{i0}}{\eta_1} \sin\theta_i \left(\sin(\beta_1 z \cos\theta_i) \right) e^{-j\beta_1 x \sin\theta_i}\, \hat{z}$$

1)

$$E_y = -2j\, E_{i0} \sin(\beta_1 z \cos\theta_i)\, e^{-j\beta_1 x \sin\theta_i}\, \hat{y}$$

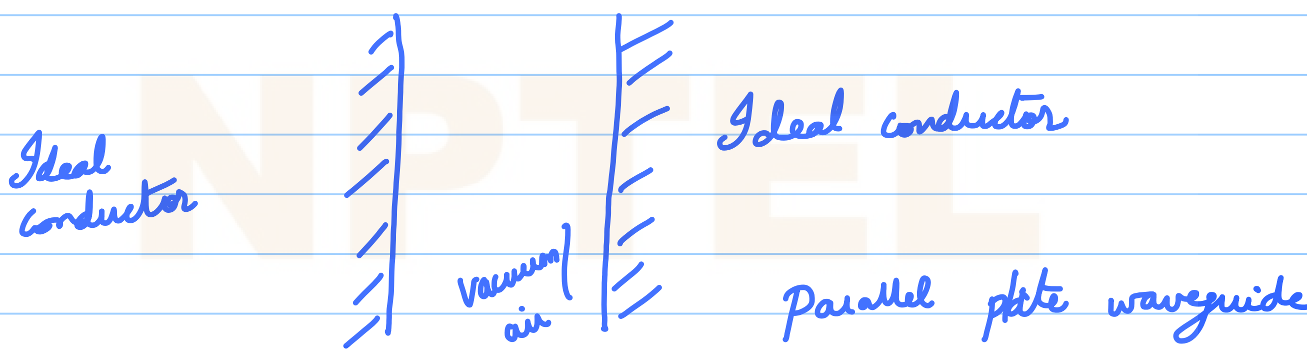
2) The zeros of the electric field, are located at,

$$\beta_1 z \cos\theta_i = m\pi$$

$$\Rightarrow \quad z = \frac{m\pi}{\beta_1 \cos\theta_i}$$

$$\beta_1 = \frac{2\pi}{\lambda_1} \quad , \quad z = \frac{m\lambda_1}{2\cos\theta_i}$$

3)



4) Construct a parallel plate waveguide with distance b/w the plates being "d"

Integer

$$\cos\theta_i = \frac{m\lambda_1}{2d}$$

$$|\;| \leq 1$$

There are a finite number of angles for a given "d".

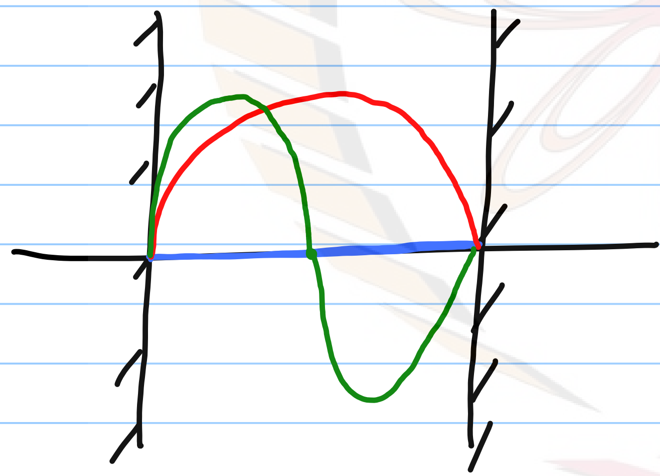
5)

$$-2j\,E_{i0}\,\sin(\beta_1 z\cos\theta_i)\,e^{-j\beta_1 x\sin\theta_i}\,\hat{y}$$

$$= -2j\,E_{i0}\,\sin\left(\beta_1 z\,\frac{m\lambda_1}{2d}\right)\,e^{-j\beta_1 x\sqrt{1-\left(\frac{m\lambda_1}{2d}\right)^2}}\;\text{''}\,\hat{y}\,\text{''}$$

6) Let $\beta_w = \beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}$

7) Propagation constant in the waveguide is different from the homogeneous bulk medium.

8) 

$m = 0$

$m = 1$ (Max. e-field at centre of waveguide)

$m = 2$ (Centre has no e-field)

Modes of the ||el plate waveguide

9) $$\beta_w = \beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}$$

$\beta_1 \rightarrow 2\pi/\lambda_1 = \dfrac{2\pi f_1}{v_1}$

$$\Rightarrow \quad \beta_1 \geq \frac{m\pi}{d}$$

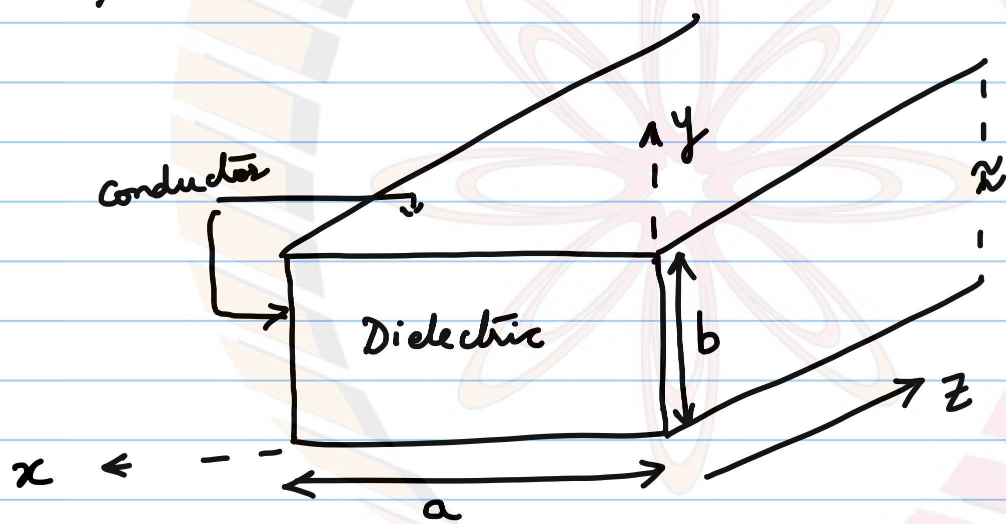
$$\Rightarrow \quad \frac{2\pi}{\lambda_1} \geq \frac{m\pi}{d} \quad \text{or} \quad \boxed{\lambda_1 \leq \frac{2d}{m}}$$ Cut-off wavelength

$$\text{or} \quad \boxed{f \geq \frac{m v_1}{2d}}$$ Cut-off frequency.

Rectangular waveguides

1) Key ideas/results are important.

2)



3) If $H_z = 0$; $E_z \neq 0$

TM polarization (Transverse Magnetic)

4) $E_z(x, y, z)$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} + \omega^2 \mu\varepsilon E_z = 0$$

Method of separation of variables :-

$$E_z(x,y,z) = X(x)\ Y(y)\ Z(z)$$

$$YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + XY\frac{d^2Z}{dz^2} + \omega^2\mu\varepsilon\, XYZ = 0$$

Dividing both sides by XYZ,

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + \omega^2\mu\varepsilon = 0$$

$$\Rightarrow \frac{1}{X}\frac{d^2X}{dx^2} + A^2 = 0$$

or
$$\boxed{\frac{1}{X}\frac{d^2X}{dx^2} = -A^2} \tag{1}$$

$$\boxed{\frac{1}{Y}\frac{d^2Y}{dy^2} = -B^2} \tag{2}$$

$$\boxed{\frac{1}{Z}\frac{d^2Z}{dz^2} = -\beta^2} \tag{3}$$

Assume general solutions of the form,

$$\left.\begin{aligned} X &= C_1 \cos Ax + C_2 \sin Ax \\ Y &= C_3 \cos By + C_4 \sin By \end{aligned}\right\} \text{Standing wave patterns}$$

$$Z = C_5\, e^{-j\beta z} + C_6\, e^{+j\beta z} \longrightarrow \text{Travelling wave}$$

If the waveguide is infinitely long,

$$C_6 = 0$$

$$\left.\begin{array}{l} \text{at } x = 0 \\ \text{at } y = 0 \\ \text{at } x = a \\ \text{at } y = b \end{array}\right\} E_z = 0$$

at $x = 0$, $E_z = 0 \Rightarrow C_1 = 0$

at $y = 0$, $E_z = 0 \Rightarrow C_3 = 0$

at $x = a$, $E_z = 0$
$y = b$,

As of now, $E_z(x,y,z) = \left(C_2 \sin Ax\right)\left(C_4 \sin By\right)\left(C_5 e^{-j\beta z}\right)$

At $x=a$, $E_z=0 \Rightarrow A = \frac{m\pi}{a}$

At $y=b$, $E_z=0 \Rightarrow B = \frac{n\pi}{b}$

$$E_z(x,y,z) = \left[C_2 \sin \frac{m\pi}{a} x\right]\left[C_4 \sin \frac{n\pi}{b} y\right]\left[C_5 e^{-j\beta z}\right]$$

$$= C \left[\sin \frac{m\pi}{a} x\right]\left[\sin \frac{n\pi}{b} y\right]\left[e^{-j\beta z}\right]$$

$m=0$, $n=0$ is not possible

$m=0$, $n=1$ " " "

$m=1$, $n=0$ " " "

$m=1$, $n=1$ ——————

TM_{mn}

1) $$E_x = \frac{-j\omega\mu}{\omega^2\mu\varepsilon - \beta^2} \frac{\partial H_z}{\partial y} - \frac{j\beta}{\omega^2\mu\varepsilon - \beta^2} \frac{\partial E_z}{\partial x}$$

0.

$$E_x = \frac{-j\beta}{\omega^2\mu\varepsilon - \beta^2} \left(\frac{m\pi}{a}\right) C \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

2) $$E_y = \frac{j\omega\mu}{\omega^2\mu\varepsilon - \beta^2} \frac{\partial H_z}{\partial x} - \frac{j\beta}{\omega^2\mu\varepsilon - \beta^2} \frac{\partial E_z}{\partial y}$$

0

$$E_y = \frac{-j\beta}{\omega^2\mu\varepsilon - \beta^2} \left(\frac{n\pi}{b}\right) C \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

3) (i) Fields $\longrightarrow$ Patterns (Depend on m for parallel plate waveguide
m, n " rectangular waveguide)

↳ Discrete patterns

↳ Sinusoidal variations in transversal directions

(ii) $$\omega^2 \mu \varepsilon = \beta^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\beta^2 = \omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \quad \beta = \sqrt{\underline{\underline{\omega^2 \mu \varepsilon}} - \underline{\underline{\left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}}$$

$$\omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$ worst case

$$\Rightarrow \quad \omega_c = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$\omega_{input} > \omega_c$

cut-off frequency

1) $\beta_w = \beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}$

2) Phase Velocity $= \frac{\omega}{\beta_w} = \frac{\omega}{\beta_1 \sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}}$

$$= \frac{V_1}{\sqrt{1 - \left(\frac{m\lambda_1}{2d}\right)^2}}$$

3) From prior lectures,

$$\lambda_{cut-off} = \frac{2d}{m}$$

&

$$f_{cut-off} = \frac{mV_1}{2d}$$

4) $$V_p = \frac{V_1}{\sqrt{1-\left(\frac{\lambda_1}{\lambda_{cut-off}}\right)^2}} = \frac{V_1}{\sqrt{1-\left(\frac{f_{cut-off}}{f}\right)^2}}$$

5) Phase velocity is a function of frequency.
↳ Means wave dispersion

6) As $f \uparrow \infty$, $V_p = V_1$

But at all lower frequencies, $V_p > V_1$

7) Phase velocity does not convey information.

8) In the simplest case, a single tone amplitude modulation, can be achieved by 2 sinusoids. (of different frequencies & of different wavelengths)

9) $\cos\left((\omega+\Delta\omega)t - (\beta+\Delta\beta)z\right) + \cos\left((\omega-\Delta\omega)t - (\beta-\Delta\beta)z\right)$

10) $2\cos(\omega t - \beta z)\cos(\Delta\omega t - \Delta\beta z)$

11) $V_p = \dfrac{\omega}{\beta}$ $\qquad V_g = \dfrac{\Delta\omega}{\Delta\beta}$

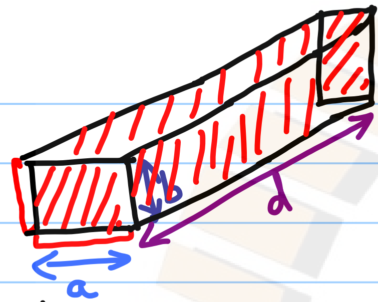
As $\Delta\omega \to 0$, $\Delta\beta \to 0$, $\quad V_g = \dfrac{d\omega}{d\beta}$

12) $V_g = \dfrac{d\omega}{d\beta} = V_1\sqrt{1-\left(\dfrac{f_{cut\text{-}off}}{f}\right)^2}$

As $f \uparrow \infty$, $\quad V_g = V_1$

For All other lower f, $\quad V_g < V_1$

$\therefore \quad V_g \leq V_1$



CAVITY

For TM_{mn} modes in rectangular waveguides that we have already seen,

$$E_{z_{forward}} = C \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-j\beta z}$$

$$E_{z_{backward}} = D \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{+j\beta z}$$

Total E-field,

$$E_{z_{Total}} = E_{z_{forward}} + E_{z_{backward}}$$

At $z=0$, $E_{z_{total}} = 0$, & at $z=d$, $E_{z_{total}} = 0$

$$\Rightarrow \quad C = -D$$

$$E_{z\,Total} = C \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)\left[e^{-j\beta z} - e^{+j\beta z}\right]$$

$$= -2j\,C \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \underline{\underline{\sin(\beta z)}}$$

At $z = d$,

$$\sin(\beta z) = \sin(\beta d) = 0$$

$$\Rightarrow \beta d = l\pi \quad , \quad l = 1, 2, 3 \cdots$$

$$\Rightarrow \beta = \frac{l\pi}{d}$$

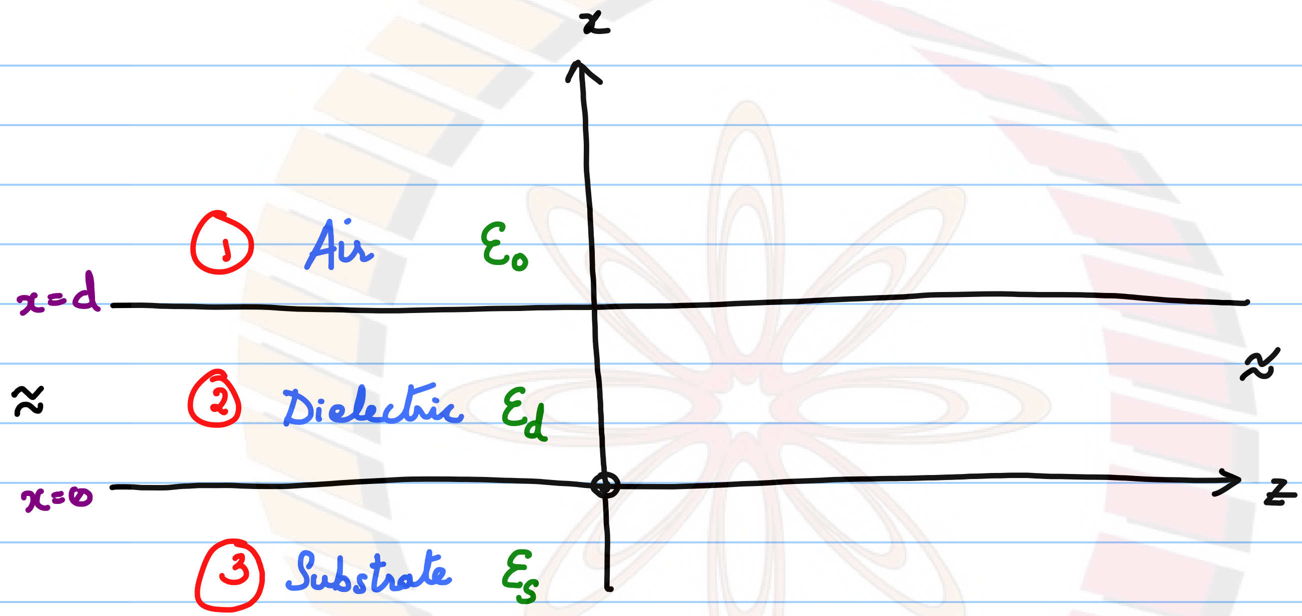
As before,

$$\omega^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2$$

$$\Rightarrow \omega = \frac{1}{\sqrt{\mu\varepsilon}} \left\{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2\right\}^{1/2}$$

Cavity Resonance Frequencies.

TM_{mnl}



TE polarization:

$$E_z = 0$$

$$H_z \neq 0$$

$$H_z = X(x)\, e^{-j\beta z}$$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}} - \beta^2 + \omega^2\mu\varepsilon = 0$$

Let $\frac{1}{X}\frac{d^2X}{dx^2} = \text{constant} = \pm k^2$

$$\Rightarrow \quad \frac{d^2X}{dx^2} = \pm k^2 X$$

Region 1

$$\frac{d^2X_1}{dx^2} = k_1^2 X_1$$

$$k_1^2 - \beta^2 + \omega^2\mu\varepsilon_0 = 0 \quad \text{——} \quad (1)$$

Region 2

$$\frac{d^2X_2}{dx^2} = -k_2^2 X_2$$

$$-k_2^2 - \beta^2 + \omega^2\mu\varepsilon_d = 0 \quad \text{——} \quad (2)$$

Region 3

$$\frac{d^2X_3}{dx^2} = k_3^2 X_3$$

$$k_3^2 - \beta^2 + \omega^2 \mu \varepsilon_s = 0 \quad \text{—— } (3)$$

$$X_1 = A_1 e^{-k_1 x} + A_2 e^{k_1 x}$$

$$\hookrightarrow A_2 \equiv 0$$

$$X_2 = A_3 \cos k_2 x + A_4 \sin k_2 x$$

$$X_3 = A_5 e^{-k_3 x} + A_6 e^{k_3 x}$$

$$\hookrightarrow \equiv 0$$

$$H_{z1} = A_1 e^{-k_1 x} e^{-j\beta z}$$

$$H_{z2} = \left[A_3 \cos k_2 x + A_4 \sin k_2 x \right] e^{-j\beta z}$$

$$H_{z3} = A_6 e^{k_3 x} e^{-j\beta z}$$

At $x = 0$,

$$H_{z2} = H_{z3} \Rightarrow \boxed{A_3 = A_6}$$

and

$$E_{y2} = E_{y3}$$

$$E_{y2} = \frac{j\omega\mu}{k_2}\left[A_3 \sin k_2 x - A_4 \cos k_2 x\right] e^{-j\beta z}$$

$$E_{y3} = \frac{j\omega\mu}{k_3} A_6 e^{k_3 x} e^{-j\beta z}$$

$$\Rightarrow \quad \frac{-A_4}{k_2} = \frac{A_6}{k_3}$$

At $x = d$,

$$H_{z1} = H_{z2}$$

$$A_1 e^{-k_1 d} = A_3 \cos k_2 d + A_4 \sin k_2 d$$

&

$$E_{y1} = E_{y2}$$

(1) − (2),

$$k_1^2 + k_2^2 + \omega^2 \mu \varepsilon_0 - \omega^2 \mu \varepsilon_d = 0$$

$$\Rightarrow k_1 = \sqrt{\omega^2 \mu (\varepsilon_d - \varepsilon_0) - k_2^2}$$

k_1 is real,

$$\Rightarrow \omega^2 \mu (\varepsilon_d - \varepsilon_0) > k_2^2 \quad \text{———} \quad (4)$$

From (2) & (3),

$$k_3^2 + k_2^2 + \omega^2 \mu \varepsilon_s - \omega^2 \mu \varepsilon_d = 0$$

k_3 is real,

$$\Rightarrow \quad \omega^2\mu\,(\varepsilon_d - \varepsilon_s) > k_2^2 \quad \text{——} \quad (5)$$

From (4), $\quad \varepsilon_d > \varepsilon_0$

From (5), $\quad \varepsilon_d > \varepsilon_s$