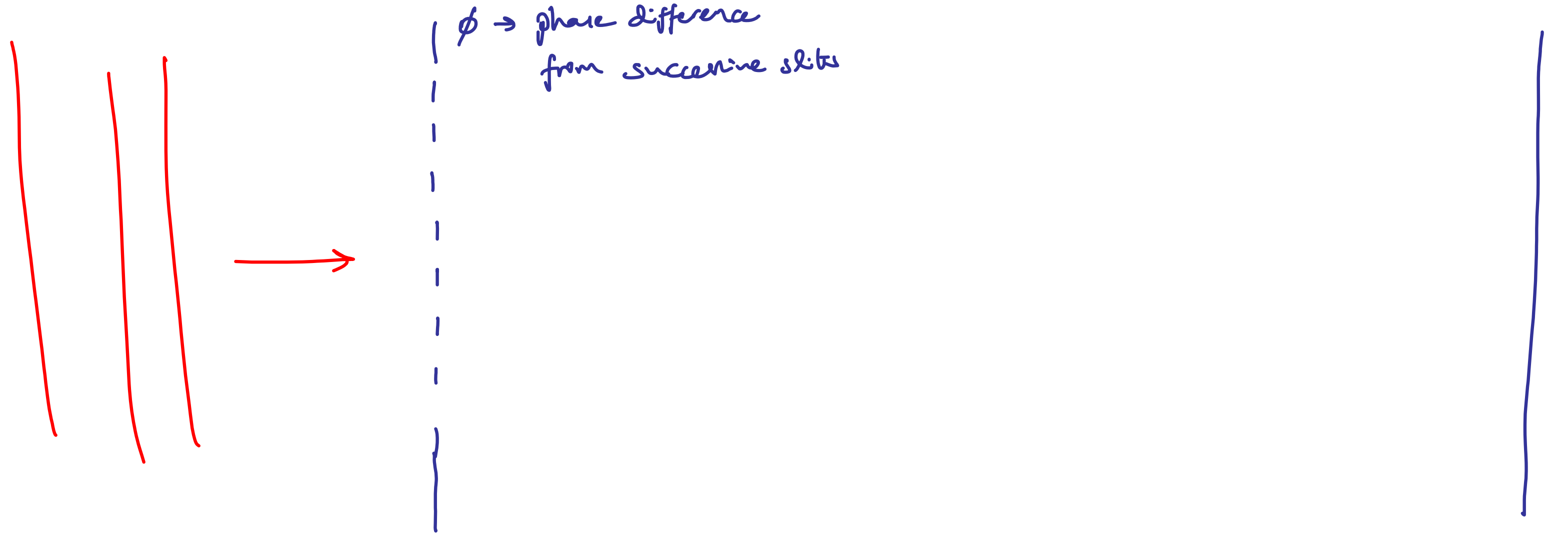
Note Title 8/6/2018

Learning Outcome: Interference of light and Coherence property of light

$M \rightarrow$ # of slits

$\phi \rightarrow$ phase difference from successive slits



Wave component

$$U_m = \sqrt{I_0}\,\exp[i(m-1)\phi]$$

where $m = 1, 2, \ldots M$

??

Observation plane

Total Wave amplitude

$$U = \sqrt{I_0}\left(1 + h + h^2 + \dots + h^{M-1}\right) \qquad \text{where } h = e^{j\phi}$$

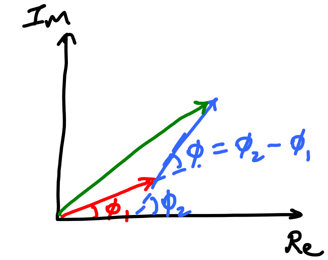
$$= \sqrt{I_0} \cdot \frac{1 - h^M}{1 - h} = \sqrt{I_0}\, \frac{1 - e^{jM\phi}}{1 - e^{j\phi}}$$

$M \to$ # of slits
$\phi \to$ phase difference from consecutive slits

??

Total Intensity

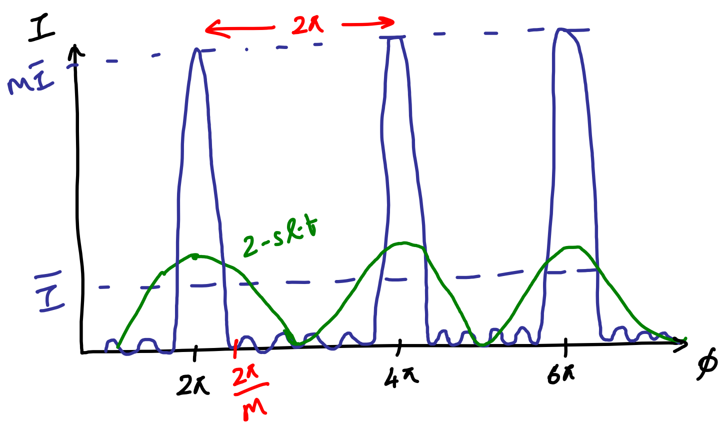
$$I = |U|^2 = I_0 \cdot \left| \frac{1 - e^{jM\phi}}{1 - e^{j\phi}} \right|^2 = I_0 \left| \frac{e^{-jM\phi/2} - e^{jM\phi/2}}{e^{-j\phi/2} - e^{j\phi/2}} \right|^2$$



$$\boxed{I = I_0 \frac{\sin^2(M\phi/2)}{\sin^2(\phi/2)}}$$

$$\frac{M\phi_{min}}{2} = \pi$$

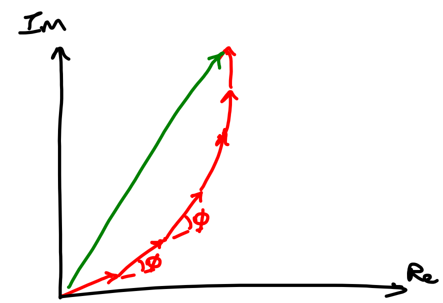
$$\phi_{min} = \frac{2\pi}{M}$$



$$\phi = \frac{2\pi}{\lambda} d \sin\theta_m$$

More # of interfering sources

$\Rightarrow$ Narrower spectral selectivity



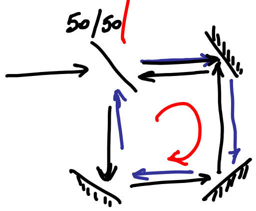
Interferometers

Common Path

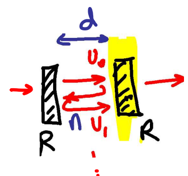
Sagnac

Gyroscope

$\phi_{cw} - \phi_{ccw}$



Fabry-Perot



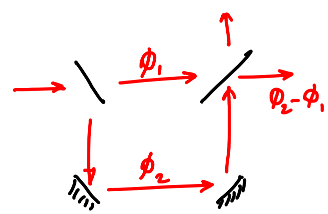
$$\Delta\phi = \frac{2\pi}{\lambda} n \cdot 2d$$

$$= 2\pi m$$

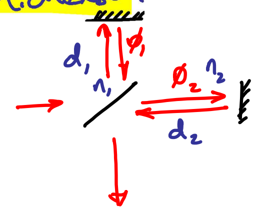
$n=1$, $\boxed{d = m \cdot \frac{\lambda}{2}}$

Differential Path

Mach-Zehnder



Michelson



$$\phi_2 - \phi_1 = \frac{2\pi}{\lambda} 2 n_2 d_2 - \frac{2\pi}{\lambda} 2 n_1 d_1$$

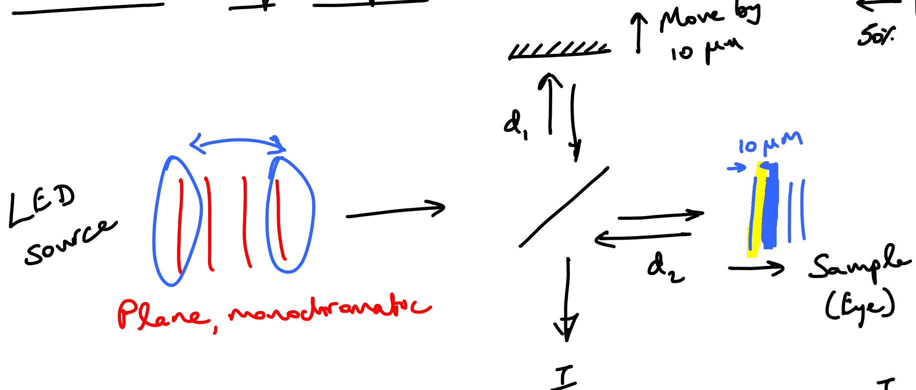
$$= \frac{2\pi}{\lambda} 2 (n_2 d_2 - n_1 d_1)$$

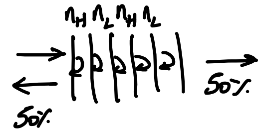
Optical path length difference

Constructive Interference, $\phi_2 - \phi_1 = 2\pi m$

If $n_1 = n_2 = 1$, $\boxed{d_1 - d_2 = \frac{m\lambda}{2}}$

Coherence of light:





$$\Delta\phi = \frac{\cancel{2\pi}}{\lambda}\, 2(d_2 - d_1) = \cancel{2\pi}\, m$$

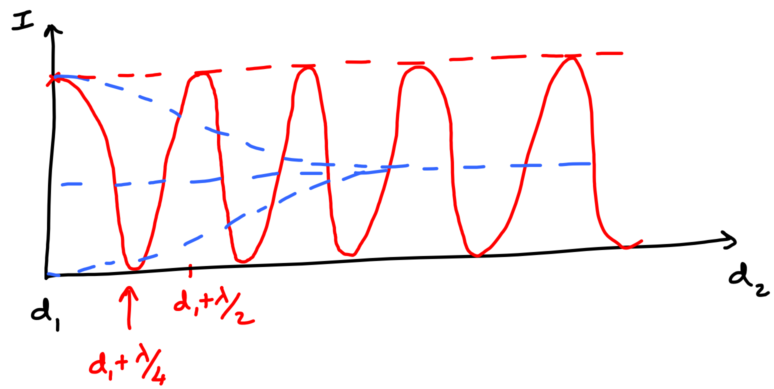
$$d_2 - d_1 = \frac{m\lambda}{2}$$

$$\langle U_1^*(t)\; U_2(t)\rangle$$

Auto correlation $G(\tau)$ $\langle U_1^*(t)\; U_1(t+\tau)\rangle$

$$U_0\, e^{-j\omega_0 t}\; U_0\, e^{j\omega_0(t+\tau)}$$

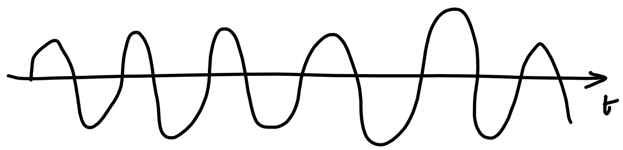
$$= U_0^2\, e^{j\omega\tau}$$



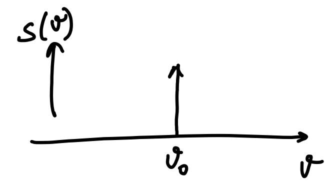
Degree of coherence $g(\tau) = \dfrac{G(\tau)}{\langle U^*(t)\, U(t)\rangle}$

$$0 \le |g(\tau)| \le 1$$

Monochromatic



$\Rightarrow$

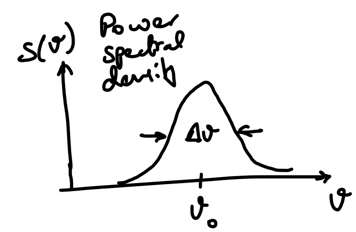


Weiner - Khinchin Theorem

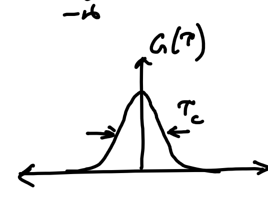
$$S(\nu) = \int_{-\infty}^{\infty} G(\tau)\exp(-j2\pi\nu\tau)\,d\tau$$

Non-ideal (practical)

$\Rightarrow$



$\Rightarrow$



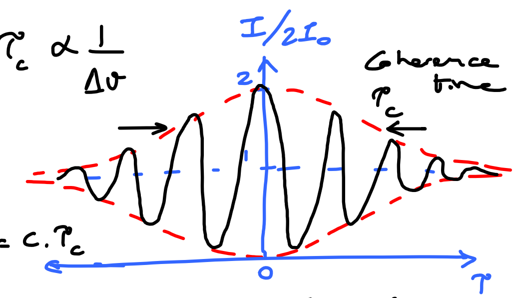
	$\Delta\nu$ (Hz)	τ_c	l_c
Sunlight (0.4 - 0.7 μm)	3.75×10^{14}	2.67 fs	800 nm
Semiconductor LED (λ_0 = 1 μm, $\Delta\lambda$ = 50 nm)	1.5×10^{13}	67 fs	20 μm
Laser Diode (λ_0 = 1 μm, $\Delta\lambda$ = 1 nm)	3×10^{11}	3.3 ps	1 mm
He-Ne Laser (λ_0 = 0.633 μm)	1×10^{6}	1 μs	300 m

OCT

LIGO
6 km long

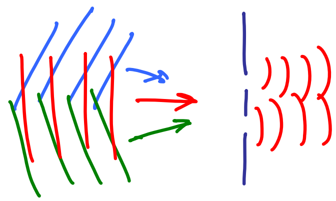
Coherence time $\tau_c \propto \frac{1}{\Delta\nu}$

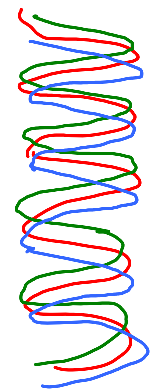
Longitudinal coherence length $l_c = c.\tau_c$

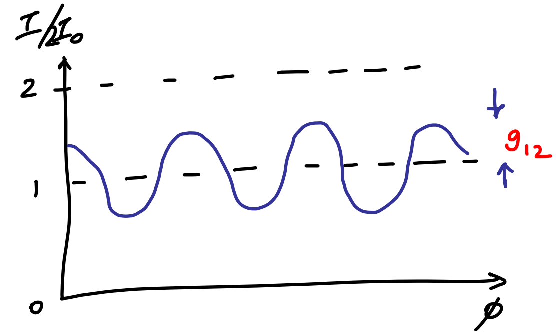


$$\text{Visibility} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Spatial Coherence





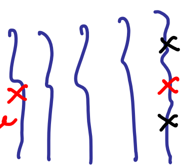


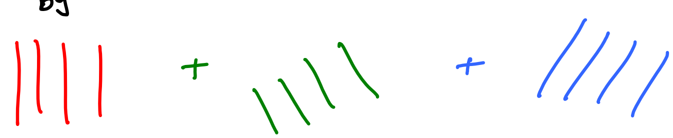
Spatial coherence function $= \langle U^*(r_1)\, U(r_2) \rangle$

$\boxed{\rho_c = \frac{\lambda}{\theta_s}}$ Angles subtended by source

Temporal coherence

$\tau_c \propto \frac{1}{\Delta \nu}$



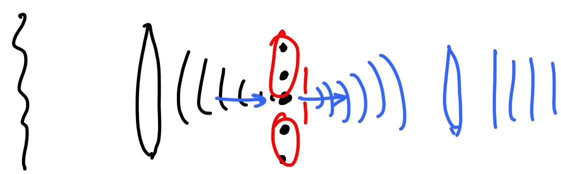


$$I = \langle |U_1 + U_2|^2 \rangle = \langle |U_1|^2 \rangle + \langle |U_2|^2 \rangle + \langle U_1^* U_2 \rangle + \langle U_1 U_2^* \rangle$$

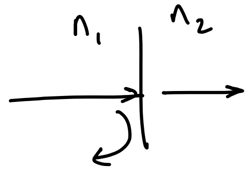
$$= I_1 + I_2 + G_{12} + G_{12}^*$$

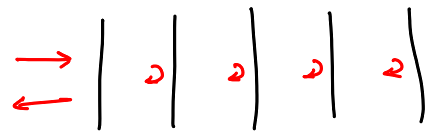
$$= I_1 + I_2 + 2\,\mathrm{Re}\{G_{12}\} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,\mathrm{Re}\{g_{12}\}$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\,|g_{12}| \cos \Delta\phi$$



Example 1: Interference in thin films (multiple layers)





$\eta = \sqrt{\frac{\mu}{\epsilon}}$

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

$\sqrt{\epsilon_r} = n$

If $n_2 > n_1$ r is negative

π phase shift