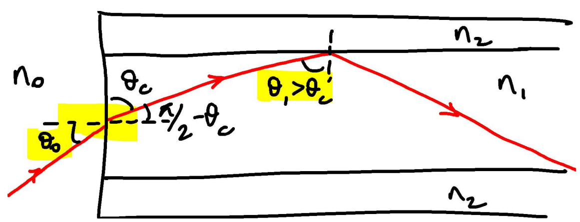
Numerical Aperture



What is the largest value of θ_0 ?

$\rightarrow$ $\theta_1 = \theta_c$

$$n_0 \sin\theta_0 = n_1 \sin\left(\frac{\pi}{2} - \theta_c\right) = n_1 \cos\theta_c$$

$$= n_1\sqrt{1 - \sin^2\theta_c} = n_1\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

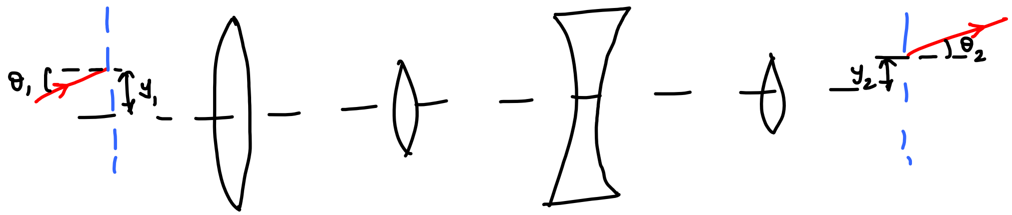
If $n_0 = 1$, $\boxed{\sin\theta_0 = \sqrt{n_1^2 - n_2^2}}$

Ray optics

Dispersion in prism

Optical system design

Hubble Telescope



$$y_2 = A y_1 + B \theta_1$$

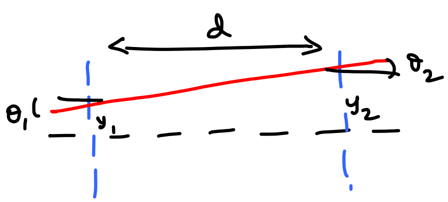
$$\theta_2 = C y_1 + D \theta_1$$

$$\Rightarrow \begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

$\downarrow$

Ray Matrix

$$M = M_N M_{N-1} \cdots M_1$$



$$\theta_2 = \theta_1$$

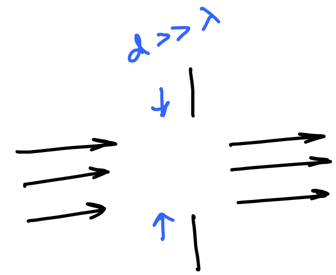
$$y_2 = y_1 + d\theta_1 \Rightarrow \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

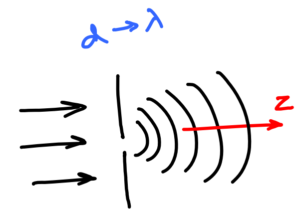
Paraxial approximation

$$\sin\theta \simeq \theta \Rightarrow n_1 \theta_1 = n_2 \theta_2$$

Learning Outcome: Identify the limits of ray optics and the fundamental concepts of wave optics

Diffraction of light





Pebble dropped in water

Sound from a loudspeaker

Wave Equation $$\nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0$$

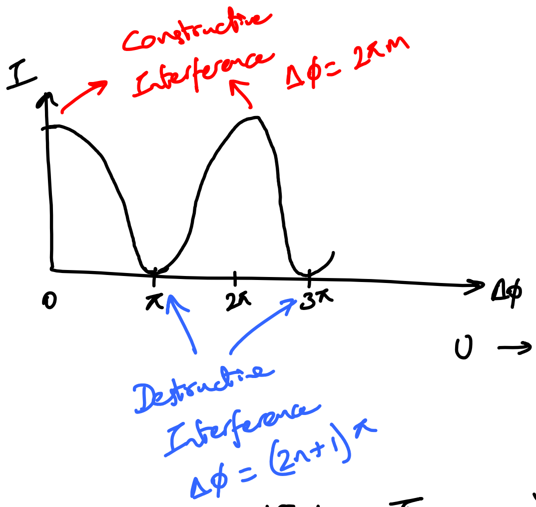
$$\nabla^2 \to \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

In EM
$U \to E/H$

Time-periodic signals, $$U(x, y, z, t) = U(x, y, z)\, e^{j\omega t}$$

$$k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}$$

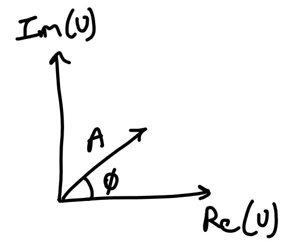
$$\frac{\partial}{\partial t} \to j\omega \qquad \frac{\partial^2}{\partial t^2} \to -\omega^2 \Rightarrow \boxed{\nabla^2 U + k^2 U = 0}$$

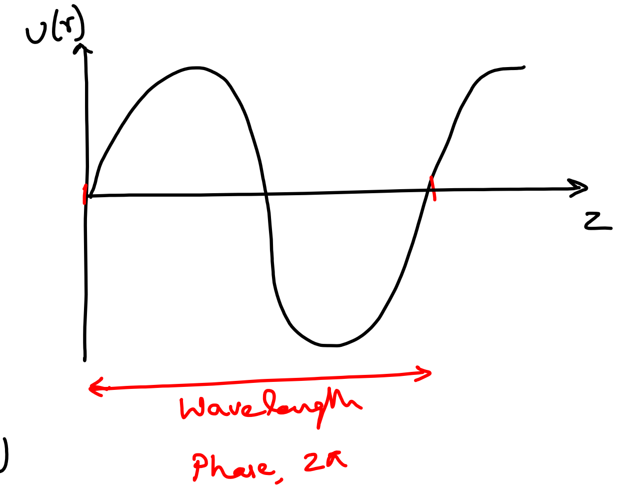


$$U(r) = A(r)\, e^{-jkz}\, e^{j\omega t}$$

Phase

$U \to$ Phasor $\to A e^{j\phi}$



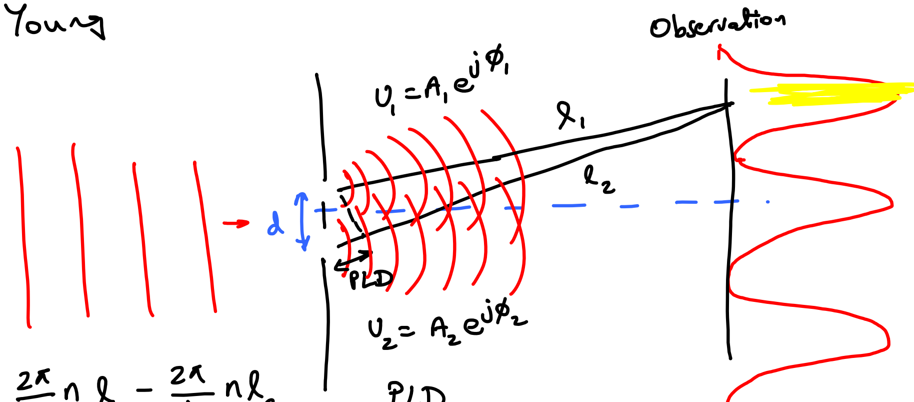


1801, Thomas Young

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_1 - \phi_2)$$

If $I_1 = I_2 = I_0$

$$\boxed{I = 2I_0\left(1 + \cos\Delta\phi\right)}$$



$$\phi_1 - \phi_2 = \frac{2\pi}{\lambda} n l_1 - \frac{2\pi}{\lambda} n l_2 = \frac{2\pi}{\lambda} n (l_1 - l_2)$$

PLD

$$I = |U|^2 = |U_1 + U_2|^2$$

$$= |U_1|^2 + |U_2|^2 + U_1 U_2^* + U_1^* U_2$$

$$= I_1 + I_2 + \sqrt{I_1 I_2}\, e^{j(\phi_1 - \phi_2)} + \sqrt{I_1 I_2}\, e^{-j(\phi_1 - \phi_2)}$$