# EE 5500 – Introduction to Photonics
# Quiz II

60 minutes, 20 points, closed book

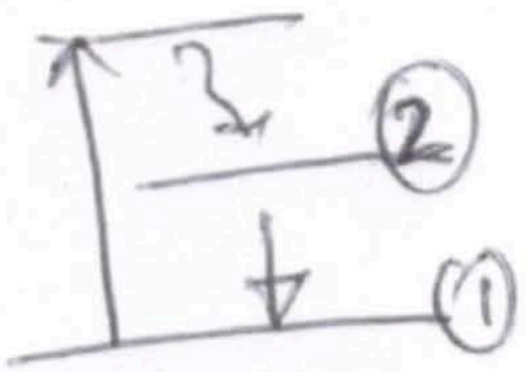
**Remember…**

- Make reasonable assumptions wherever necessary, but you should show all steps and justify assumptions for full credit. Use of figures may attract bonus points.
- Final answers will be graded ONLY if they are entered in the space provided.

## Objective Type Questions (5 points)

1. Which of these is/are (maybe more than one) necessary condition(s) for laser operation?

   (a) Gain > loss  (b) $N_2 > N_1$  (c) long $T_2$  (d) stimulated emission




Gain need not be greater than loss

$N_2$ need not be greater than $N_1$

Long $T_2$ is desirable, not necessary

Stimulated emission is necessary for laser action

2. An Er-doped fiber laser pumped at 980 nm and operating at 1550 nm can be characterized as

   (a) 2-level  (b) 3-level  (c) 4-level  (d) Cannot say

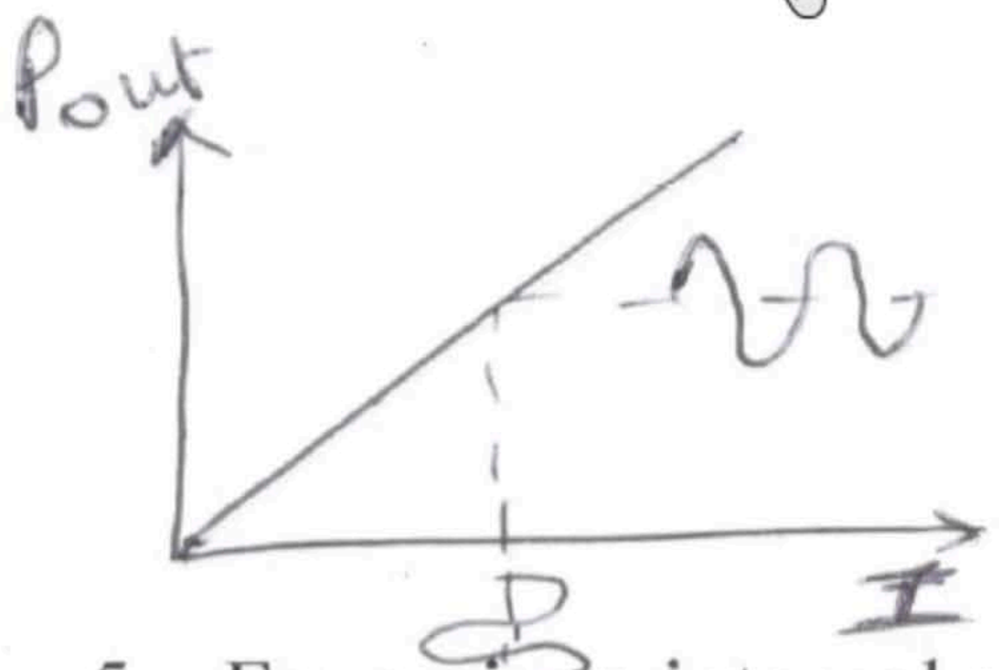
3. The average lifetime spent by a photon in a Fabry Perot laser cavity with loss=10 $cm^{-1}$ & n=3

(a) 1 ps (b) 10 ps (c) 100 ps (d) 1 ns

$$\tau_{ph} = \frac{1}{v_g \alpha_r} = \frac{1}{\frac{3\times10^{8}}{3} \times 10 \times 100} = 10^{-11}\ s \text{ or } \underline{10\ ps}$$

4. The modulation bandwidth of a LED does NOT depend on

(a) drive current (b) heterostructure (c) recombination lifetime (d) any of these




$$\boxed{f_{3dB} \propto \frac{1}{\tau_c}}$$ ⇒ depends on recombination lifetime

NOT dependent on drive current

heterostructure helps to reduce recombination lifetime

5. For a given internal efficiency, the responsivity of a photodiode scales as

(a) $1/\lambda$ (b) $\sqrt{\lambda}$ (c) $\lambda^2$ (d) $\lambda$

$$R = \frac{I_p}{P_{in}} = \frac{\eta q}{h\nu} = \frac{\eta \lambda(\mu m)}{1.24}$$

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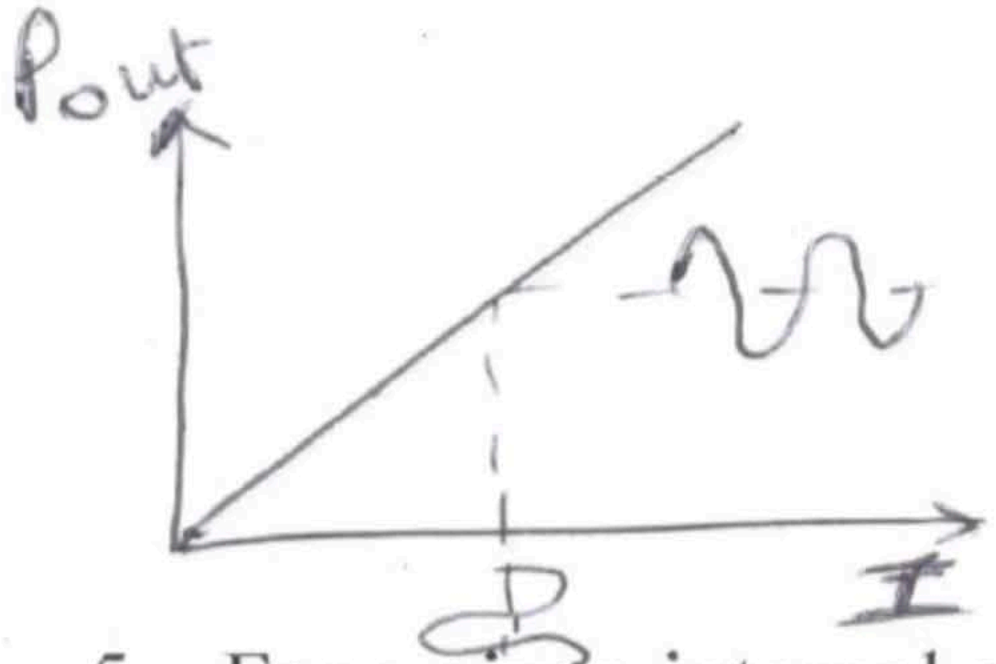


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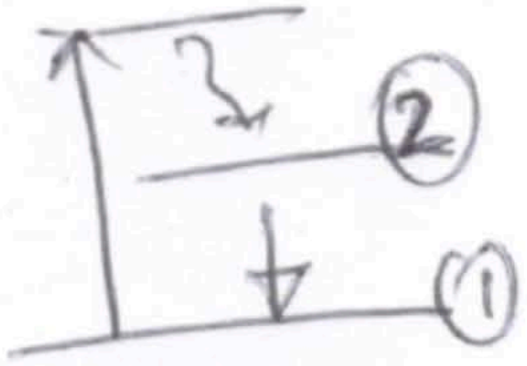
$$\boxed{R \propto \lambda}$$

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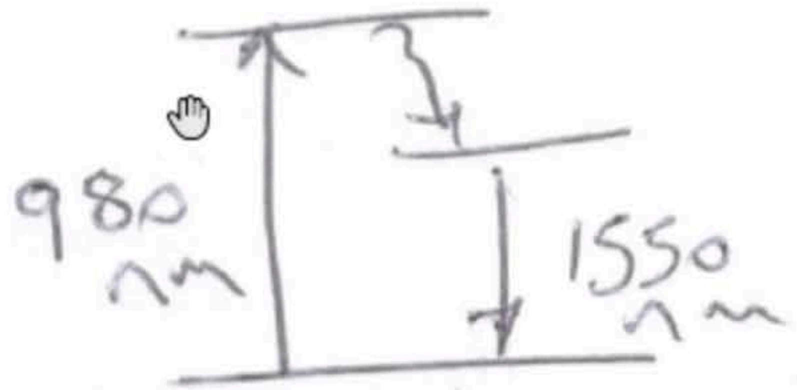


$\Rightarrow$ 3-level laser

3. The average lifetime spent by a photon in a Fabry Perot laser cavity with loss=10 cm$^{-1}$ & n=3

   (a) 1 ps (b) 10 ps (c) 100 ps (d) 1 ns

$$\tau_{ph} = \frac{1}{v_g \alpha_r} = \frac{1}{\frac{3\times10^8}{3}\times 10\times 100} = 10^{11}\ \text{s} \quad \text{or} \quad \underline{10\ \text{ps}}$$

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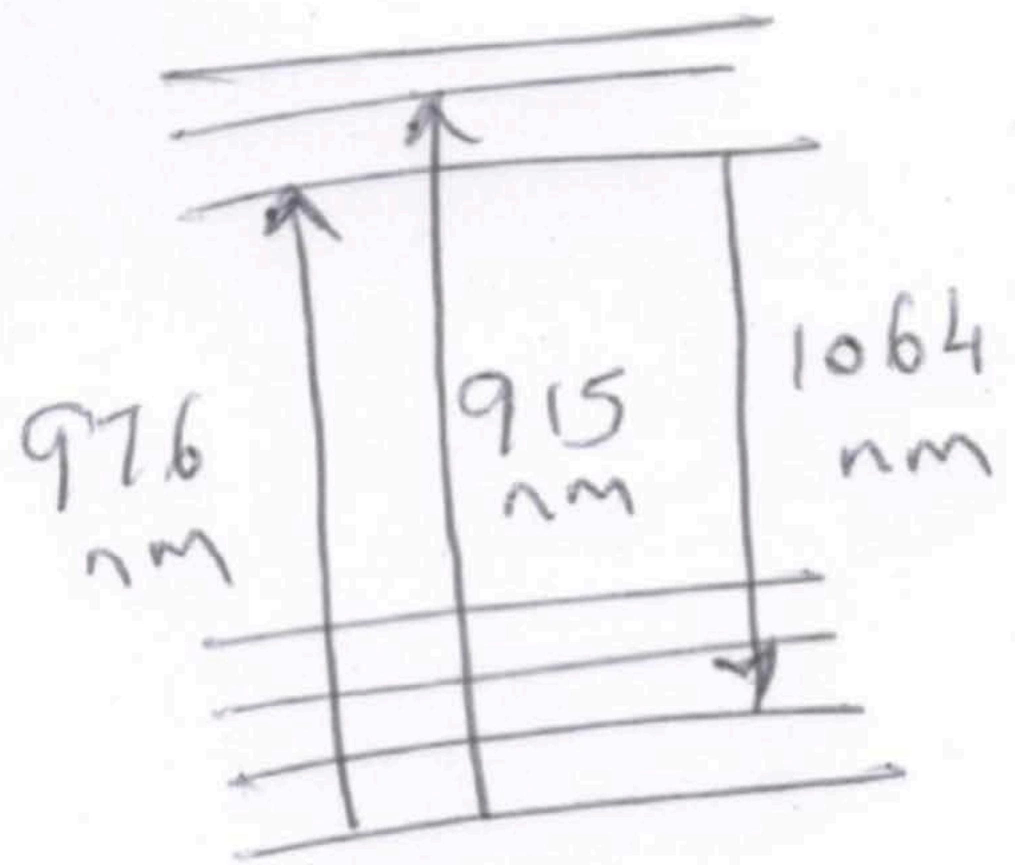
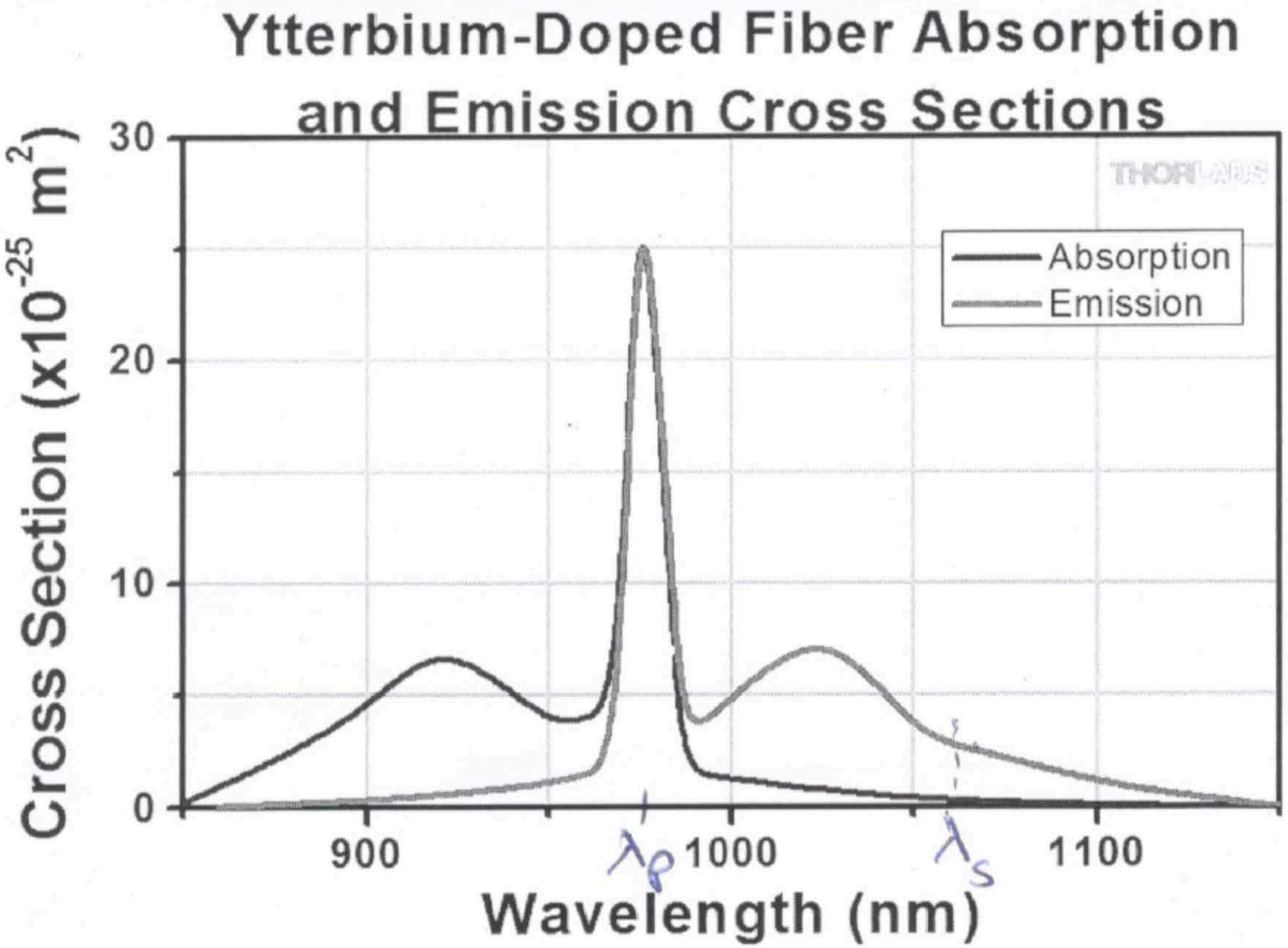
6. Suppose you are asked to design an optical amplifier at 1064 nm wavelength using a Yb-doped silica optical fiber, whose doping concentration is $3 \times 10^{24}$ $m^{-3}$ and its absorption and emission cross-sections are provided below.

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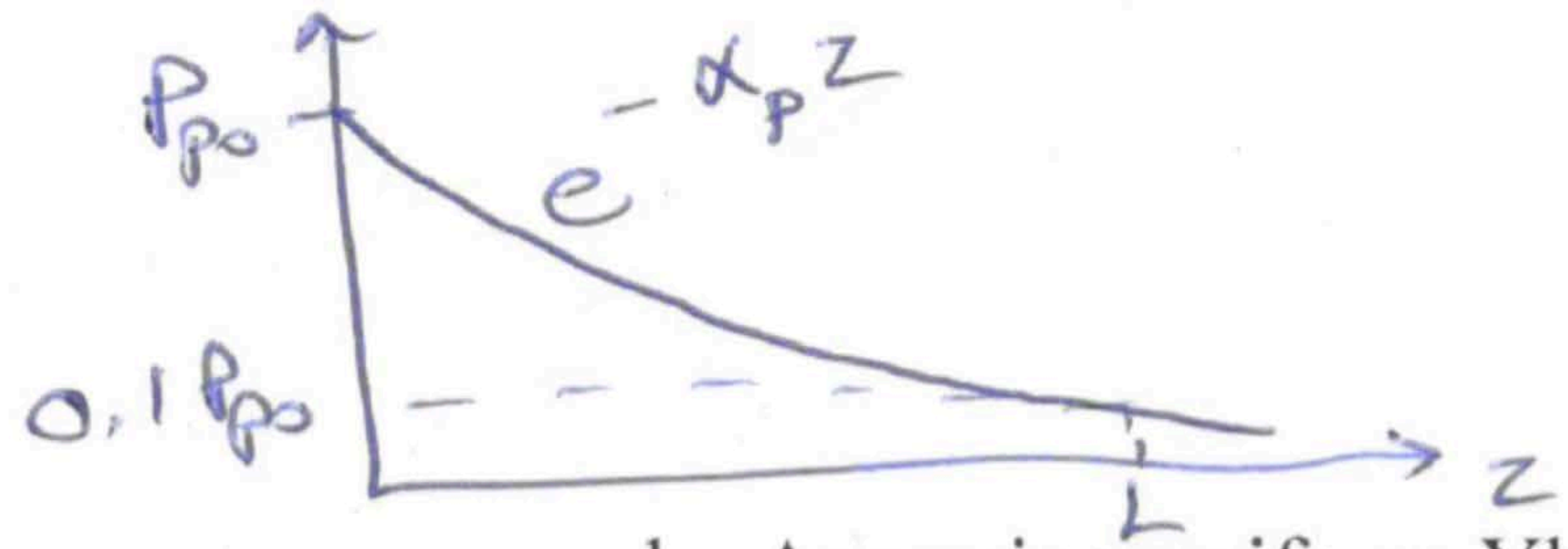
915 nm & 976 nm. The latter is preferred since the absorption cross-section is 3x higher.

b. Starting from appropriate rate equations, derive an expression for the small signal gain of the amplifier. (2 pts)

$$\frac{dP_s}{dz} = (N_2 \sigma_e^s - N_1 \sigma_a^s) P_s \Rightarrow G = \exp\left[\int_0^L (N_2 \sigma_e^s - N_1 \sigma_a^s)\, dz\right]$$

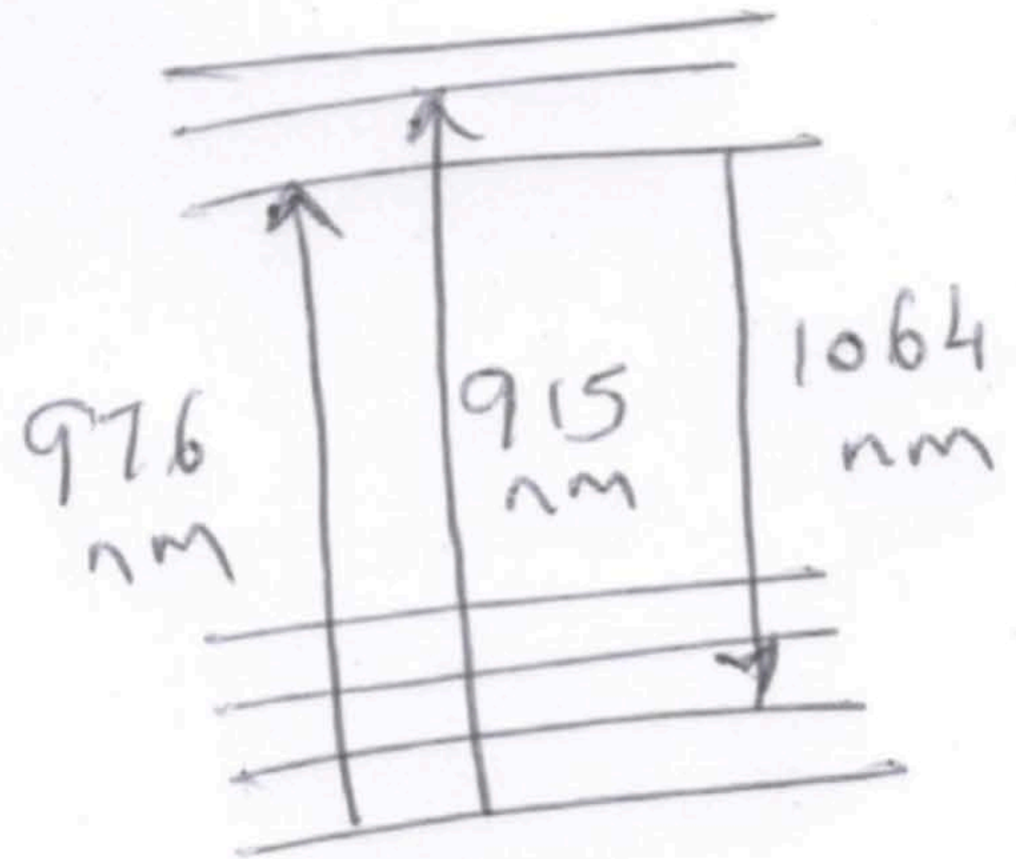
c. Estimate the length of Yb fiber required to absorb 90% of the launched pump radiation. (2 pts)

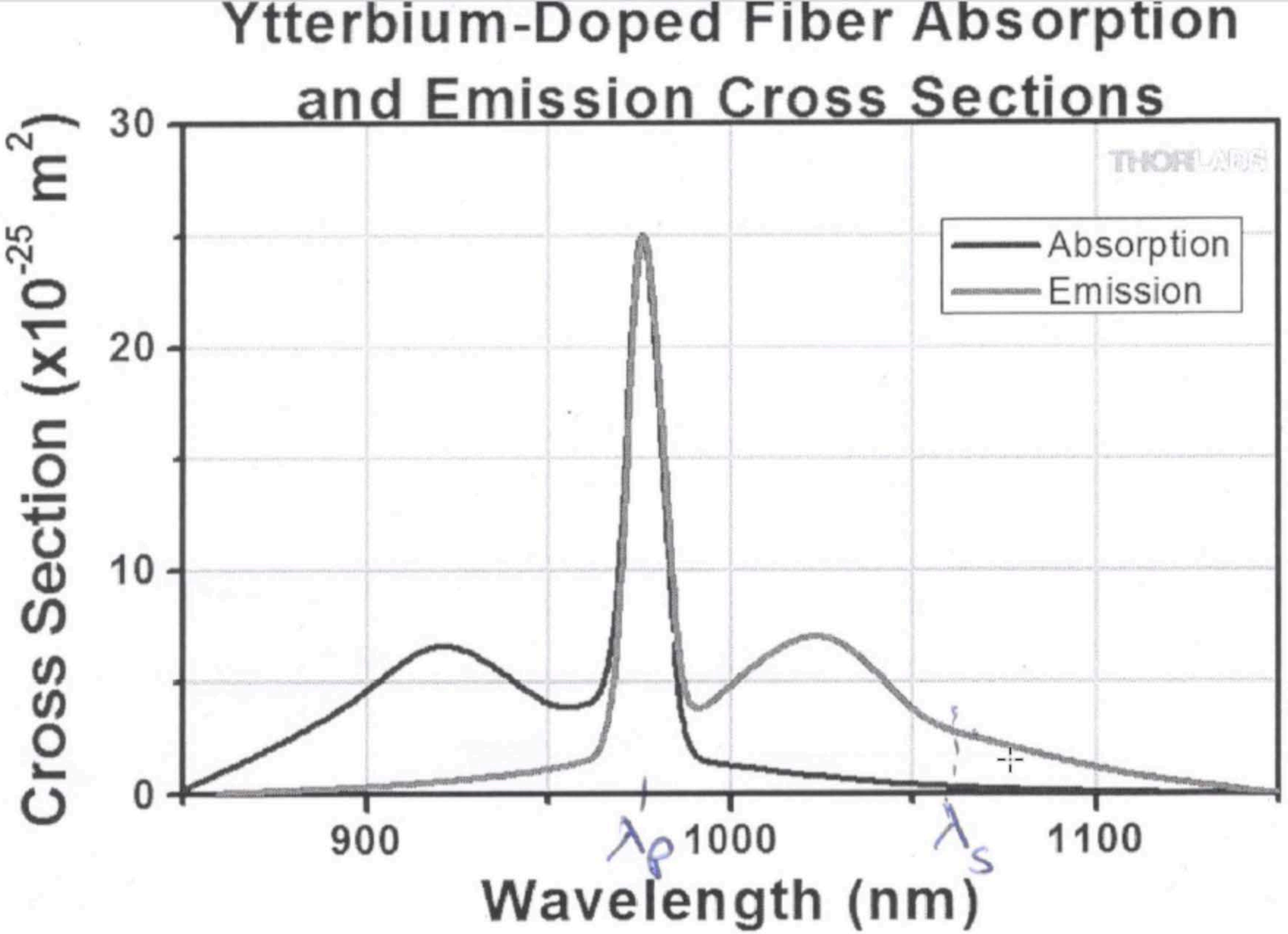



$$\alpha_p = N_a \sigma_a^p = 3 \times 10^{24} \times 2.5 \times 10^{-24} = 7.5\ m^{-1}$$

$$e^{-\alpha_p L} = 0.9 \Rightarrow L = \frac{1}{7.5} \ln(0.9) = \underline{14\ mm}$$

d. Assuming uniform Yb ion excitation of 50% along the above length of fiber, what is the small-signal gain that could be achieved? (2 pts)

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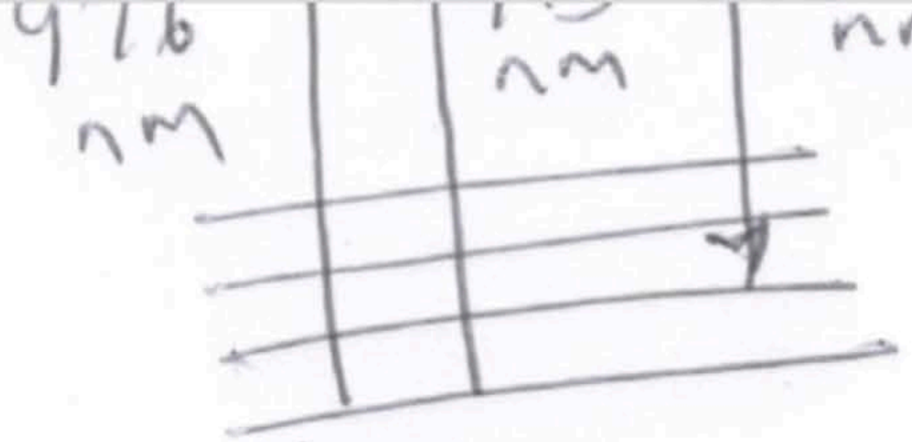


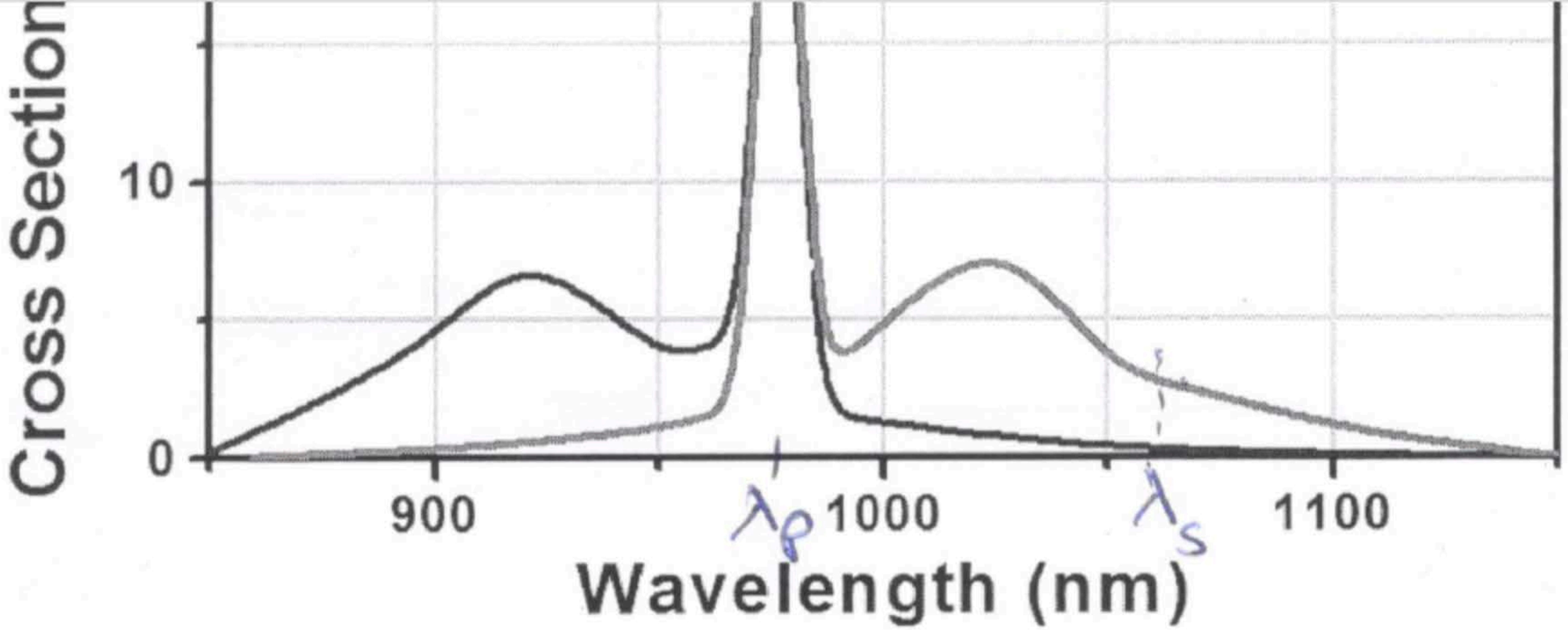


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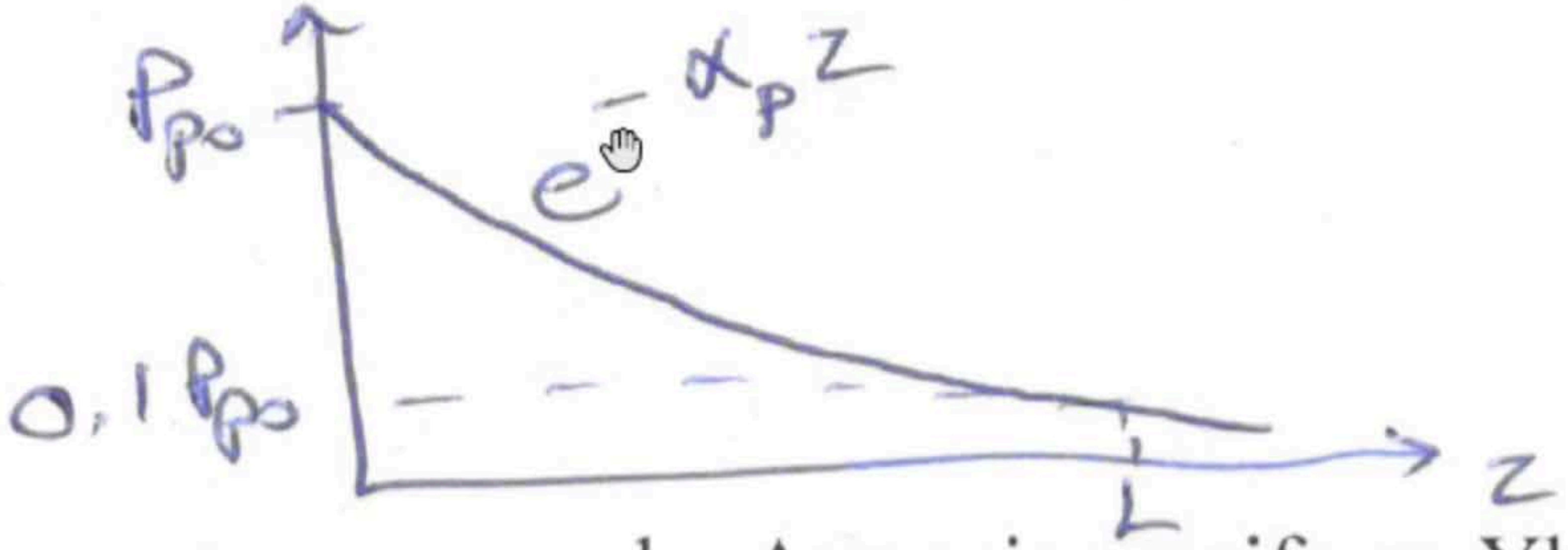


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Assuming 50% excitation, $N_1 = N_2 = 0.5N_a$

at 1064 nm ($\lambda_s$) $\sigma^s = 3\times10^{-25}$ $m^2$

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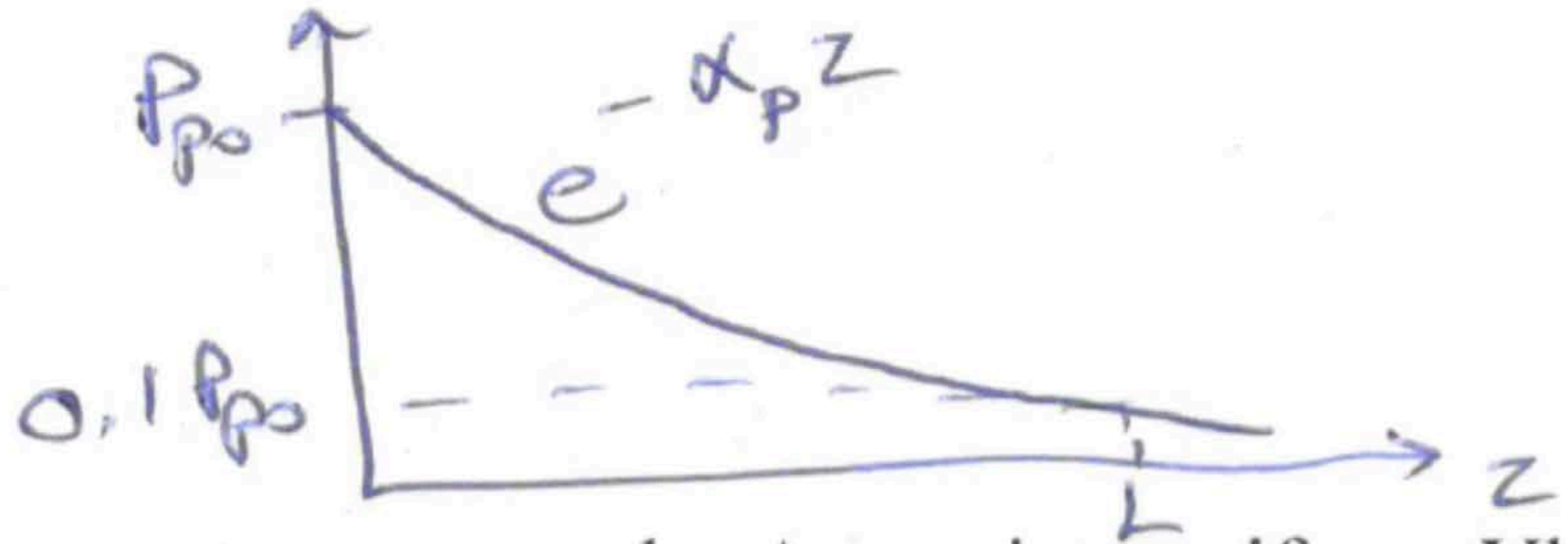


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At 1064 nm ($\lambda_s$), $\sigma_e^s = 3 \times 10^{-25}\ m^2$

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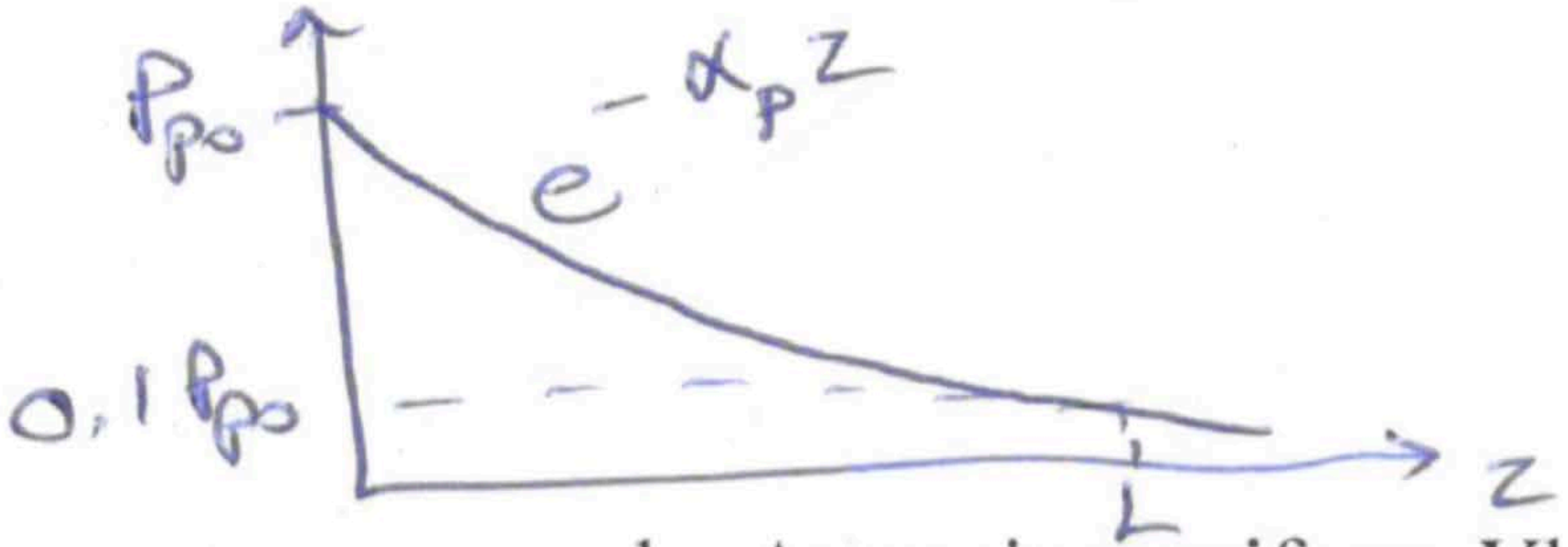


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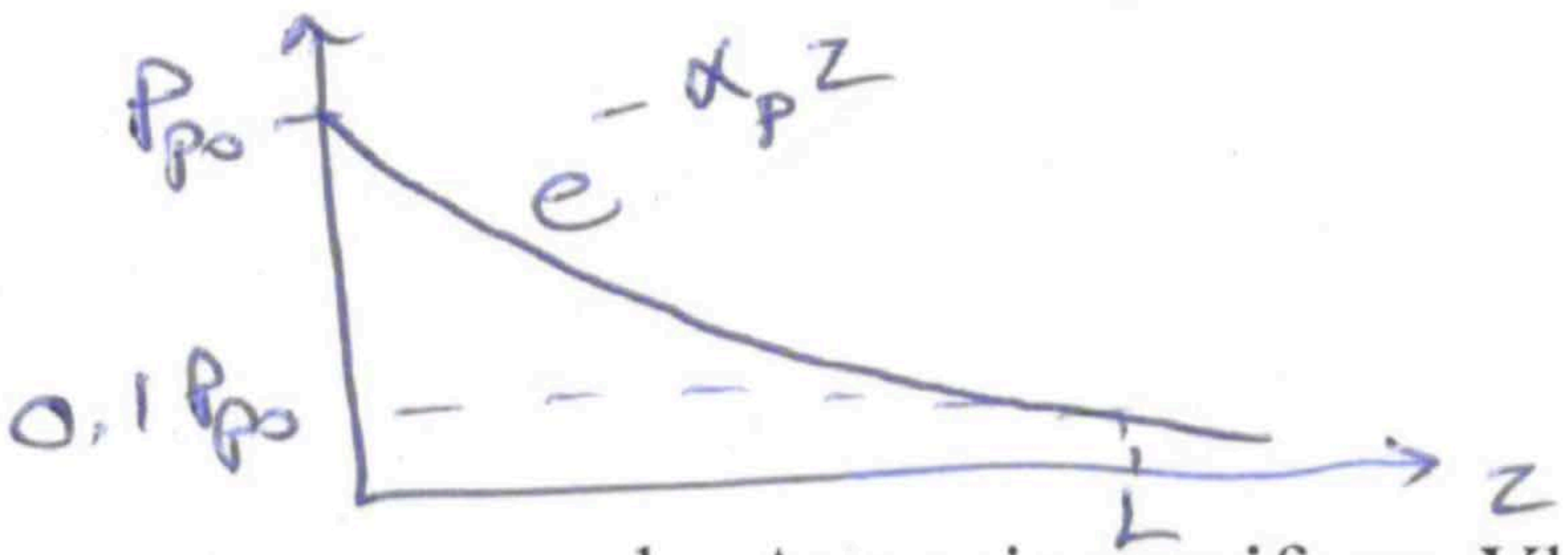


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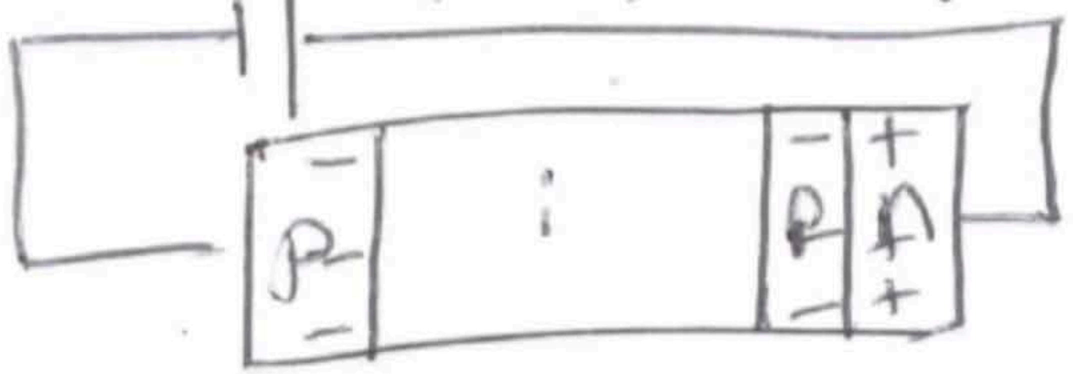
$$= 1.006$$




Add p-layer between n & i layers

$\Rightarrow$ most of $V_b$ drops across p-n junction

d. If the width of the multiplication region is 2 μm and the voltage drop across it is 50 V, what is the multiplicative gain (M) that could be achieved in the APD? (2 pts)

E-field in multiplication region $= \dfrac{50\text{ V}}{2\ \mu\text{m}} = 2.5 \times 10^5\ \text{V/cm}$

$\Rightarrow \alpha_e = 2 \times 10^4\ \text{cm}^{-1}$

**Useful Constants:**

Planck's constant (h) = 6.6256 x $10^{-34}$ J.s
Boltzmann's constant ($k_B$) =1.38 x $10^{-23}$ J/K
Electric charge (q) = 1.602 x $10^{-19}$ C
Velocity of light (c) = 3 x $10^8$ m/s

**Useful Formulae**

1. $M = (1–k) / \{\exp [-(1-k)\alpha_e w_m] – k\}$
2. $F = k.M + (1 - k).(2 - 1/M)$

$\alpha_e W_M = 4$



$M = \dfrac{0.5}{e^{-2} - 0.5} = -1.37$

a. Assuming negligible contribution from RC time constant and a saturation drift velocity of $10^5$ m/s for 5 V reverse bias, what is the maximum width of the intrinsic region allowed to support the above bandwidth? (2 pts)

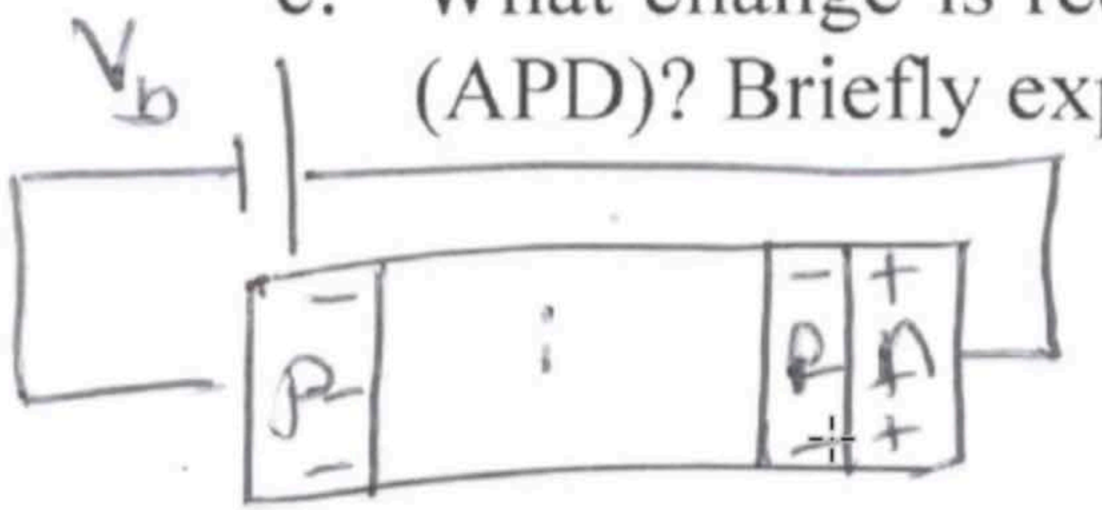
$$f_{3dB} = \frac{1}{2\pi(\tau_{tr} + \cancel{\tau_{RC}})} \quad \text{where } \tau_{tr} = \frac{W}{V_{dr}} = \frac{1}{2\pi . f_{3dB}} \Rightarrow W = \frac{V_{dr}}{2\pi \times f_{3dB}} = 6.4\ \mu m$$

b. For the above structure, what is the expected responsivity? (2 pts)

$$\mathcal{R} = \frac{\eta \lambda}{1.24} \quad \text{where } \eta = (1-R_f)\zeta(1-e^{-\alpha W}) \simeq 0.8$$

$$\Rightarrow \mathcal{R} = 1\ A/W$$

c. What change is required in the above structure to realize an avalanche photodiode (APD)? Briefly explain with the help of appropriate schematic diagram. (1 pt)

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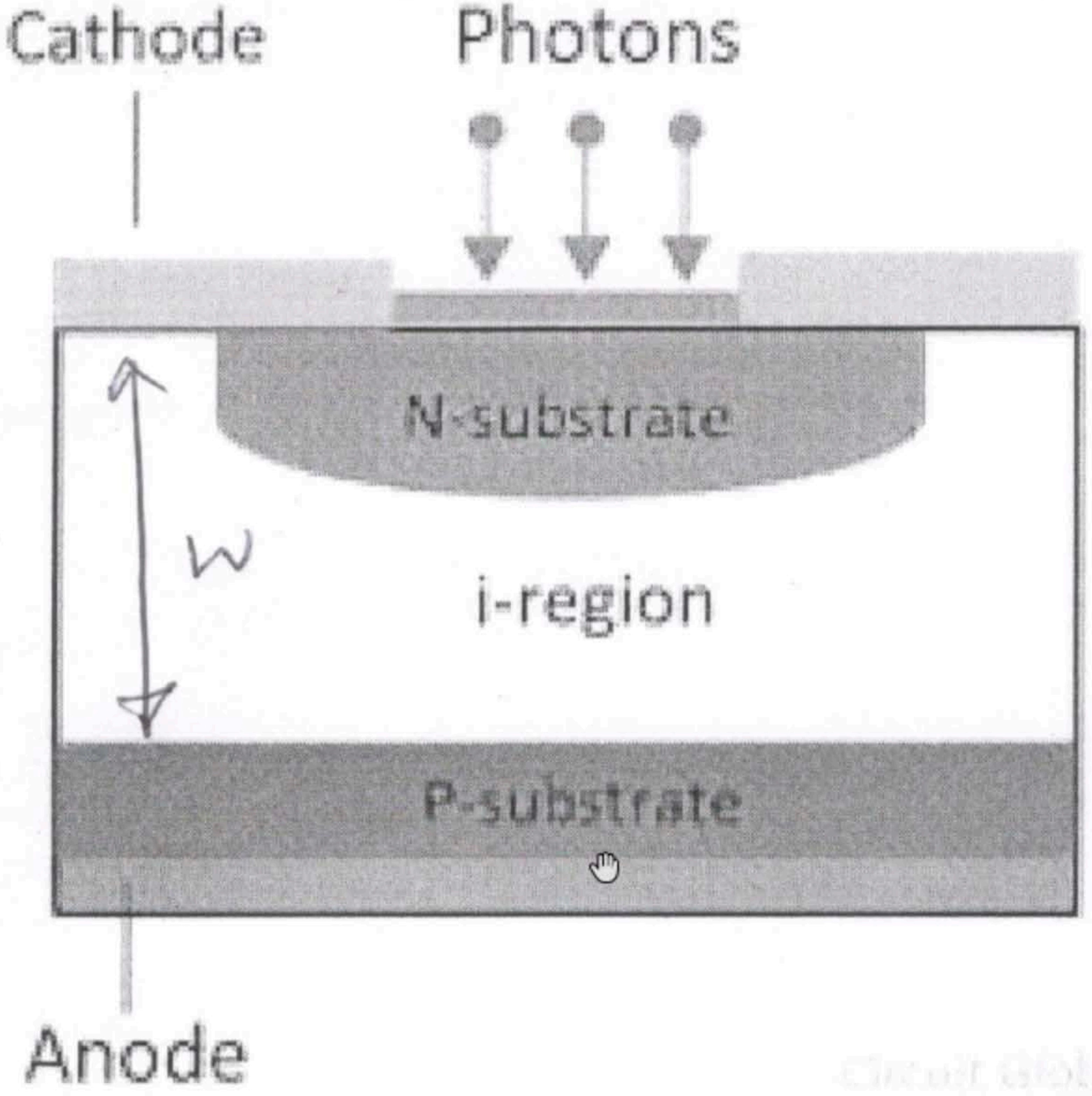
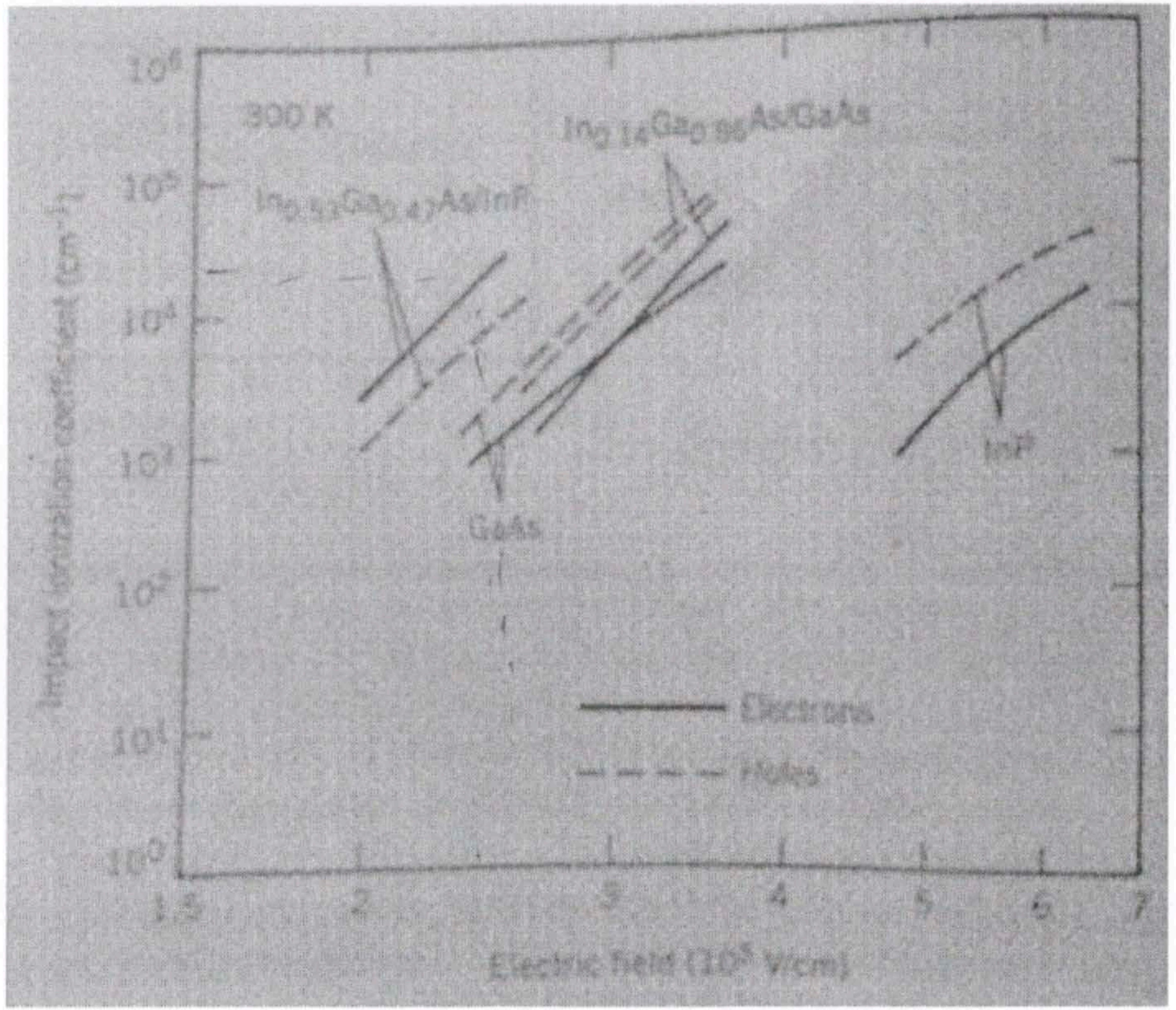
E-field in multiplication region = $\frac{50\ V}{2\ \mu m}$ = $2.5 \times 10^5$ V/cm

7. You have been asked to design a PIN photodiode with a bandwidth of 2.5 GHz for an optical fiber communication system using $In_{0.53}Ga_{0.47}As$ material ($k = 0.5$). The operation conditions include: photo-electron conversion efficiency of 0.8, anti-reflection coated facet, and an absorption coefficient of $0.5 \times 10^5$ $cm^{-1}$ at 1550 nm.

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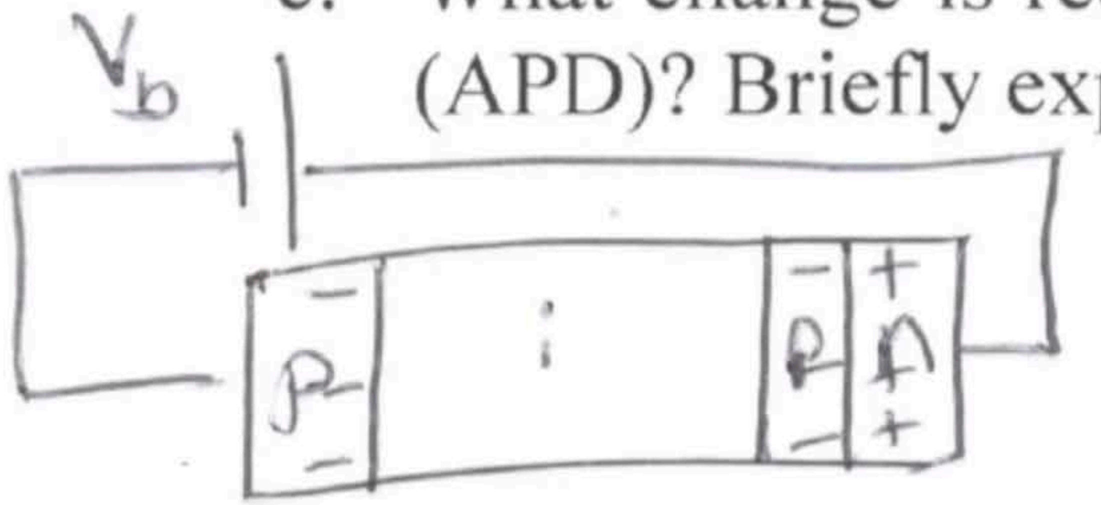


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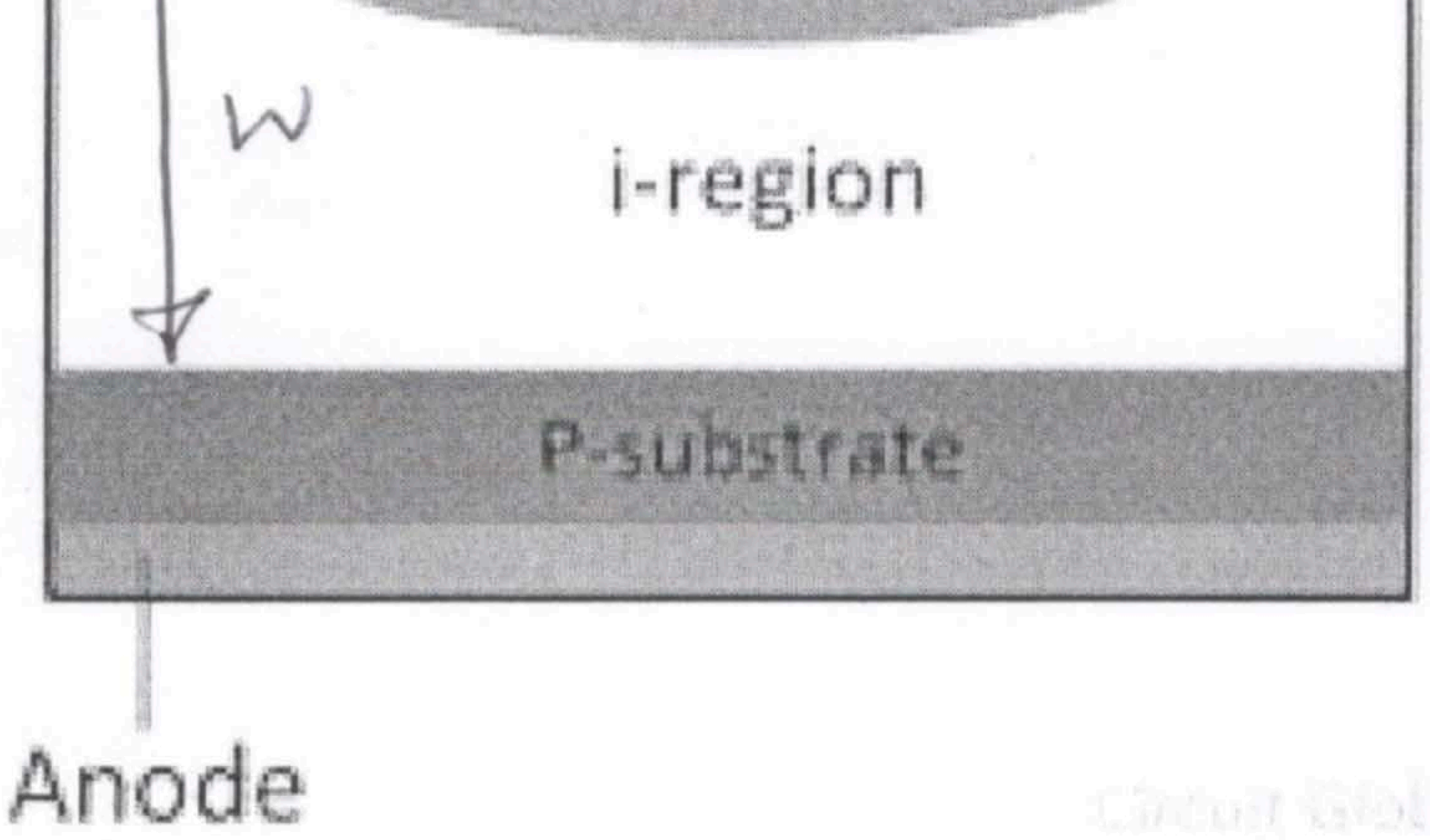


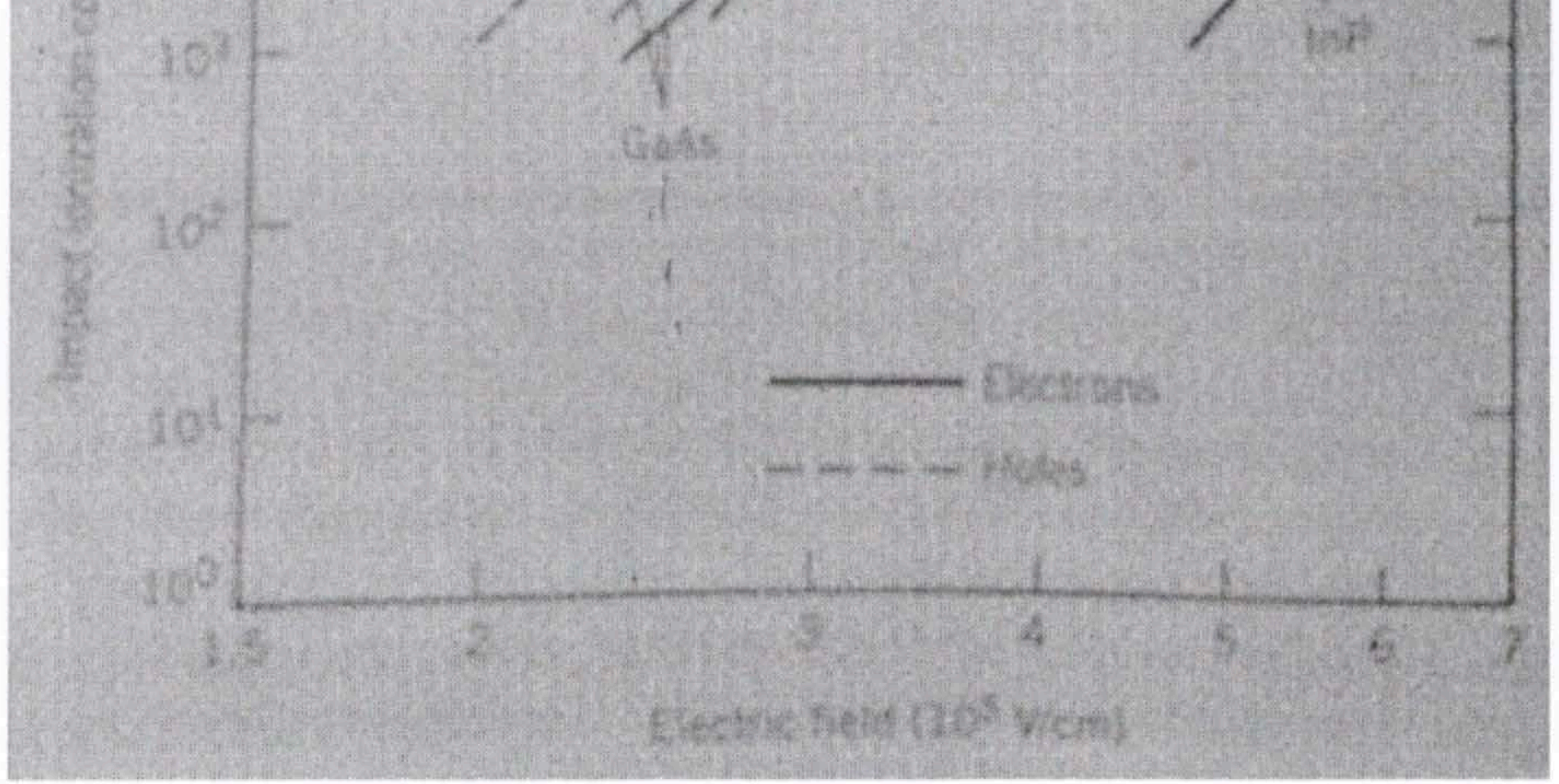


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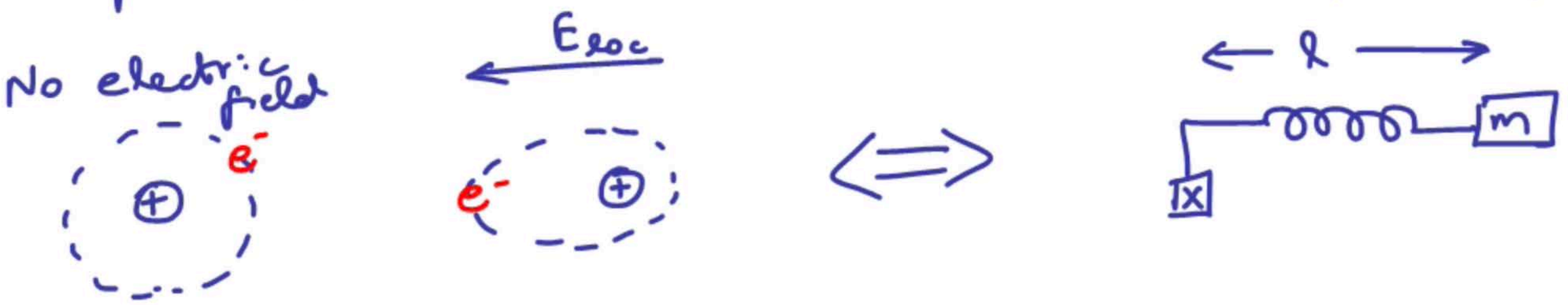
**Learning Outcome:** Identify the fundamental principles for photon/light manipulation

How does light propagate in a medium?




Displacement is proportional to permittivity ($\epsilon$)

Equation of motion,
$$m \cdot \frac{d^2 l}{dt^2} + d \cdot \frac{dl}{dt} + s \cdot l = -e E_{loc}$$

For time-periodic excitation $\left(e^{j\omega t}\right)$

photon / light manipulation

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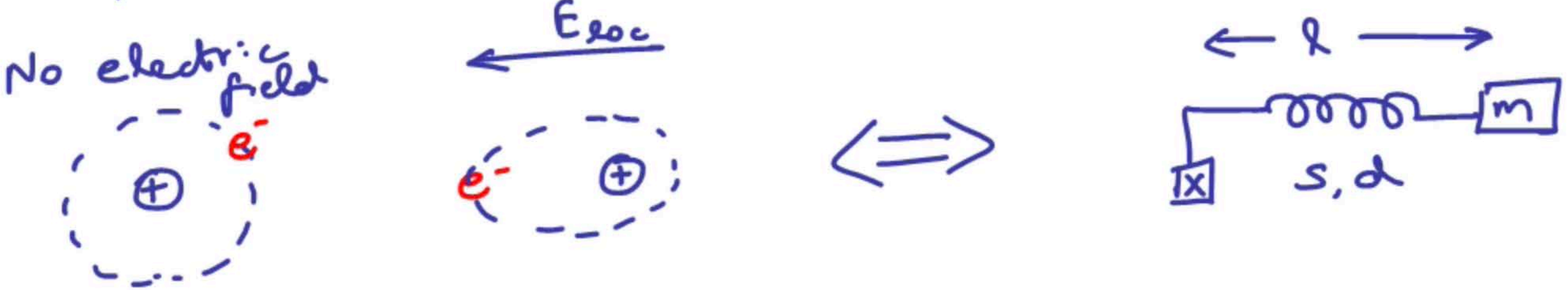


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Equation of motion, $m \cdot \frac{d^2 \ell}{dt^2} + d \cdot \frac{d\ell}{dt} + s \cdot \ell = -e E_{loc}$

$\epsilon = \epsilon' - j\epsilon''$

For time-periodic excitation $\left(e^{j\omega t}\right)$, $\ell = \frac{-e/m \, E_{loc}}{\omega_0^2 - \omega^2 + j\omega\Gamma}$

resonance freq. $\omega_0 = \sqrt{s/m}$

damping coeff. $d/m$

Equation of motion, $m \cdot \frac{d^2x}{dt^2} + d \cdot \frac{dx}{dt} + \ldots$

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resonance freq.

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$d/m$

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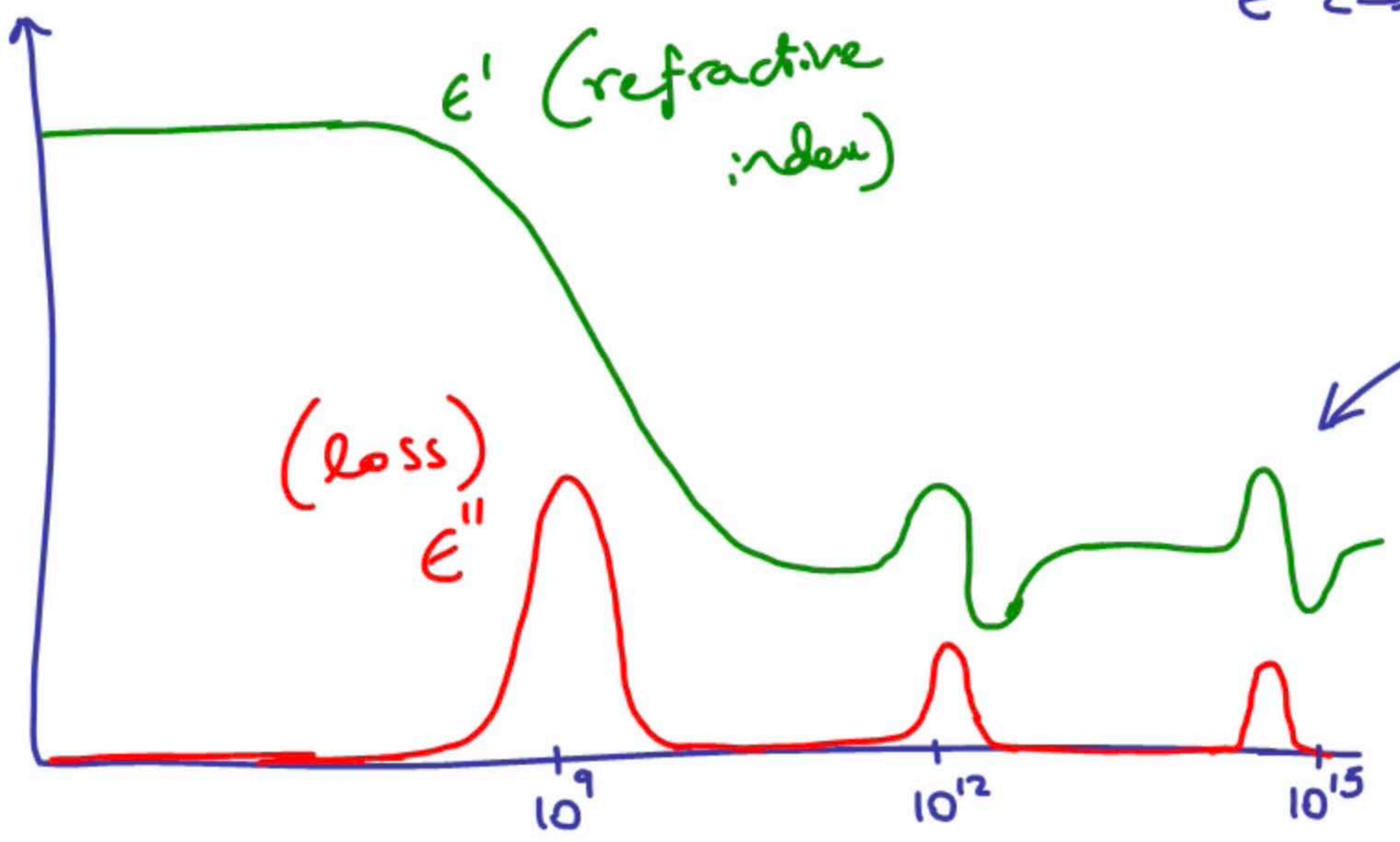
$\epsilon' \Longleftrightarrow \epsilon''$ (Kramers-Kronig relation)




Electronic resonance

Frequency

Equation of motion, $m \cdot \frac{\ldots}{dt^2} + d \cdot \frac{\ldots}{dt} + s \cdot \ldots$

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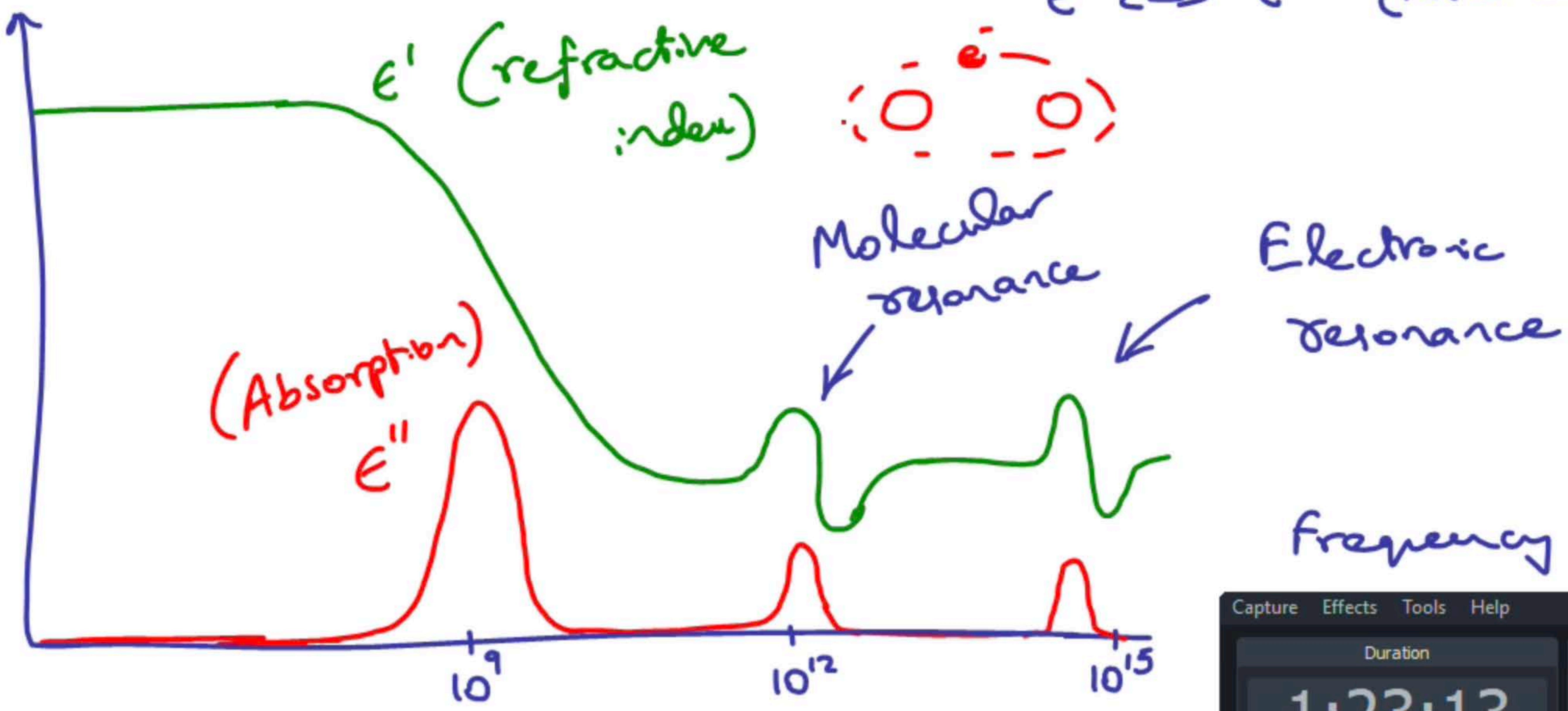

resonance freq. ← $\omega_0^2 - \omega^2 + j\omega$ damping coeff.

$\omega_0 = \sqrt{\frac{s}{m}}$ $d/m$

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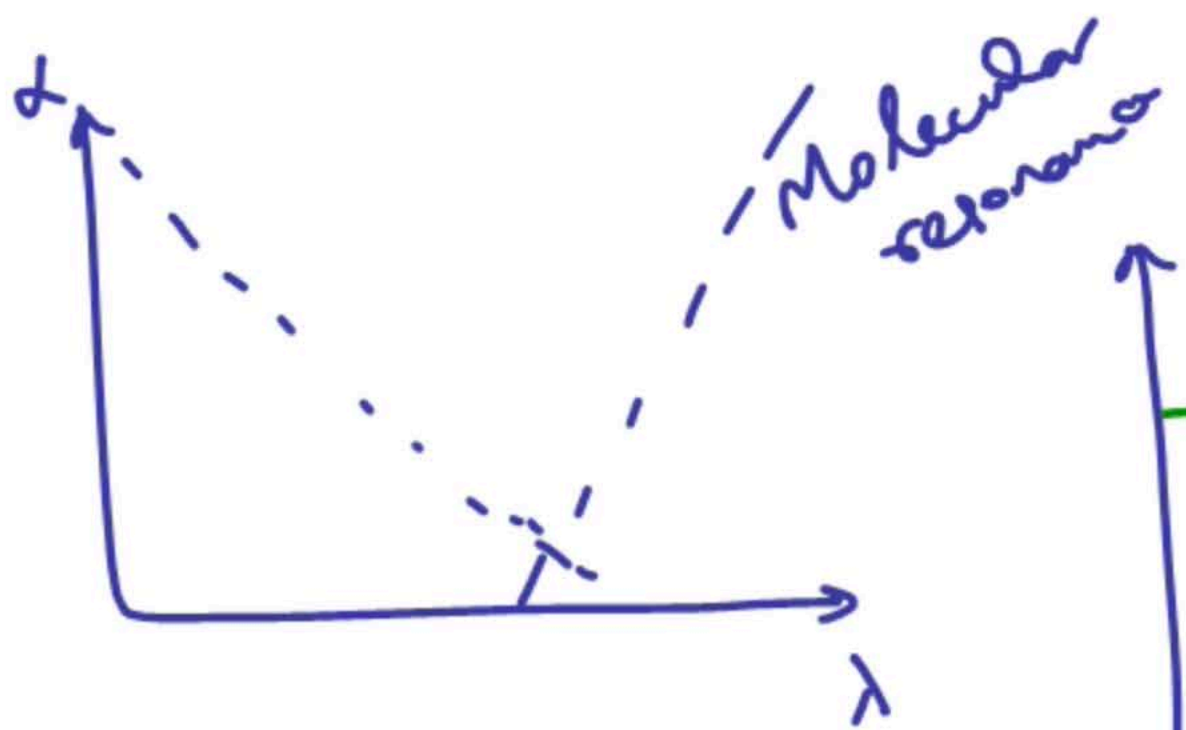
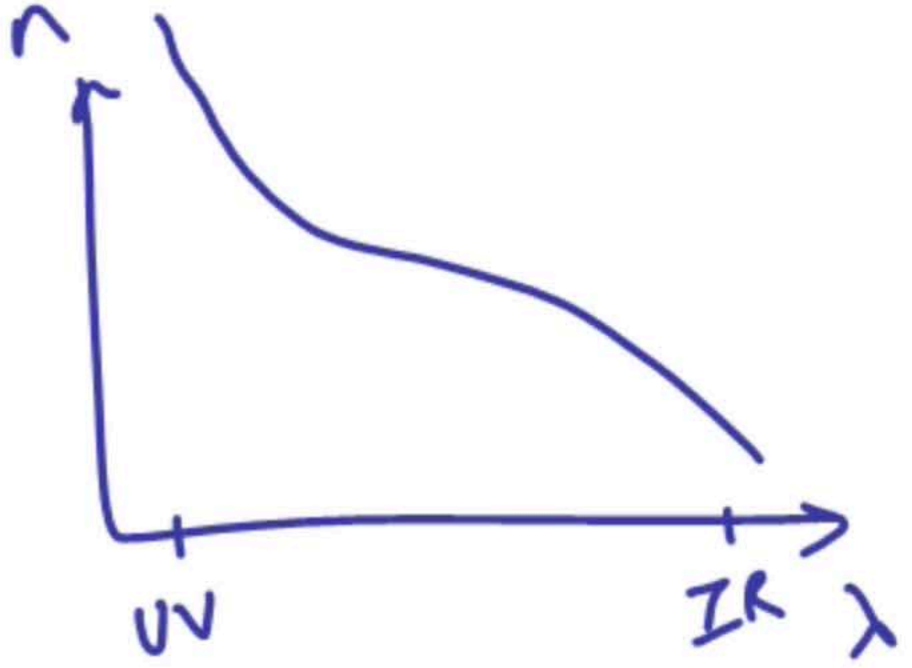
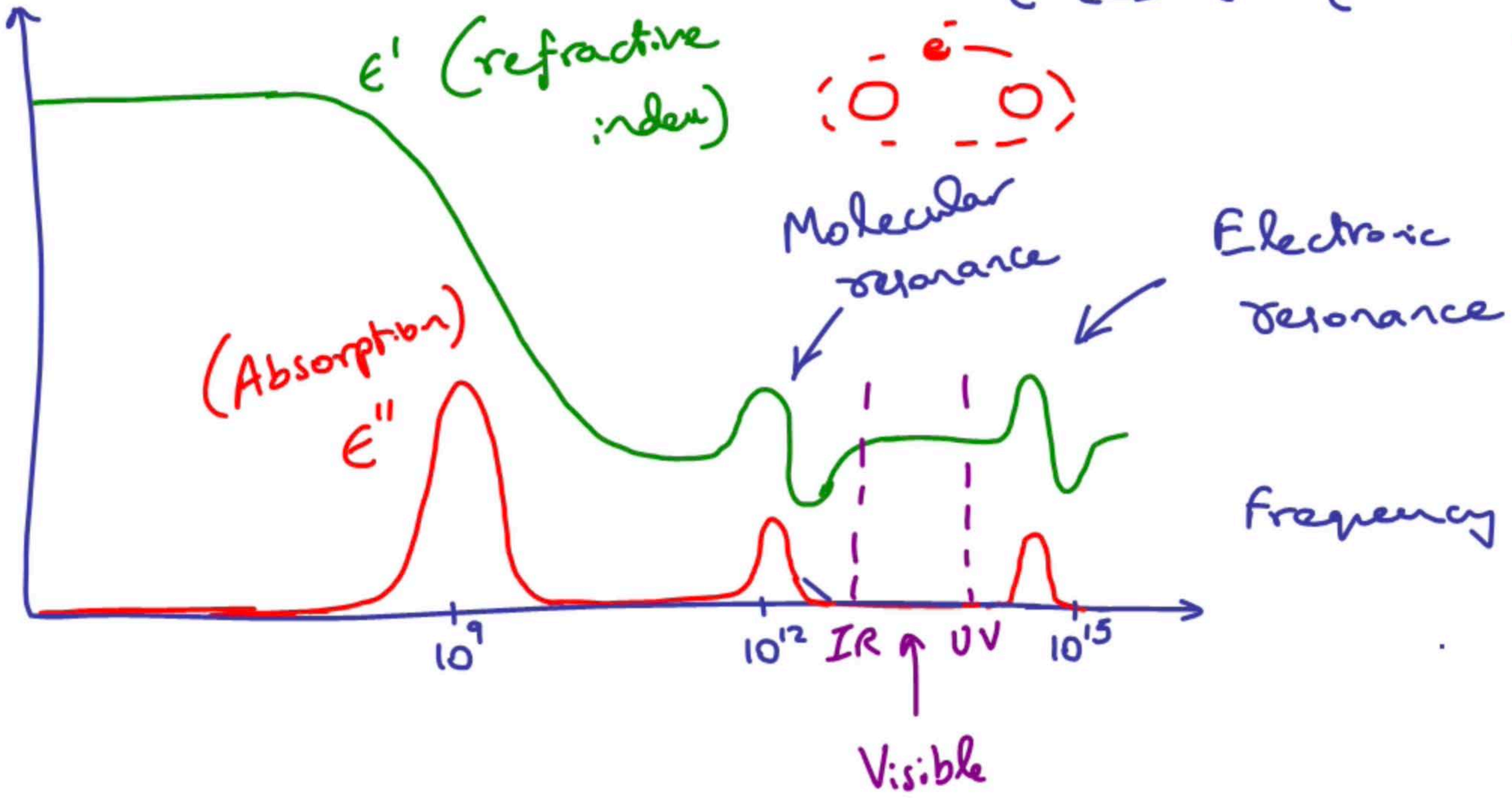
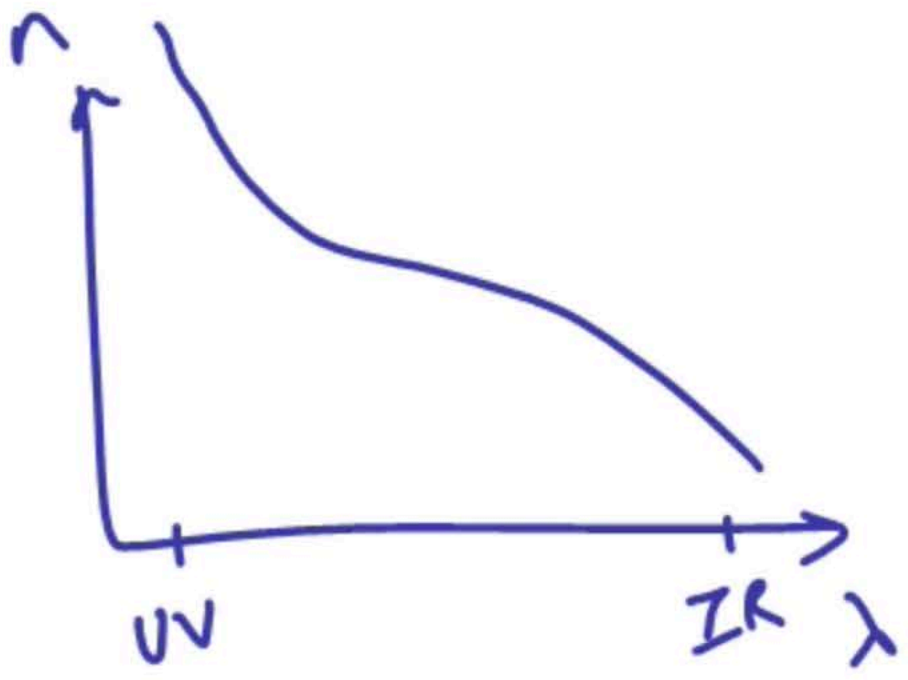


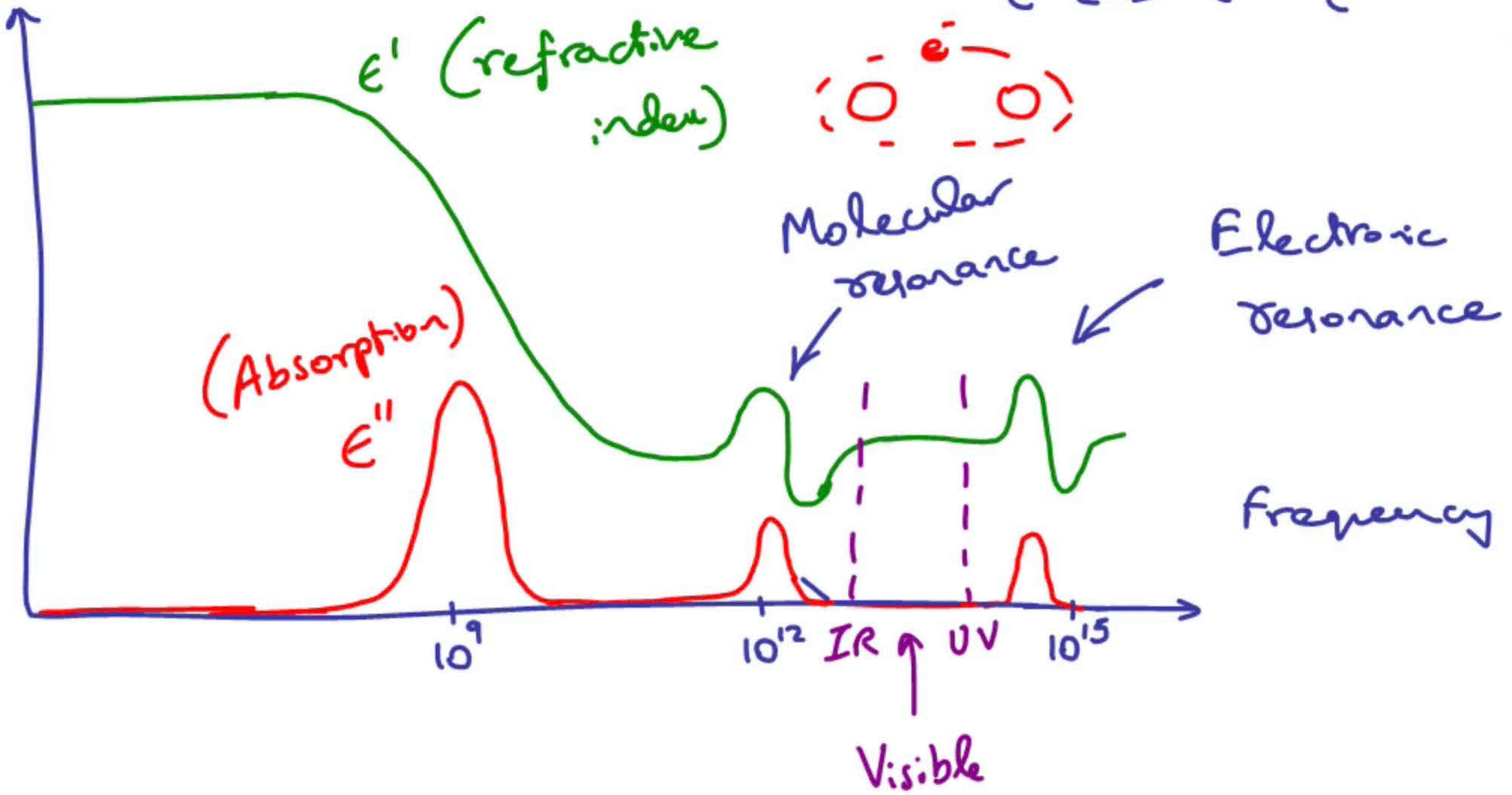

$\epsilon' \Longleftrightarrow \epsilon''$ (Kramers-Kronig relation)






$\epsilon' \longleftrightarrow \epsilon''$

relation)



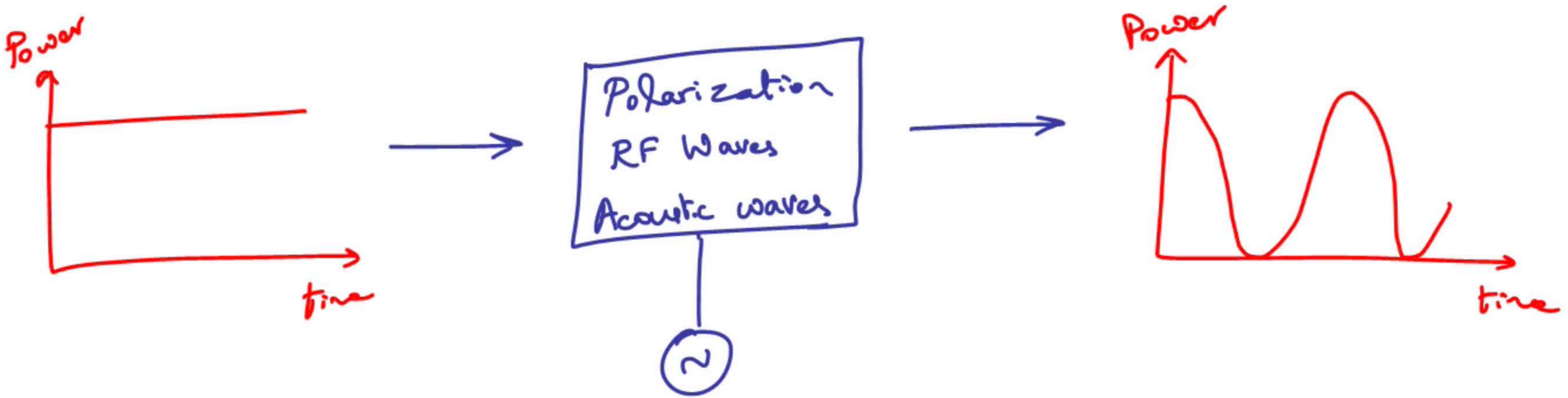

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

Electronic susceptibility

$$= \epsilon_0 \vec{E} + \vec{P}$$

Polarization




① Polarization : For an EM wave propagating in +z direction

$$\vec{E}(x, y, z, t) = \left(\hat{a}_x E_x + \hat{a}_y E_y e^{j\phi}\right) e^{j(\omega t - \beta z)}$$

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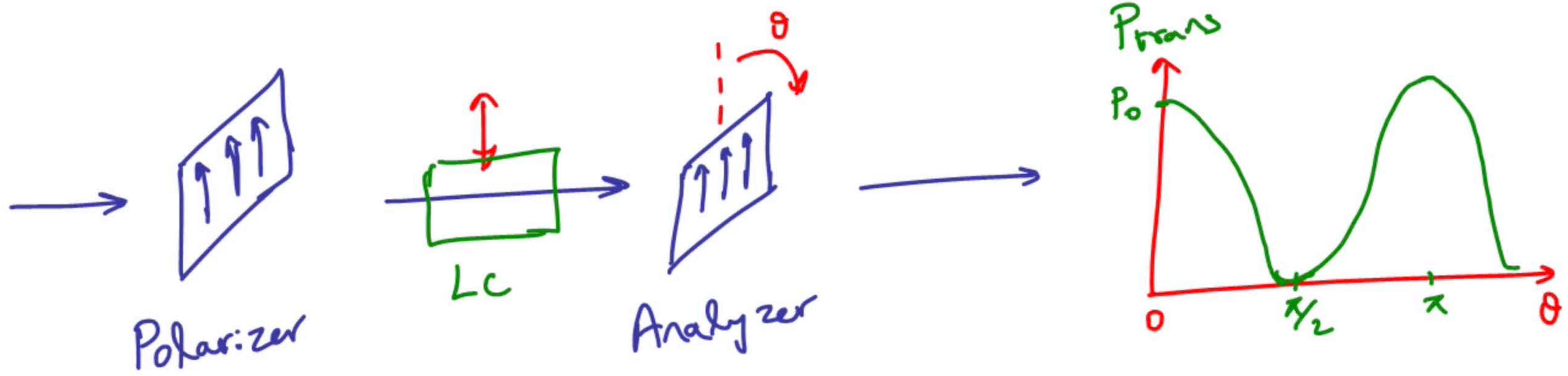
If $\phi = 0 \Rightarrow$ Linear polarization

$(E_x = E_y \Rightarrow \theta = 45^\circ)$

If $\phi = \pm \pi/2 \Rightarrow$ Circular polarization

$E_x = E_y$

Otherwise $\Rightarrow$ Elliptical polarization




Malus's law $\qquad P_0 \cos^2 \theta$