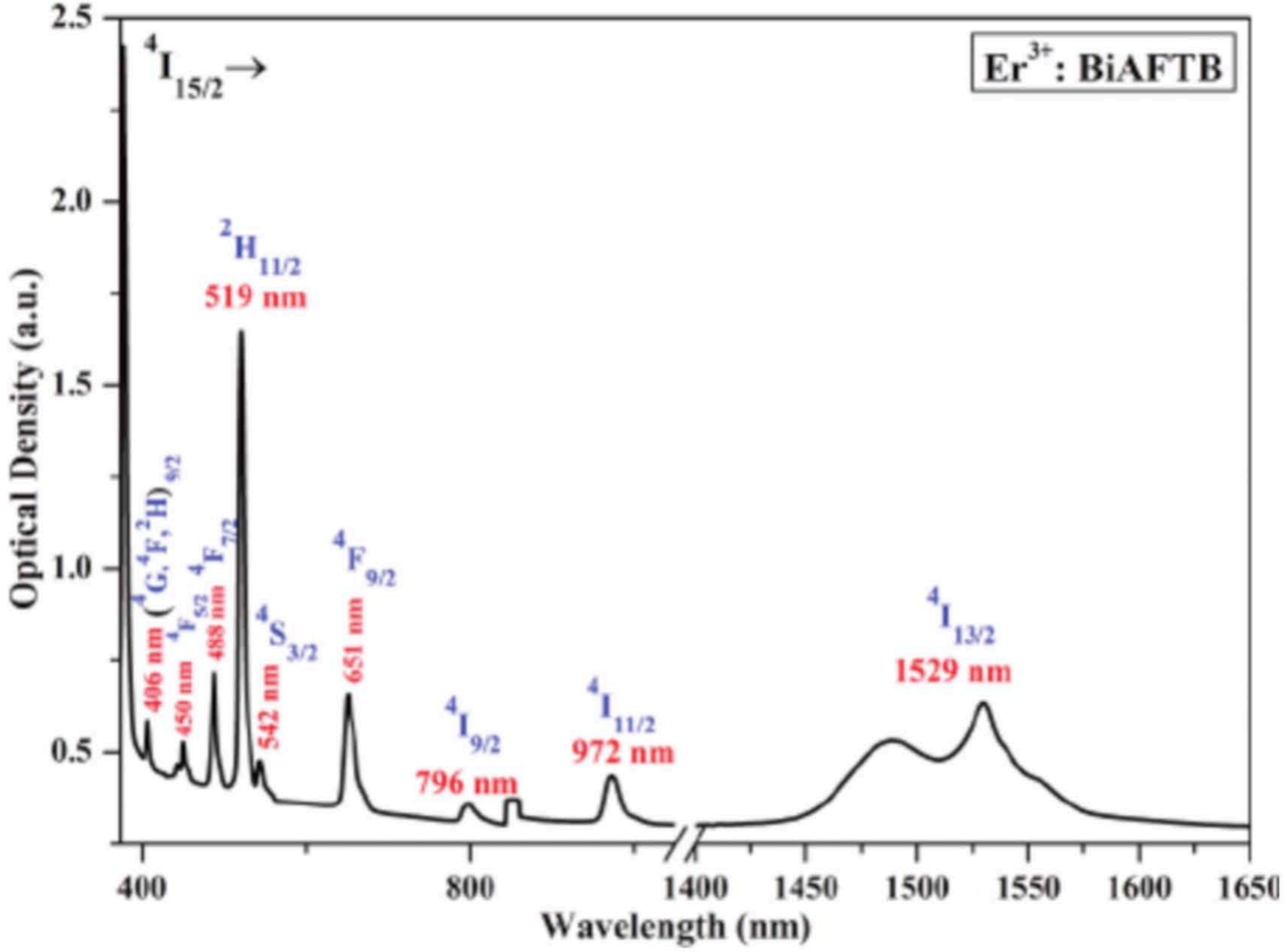
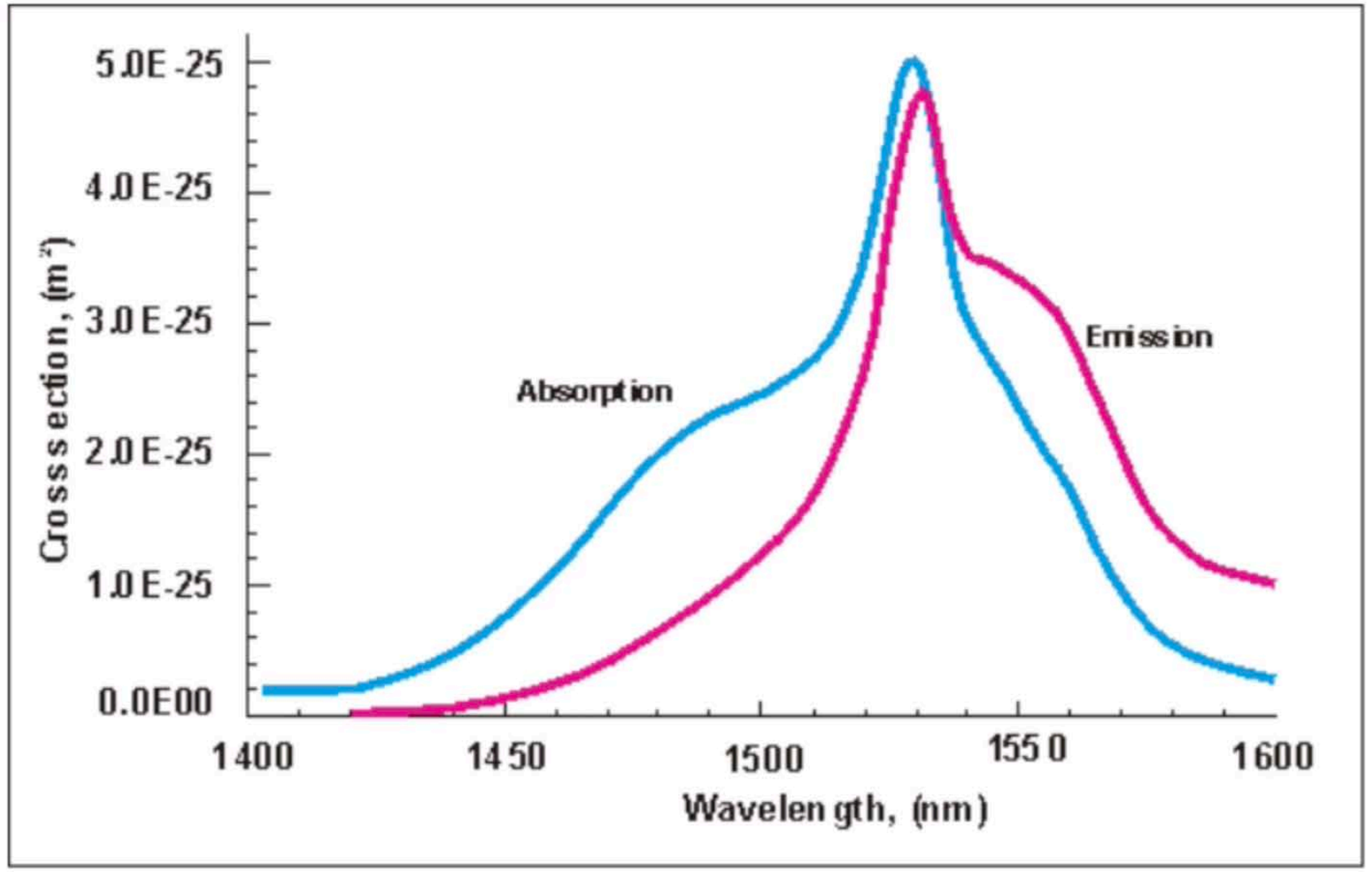
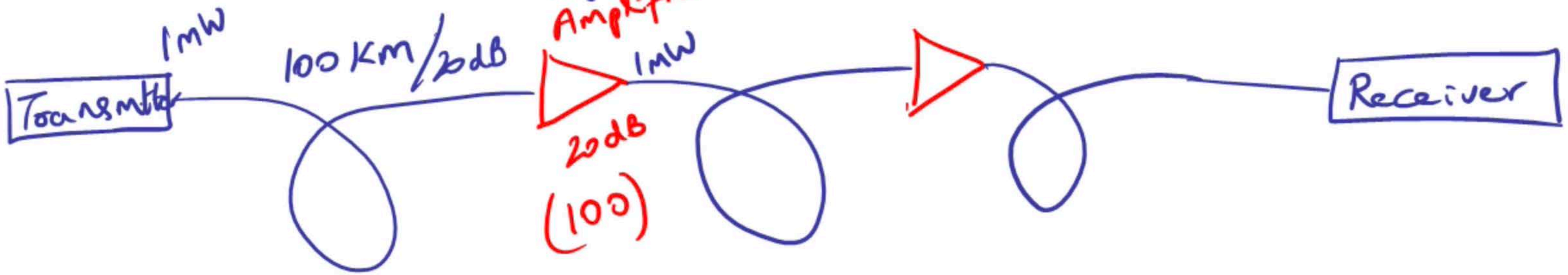

Er3+: BiAFTB
4I15/2→
2H11/2
519 nm
4F7/2
488 nm
4F5/2
450 nm
4G, 4F, 2H)9/2
406 nm
4S3/2
542 nm
4F9/2
651 nm
4I9/2
796 nm
4I11/2
972 nm
4I13/2
1529 nm
Optical Density (a.u.)
Wavelength (nm)
2.5
2.0
1.5
1.0
0.5
400
800
1400
1450
1500
1550
1600
1650


5.0E-25
4.0E-25
3.0E-25
2.0E-25
1.0E-25
0.0E00
Cross section, (m²)
Absorption
Emission
1400
1450
1500
1550
1600
Wavelength, (nm)

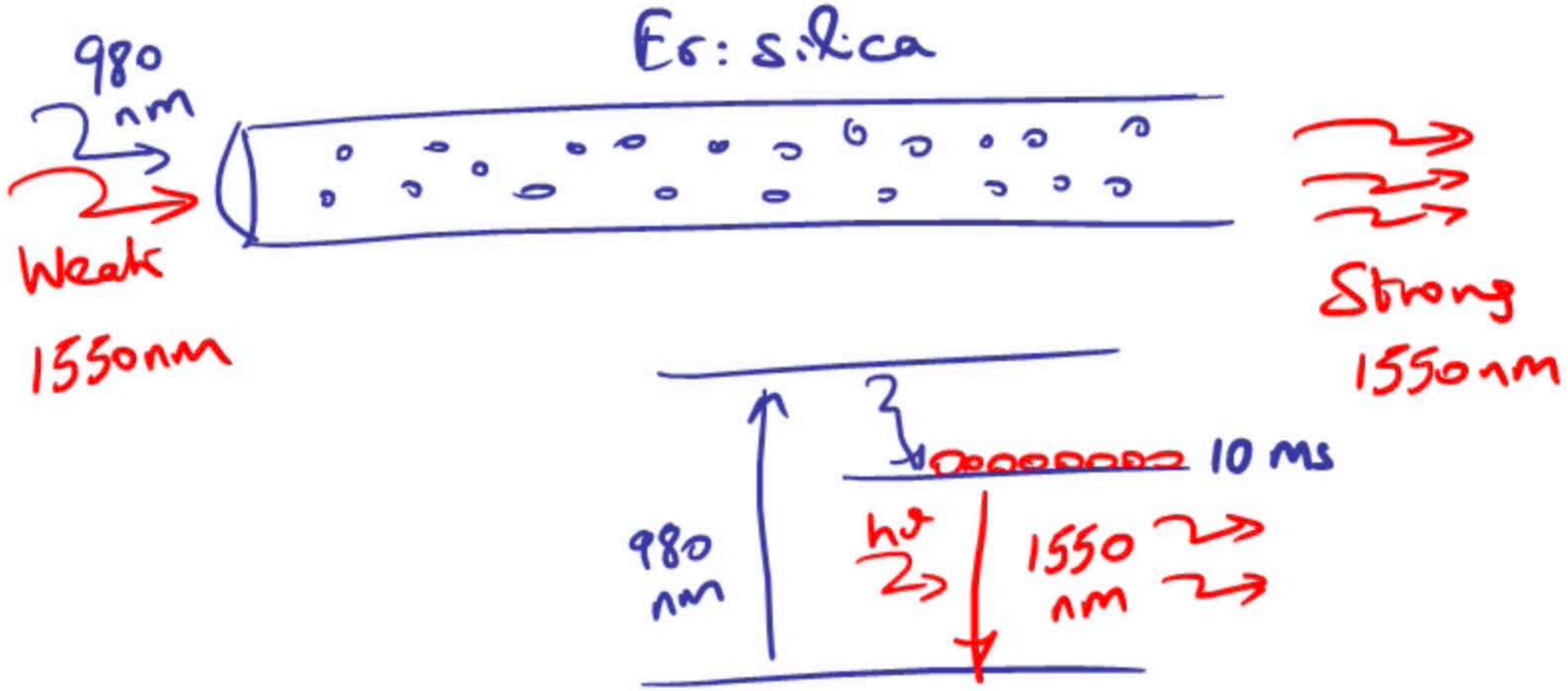
Example: Optical Fiber Communication System

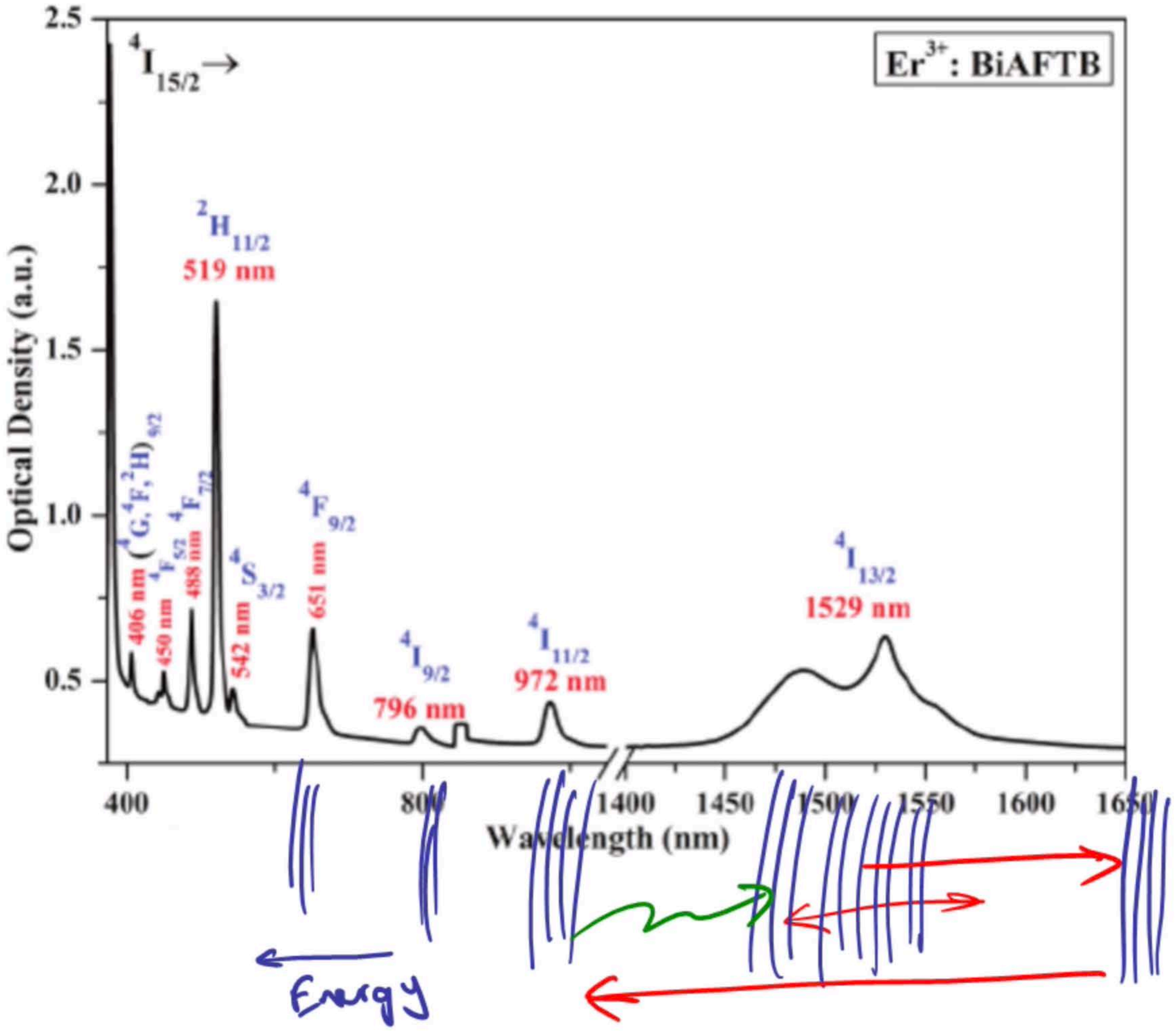
Noise Figure $= \frac{(SNR)_{IN}}{(SNR)_{OUT}}$



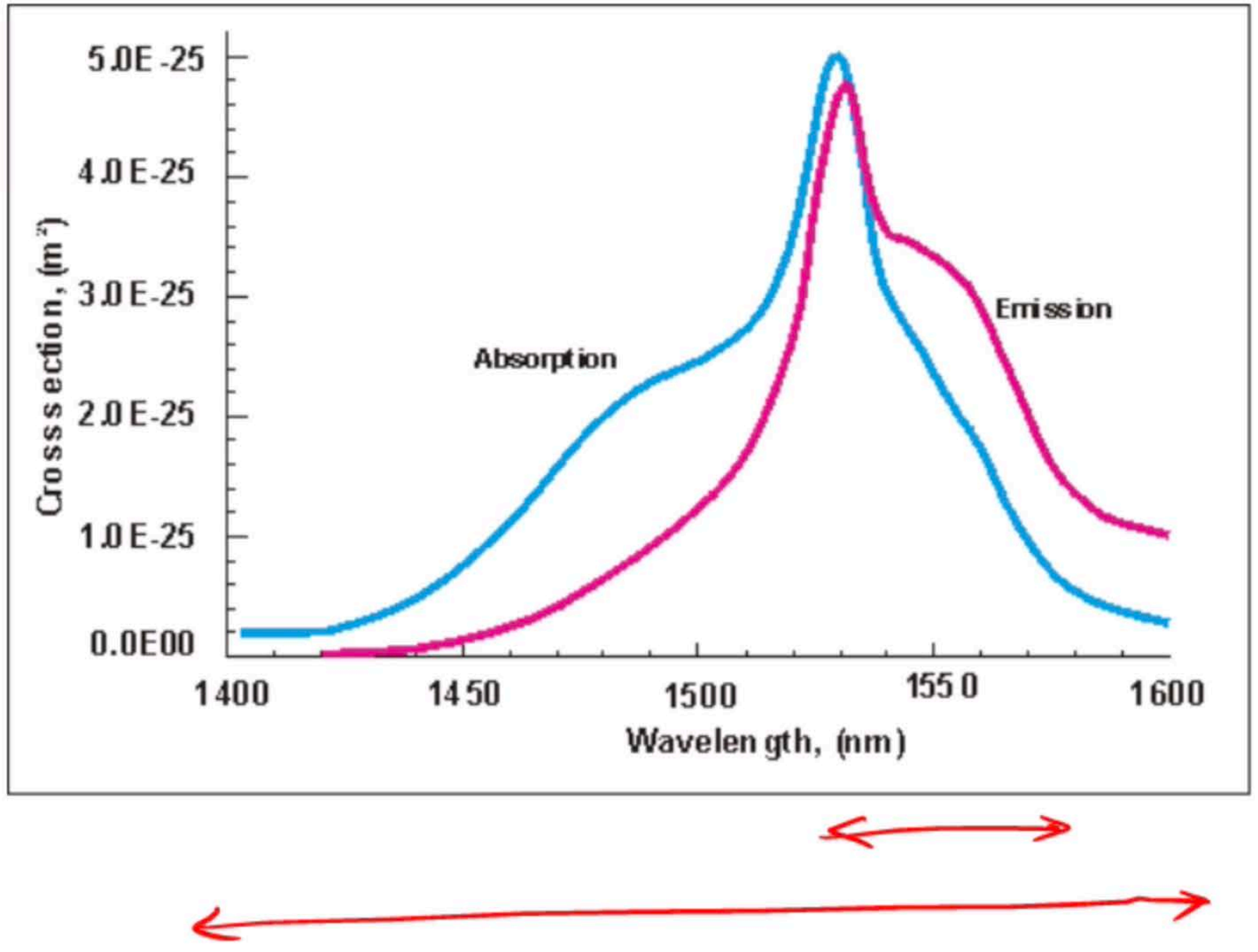
Attenuation 0.2 dB/km

@ $\lambda = 1550$ nm





$^4I_{15/2} \rightarrow$
Er^{3+}: BiAFTB
$^2H_{11/2}$
519 nm
$^4G, ^4F, ^2H)_{9/2}$
406 nm
$^4F_{5/2}$
450 nm
$^4F_{7/2}$
488 nm
$^4S_{3/2}$
542 nm
$^4F_{9/2}$
651 nm
$^4I_{9/2}$
796 nm
$^4I_{11/2}$
972 nm
$^4I_{13/2}$
1529 nm
Optical Density (a.u.)
Wavelength (nm)
Energy



Absorption
Emission
Cross section, (m²)
Wavelength, (nm)

$$N W_i = R - \frac{1}{2\tau_{21}}(N + N_a) \quad \Rightarrow \quad N = \frac{2\tau_{21} R - N_a}{2\tau_{21} W_i + 1}$$

$$\boxed{N = \frac{N_0}{1 + \tau_s W_i}}$$

where $\tau_s = 2 t_{sp}$

$N_0 = 2 R t_{sp} - N_a$

$W_i \to \phi\sigma$

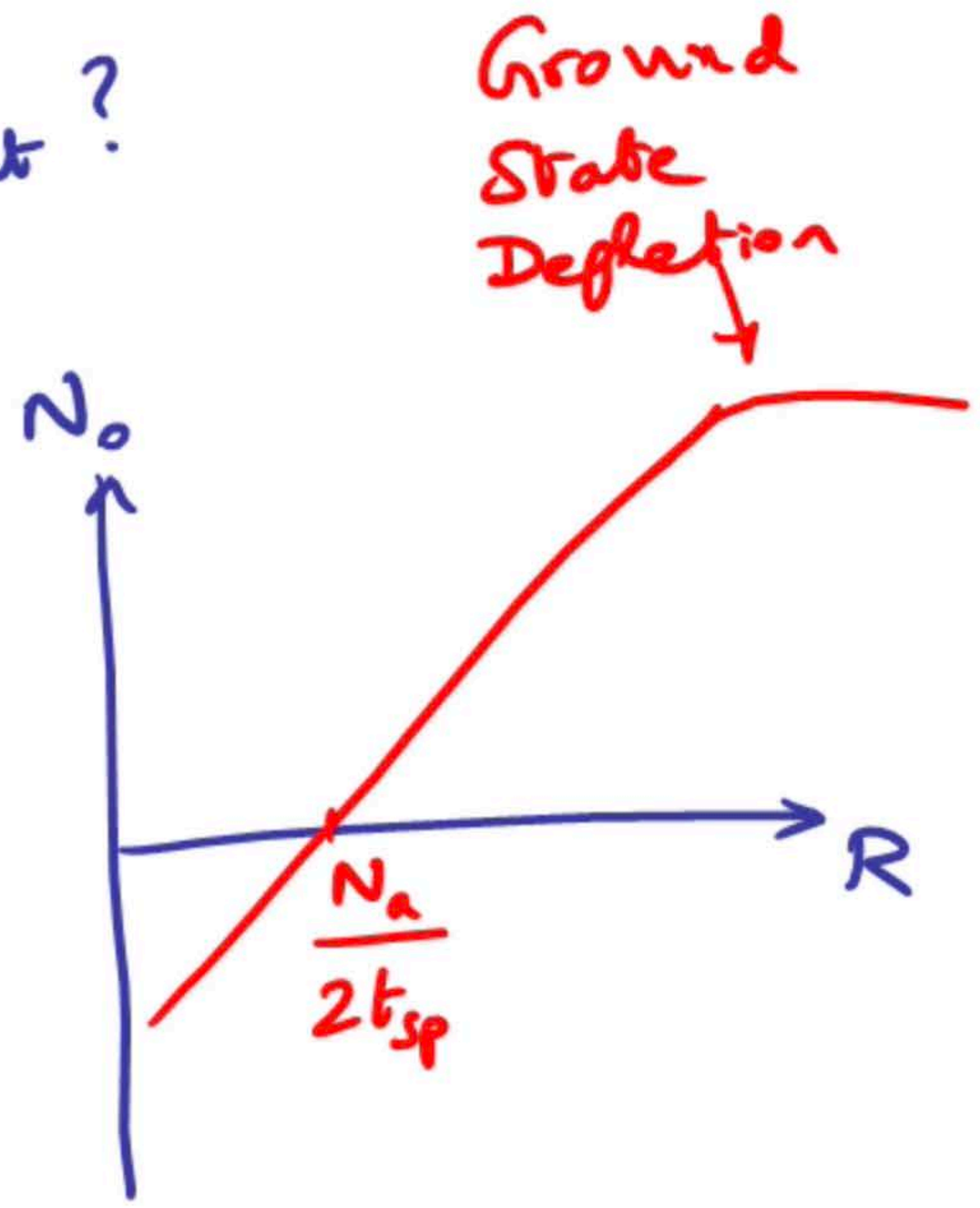
flux density of incoming signal photons

Example:

For Erbium, $t_{sp} = 10\ ms$, $N_a = 1 \times 10^{24}\ m^{-3}$

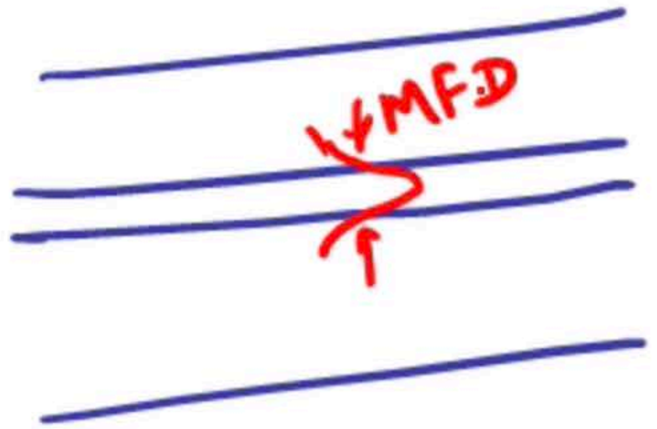
(a) What is the minimum pump power density required to achieve inversion $(N > 0)$?

(b) What is the saturation flux density? ϕ_{sat}?



(a) $$2 R t_{sp} - N_a > 0$$

$$R_{min} = \frac{N_a}{2 t_{sp}}$$



$$\text{Pump power density} = \frac{1.24}{0.98} \times 1.602\times10^{-19} \times \frac{1\times10^{24}}{2\times10^{-2}} \times 1\,\text{m}$$

length of gain medium

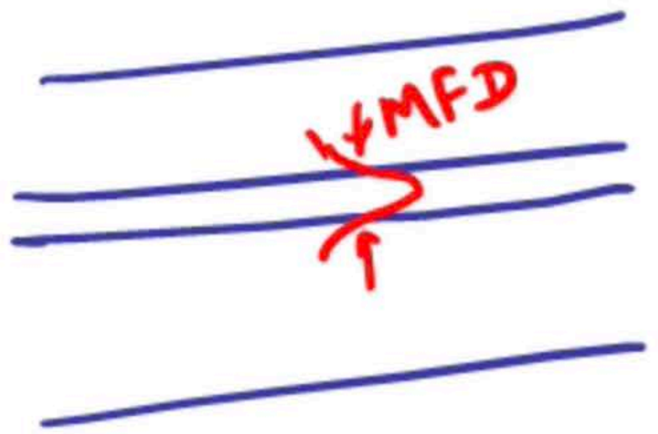
$$= 10^{7}\ \text{W/m}^2$$

Effective area of Er:silica fiber $= 100\ \mu\text{m}^2$

$$\text{Pump power} = 10^{7} \times 100\times10^{-12} = \underline{1\ \text{mW}}$$

$$\text{Pump power density} = \frac{1.24}{0.98} \times 1.602\times10^{-19} \times \frac{1\times10^{24}}{2\times10^{-2}} \times \underbrace{1\,m}_{\text{length of gain medium}}$$

$$= 10^{7}\ W/m^{2}$$



Effective area of Er:silica fiber $= 100\ \mu m^2$

$$\text{Pump power} = 10^{7} \times 100\times10^{-12} = \underline{1\ mW}$$

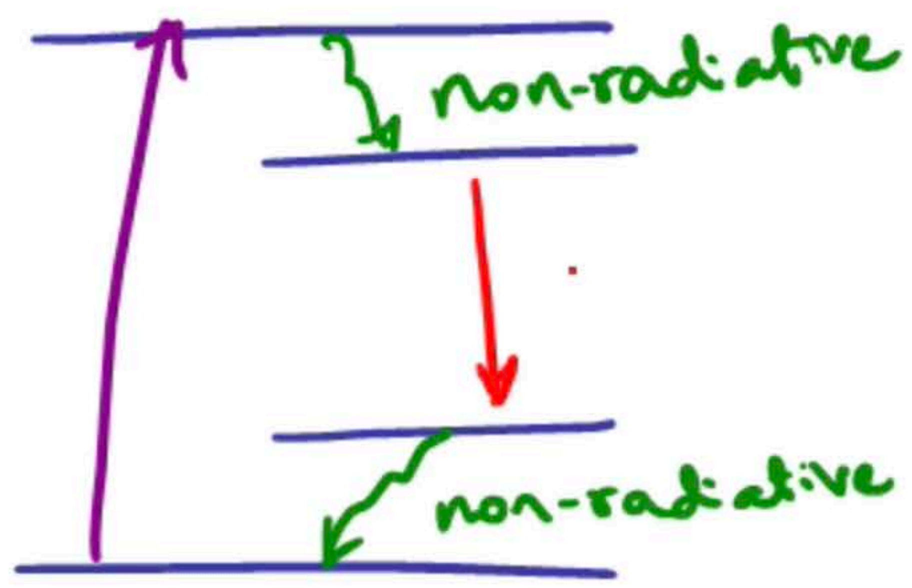
(b)

$$N = \frac{N_0}{1+\tau_s W_i} = \frac{N_0}{1+\tau_s \phi_s \sigma_e} = \frac{N_0}{1+\phi_s/\phi_{sat}}$$

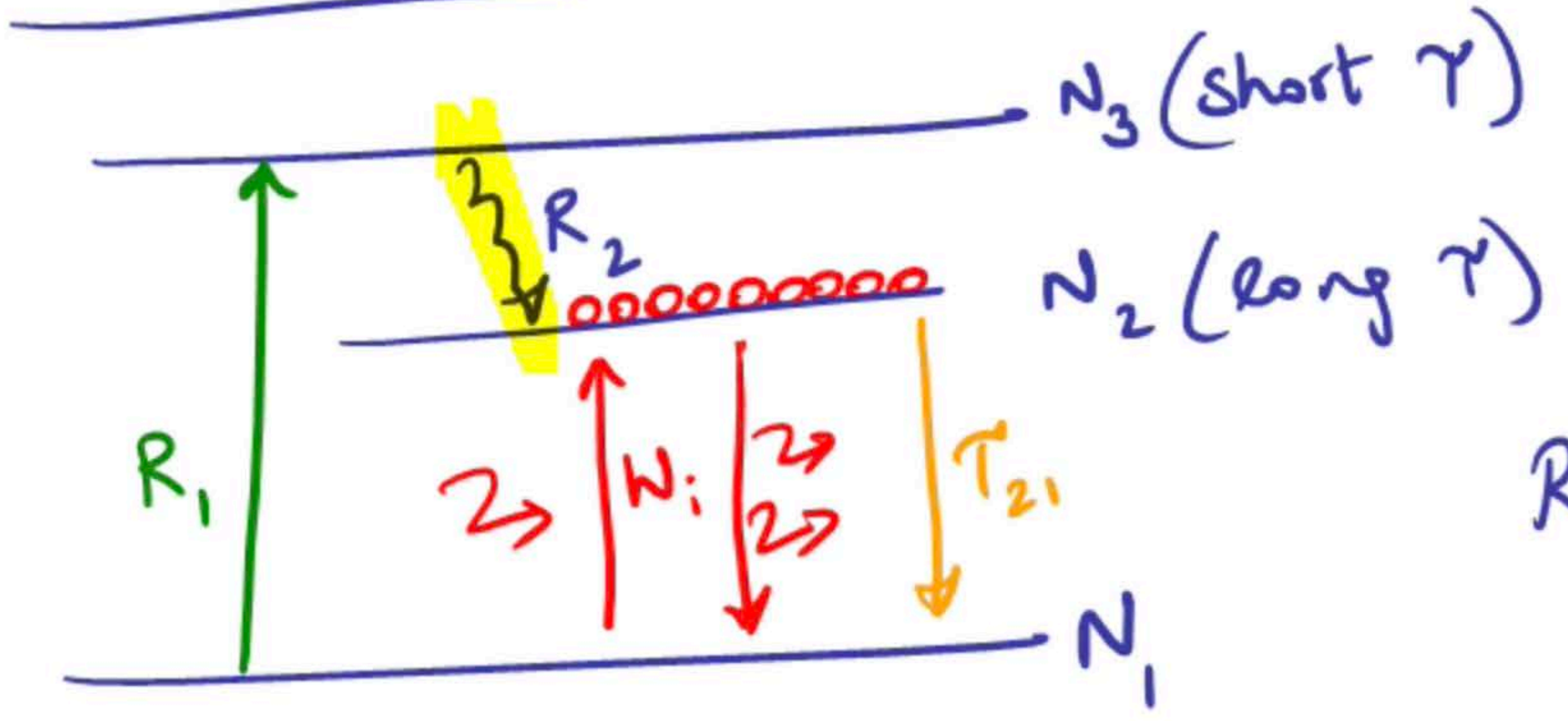
$$P_{sat} = \phi_{sat} \cdot A \cdot h\nu_s$$
$$= 1\ mW$$

$$\phi_{sat} = \frac{1}{\tau_s \sigma_e} = \frac{1}{2\times10^{-2} \times 6.5\times10^{-25}} = \underline{7.7\times10^{25}}\ \text{photons}/m^2\text{-sec}$$

Four-level system



Three-level system:



$\longrightarrow \quad R_1 = R_2 = R$

Rate equation (@ steady state) $\quad \dfrac{dN_2}{dt} = 0$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i = 0$$

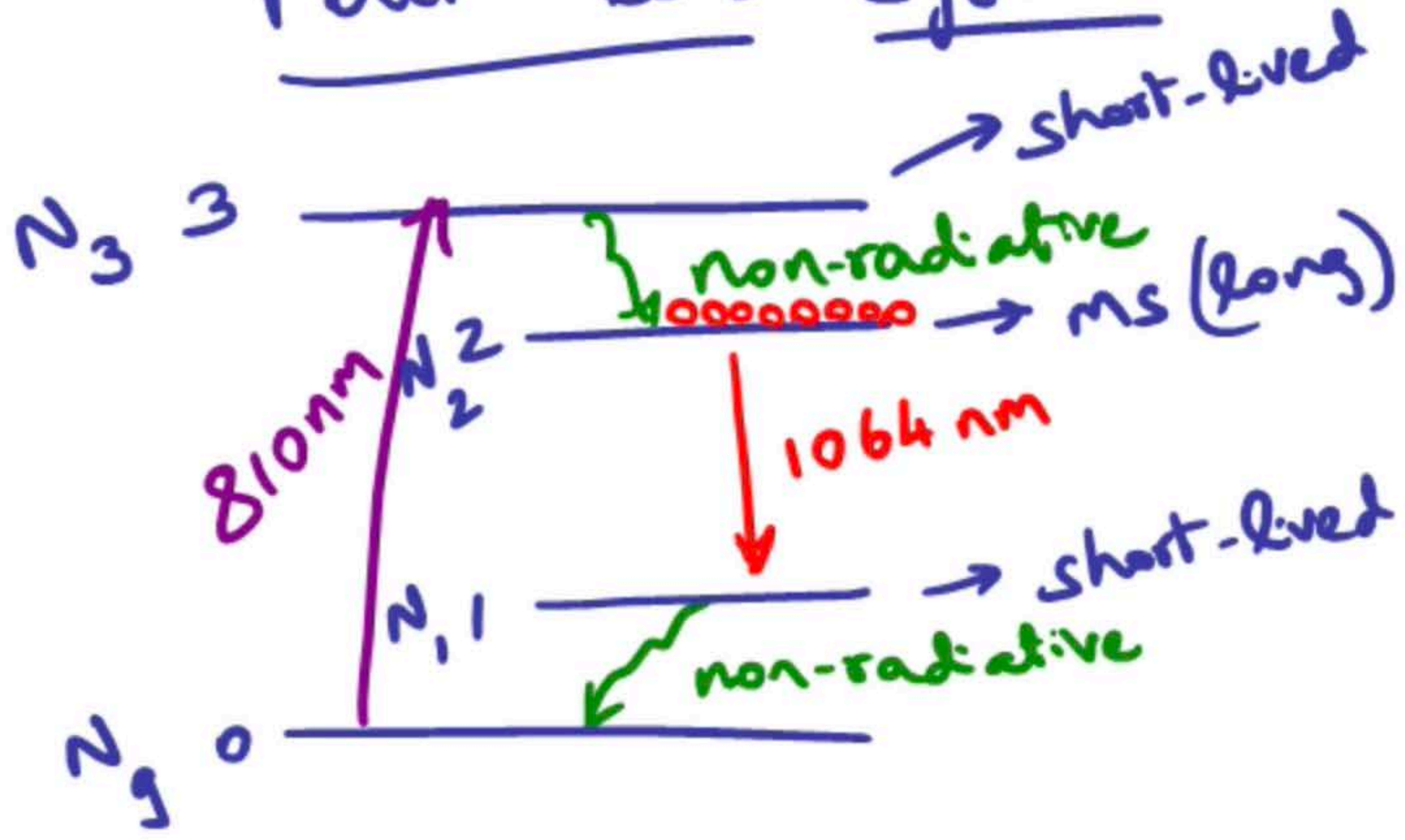
$$(N_2 - N_1)\, W_i = R - \frac{N_2}{\tau_{21}}$$

Total number density of dopants $\quad N_a = N_1 + N_2 + \cancel{N_3}$

$$N = N_2 - N_1$$

$$= 2N_2 - N_a \Rightarrow N_2 = \frac{1}{2}(N + N_a) \longrightarrow \quad N W_i = R - \frac{1}{2\tau_{21}}(N + N_a)$$

Four-level system



Neodymium

$$N_a = N_g + \cancel{N_1}^{\,0} + N_2 + \cancel{N_3}^{\,0}$$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{dN_1}{dt} = -\cancel{R_1}^{\,0} - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$

Under steady state conditions, $\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0$

$$\boxed{N = \frac{N_0}{1 + \tau_s W_i}}$$

$$N_0 = N_a \cdot \frac{t_{sp} W}{1 + t_{sp} W}$$

$W \rightarrow$ Pump transition probability ($R = W N_g$)

$$\tau_s = \frac{t_{sp}}{1 + t_{sp} W}$$

For weak pumping

$W << 1/t_{sp}$

$$N_0 = N_a t_{sp} W$$

$$\tau_s = t_{sp}$$