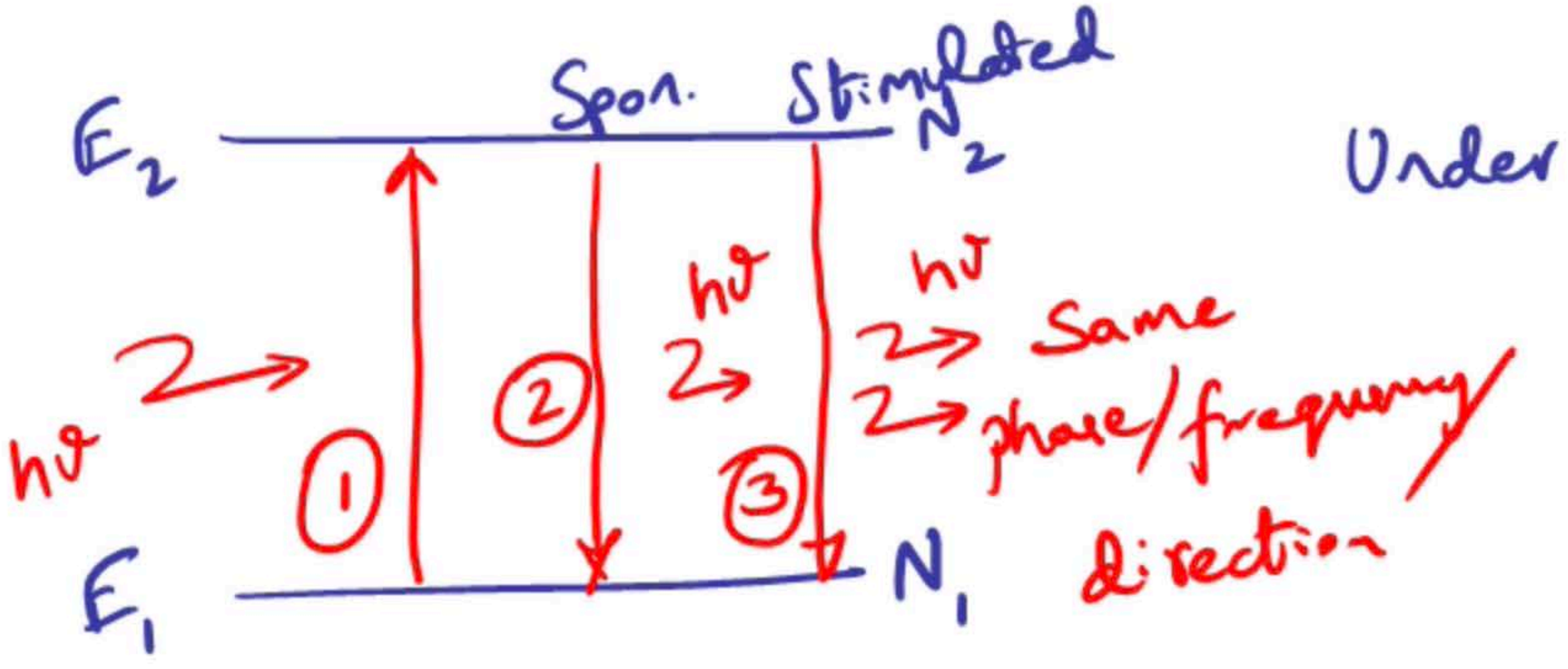
Note Title 8/27/2018

Learning Objective: Identify the fundamental principles of photon interaction w/ atoms

Analyze light generation and amplification



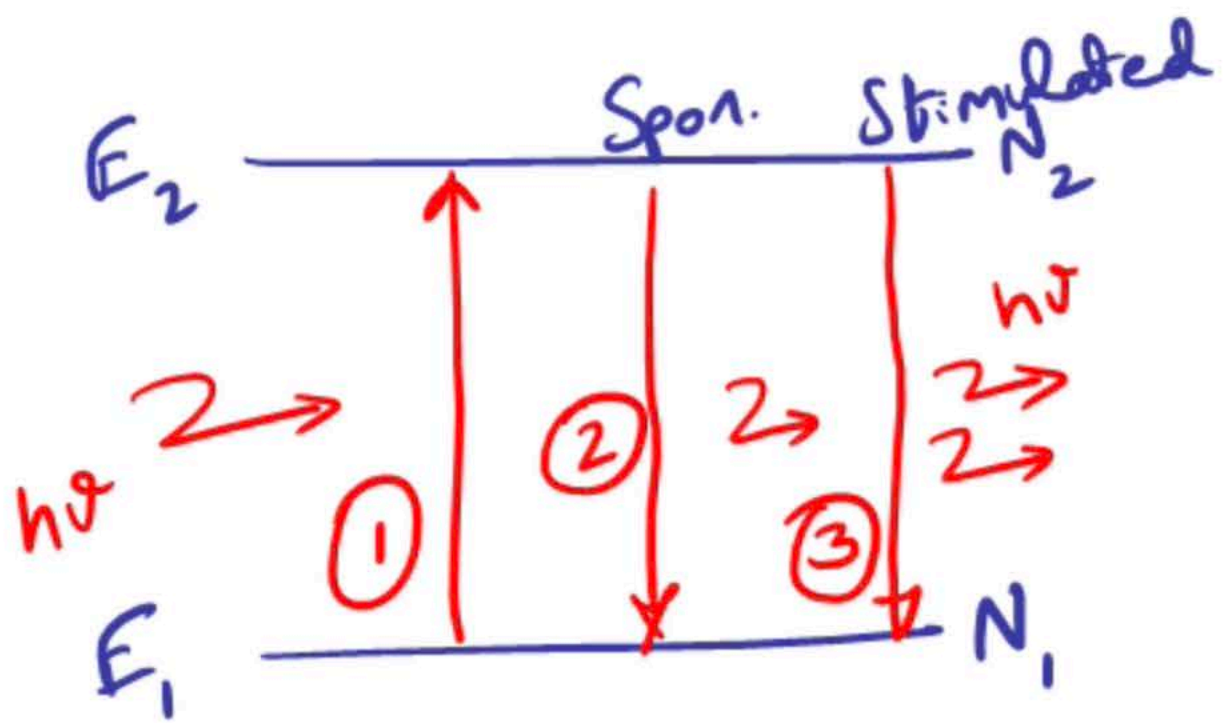
Under steady state conditions,

$$R_{abs} = R_{spon} + R_{stim}$$

$$B' N_1 \rho_{abs} = A N_2 + B N_2 \rho_{em}$$

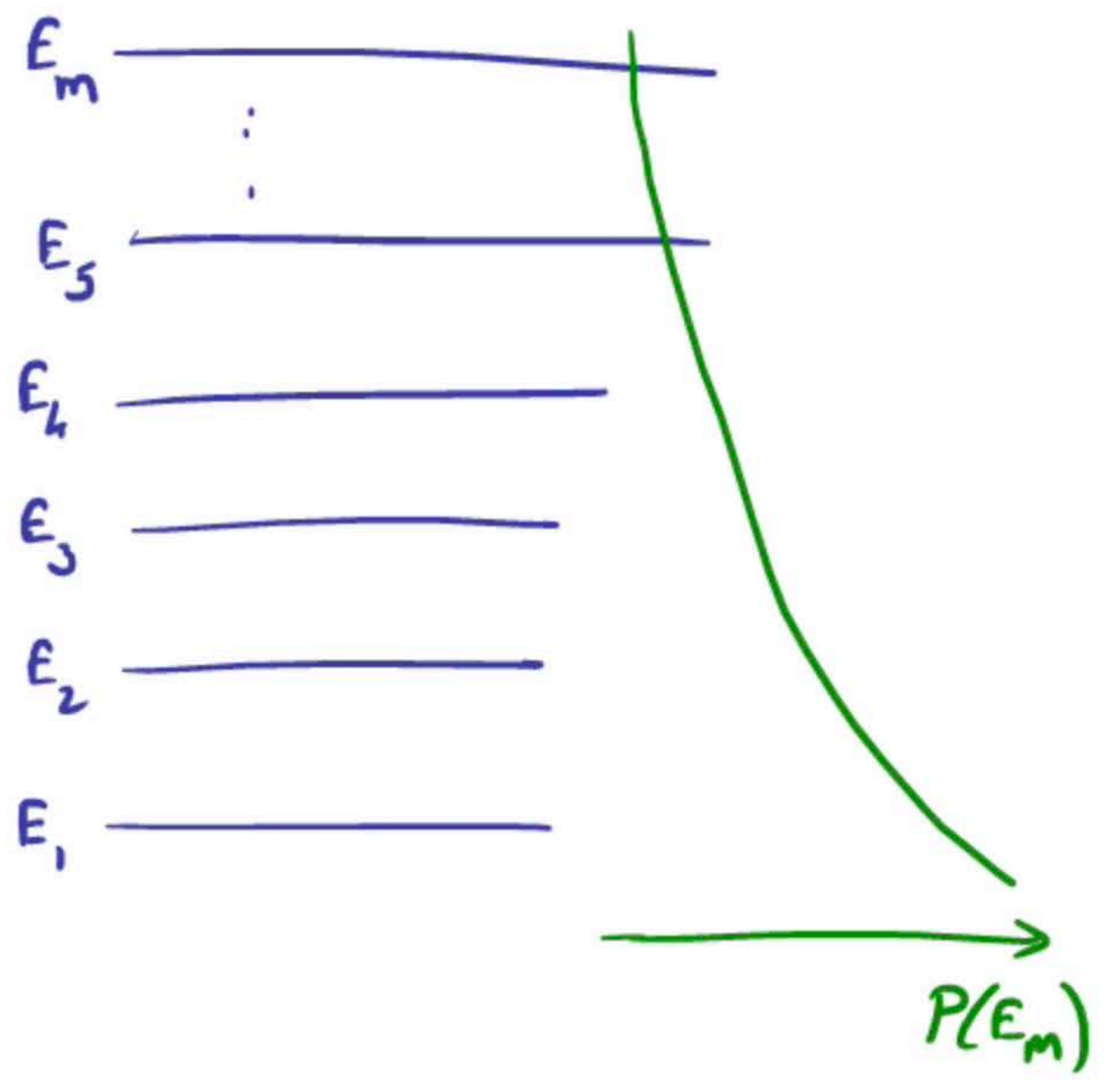
If $\rho_{em} = \rho_{abs}$, $\quad \rho_{em} = \dfrac{A N_2}{B' N_1 - B N_2} = \dfrac{A/B}{\frac{B'}{B} \cdot N_1/N_2 - 1}$

$\Rightarrow$ Similar to Planck's ρ_{em} for blackbody radiation



Under thermal equilibrium conditions.

$$P(E_m) \propto \exp\left(-\frac{E_m}{K_B T}\right)$$



$K_B \rightarrow$ Boltzmann const.

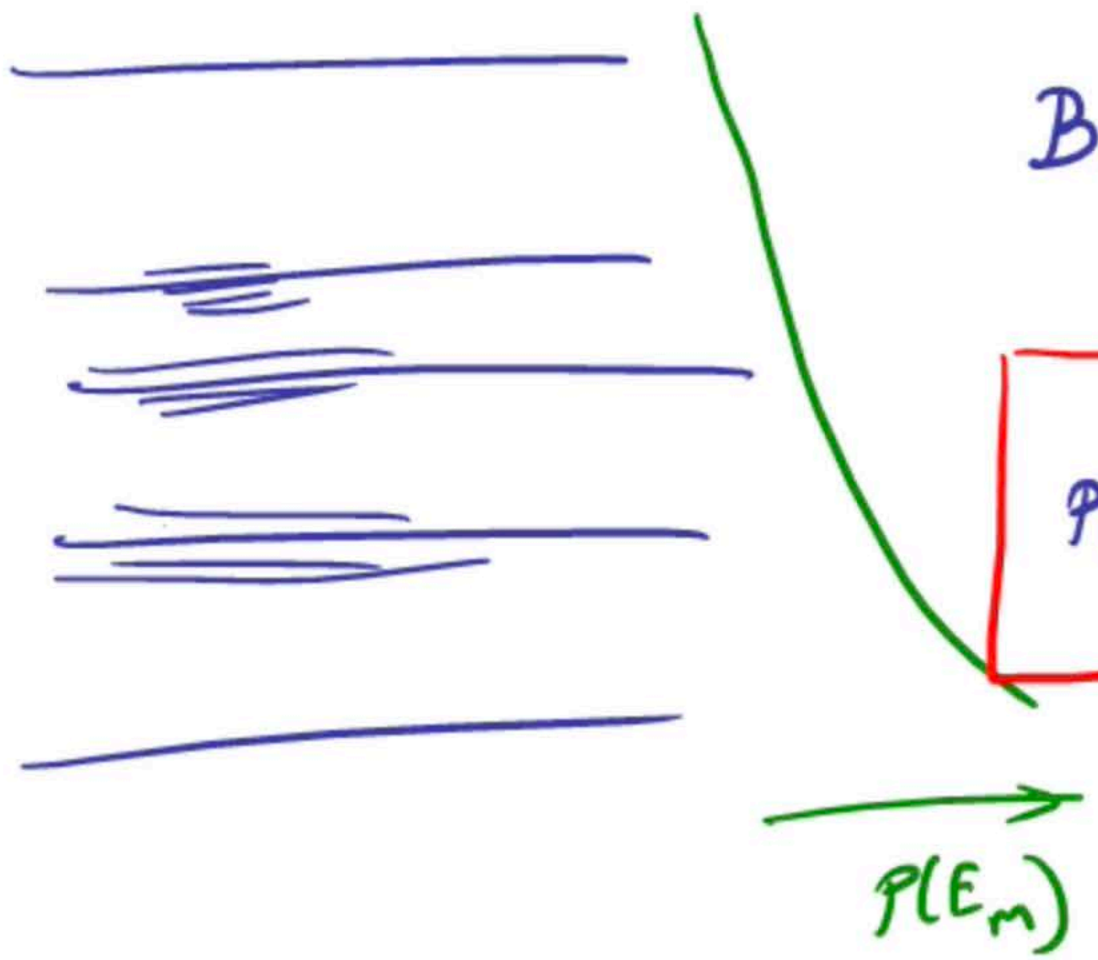
If there are N number of atoms

$$\frac{N_m}{N} = P(E_m) \Rightarrow \frac{N_2}{N_1} = \exp\left(-\frac{E_2 - E_1}{K_B T}\right)$$

$$= \exp\left(-\frac{\Delta E}{K_B T}\right)$$

Collection of atoms

Dilute gas of atoms

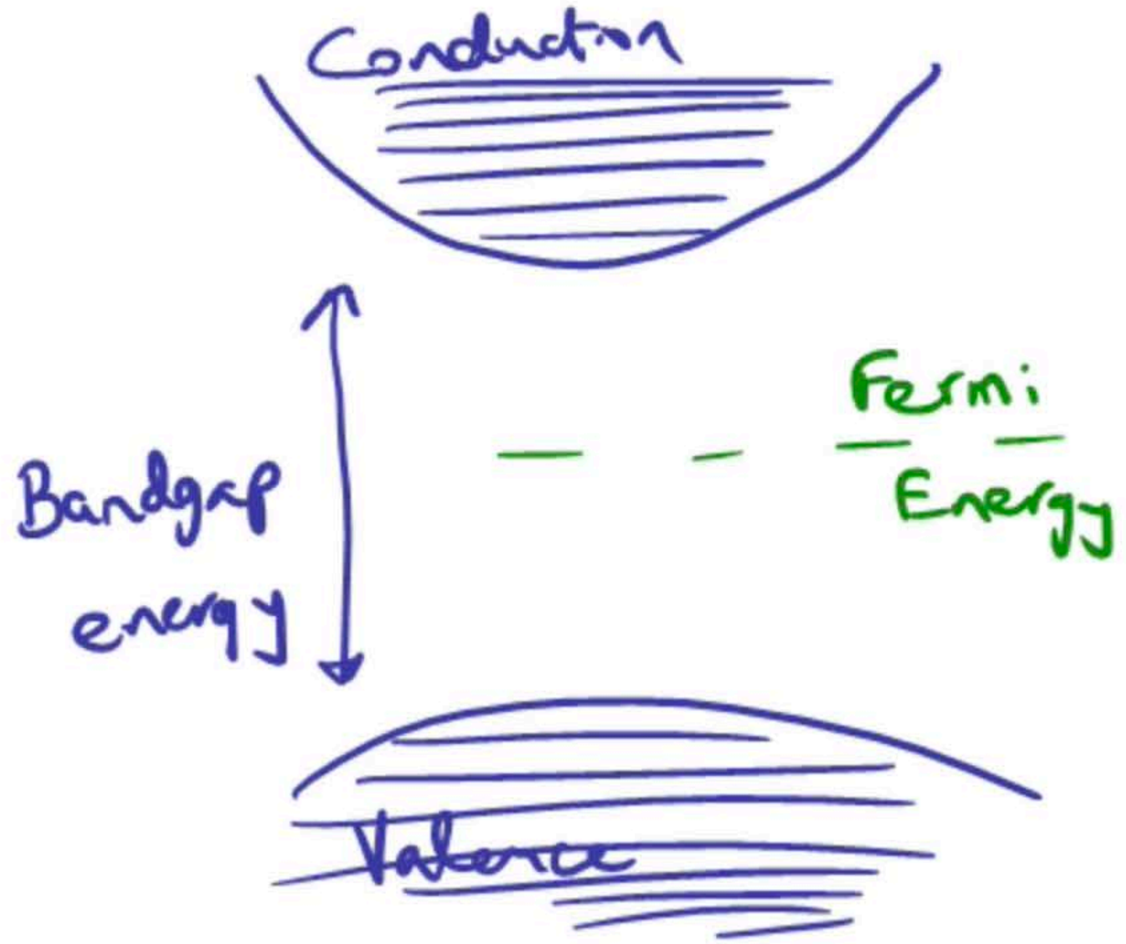


Boltzmann distribution

$$\boxed{p(E_m) = \exp\left(-\frac{E_m}{k_B T}\right)}$$

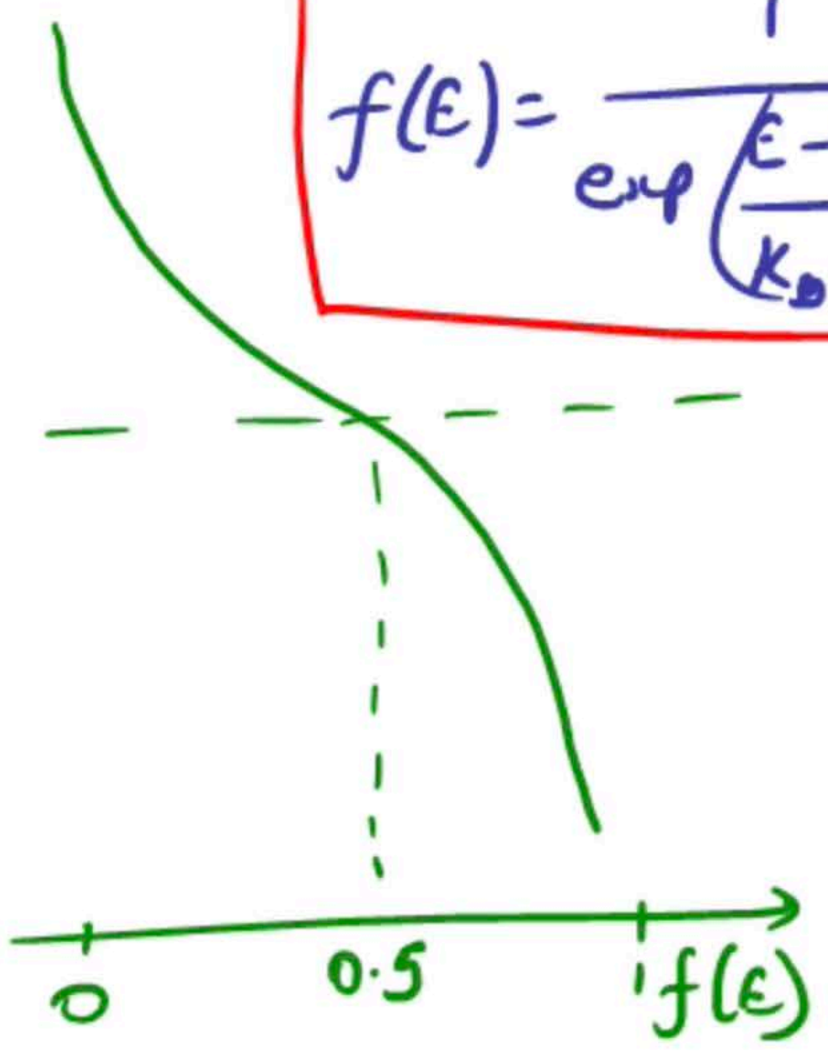
periodic arrangement of atoms w/ overlapping wavefunctions

(Semiconductors)



Fermi-Dirac

$$\boxed{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{k_B T}\right) + 1}}$$

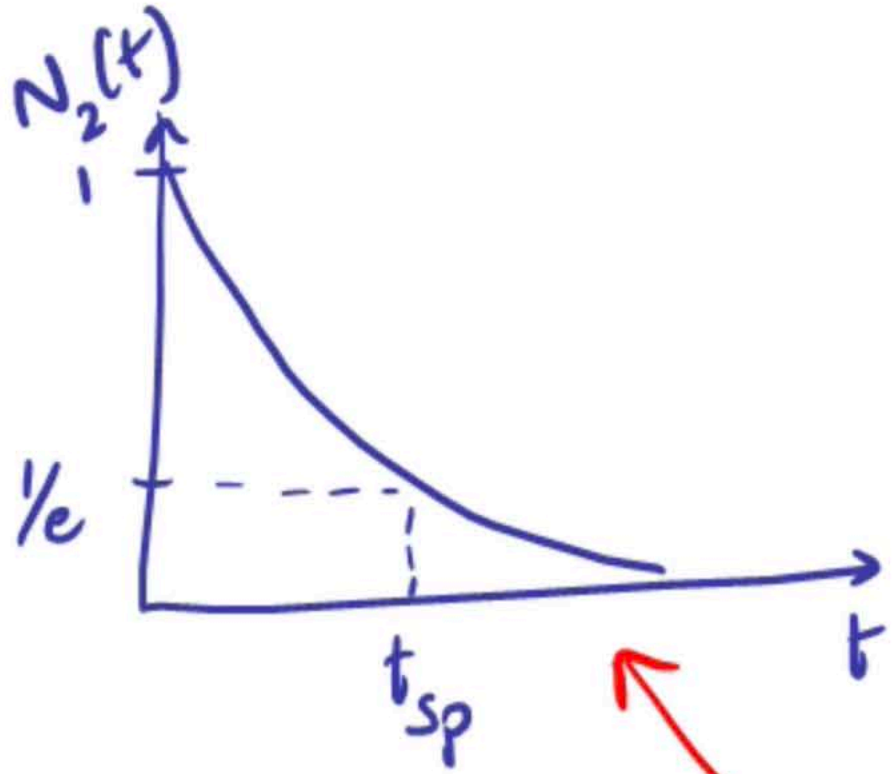


4 / 5

$$P_{em} = \frac{A/B}{B'/B \, \exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

provided $A/B = 8\pi h \frac{\nu^3}{c^3}$ & $B' = B$

$$A = P_{sp} = \frac{1}{t_{sp}} \qquad B = \frac{\lambda^3}{8\pi h \, t_{sp}}$$



# of atoms emitting simultaneously

$$\Delta N_2 = N_2 \cdot (P_{sp} \cdot \Delta t)$$

$$\frac{dN_2}{dt} = -N_2 P_{sp}$$

$$\boxed{N_2(t) = N_2(0)\exp(-P_{sp} t)}$$

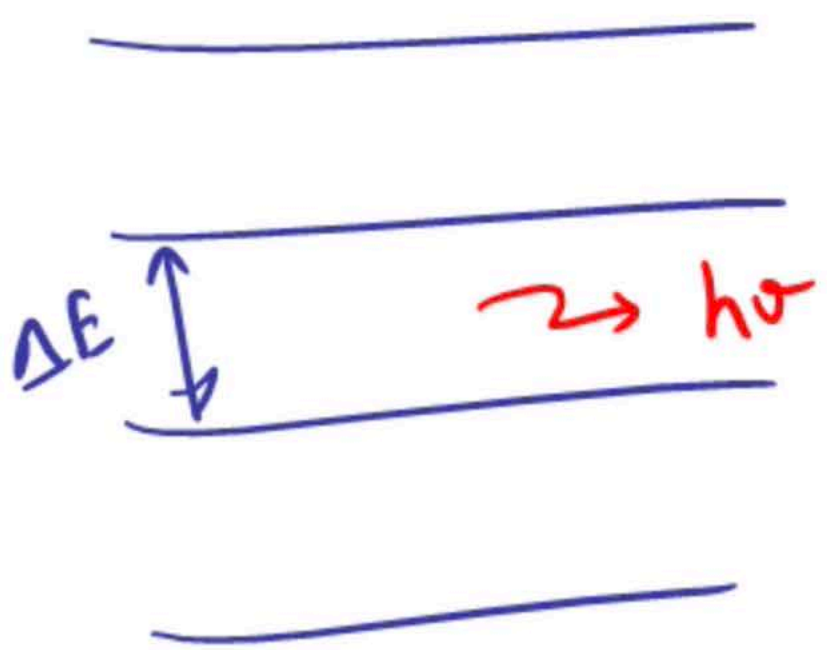
$$P_{sp} = \frac{c}{V}\sigma(\nu)$$

Transition cross-section (cm^2)

$$\boxed{P_{em} = \frac{A/B}{\frac{B'}{B}\exp\left(\frac{h\nu}{K_B T}\right) - 1}}$$

<u>Observation 1:</u> $K_B T > \Delta E$

$$R_{spon} >> R_{stim}$$ (Thermal light sources)

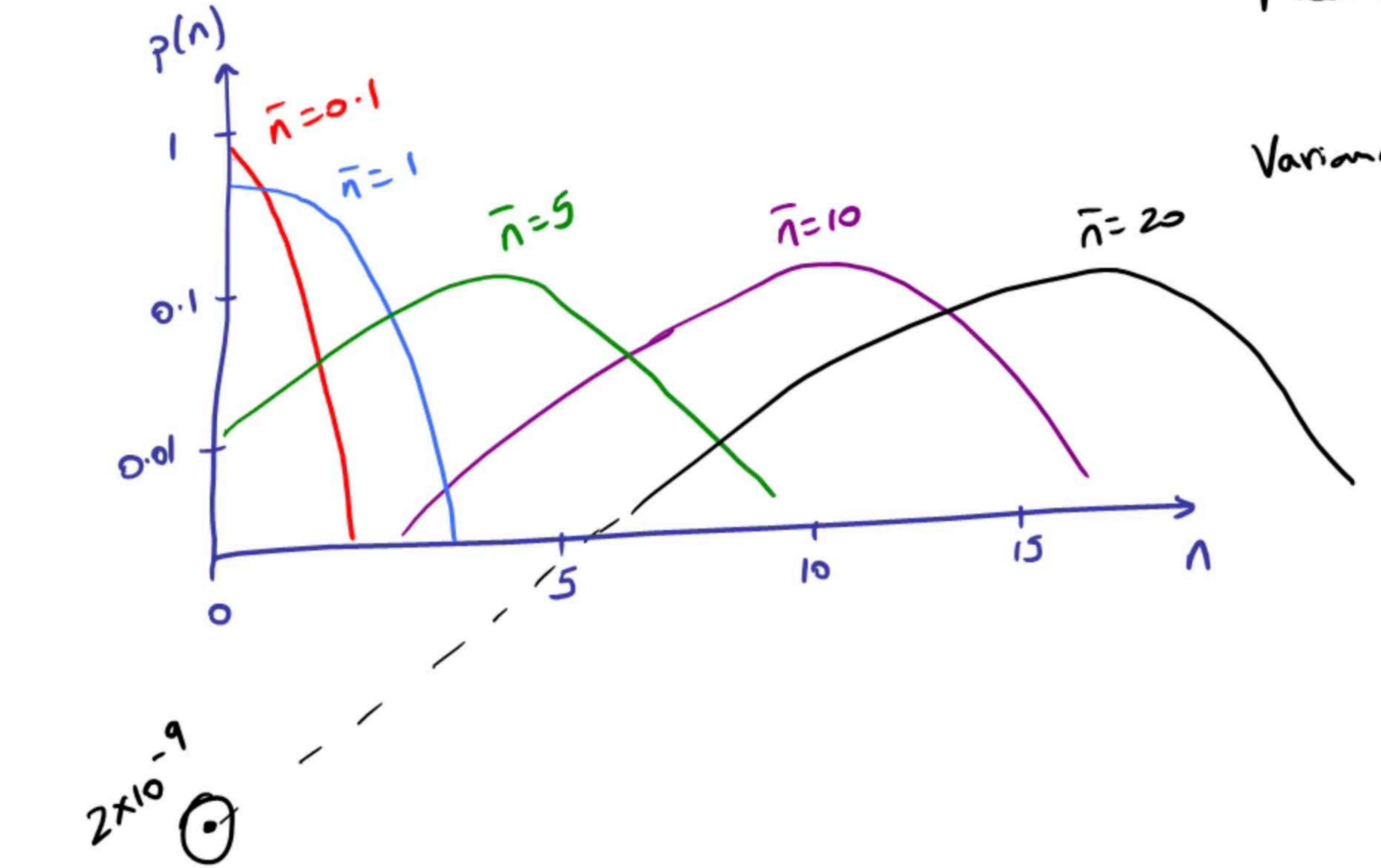


Probability of finding 'n' photons emitted w/ energy 'hν' $\Big\}$ $p(n) = \left[\exp\left(-\frac{h\nu}{K_B T}\right)\right]^n$

Since $\sum_{n=0}^{\infty} p(n) = 1 \Rightarrow$ $\boxed{p(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n}$

Bose-Einstein distribution

$$p(n) = \frac{(\bar{n})^n \exp(-\bar{n})}{n!}$$



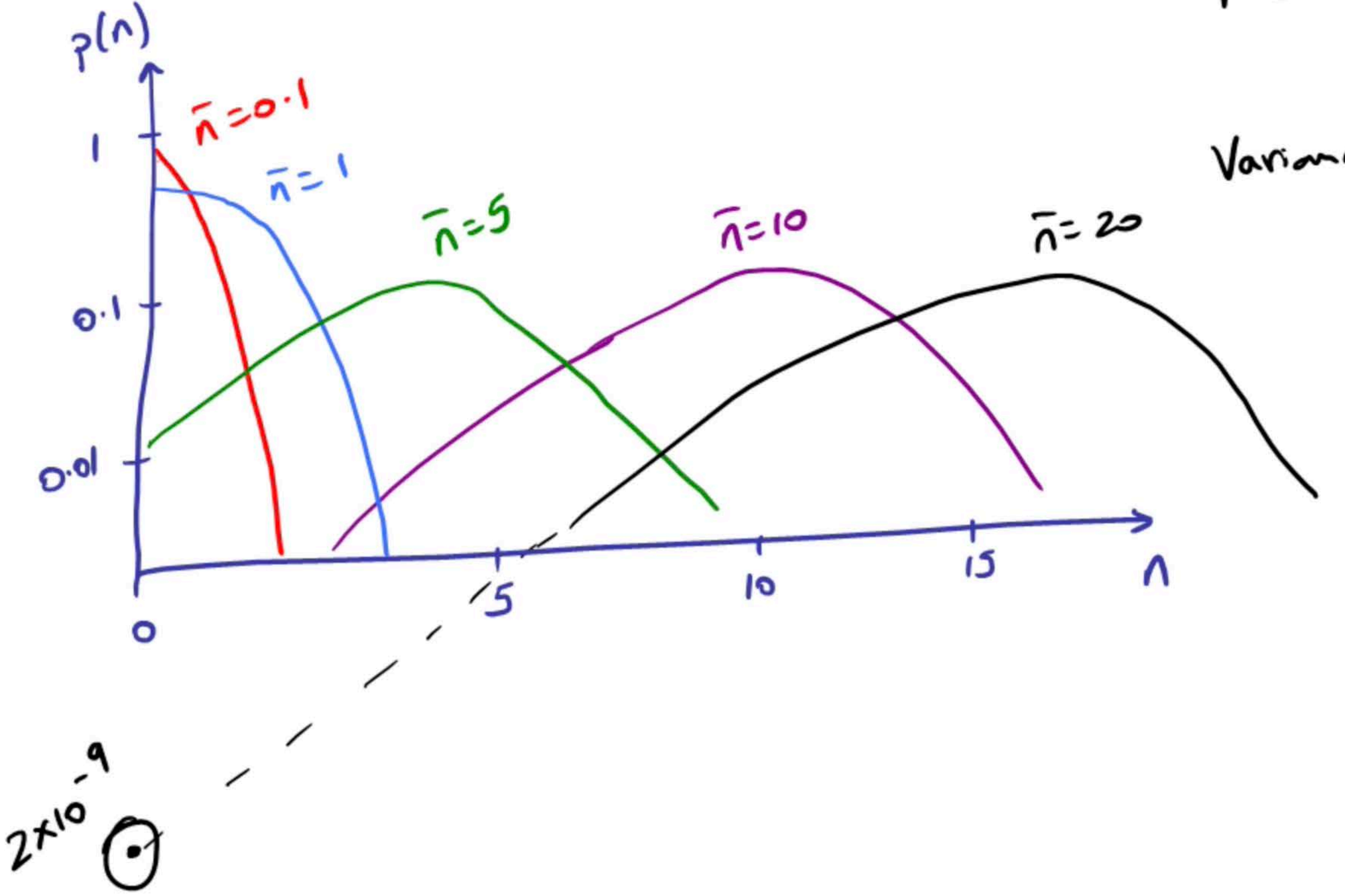
Mean, $\bar{n} = \sum_{n=0}^{\infty} n \cdot p(n)$

Variance, $\sigma_n^2 = \sum_{n=0}^{\infty} (n - \bar{n})^2 p(n)$

$= \bar{n}$ (Mean)

$$SNR = \frac{(\text{Mean})^2}{\text{Variance}} = \bar{n}$$

$$p(n) = \frac{(\bar{n})^n \exp(-\bar{n})}{n!}$$



Mean, $\bar{n} = \sum_{n=0}^{\infty} n . p(n)$

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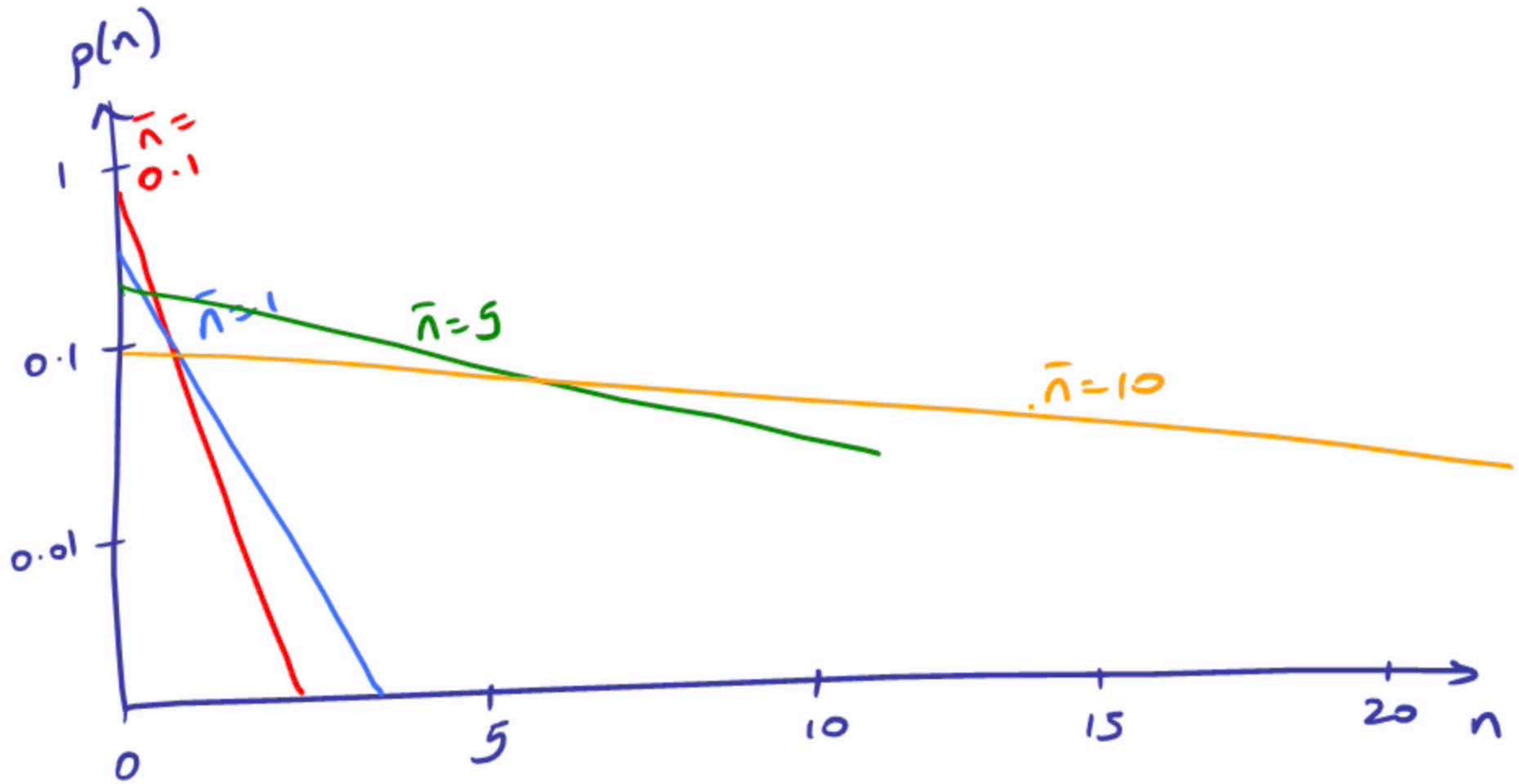
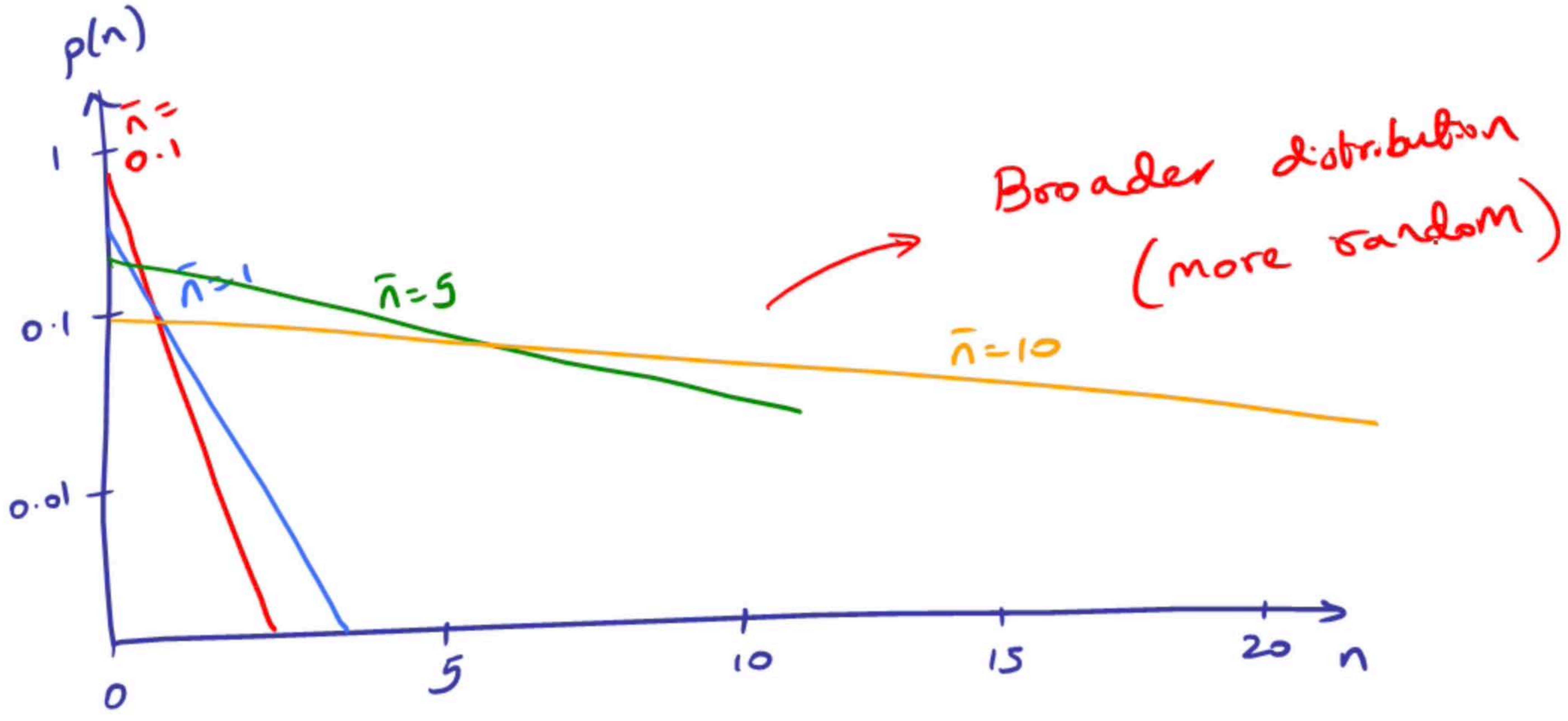
$$\text{SNR} = \frac{(\text{Mean})^2}{\text{Variance}} = \bar{n}$$

$$\boxed{p(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n}$$

where $\bar{n} = \dfrac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$



$$\boxed{p(n) = \frac{1}{\bar{n}+1}\left(\frac{\bar{n}}{\bar{n}+1}\right)^n}$$ where $$\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$



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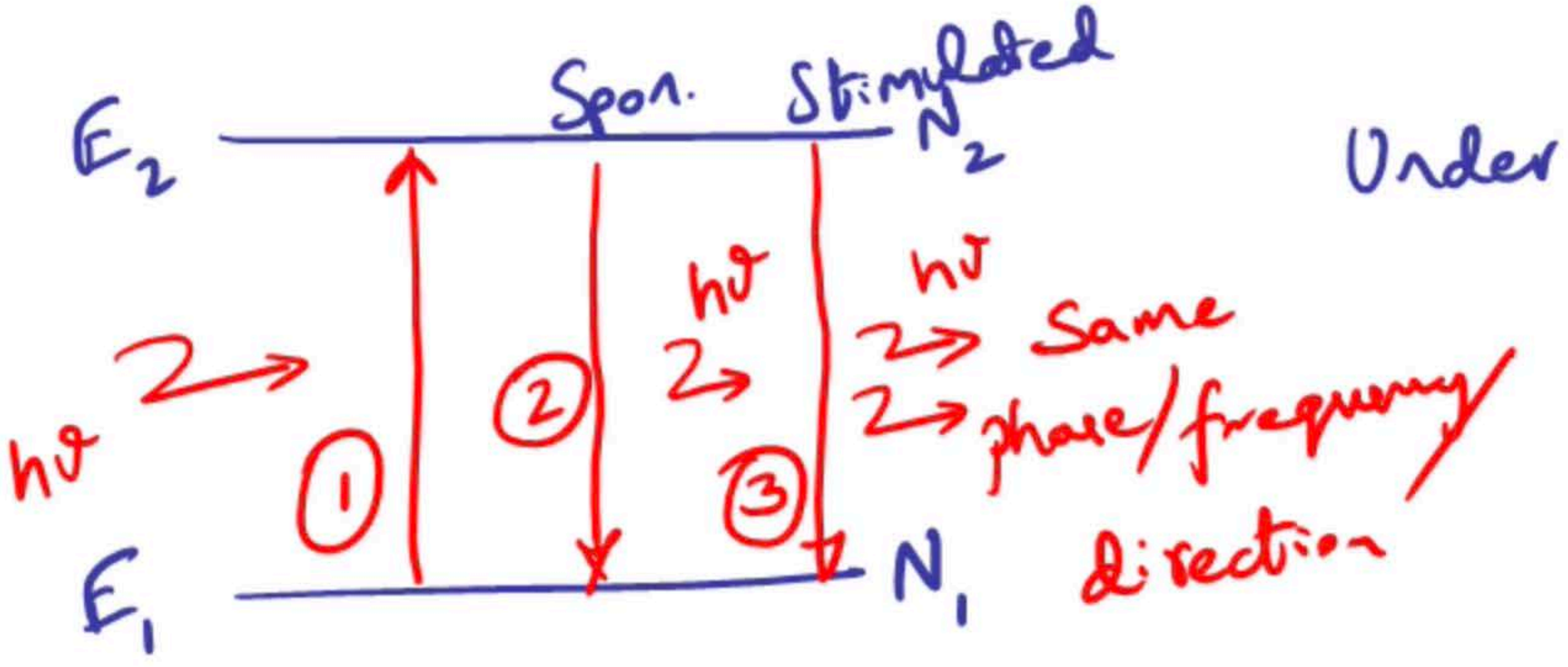

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$\Rightarrow$ Similar to Planck's ρ_{em} for blackbody radiation

Observation #2

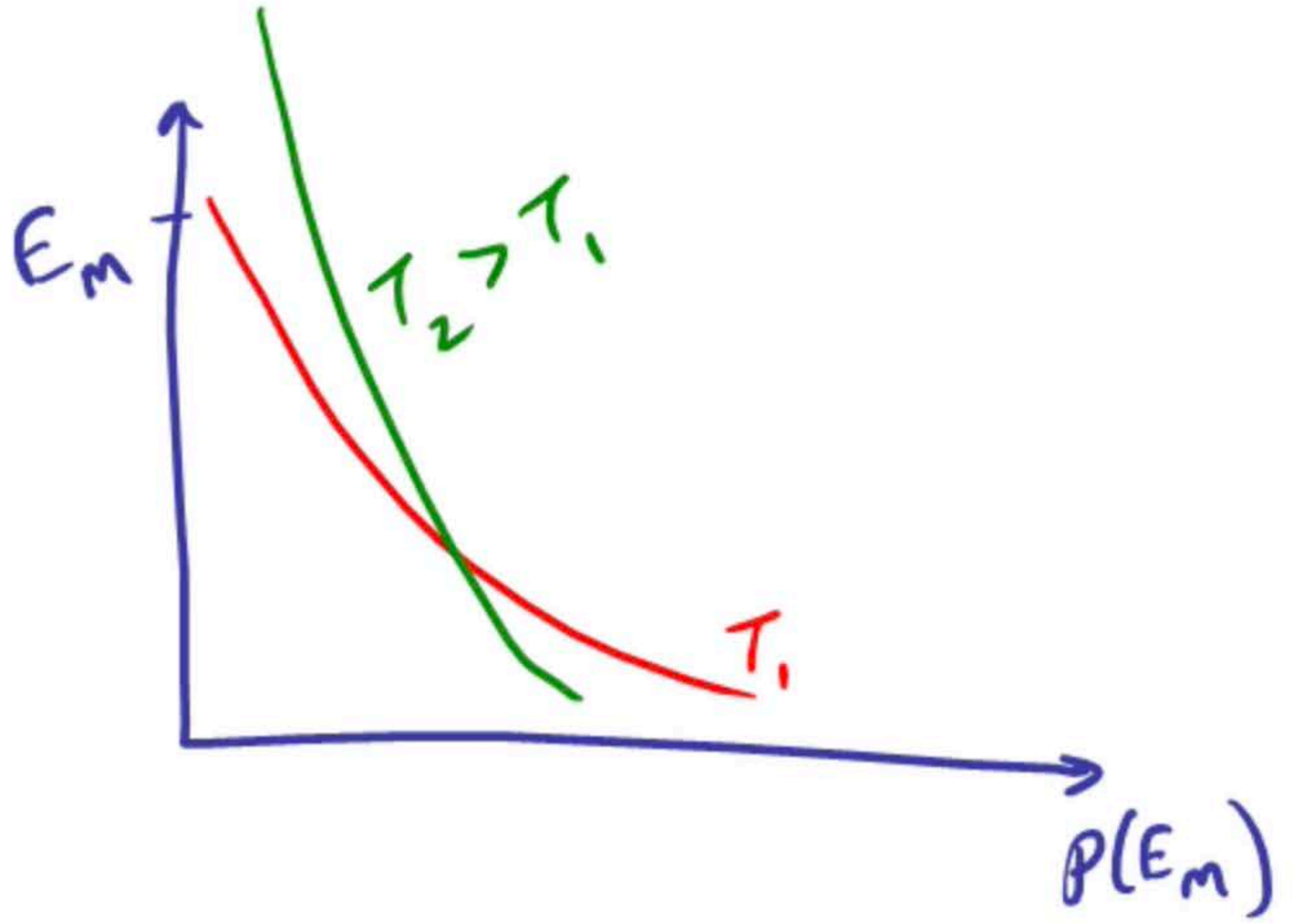
For optical radiation, $h\nu \simeq 1\,eV$

$k_B T = 25.6\ meV$ (room temperature)

$$\rho_{em} = \frac{A/B}{\frac{B'}{B}\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

$h\nu \gg k_B T$

$\rho_{em} \ll 1 \quad (R_{spon} \gg R_{stim})$



$\Rightarrow$ All sources that rely on stimulated emission (e.g. lasers) should operate well above thermal equilibrium.

Observation #3

For $R_{stim} > R_{spon}$

$N_2 > N_1$ (population inversion)

$\Rightarrow$ external pumping/excitation