EE 5500 – Introduction to Photonics
Quiz I

50 minutes, 15 points, closed book

Remember…

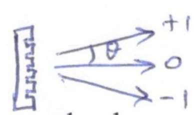
- Make reasonable assumptions wherever necessary, but you should show all steps and justify assumptions for full credit. Use of figures may attract bonus points.
- Final answers will be graded ONLY if they are entered in the space provided.

Objective Type Questions (5 points)

1. For a diffraction grating illuminated with white light, which colour would get diffracted more

   (a) blue (b) green (c) red (d) Cannot say

For a diffraction grating, $2d\sin\theta = m\lambda \Rightarrow \theta \propto \lambda$

$\Rightarrow$ 

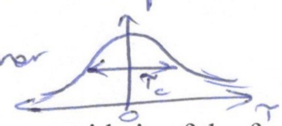
$\Rightarrow$ More diffraction for longer wavelengths

2. The temporal coherence of a light beam can be unambiguously measured using

   (a) Mach-Zehnder (b) Michelson (c) Double slit (d) All of the above

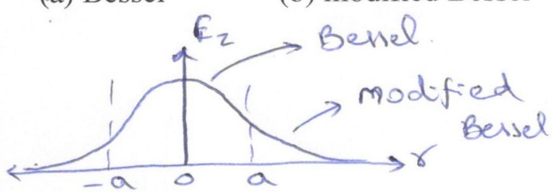
Temporal coherence, τ_c corresponds to FWHM of autocorrelation

$\Rightarrow$ Michelson/MZ would work; but former is simpler



3. The solution for E_z (propagation comp.) in the core of a dielectric waveguide is of the form

   (a) Bessel (b) modified Bessel (c) sine (d) cosine



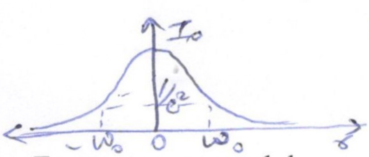
$\nabla^2\vec{E} + k^2\vec{E} = 0$

Solution in cylindrical coordinates is Bessel in core region

4. The probability of detecting a photon at a point on a circle whose radius is twice the waist radius (W_0) of a Gaussian beam is proportional to

   (a) $1/e$ (b) $1/e^2$ (c) $1/e^4$ (d) $1/e^8$

Probability of finding a photon within an area $p(r)\,dA \propto I(r)\,dA$



$I(r) = I_0 \exp\left(-\frac{2r^2}{w_0^2}\right)$

If $r = 2w_0$

$I(r) \propto 1/e^8$

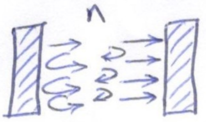
5. For a nanosecond laser at 1 um wavelength emitting pulses with picoJ energy, the mean number of photons within one pulse is 5 Million

$$\text{Mean \# of photons per pulse} = \frac{\text{Pulse energy}}{\text{light energy } (h\nu)} = \frac{1\times10^{-12}}{1.24\times1.602\times10^{-19}} \simeq \underline{5\times10^{6}}$$

For $\lambda = 1\mu m$, $h\nu = 1.24$ eV

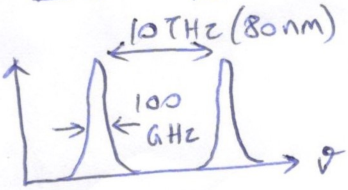
Quantitative Problems (10 points)

6. It is required to resolve two frequency components ν_1 and ν_2 spaced 100 GHz apart using a Fabry-Perot interferometer whose free spectral range corresponds to 80 nm around 1550 nm wavelength. (6 pts)
   a. From first principles, derive an expression for the transmission of the Fabry Perot interferometer. Assume negligible loss due to scattering or absorption in the cavity.
   b. Determine the construction of the interferometer (cavity length, mirror reflectivity) such that it meets the above requirements and plot the corresponding transmission as a function of frequency (clearly label all relevant quantities)
   c. Assuming we use dielectric mirrors for the above interferometer, what should be the structure of the mirrors. Use $n_H = 2.32$, $n_L = 1.38$.

(a) 

$$\longrightarrow I_t = \frac{I_{max}}{1+\left(\frac{2f}{\pi}\right)^2 \sin^2(\phi/2)}$$ where $f = \frac{\pi\sqrt{R}}{1-R}$

$$\phi = \frac{2\pi}{\lambda}\cdot 2nL$$

(b) 

Finesse, $f = \frac{\Delta\vartheta_{FSR}}{\delta\vartheta_{FWHM}} = \frac{10\times10^{12}}{100\times10^{9}} = 100$

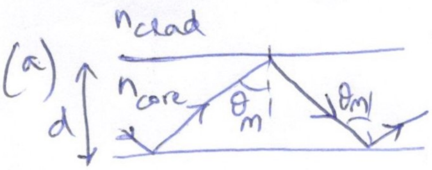
$\Rightarrow R = \underline{96.6\%}$

Assuming $n=1$, $\Delta\vartheta_{FSR} = \frac{c}{2L} \Rightarrow L = \frac{3\times10^8}{2\times10\times10^{12}} = \underline{15\ \mu m}$

(c) 

$d_H = \frac{1550\times10^{-9}}{4\times2.32} = 168\ nm$ $d_L = 280\ nm$ $N = 4$

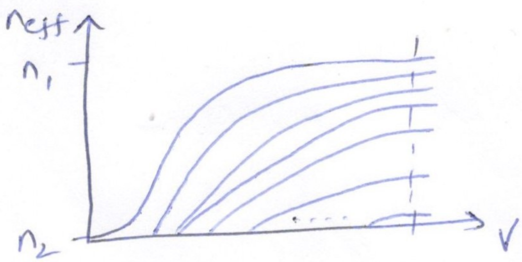
7. Suppose you need to send information over a step-index multimode optical fiber through a laser beam launched into the fiber. (4 pts)
   a. Neglecting the phase change upon reflection at the core-cladding boundary, derive an expression for the phase matching condition of the different modes.
   b. Determine the highest bandwidth that can be supported by the optical fiber with the following parameters: $n_{core} = 1.453$, $n_{clad} = 1.450$, core dia = 50 microns, length = 100 m. Hint: find the inter-modal delay between the different modes

(a) 

$$2K_x d = 2\pi m$$

$$2\frac{2\pi}{\lambda} n_1 d\cos\theta_m = 2\pi m \Rightarrow \boxed{\cos\theta_m = \frac{m\lambda}{2n_1 d}}$$

(b) For a multimode fiber



Highest BW $= \frac{1}{\Delta\tau_{IM}}$

where $\Delta\tau_{IM}$ = Propagation time for fundamental mode − Prog. time for highest order mode

$$\simeq \frac{L}{c}\cdot n_1 - \frac{L}{c}\cdot n_2 = \frac{L}{c}(n_1 - n_2) = \underline{1\ ns}$$

Useful Constants:
Planck's constant (h) = 6.6256×10^{-34} J.s,
Boltzmann's constant (k_B) = 1.38×10^{-23} J/K
Electric charge (q) = 1.602×10^{-19} C,
Velocity of light (c) = 3×10^8 m/s

Useful Formulae
1. MFD = 2a . [$0.65 + (1.879 / V^{1.5}) + (2.7 / V^6)$]
2. Finesse, F = $\pi\sqrt{R} / (1-R)$
3. Refl. coeff, R = $1 - (n_H/n_L)^{2N} / 1 + (n_H/n_L)^{2N}$

If $\Delta\tau_{IM} = 1\,ns$, highest BW = $\underline{1\ GHz}$