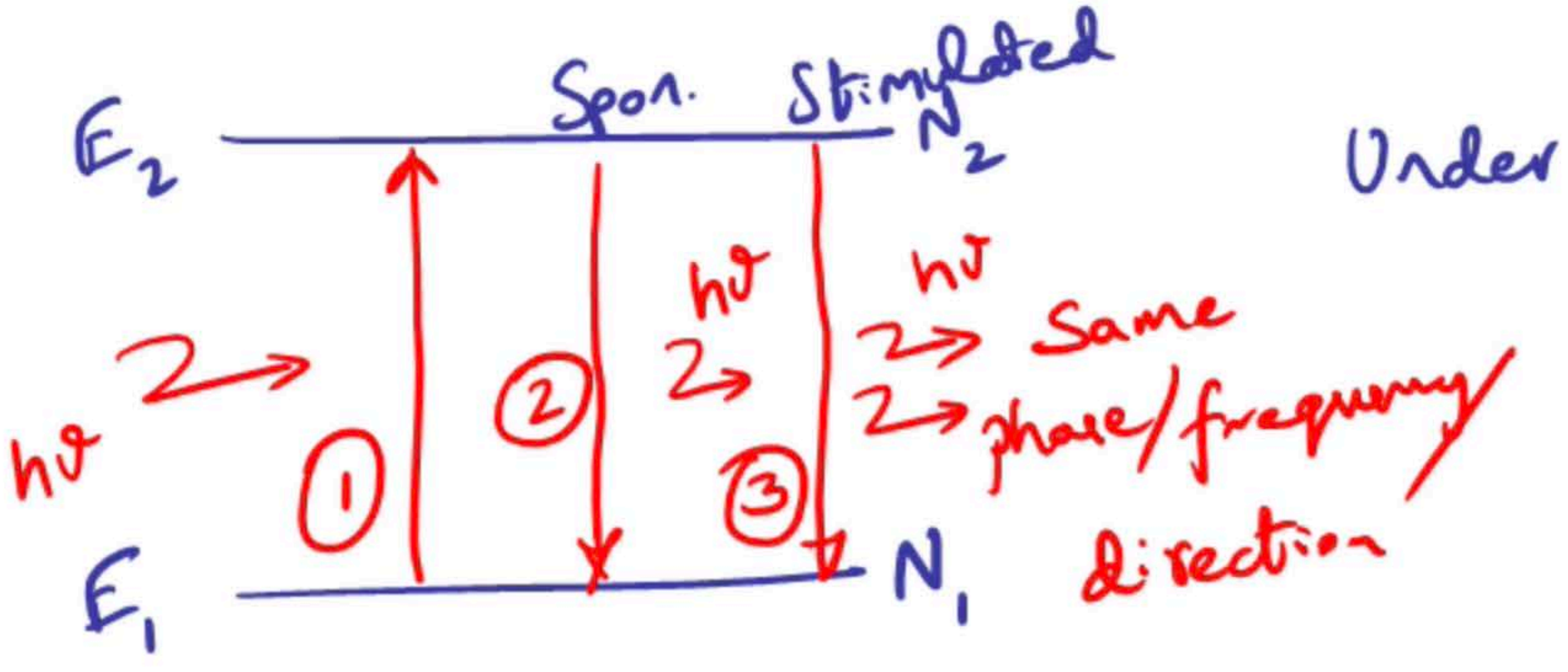
Learning Objective: Identify the fundamental principles of photon interaction w/ atoms

Analyze light generation and amplification



Under steady state conditions,

$$R_{abs} = R_{spon} + R_{stim}$$

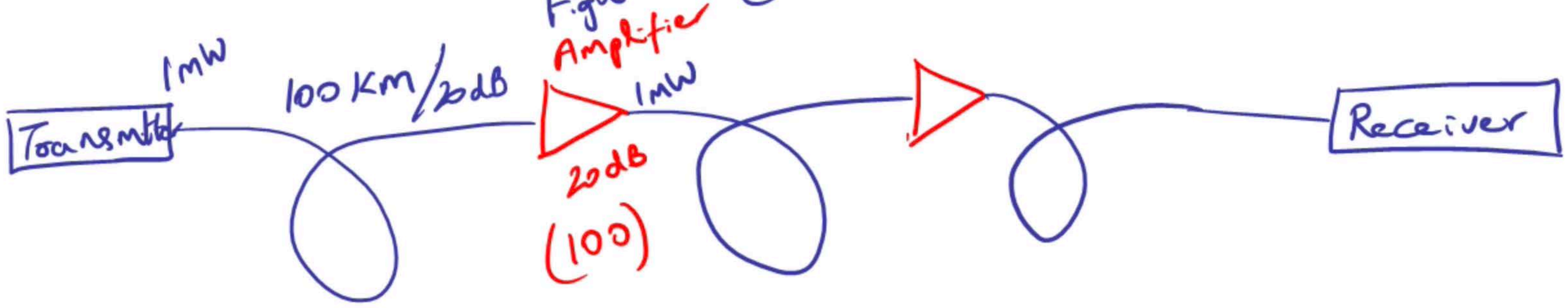
$$B' N_1 \rho_{abs} = A N_2 + B N_2 \rho_{em}$$

If $\rho_{em} = \rho_{abs}$, $\quad \rho_{em} = \dfrac{A N_2}{B' N_1 - B N_2} = \dfrac{A/B}{\frac{B'}{B} \cdot \frac{N_1}{N_2} - 1}$

$\Rightarrow$ Similar to Planck's ρ_{em} for blackbody radiation

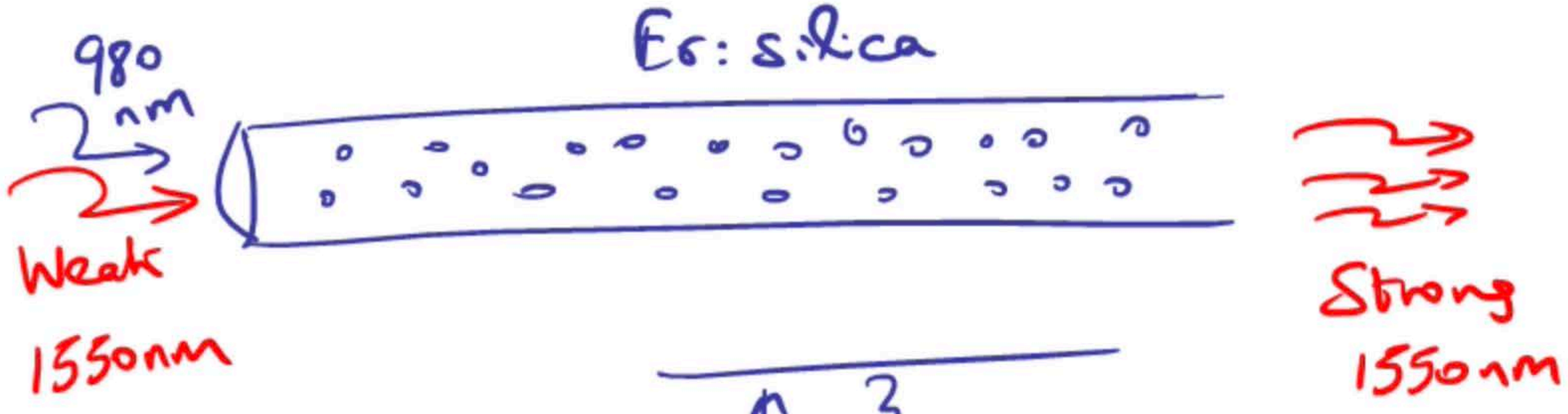
Example: Optical Fiber Communication System

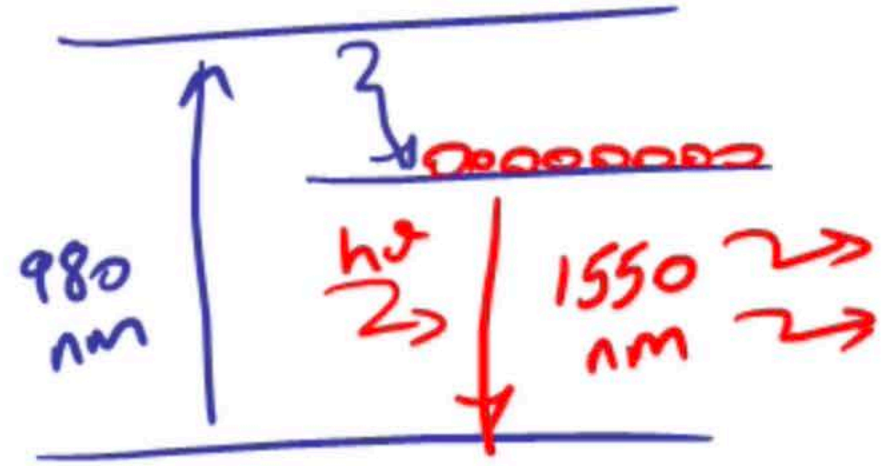
$$\text{Noise figure} = \frac{(SNR)_{IN}}{(SNR)_{OUT}}$$

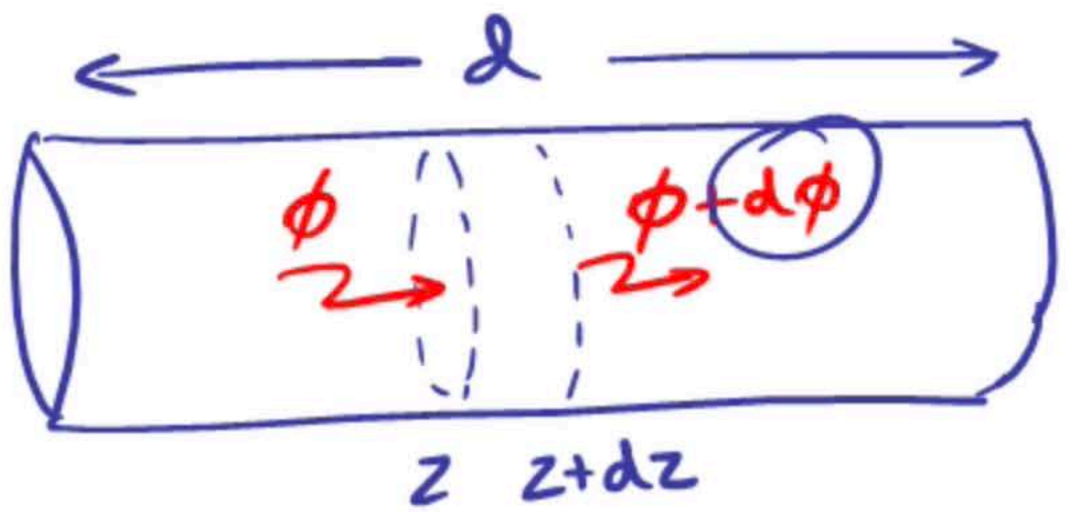


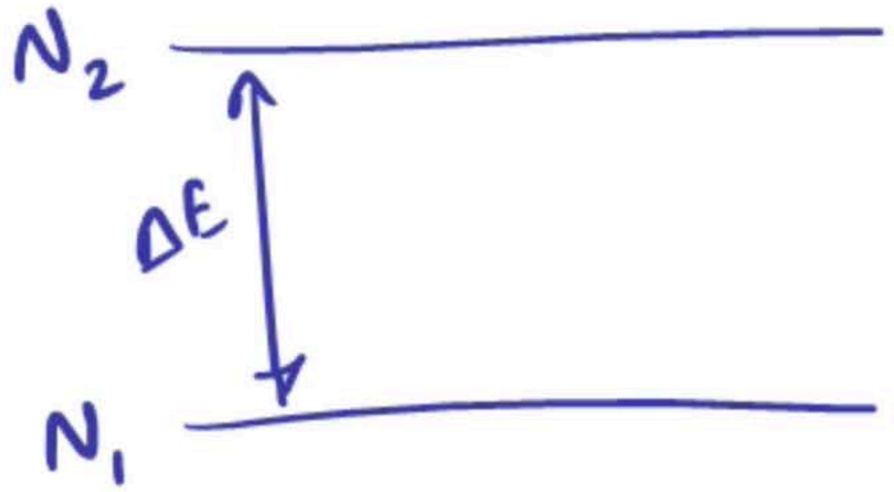
Attenuation 0.2 dB/km

@ $\lambda = 1550$ nm









Assume spontaneous emission is negligible

Absorption @ rate $N_1 W_i$ where $W_i = \phi\, \sigma(\nu)$

Transition cross-section

$$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}}\, g(\nu)$$

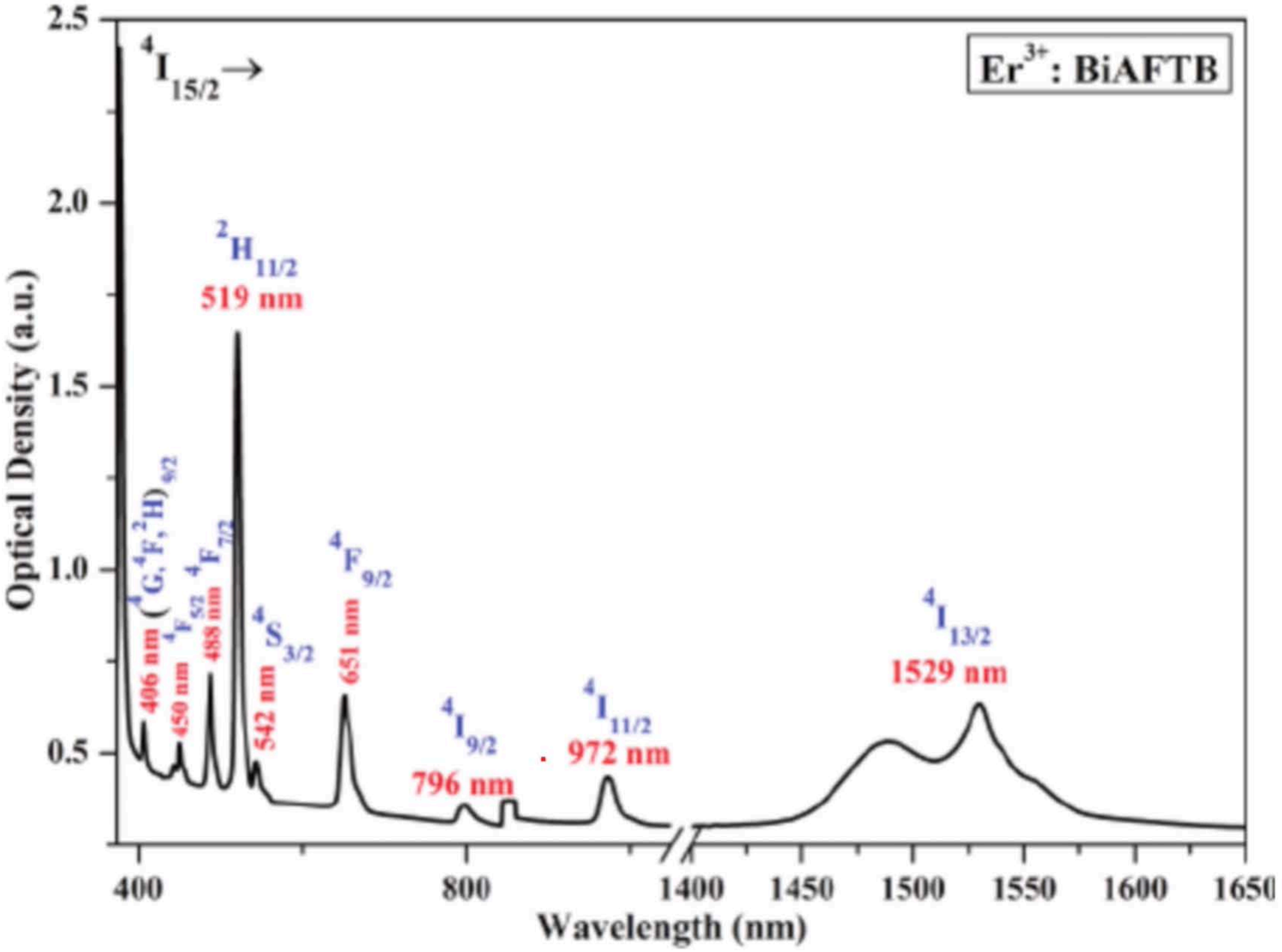
Stimulated emission @ rate $N_2 W_i$

Net Flux, $d\phi = N \cdot W_i\, dz$ where $N = N_2 - N_1$

$N\sigma$

$$\frac{d\phi}{dz} = N \cdot \phi(z) \cdot \sigma(\nu) \Rightarrow \phi(z) = \phi(0) \cdot \exp[\gamma(\nu) \cdot z]$$

$$\boxed{\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[\gamma(\nu) \cdot d]}$$



Er3+: BiAFTB
4I15/2→
2H11/2
519 nm
(4G, 4F, 2H)9/2
406 nm
4F5/2
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542 nm
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651 nm
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796 nm
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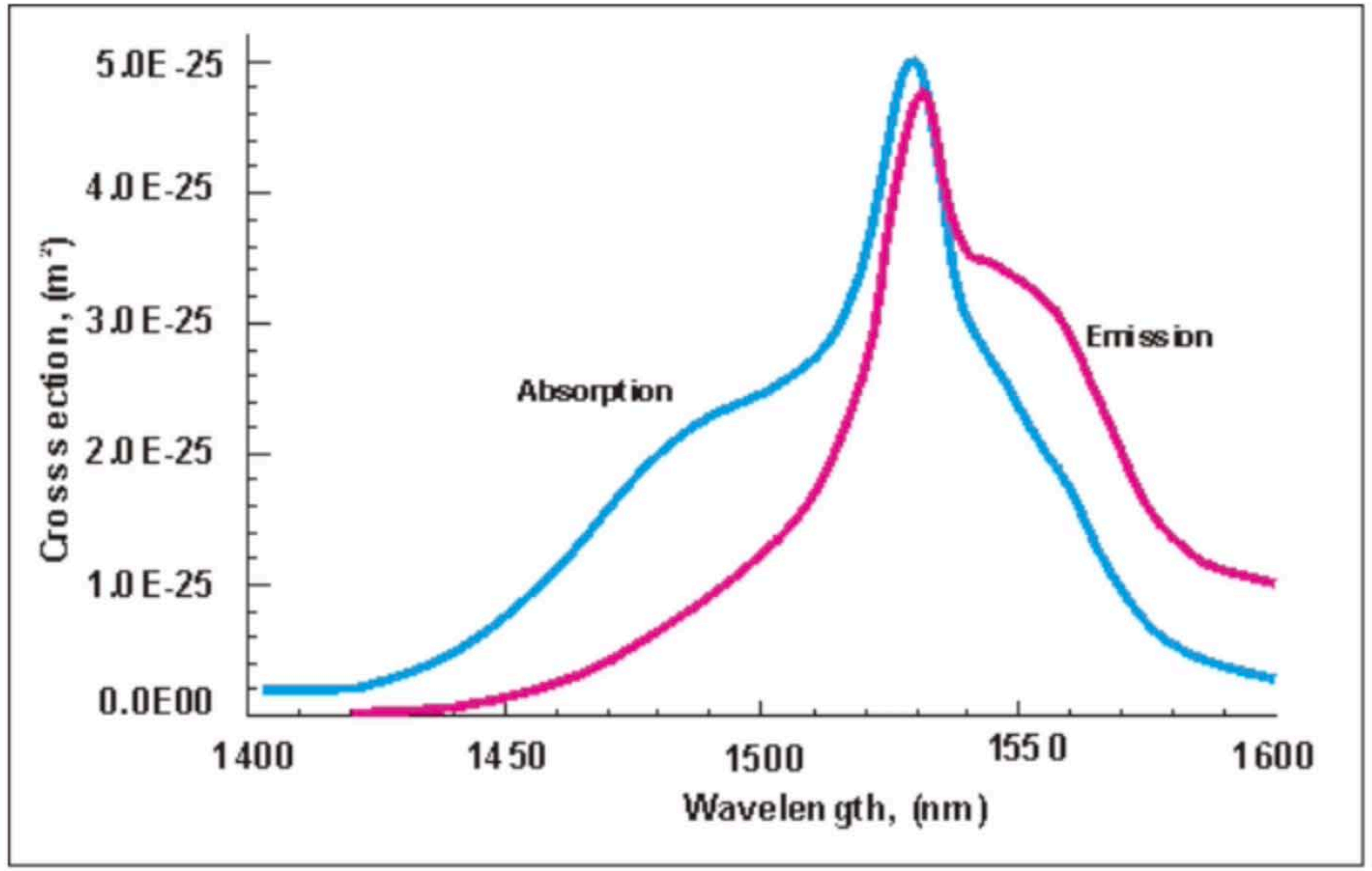
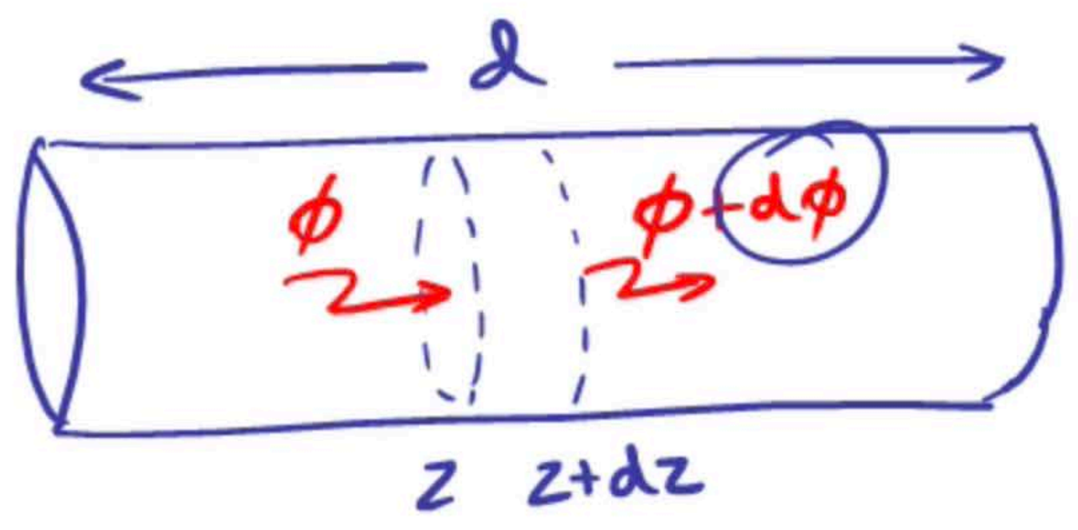
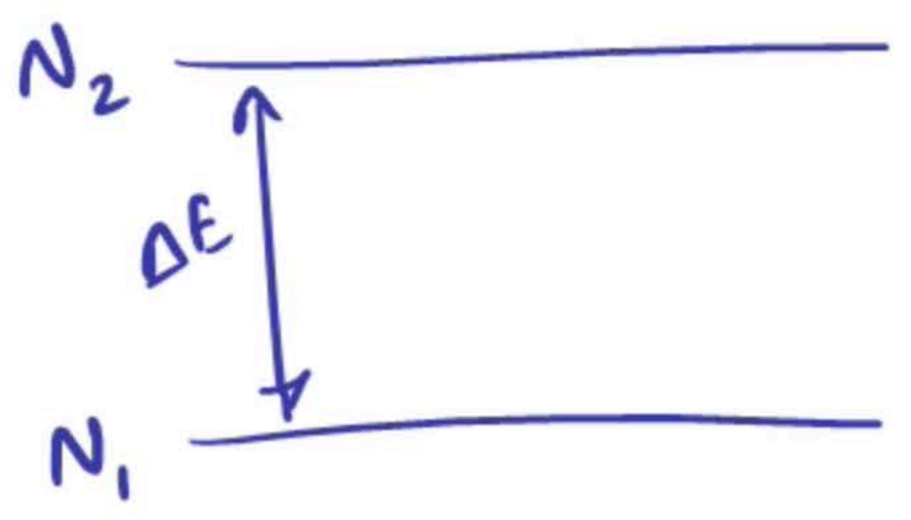
972 nm
4I13/2
1529 nm
Optical Density (a.u.)
Wavelength (nm)



Absorption
Emission
Cross section, (m2)
Wavelength, (nm)





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At $\lambda = 1550$ nm,

$$\sigma_e = 6.5 \times 10^{-25} \; m^2$$

If $N_t = 1 \times 10^{24} \; m^{-3}$, 80% of Er ions are in excited state

$$\Rightarrow N_2 = 0.8 \times 10^{24} \; m^{-3}$$

$$N_1 = 0.2 \times 10^{24} \; m^{-3}$$

$$N = N_2 - N_1 = 0.6 \times 10^{24} \; m^{-3}$$

$$\Rightarrow \gamma_{1550} = 0.6 \times 10^{24} \times 6.5 \times 10^{-25}$$

$$\simeq \underline{\underline{0.4 \; m^{-1}}}$$

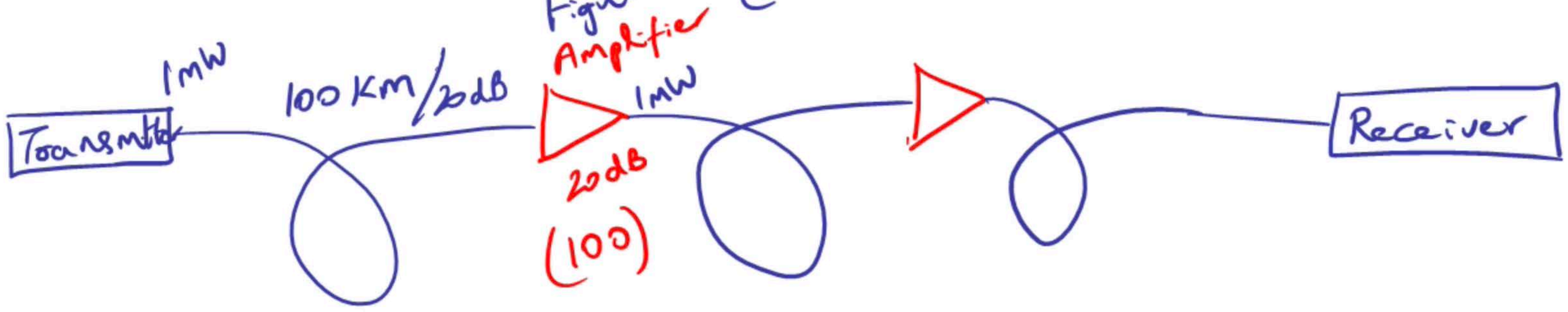
Gain = 20 dB (100)

$$100 = \exp\left[0.4\, d\right]$$

$$d = \frac{1}{0.4} \ln(100) = \underline{\underline{11.5 \; m}}$$

Example: Optical Fiber Communication System

$$\text{Noise Figure} = \frac{(SNR)_{IN}}{(SNR)_{OUT}}$$



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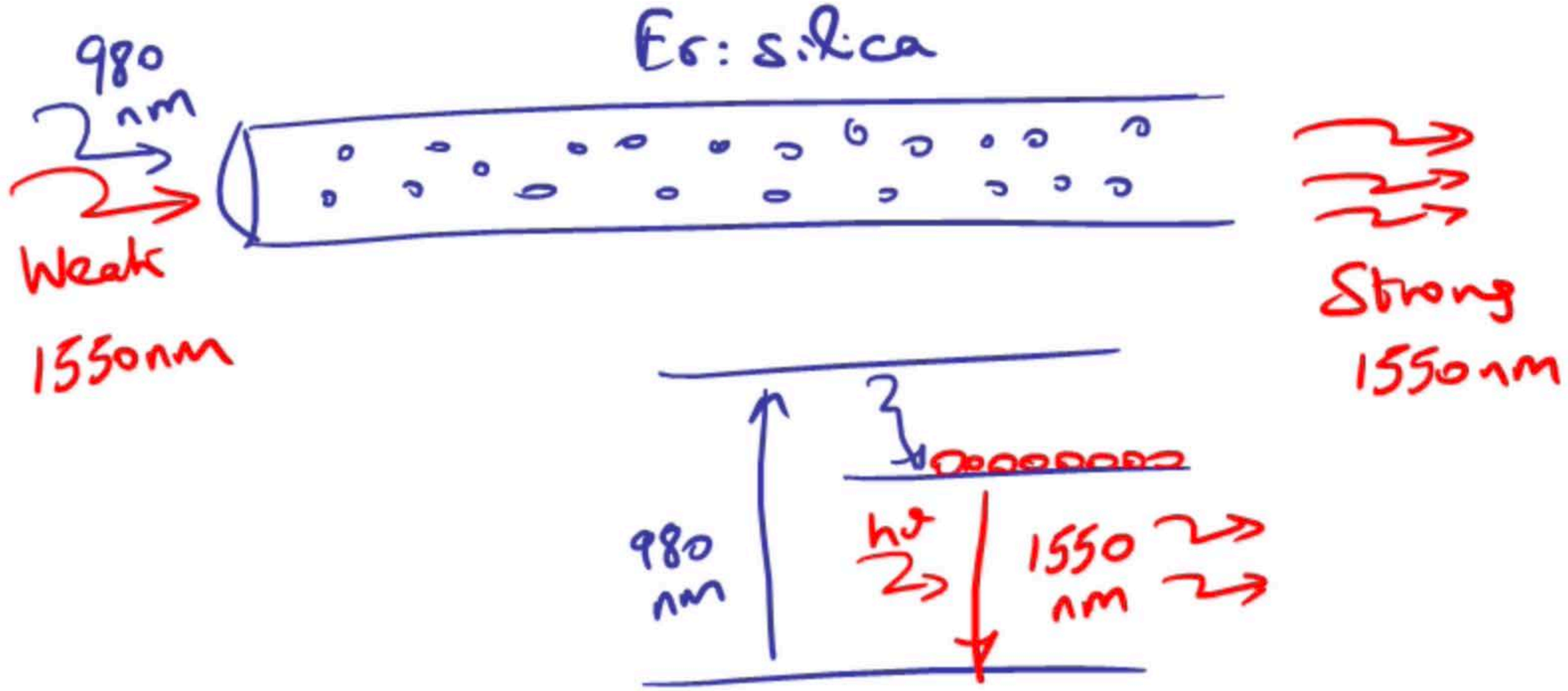
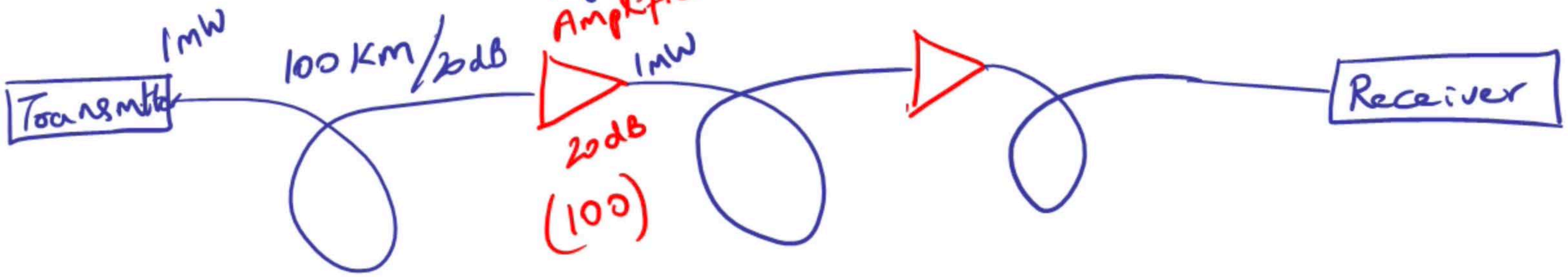


Example: Optical Fiber Communication System

Noise figure $= \dfrac{(SNR)_{IN}}{(SNR)_{OUT}}$



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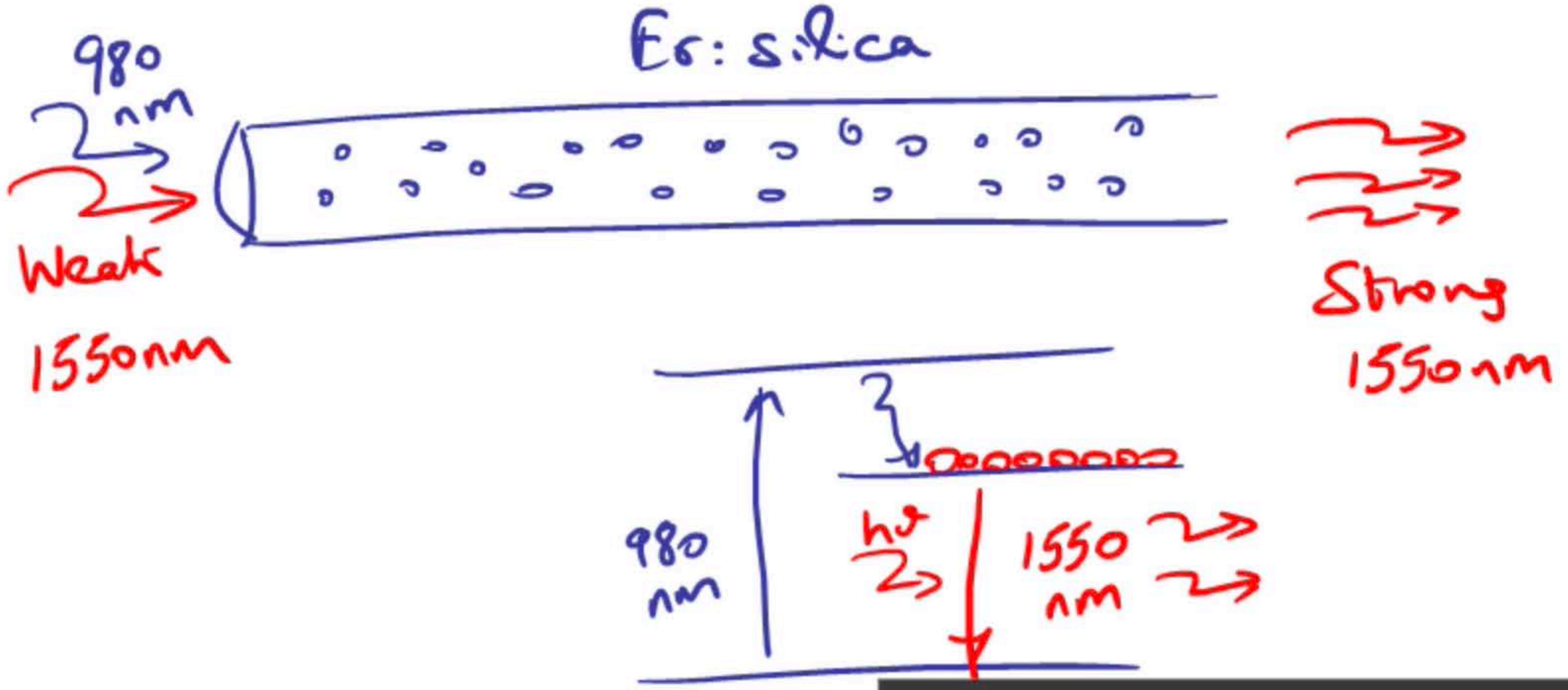
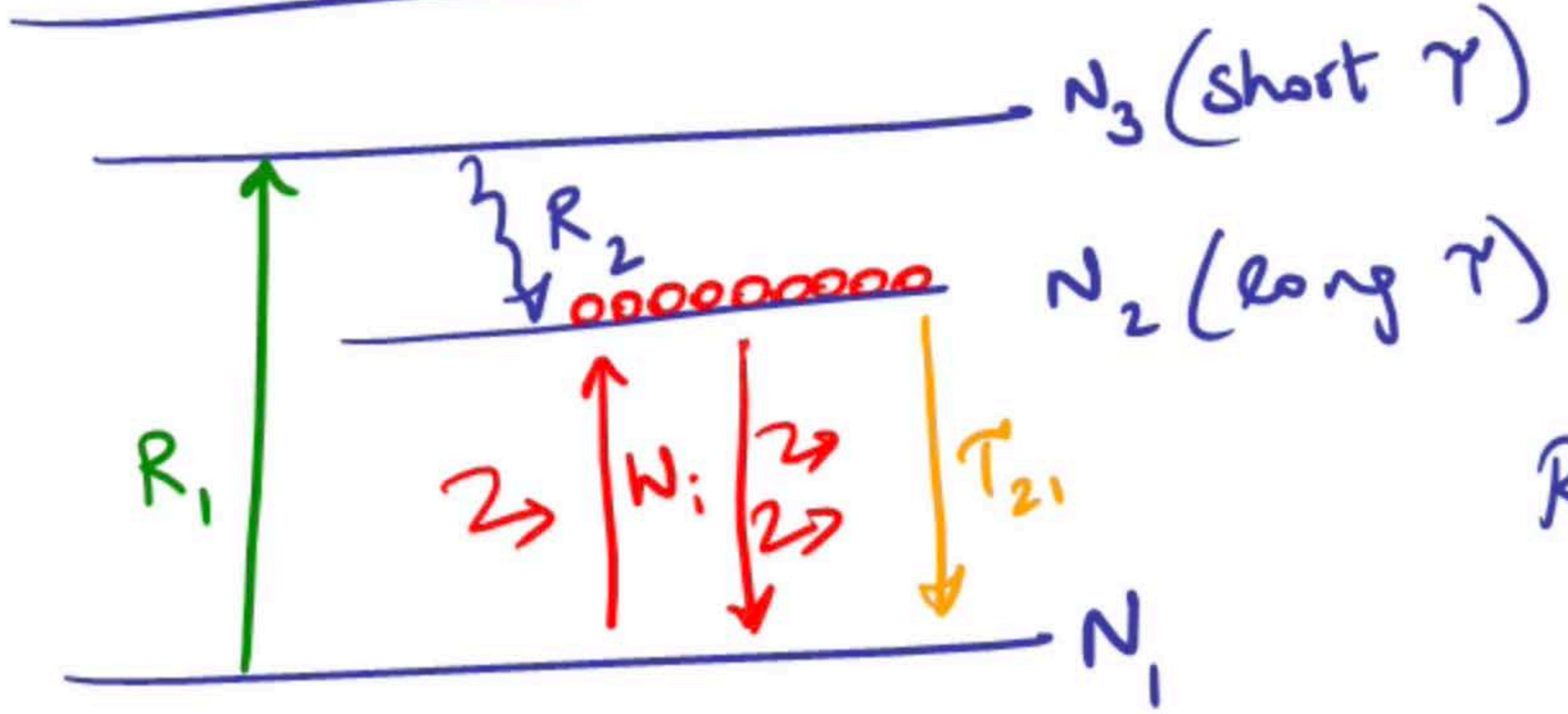


Three - level system:



$\longrightarrow R_1 = R_2 = R$

Rate equation (@ steady state) $\frac{dN_2}{dt} = 0$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_i + N_1 W_i = 0$$

$$(N_2 - N_1) W_i = R - \frac{N_2}{\tau_{21}}$$

Total number density of dopants

$$N_a = N_1 + N_2 + \cancel{N_3}$$

$$N = N_2 - N_1$$

$$= 2N_2 - N_a \Rightarrow N_2 = \frac{1}{2}(N + N_a)$$

$$\longrightarrow N W_i = R - \frac{1}{2\tau_{21}}(N + N_a)$$

$$N W_i = R - \frac{1}{2\tau_{21}}(N+N_a) \quad \Rightarrow \quad N = \frac{2\tau_{21}R - N_a}{2\tau_{21}W_i + 1}$$

$$\boxed{N = \frac{N_0}{1+\tau_s W_i}}$$

where $\tau_s = 2t_{sp}$

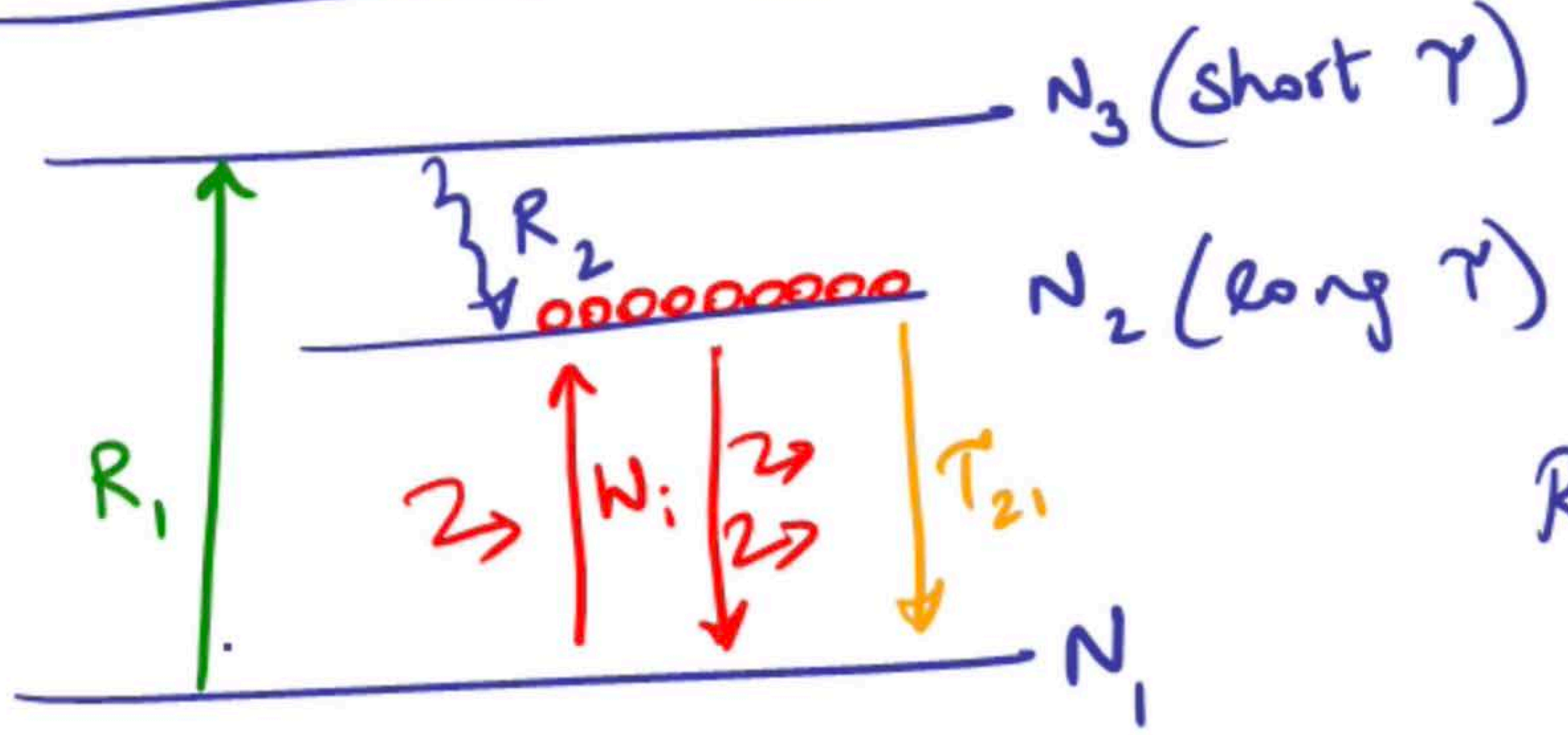
$N_0 = 2Rt_{sp} - N_a$

Gain Saturation

$W_i \rightarrow \phi\sigma$

flux density of incoming signal photons

Three - level system:



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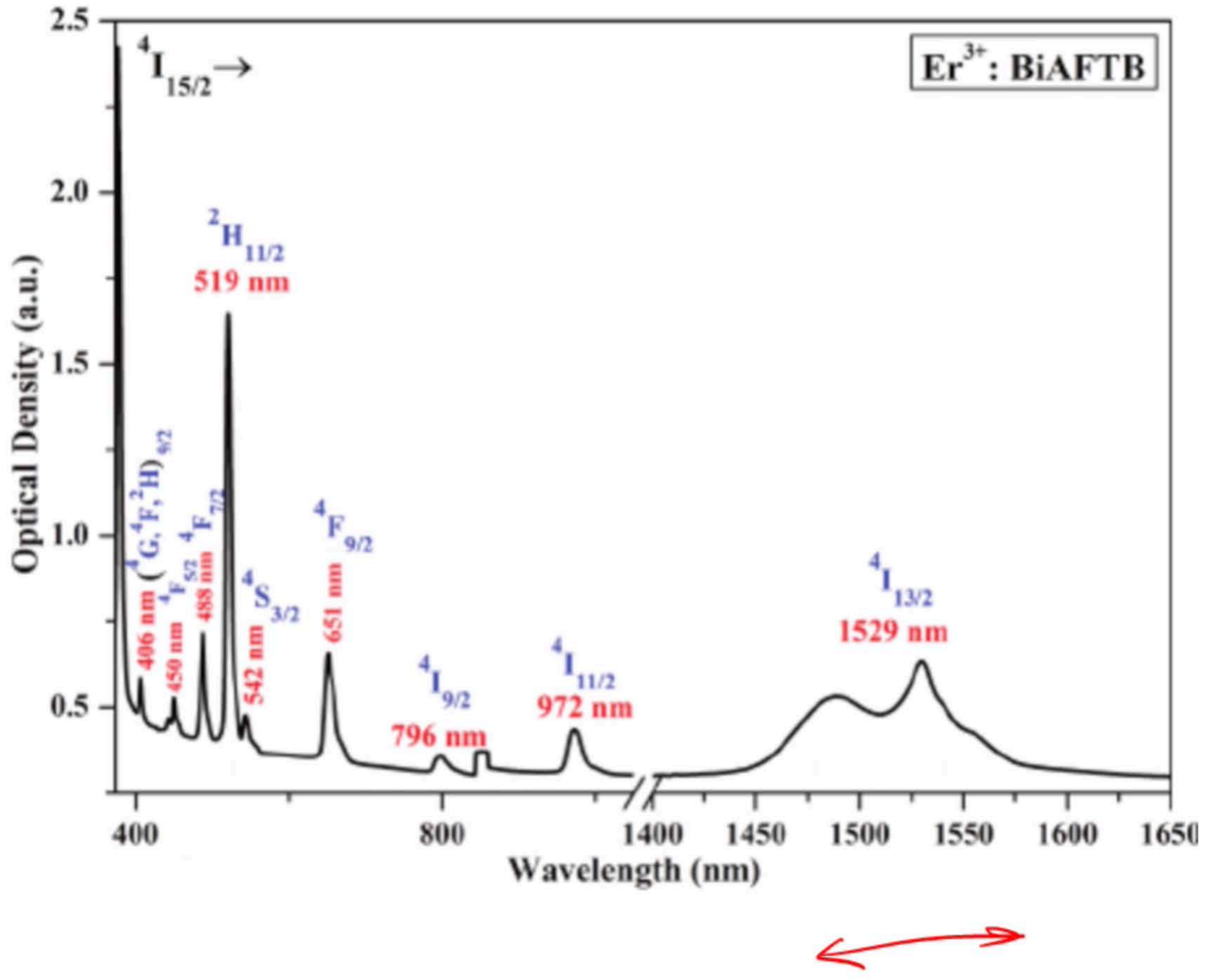

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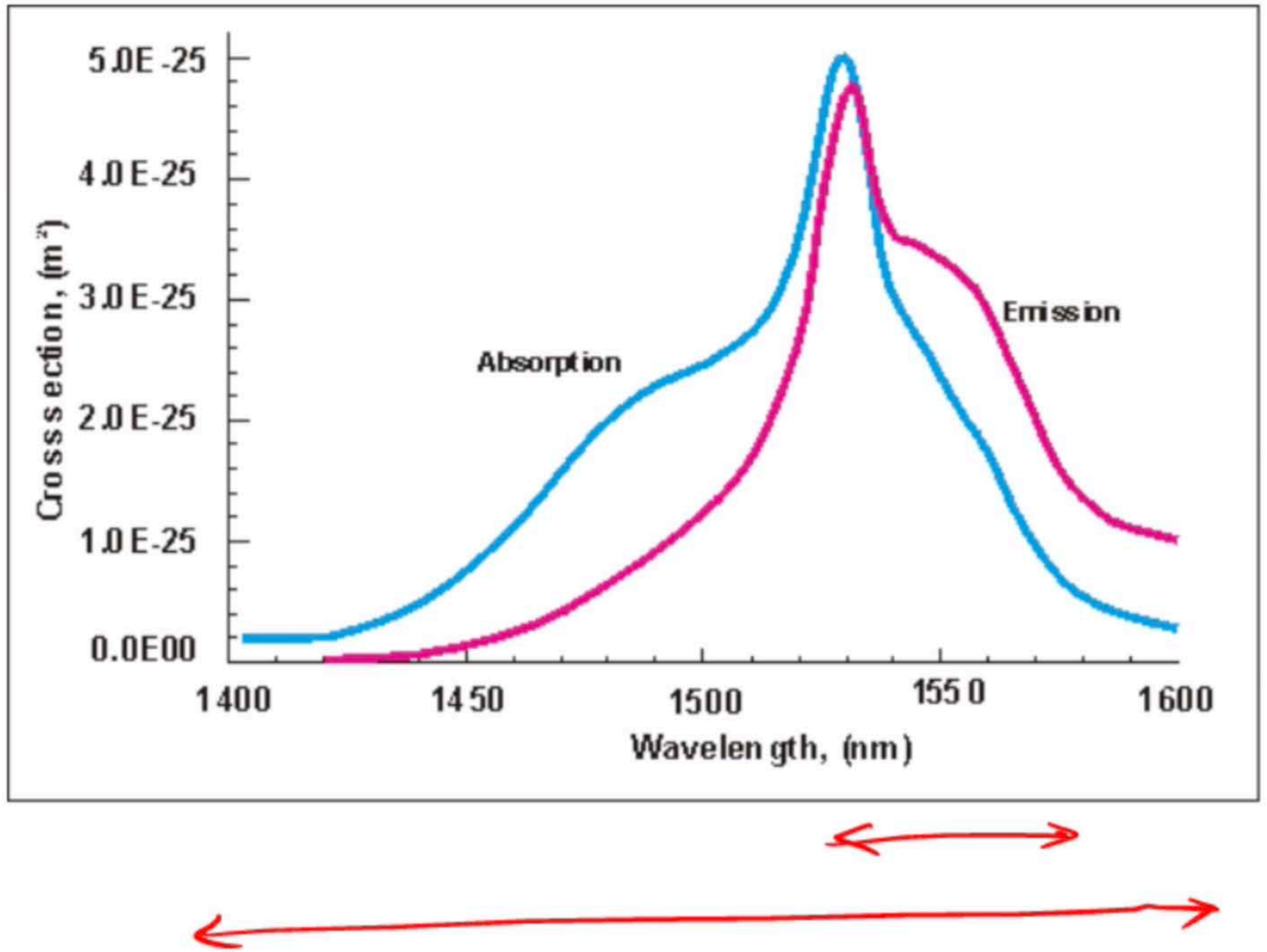

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Wavelength, (nm)
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