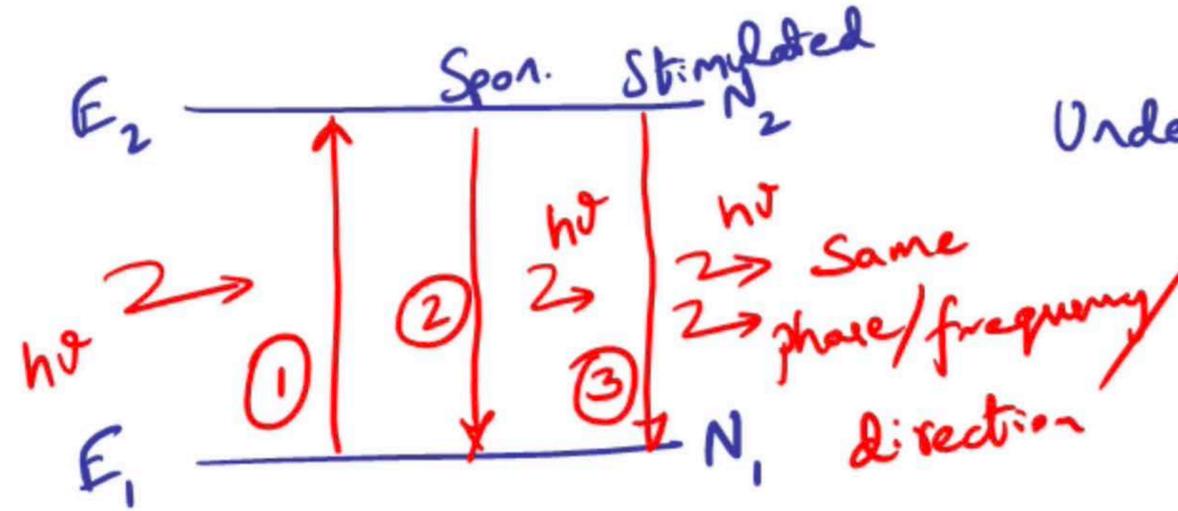


Analyze **light generation** and **amplification**



Under steady state conditions,

$$R_{abs} = R_{spont} + R_{stim}$$

$$B' N_1 P_{abs} = A N_2 + B N_2 P_{em}$$

If $P_{em} = P_{abs}$,

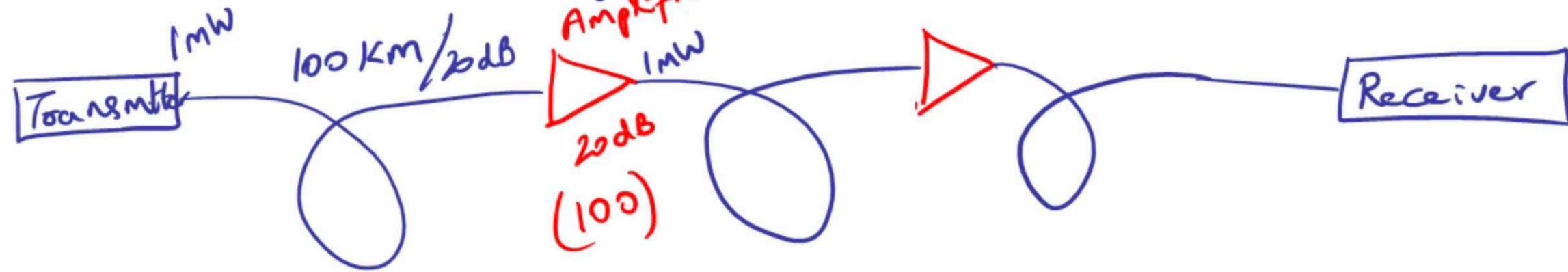
$$P_{em} = \frac{A N_2}{B' N_1 - B N_2} = \frac{A/B}{B'/B \cdot \frac{N_1}{N_2} - 1}$$

\Rightarrow Similar to Planck's P_{em} for blackbody radiation

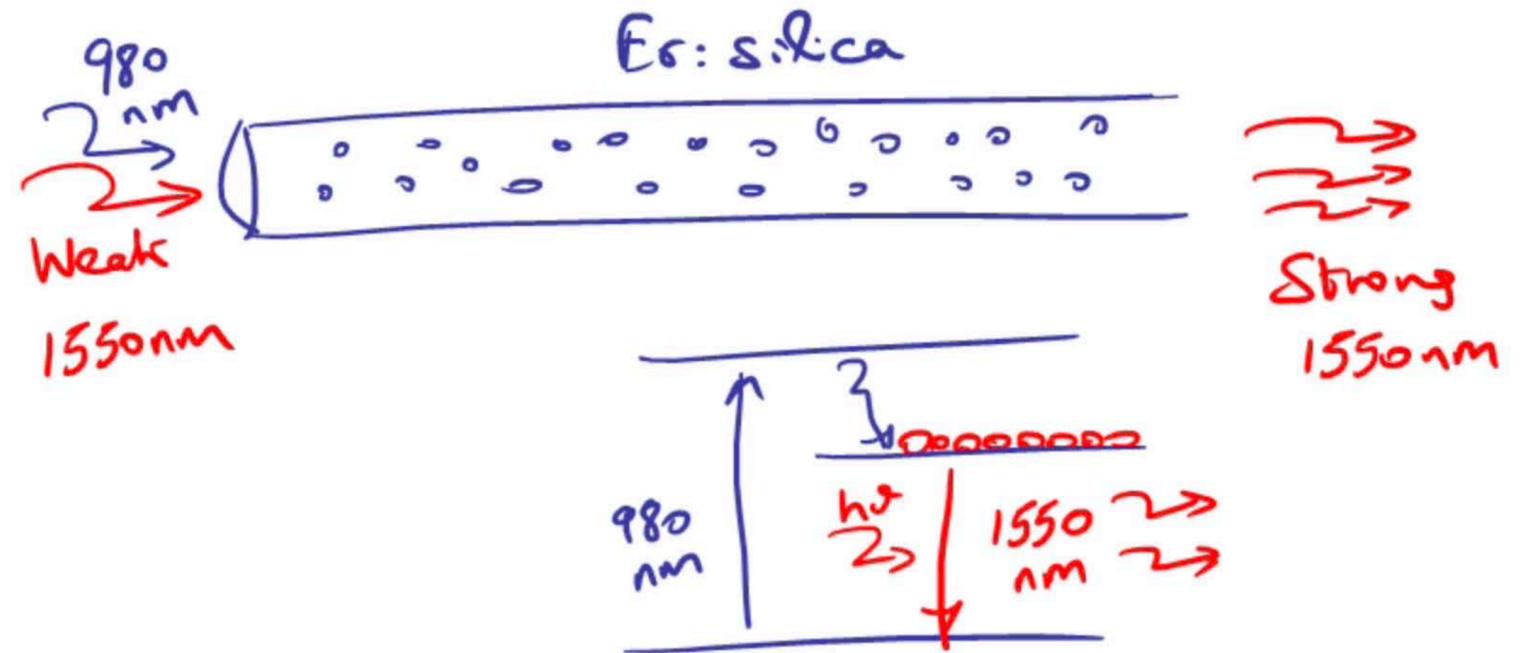
Example:

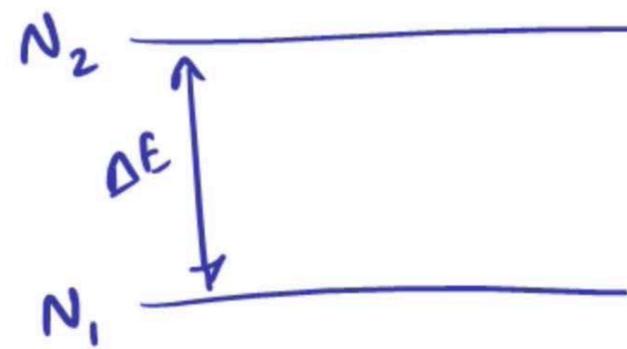
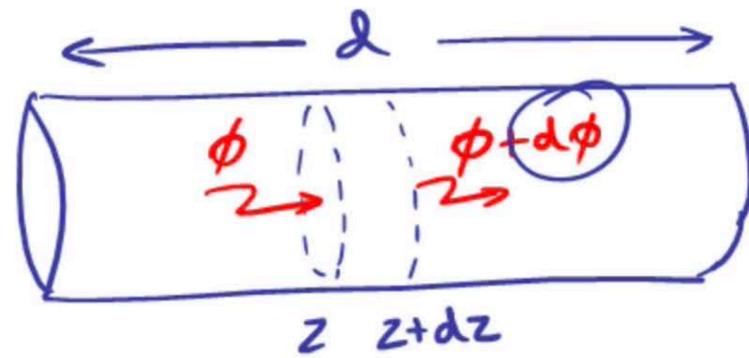
Optical Fiber Communication System

$$\text{Noise Figure} = \frac{(SNR)_in}{(SNR)_out}$$



Attenuation 0.2 dB/km
Q $\lambda = 1550 \text{ nm}$





Assume
spontaneous emission
is negligible

Absorption @ rate $N_1 W_i$

where $W_i = \phi \sigma(\nu)$

Stimulated emission @ rate $N_2 W_i$

Transition
cross-section

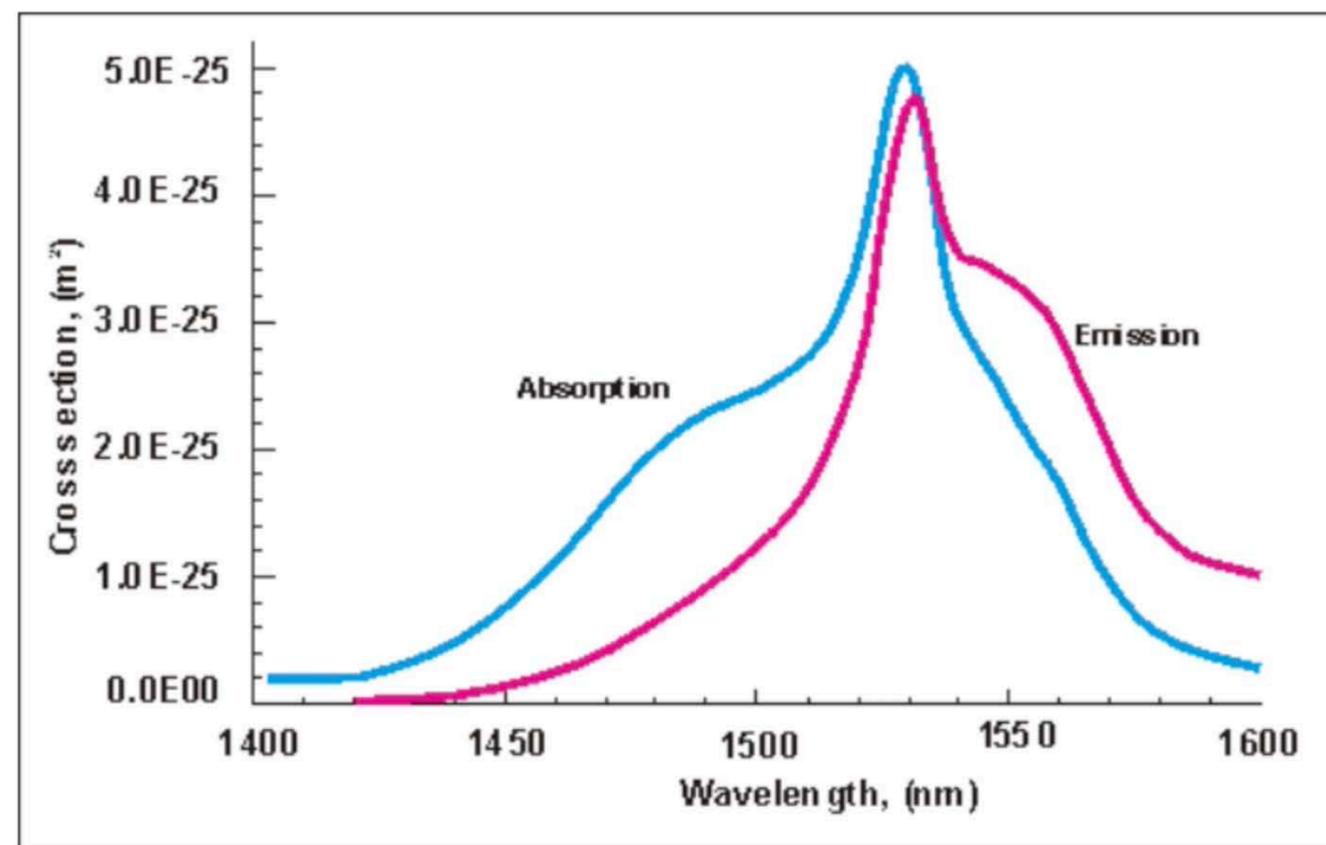
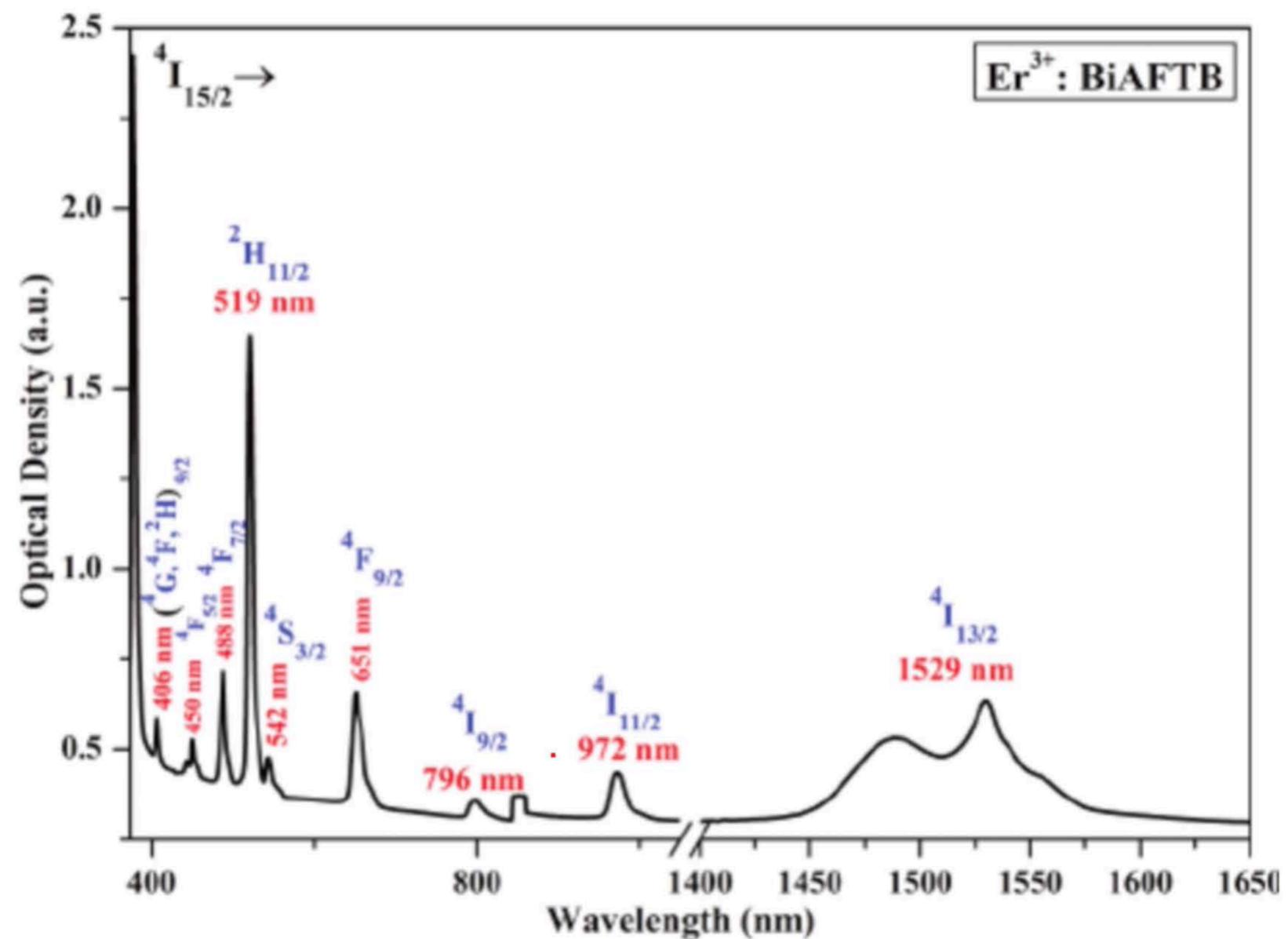
$$\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}} g(\nu)$$

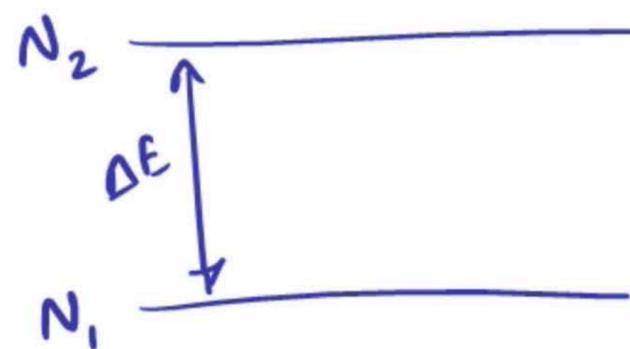
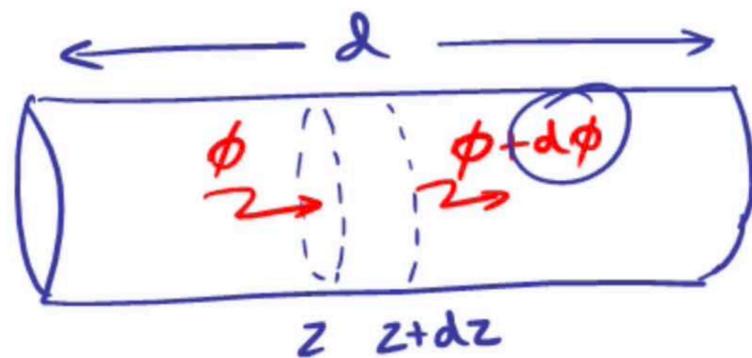
Net Flux, $d\phi = N \cdot W_i dz$ where $N = N_2 - N_1$

$N\sigma$

$$\frac{d\phi}{dz} = N \cdot \phi(z) \cdot \sigma(\nu) \Rightarrow \phi(z) = \phi(0) \cdot \exp[\gamma(\nu) \cdot z]$$

$$\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[\gamma(\nu) \cdot d]$$





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$$\frac{d\phi}{dz} = N \cdot \phi(z) \cdot \sigma(\nu) \Rightarrow \phi(z) = \phi(0) \cdot \exp[r(\nu) \cdot z]$$

$$\text{Gain} = \frac{\phi(d)}{\phi(0)} = \exp[r(\nu) \cdot d]$$

At $\lambda = 1550 \text{ nm}$,

$$\sigma_e = 6.5 \times 10^{-25} \text{ m}^2$$

If $N_t = 1 \times 10^{24} \text{ m}^{-3}$, 80% of Er ions are in excited state

$$\Rightarrow N_2 = 0.8 \times 10^{24} \text{ m}^{-3}$$

$$N_1 = 0.2 \times 10^{24} \text{ m}^{-3}$$

$$N = N_2 - N_1 = 0.6 \times 10^{24} \text{ m}^{-3}$$

Gain = 20 dB (100)

$$100 = \exp[0.4 d]$$

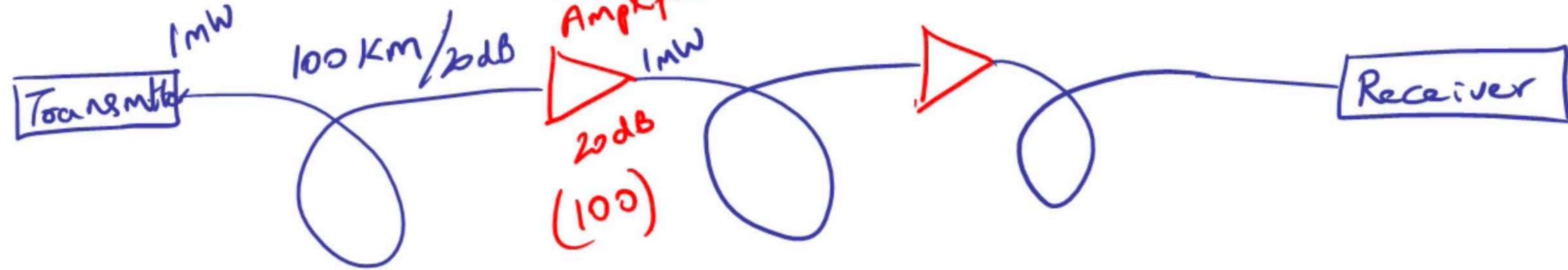
$$d = \frac{1}{0.4} \ln(100) = \underline{\underline{11.5 \text{ m}}}$$

$$\Rightarrow \gamma_{1550} = 0.6 \times 10^{24} \times 6.5 \times 10^{-25} \\ \approx \underline{\underline{0.4 \text{ m}^{-1}}}$$

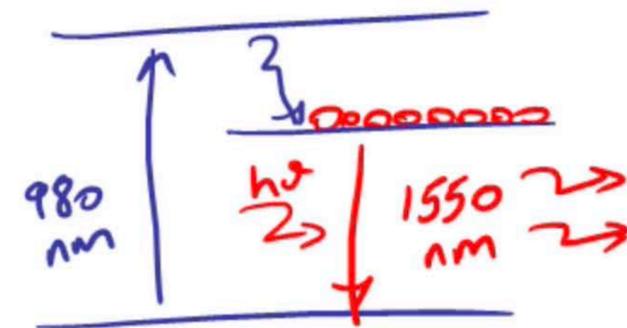
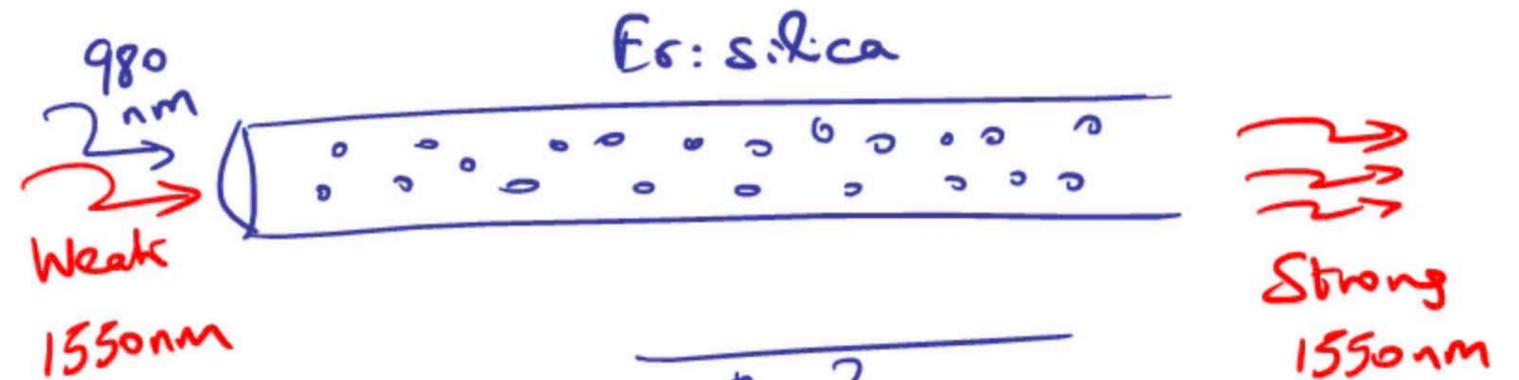
Example:

Optical Fiber Communication System

Noise Figure = $\frac{(SNR)_{in}}{(SNR)_{out}}$



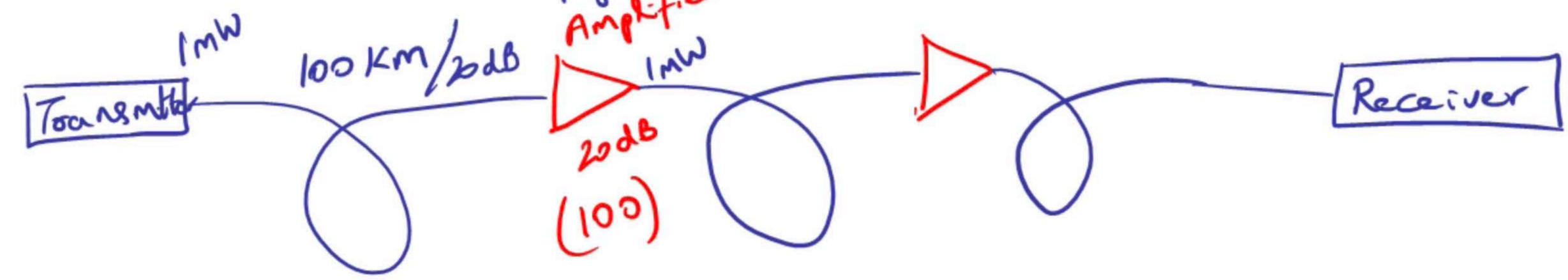
Attenuation 0.2 dB/km
Q $\lambda = 1550 \text{ nm}$



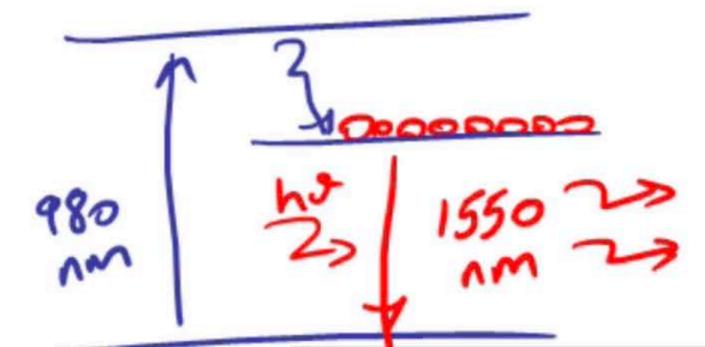
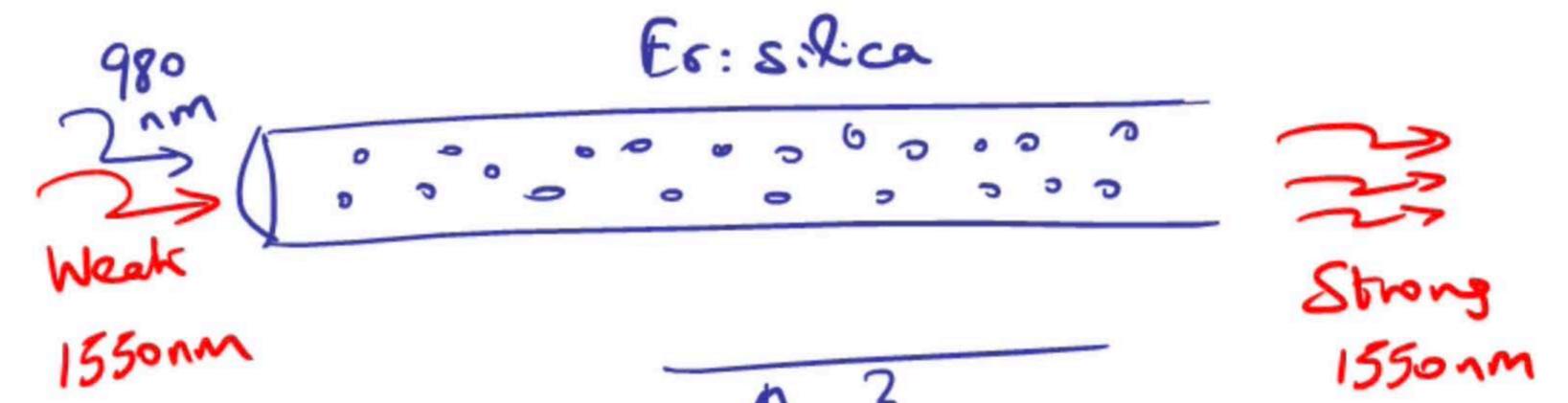
Example:

Optical Fiber Communication System

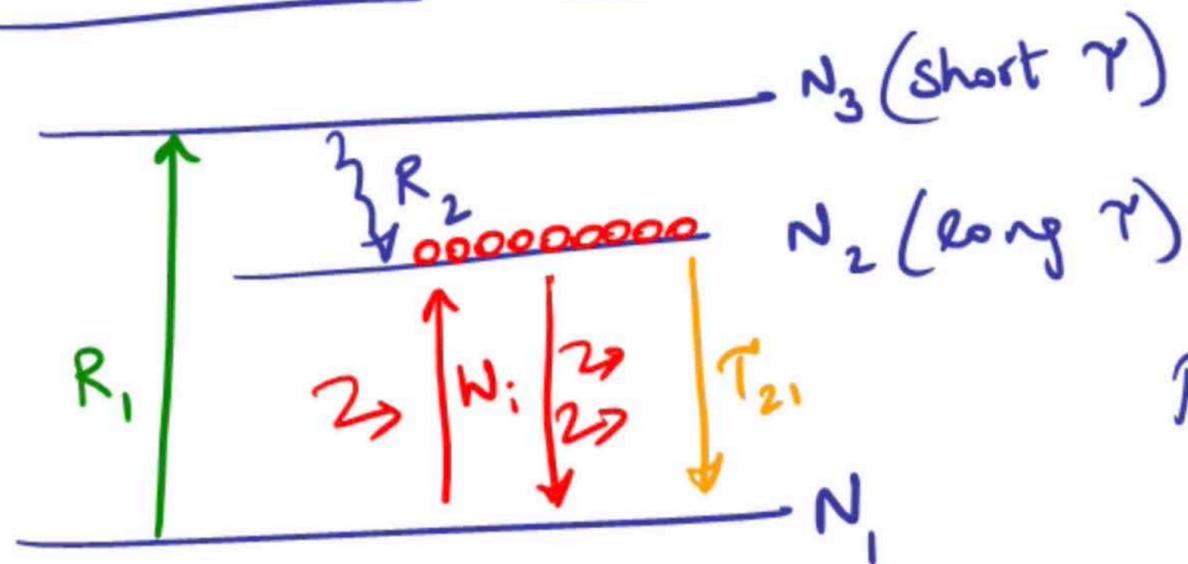
Noise Figure = $\frac{(SNR)_{in}}{(SNR)_{out}}$



Attenuation $0.2 dB/km$
 $Q \lambda = 1550 nm$



Three-level system:



$$R_1 = R_2 = R$$

Rate equation (⊙ steady state) $\frac{dN_2}{dt} = 0$

Total number density of dopants

$$N_a = N_1 + N_2 + \cancel{N_3}$$

$$N = N_2 - N_1$$

$$= 2N_2 - N_a \Rightarrow N_2 = \frac{1}{2}(N + N_a)$$

$$\frac{dN_2}{dt} = R - \frac{N_2}{\tau_{21}} - N_2 W_1 + N_1 W_1 = 0$$

$$(N_2 - N_1) W_1 = R - \frac{N_2}{\tau_{21}}$$

$$N W_1 = R - \frac{1}{2\tau_{21}}(N + N_a)$$

$$N W_i = R - \frac{1}{2\tau_{21}} (N + N_a)$$

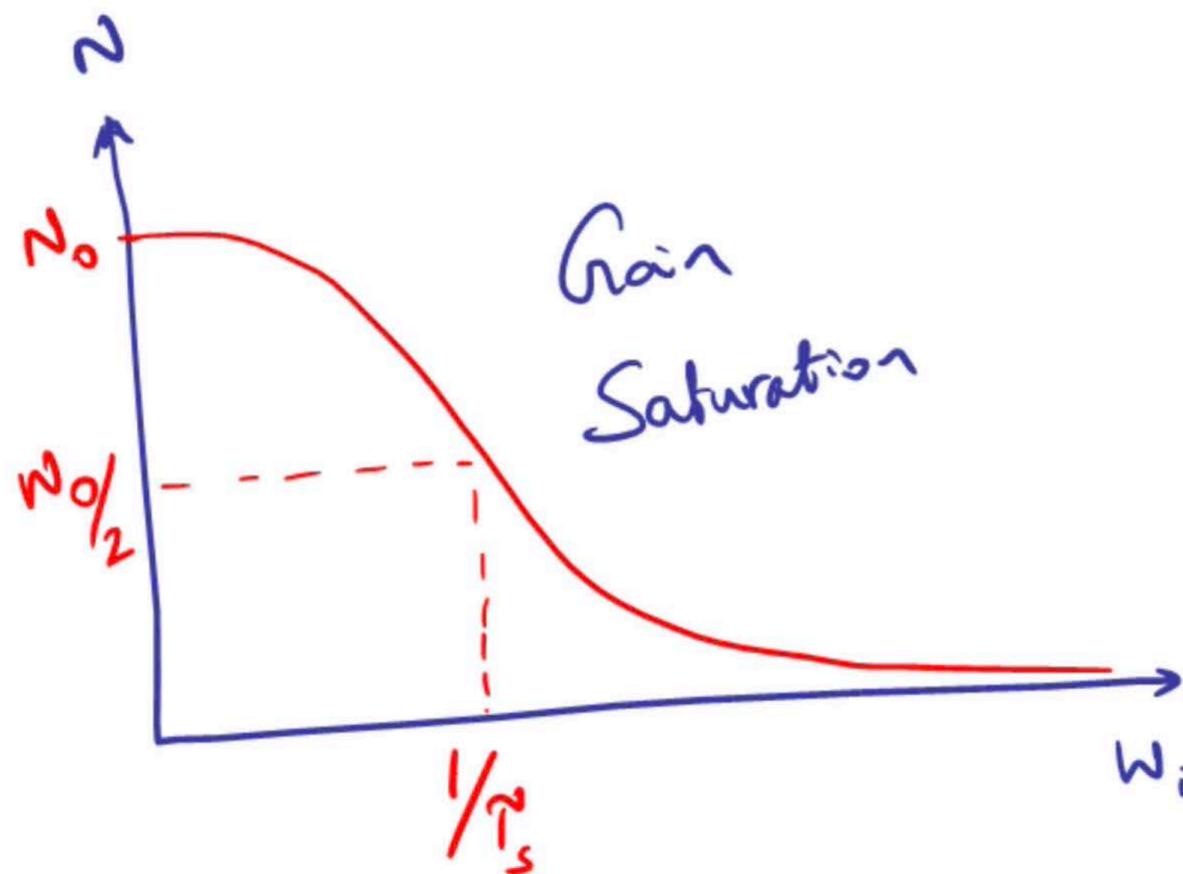
\Rightarrow

$$N = \frac{2\tau_{21}R - N_a}{2\tau_{21}W_i + 1}$$

$$N = \frac{N_0}{1 + \tau_s W_i}$$

where $\tau_s = 2t_{sp}$

$$N_0 = 2Rt_{sp} - N_a$$



flux density of incoming signal photons

$W_i \rightarrow \phi \sigma$