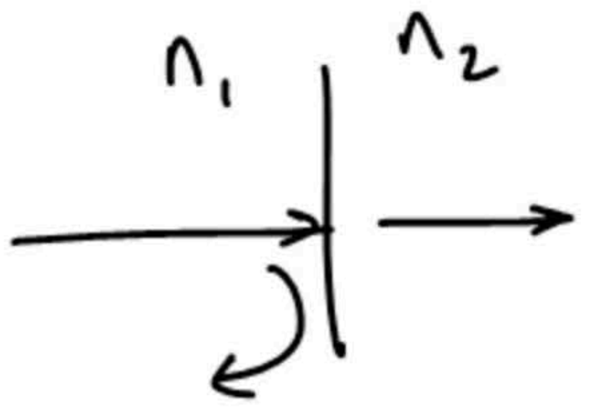
Example 1: Interference in thin films (multiple layers)

(a)



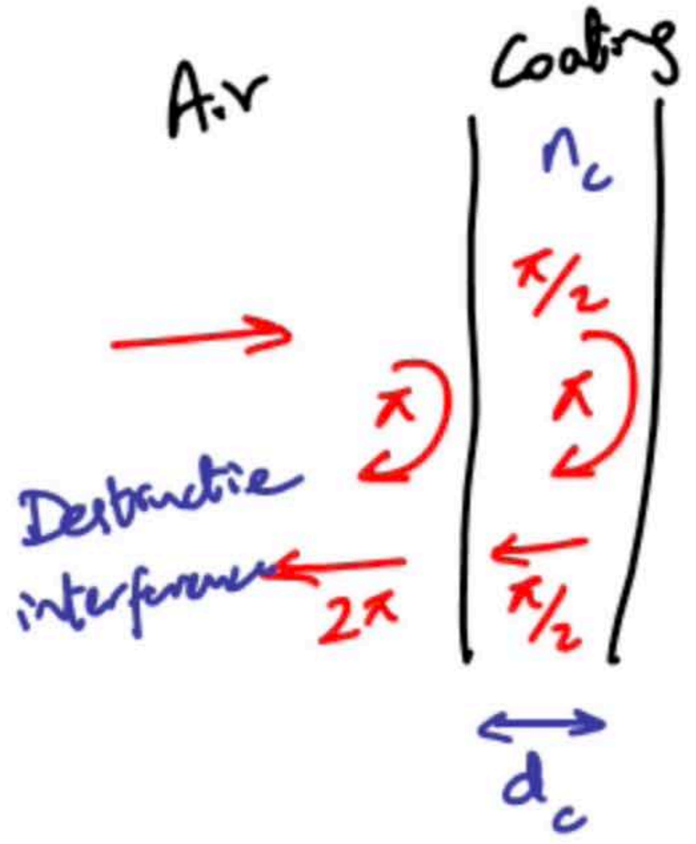
$\eta = \sqrt{\frac{\mu}{\epsilon}}$

$r = \frac{n_1 - n_2}{n_1 + n_2}$

$\sqrt{\epsilon_r} = n$

If $n_2 > n_1$ r is negative

π phase shift

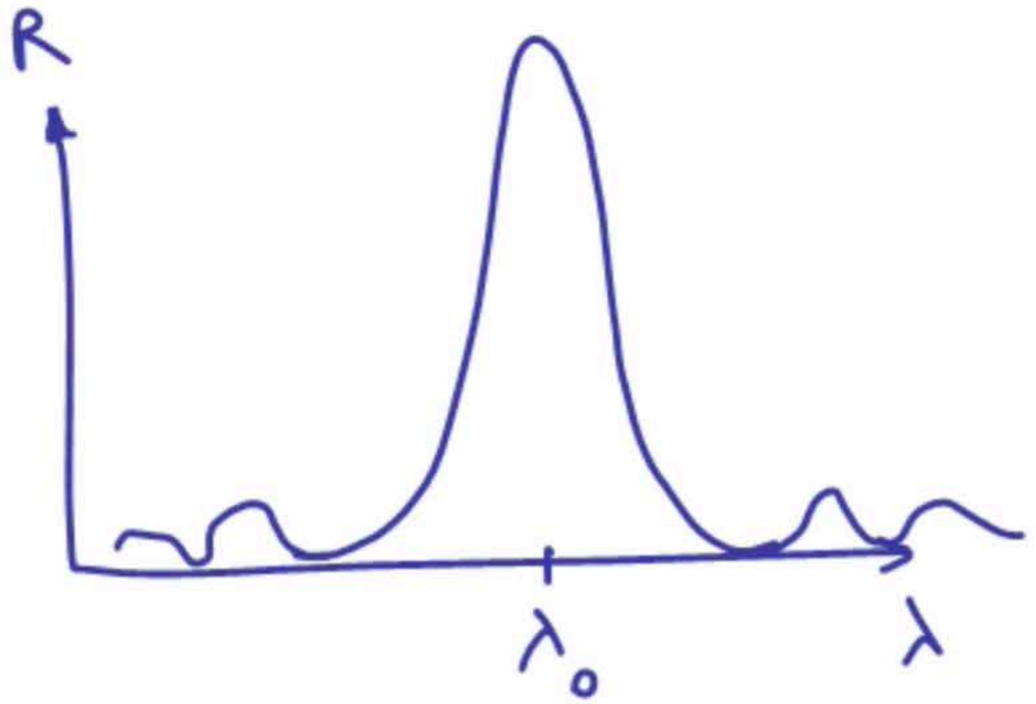


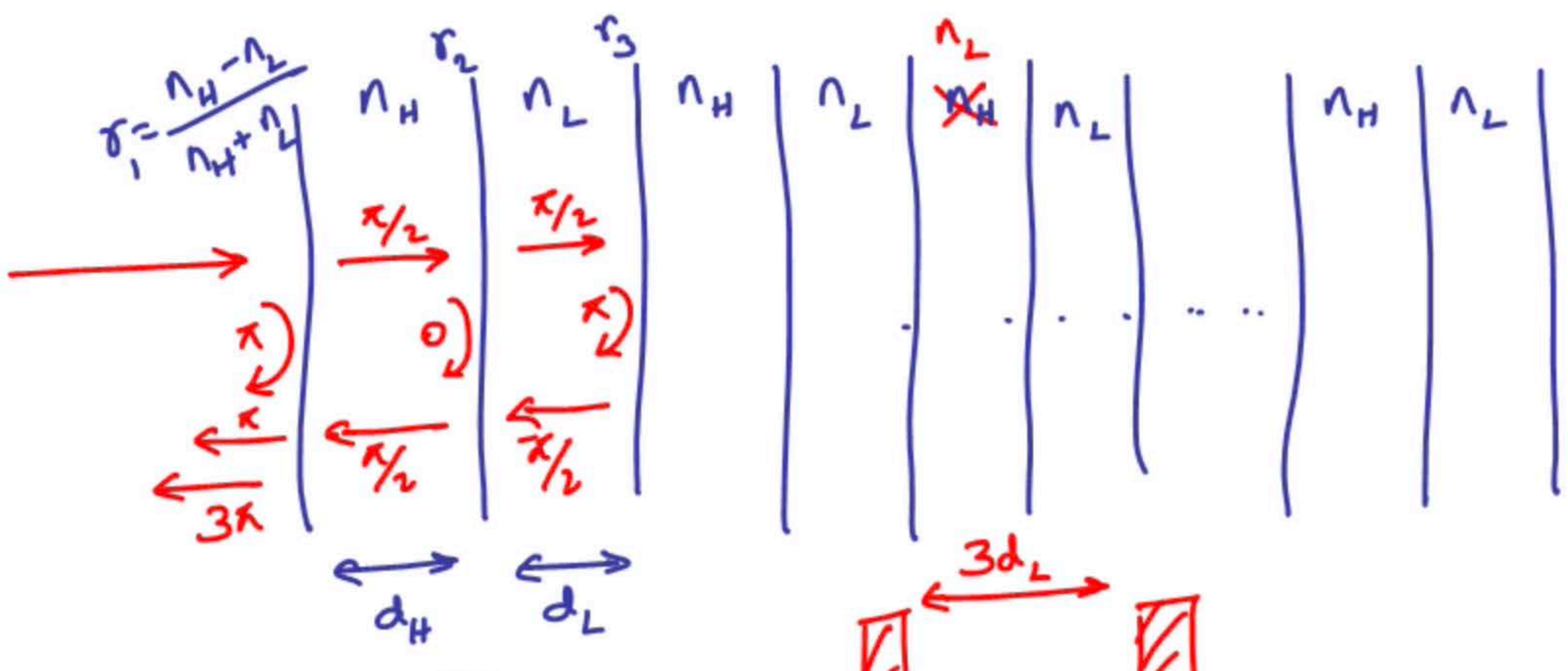
$n_c = \sqrt{n_{air} \cdot n_{glass}}$

$= \sqrt{n_{glass}}$

$\frac{2\pi}{\lambda} \cdot n_c \cdot d_c = \frac{\pi}{2} \Rightarrow \boxed{d_c = \frac{\lambda}{4 n_c}}$

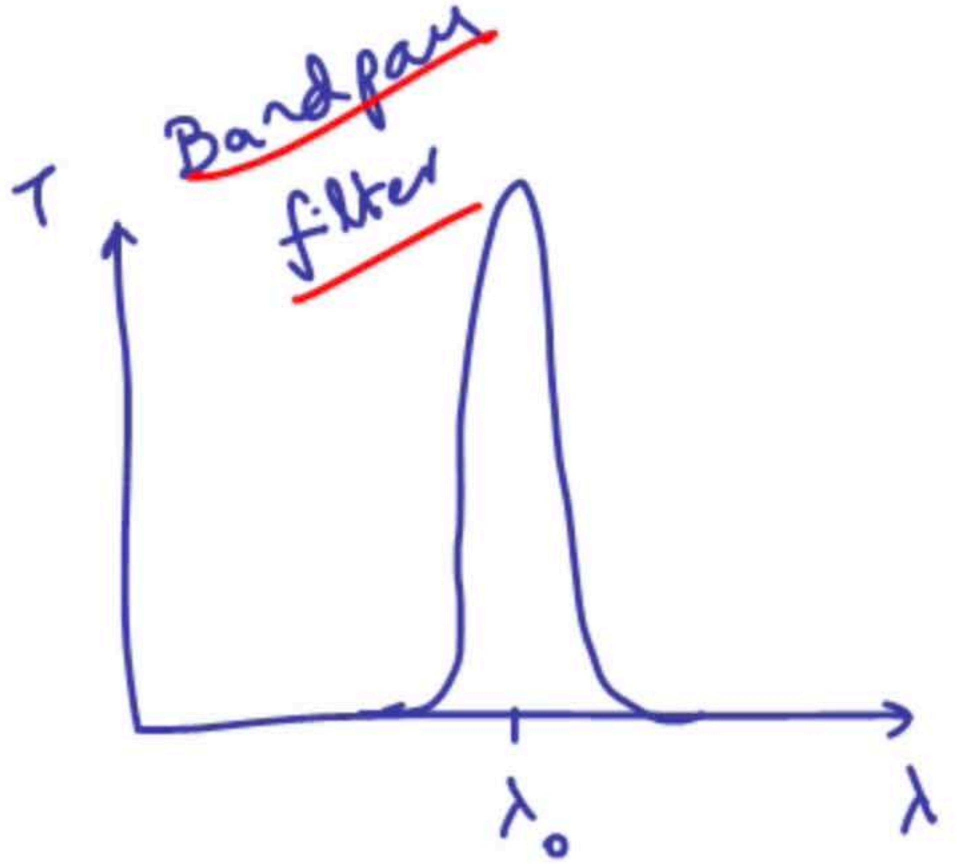
(b) How can we extend this principle to design a high reflectivity (>99%) mirror?





$$d_H = \frac{\lambda}{4n_H} \qquad d_L = \frac{\lambda}{4n_L}$$

$$R = \frac{1 - \left(\frac{n_H}{n_L}\right)^{2N}}{1 + \left(\frac{n_H}{n_L}\right)^{2N}}$$



If $\lambda = 532\,nm$, $d_H \simeq 100\,nm$, $d_L \simeq 80\,nm$

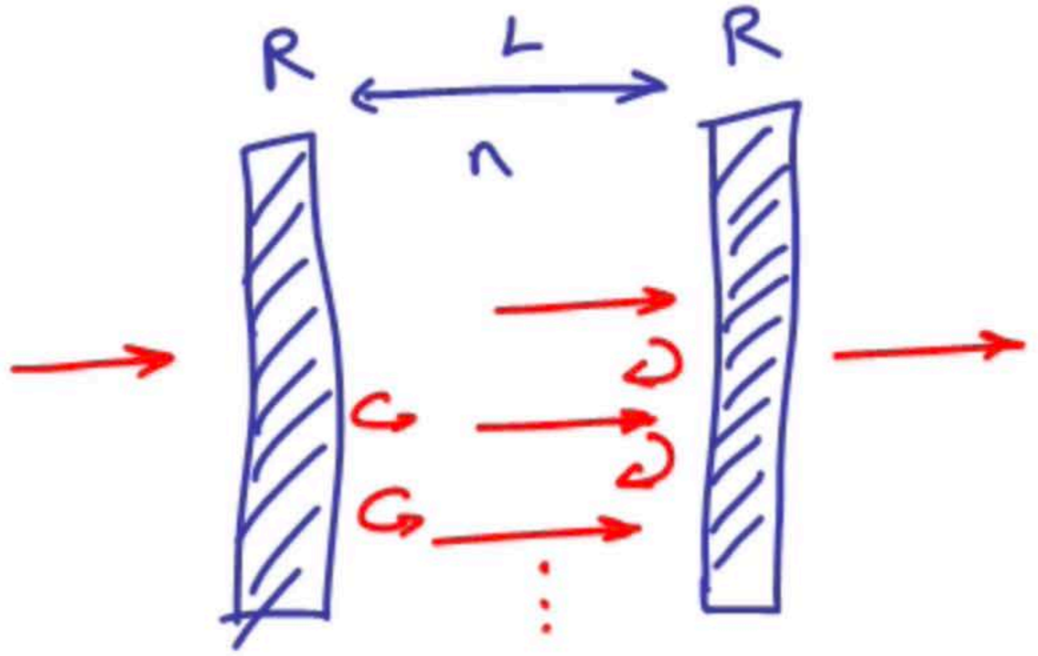
$n_H = 2.32$ (ZnS)

$n_L = 1.38$ (MgF_2)

$N \rightarrow$ Number of pairs of H-L regions

(c) Fabry Perot Etalon/Cavity:

$f = 1000$



$h = R e^{j\phi}$ $\qquad \phi = \frac{2\pi}{\lambda} n . 2L$

$$I = |U|^2 = |U_1 + U_2 + U_3 + \dots|^2$$

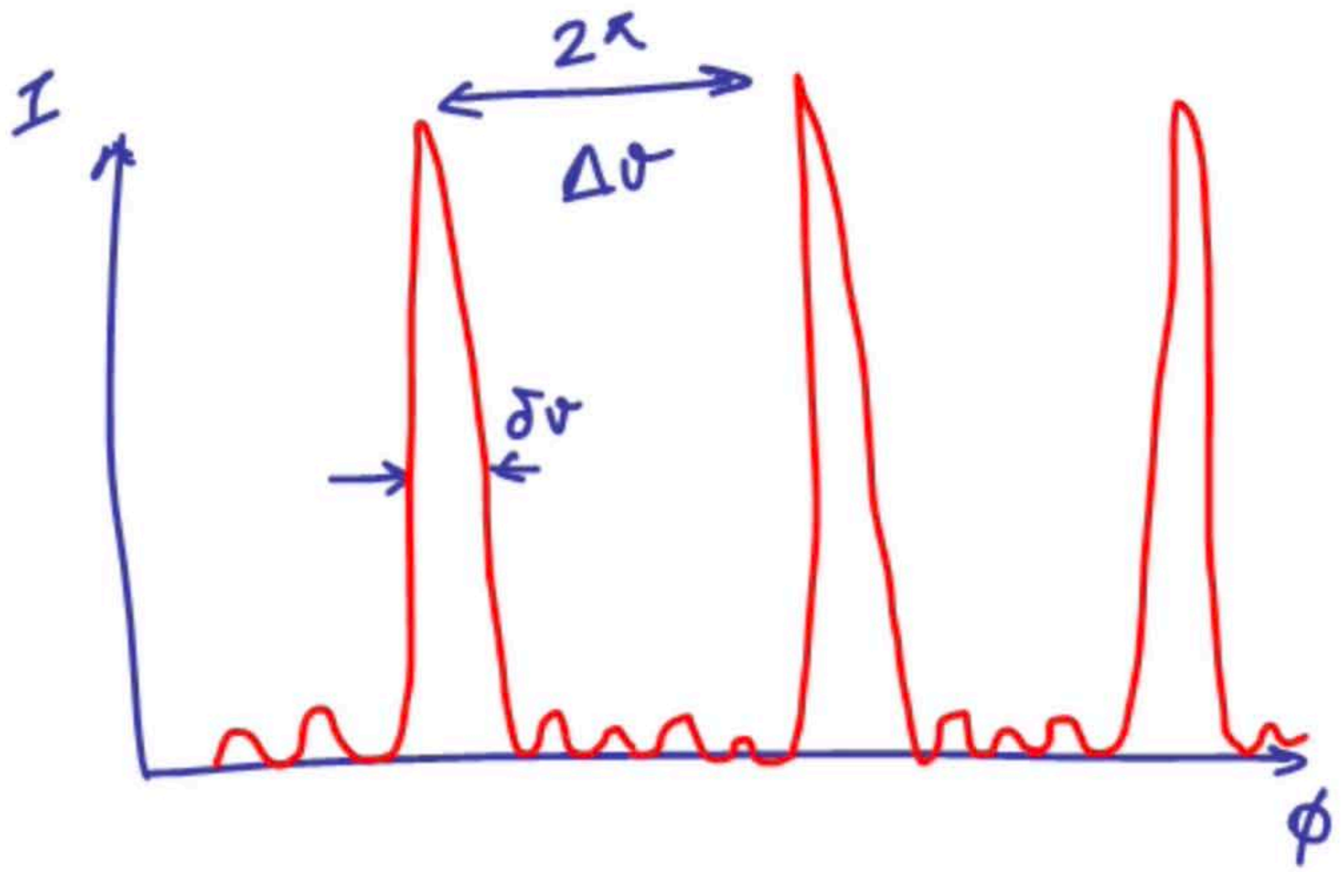
$$= |U_0| \, |1 + h + h^2 + \dots|^2$$

$$U = \frac{\sqrt{I_0}}{1 - h}$$

$$\Delta\phi = \frac{2\pi}{\lambda_1} n . 2L - \frac{2\pi}{\lambda_2} n . 2L$$

$$= \frac{\cancel{2\pi}}{c} \nu_1 n . 2L - \frac{\cancel{2\pi}}{c} \nu_2 n . 2L = \cancel{2\pi}$$

$$\boxed{\Delta \nu = \frac{c}{2nL}}$$



$$I = \frac{I_0}{(1-R)^2 + 4R \sin^2(\phi/2)}$$

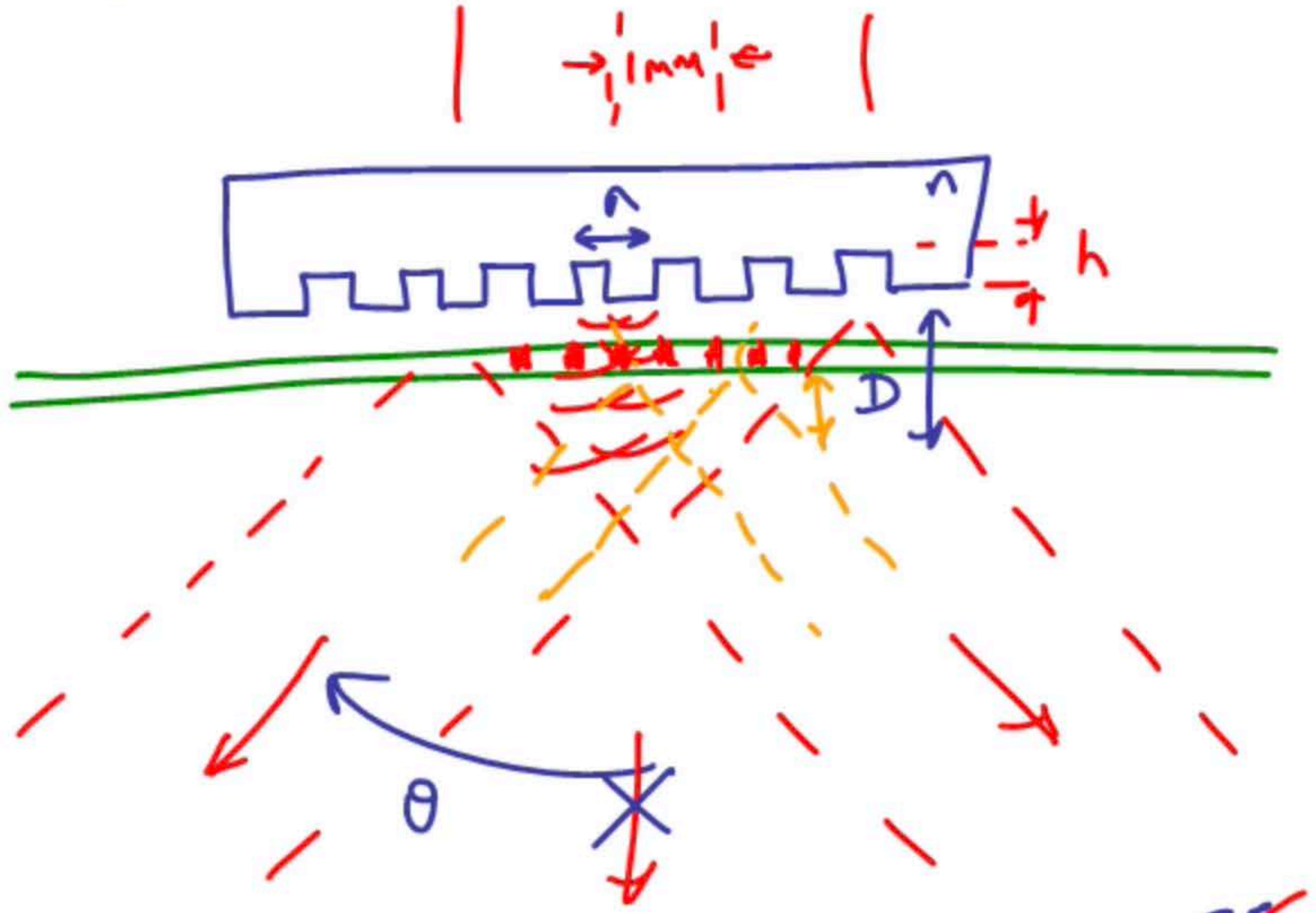
$$\boxed{I = \frac{I_{max}}{1 + \left(\frac{2f}{\pi}\right)^2 \sin^2(\phi/2)}}$$

where $I_{max} = \frac{I_0}{(1-R)^2}$

$$f = \frac{\Delta\nu}{\delta\nu}$$

$R = 99.9\%$ ← Finesse $f = \frac{\pi\sqrt{R}}{1 - R}$

Example 2: Design a diffractive element for first order interference



$\lambda_B = 2n\Lambda$

$$\Delta\phi = \frac{2\pi}{\lambda} nh - \frac{2\pi}{\lambda} \cdot h = \frac{2\pi}{\lambda} h(n-1) = \pi$$

(a) $$\boxed{h = \frac{\lambda}{2(n-1)}}$$

$n = 1.5$

$\lambda = 250\ nm$

$\Rightarrow h = 250\ nm$

(b) $$\Delta\phi = \frac{2\pi}{\lambda} PLD + \pi = 2\pi$$

$\frac{\Lambda}{2} \sin\theta$

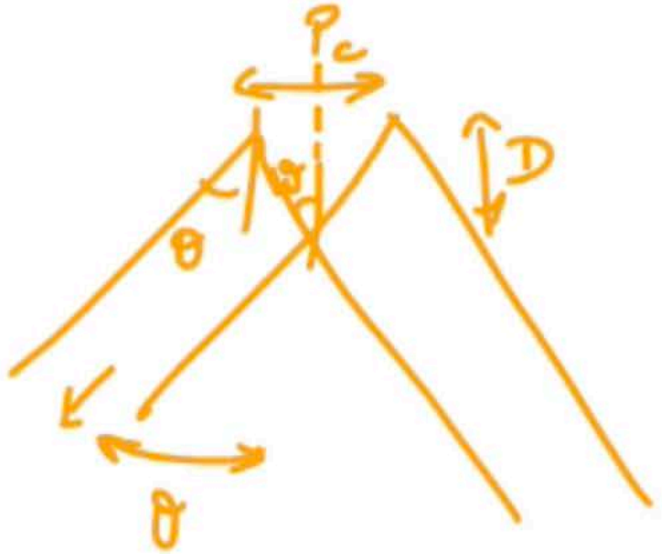
$$\sin\theta = \frac{\lambda}{\Lambda}$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{\Lambda}\right)$$

If $\Lambda = 1\ \mu m$

$\theta \simeq 14^\circ$

(c) $P_c = 1\ mm$



$$\tan\left(\frac{\pi}{2} - \theta\right) = \frac{D}{P_c/2}$$

$$\cot\theta = \frac{2D}{P_c}$$

$$D = \frac{P_c}{2} \cot\theta = 1.95\ mm$$