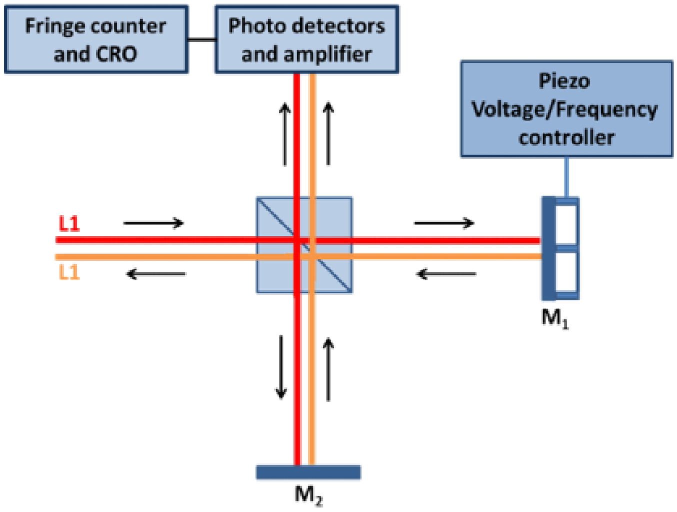
Tutorial 2

1. What should be the thickness and refractive index of an anti-reflecting film coated over a glass plate of refractive index 1.6 expected to work at 650 nm. Compare the reflectivity of crown glass (n= 1.5) with flint glass (n= 1.7) in air.

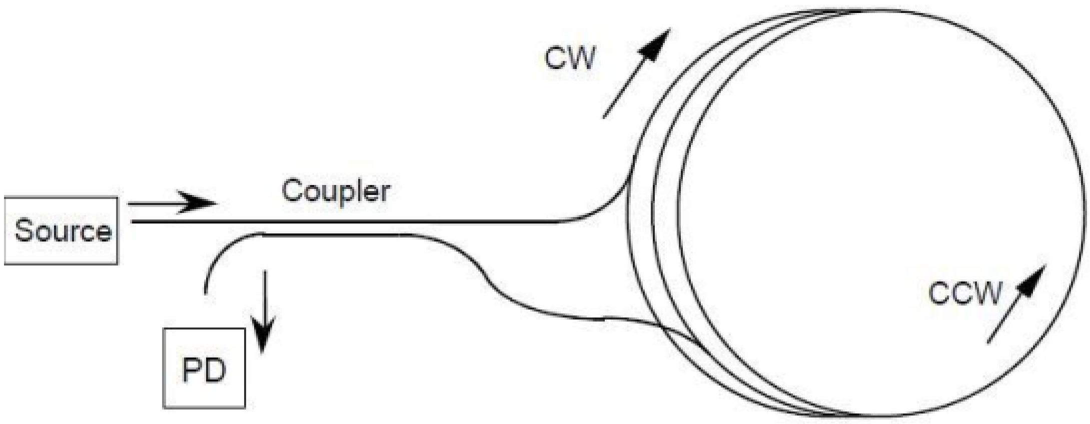
2. A wavelength tunable Fabry-Perot optical bandpass filter is required to tune across a wavelength band of 100 nm at central wavelength of 1550nm, and it is desired that there should be only one resonant mode of the filter at a time within this tuning range. Assuming air medium in the cavity, determine the length of the cavity of the filter. If wavelength resolution of the filter has to be 0.2 nm, calculate the Finesse of the cavity and the reflectivity of the mirrors (assuming equal reflectivity for both).

3. A Michelson interferometer is modified to use two wavelengths and a piezo-electric transducer is attached to one of the mirrors M_1 for the displacement (see figure). Two similar photo detectors were used to detect the fringe pattern generated by each of the two wavelengths. Wavelength of one laser is known to be 632.8 nm. The output of the photo detectors are amplified and connected to a fringe counter. The piezo-electric transducer voltage can be varied manually from V_1 = 1 V to V_2 = 5 V which results in a change in position of the mirror M_1 causing fringes system to change. The corresponding fringe counter readings for 632.8 nm is 38 and that of the other wavelength is 45 fringes.



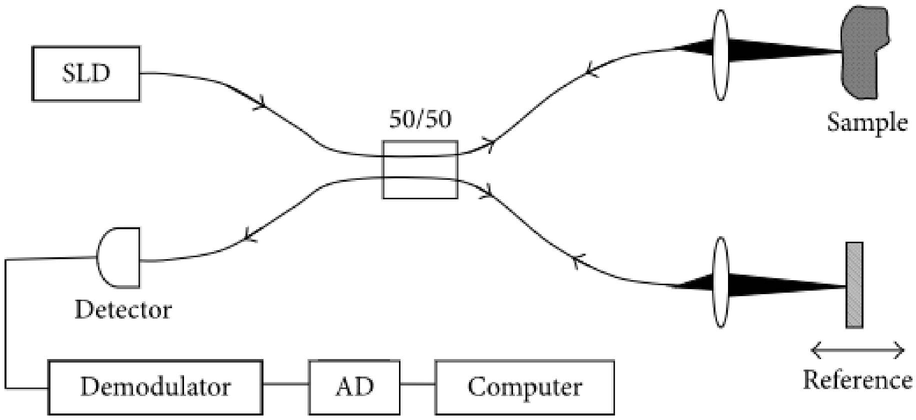
   a) Estimate the wavelength of the other laser.
   b) Estimate the distance moved by the mirror for getting the fringe count = 38 with $\lambda 1$.
   c) Calculate the error in wavelength measurement using logarithmic differentiation. (Assume the reference wavelength (632.8 nm) is known with 0.01 nm accuracy and use the above constraint on counter).
   d) Estimate the displacements required for measuring wavelengths 612 nm and 543.5 nm with an accuracy of 0.001 nm (Assume the reference wavelength is known with 0.0001 nm accuracy). Use reference wavelength as 632.8000 nm. Calculate the required displacement for measuring 612 nm and 543.5 nm.

4. A fibre optic gyroscope (FOG) is used in rotation sensing, and is based on the principle of a Sagnac interferometer. Derive the expression for the Sagnac phase shift, and determine the rate of rotation for which equal intensity will be detected in both arms.



A FOG used for detecting the Earth's rotation is designed by winding a 200 m long fiber on a 10 cm diameter coil. If the Sagnac phase shift detected is 36 micro-rad, find the measured angular rotation of the Earth. Use 1550nm wavelength.

5. The block schematic of an OCT system is shown,



The axial and transverse resolution of an OCT system are independent. The axial resolution is related to the bandwidth, or the coherence-length, of the source. For a Gaussian spectrum, the axial (depth) resolution is given by, dz

$$dz = \frac{2\ln(2)}{\pi}\frac{\lambda_o^2}{\Delta\lambda}$$

and the transverse resolution is given by,

$$dx = \frac{4\lambda}{\pi}\left(\frac{f}{d}\right)$$

f is focal length of lens, d the beam diameter and NA = d/f . The depth of focus is given by, $\pi\, dx^2/2\lambda$

a. For a given source of 870nm (in NIR region water has the lowest absorption at 830nm band) wavelength, find axial resolution if the source has a spectral width of 170nm.

b. It seems from eq2 that the transverse resolution can be arbitrary reduced by choosing the right optics. Is this the case? If not what is the catch?

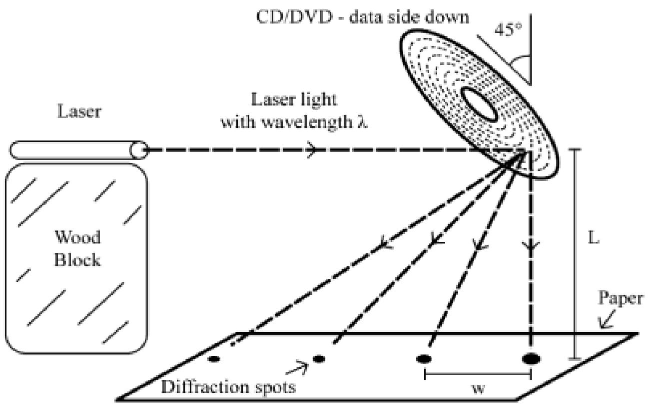
c. Why do you think a source of low coherence is particularly required for OCT ?

6. Assume we move one mirror of a Michelson interferometer through a distance of 0.233mm and we see 792 bright fringes pass by. What is the illuminating wavelength?

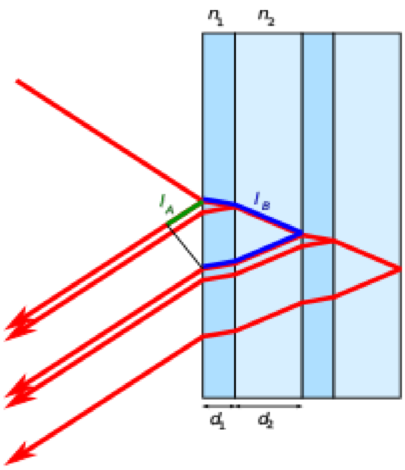
7. A light-emitting diode (LED) emits light of Lorentzian spectrum with a line-width Δv (FWHM) = 10 THz centred about a frequency corresponding to a wavelength λ_0 = 0.7 µm. Determine the line-width $\Delta\lambda_0$ (in nm), the coherence time τ_c and coherence length l_c. What is the maximum time delay within which the magnitude of the complex degree of temporal coherence $|g(\tau)|$ is greater than 0.5.

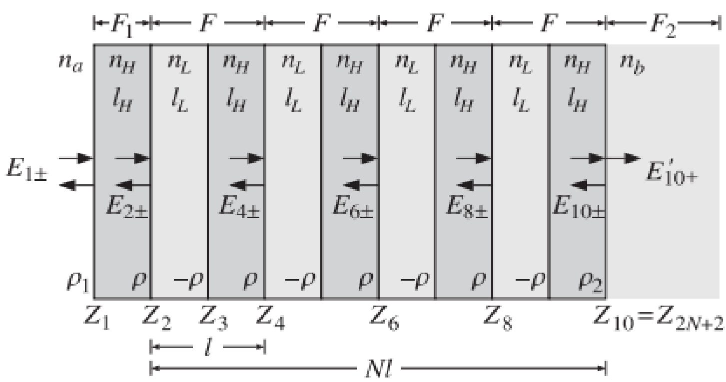
8. A phase mask used in FBG fabrication is designed to diffract UV beam in such a way that 0 order beam is suppressed to less than 5% and most of the energy is concentrated in +1 and -1 order beams. Overlapping of these two beams of order +1 and -1 produce the interference pattern. Here, how can you relate the depth of interference field to the spatial coherence of the laser source?

9. Recall the diffraction experiment from the lab session, if red laser of 650nm shine upon the CD (distance between tracks = 1.6um), where do you expect the first and second order diffraction spots (i.e find w for each case). See Fig for reference. Take L = 10 cm.



10. A dielectric mirror, also known as a Bragg mirror, is a type of mirror composed of multiple thin layers of dielectric material, typically deposited on a substrate of glass or some other optical material.



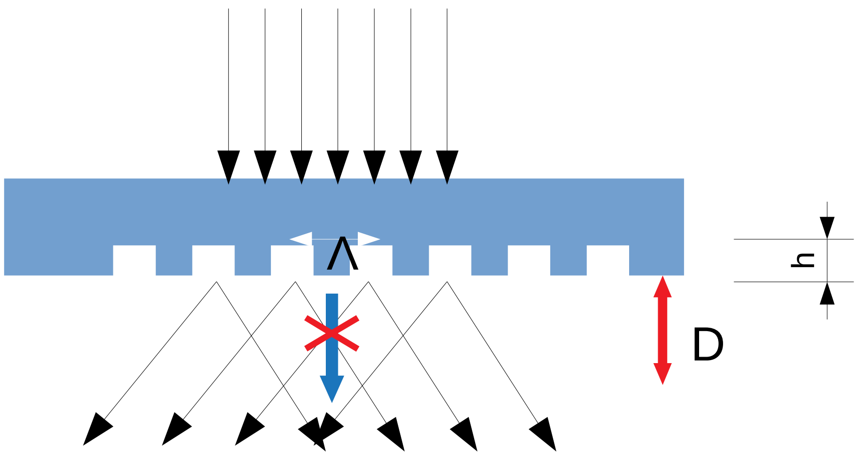


Thin layers with a high refractive index n1 are interleaved with thicker layers with a lower refractive index n2 (See fig). The path lengths I_A and I_B differ by exactly one wavelength, which leads to constructive interference.
The reflection is given by, (see second fig)

$$\Gamma_1 = \frac{Z_1 - \eta_a}{Z_1 + \eta_a} = \frac{1 - \left(\frac{n_H}{n_L}\right)^{2N} \frac{n_H^2}{n_a n_b}}{1 + \left(\frac{n_H}{n_L}\right)^{2N} \frac{n_H^2}{n_a n_b}}$$

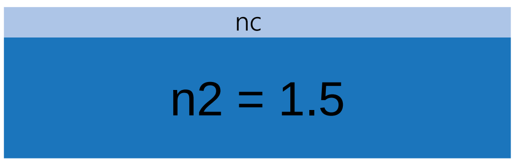
The reflectance is $|\Gamma|^2$
For N identical bilayers of low and high index, Total number of layers is M = 2N +1.
a. Show the phase additions that give reflectivity to the structure (read appendix for some extra information).
b. For nine layers (M = 9) and $n_H = 2.32$, $n_L = 1.38$, and n_a = nb = 1. Find the reflectance.

11. A diffractive element (transmission grating with periodic etching) of first order needs to be designed as shown.



a. What should be the h, so that the power in the zeroth order diffraction is zero ? (Take n = 1.5, wavelength = 248 nm and grating period, Λ = 1um.
b. Keeping (a) in mind, find the angle at which first order diffraction pattern is formed.
c. If the incident beam has a spatial coherence, ρc of 1mm. Find D.

12. Consider the following cases with multi-layer interferences,
a. For the image shown below, what should be the thickness of the coating (d_c) so as to be used as an anti-reflection coating.

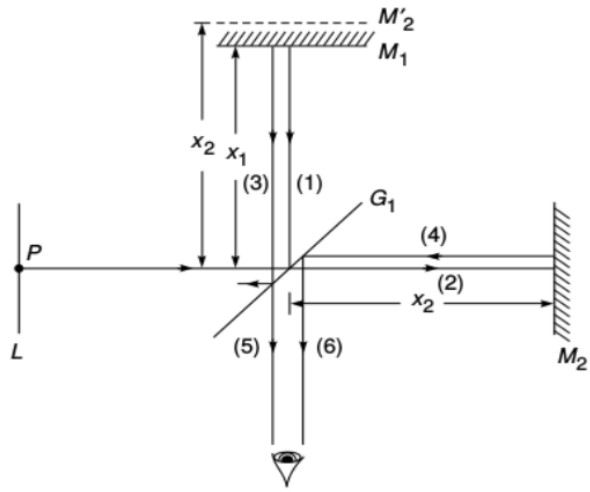


b. Now, consider the case in which we have to design a dielectric mirror. Find the thickness of High and low index layers which satisfies that reflection criteria. (Take n_H = 2.32 and n_L = 1.38 for a wavelength of 1um)
c. Use the FP etalon to build a filter having passband FWHM of 0.1nm at 532nm with a finesse of 1000.

Appendix

Here are some additional questions and more informations on questions discussed above. Do read it on at your leisure, you may find something interesting/fascinating.

Q: A Michelson Interferometer could be used in the measurement of two closely spaced wavelengths. Say we have a sodium lamp emitting two closely spaced wavelengths 589nm and 589.6nm. The interferometer is first set corresponding to the zero path difference. Near d = 0, both the fringe patterns will overlap. If mirror M1 is moved away from (or toward) plate through a distance d, then the maxima corresponding to one wavelength will not, in general, occur at the same angle as the other. Find the minimum distance d for which the fringe system disappears. Find the distance for the mirror to be moved so that the fringe system appears again.



Q5. **OCT**

Optical coherence tomography is a micrometer-scale imaging modality that permits label-free, cross-sectional imaging of biological tissue microstructure using tissue backscattering properties. Optical coherence tomography (OCT) was first developed by Fujimoto’s group at Massachusetts Institute of Technology (MIT) in 1991 that works based on tissue backscattering properties. OCT takes the advantage of short coherence length of broadband light sources to perform micrometer-scale, cross-sectional imaging of biologic tissue sample.

Advantages of OCT.

(i) quality images (OCT has demonstrated the ability to render images within a range of 1–10 μm axial resolution usually and even sub-micrometer (0.5 μm) resolution too

(ii) imaging speed (OCT can give a temporal resolution up to milliseconds

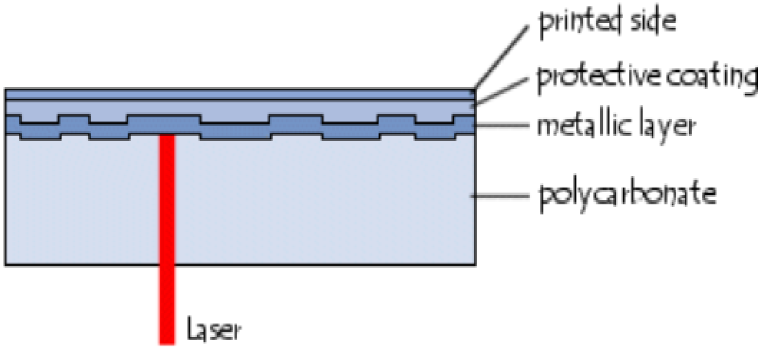
(iii) label-free imaging (iv) low cost (v) additional functionality (while a basic OCT imaging method is able to render depth-resolved structural images of the target, more sophisticated OCT imaging strategies can provide additional functional information, such as blood flow (through Doppler OCT), tissue structural arrangement (through birefringence OCT), and the spatial distribution of specific contrast agents (through molecular contrast OCT)

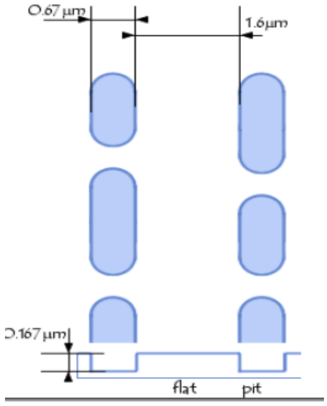
Reference: https://www.hindawi.com/journals/jme/2017/3409327/

Figure shows a generic time domain OCT system. An OCT system contains a low-coherence broadband light source. The emitted light is coupled into an interferometer. Then the light is divided into two arms: reference arm and sample arm. The reference arm transmits the light toward a reference mirror. The sample arm sends the light toward the tissue of interest. The sample arm also contains an objective lens which focuses the light onto the sample tissue (e.g., brain, retina, and carotid artery). The light which is backscattered from the tissue structures is recombined with the reference light reflected from a highly reflective (>95%) moving reference mirror, producing an interference pattern that is detected by a light detector. To reconstruct the two-dimensional (2D) or three-dimensional (3D) cross-sectional objects, the beam is scanned across the sample surface. More complex systems may include a CCD camera and diffraction grating.

Q. 9. CD Diffraction

The Compact Disc was invented by Sony and Philips in 1981. A CD is built from a plastic (polycarbonate) substrate and a fine, reflective metallic film (24-carat gold or a silver alloy). The reflective layer is then covered with an anti-UV acrylic finish, creating a protective surface for data. Finally, an additional layer may be added so that data can be written on the other side of the CD as well. This information is stored in 22188 tracks engraved in grooves.

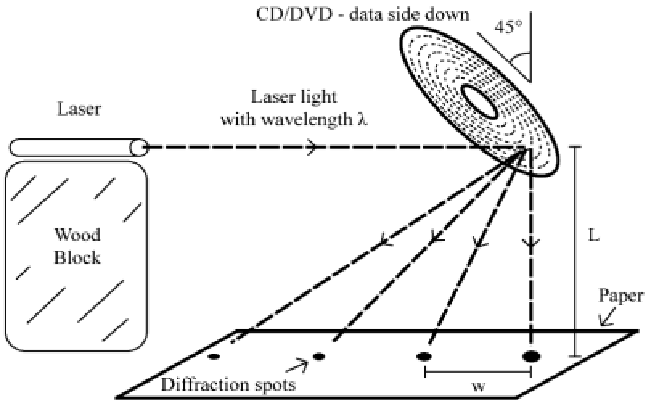




The physical track is made up of bumps 0.168um deep and 0.67um wide, with variable length. The "rings" in the spiral are spread about 1.6um apart from one another. Pits are the term for the depressions in the groove, and lands are the spaces between them.

The laser used for reading CDs has a wavelength of 780 nm when travelling through air. As the polycarbonate's refractive index is 1.55, the laser's wavelength in the polycarbonate is equal to 780/1.55 = 503nm = 0.5um. Since the depth of the groove is one quarter (1/4) the wavelength of the laser beam, a light wave reflected by a pit travels half a wavelength extra than a wave reflects by a land. This causes destructive interference between the incident light and its reflected part.

DVD increases its capacity by using higher resolution optics. DVD uses a shorter wavelength laser with red light around 650 nm compared to a CD with infrared light at 780 nm. In addition, better focusing optics allows closer tracks and smaller pits.



http://uotechnology.edu.iq/dep-laserandoptoelec-eng/laboratory/4/laser%20application/exp3.pdf

Q.10. Multi-layered structures

A dielectric mirror, also known as a Bragg mirror, is a type of mirror composed of multiple thin layers of dielectric material, typically deposited on a substrate of glass or some other optical material. By careful choice of the type and thickness of the dielectric layers, one can design an optical coating with specified reflectivity at different wavelengths of light. Alternatively, they can be made to reflect a broad spectrum of light, such as the entire visible range.

Dielectric mirrors function based on the interference of light reflected from the different layers of dielectric stack. This is the same principle used in multi-layer anti-reflection coatings, which are dielectric stacks which have been designed to minimize rather than maximize reflectivity. The thicknesses of the layers are chosen such that the path-length differences for reflections from different high-index layers are integer multiples of the wavelength for which the mirror is designed. The reflections from the low-index layers have exactly half a wavelength in path length difference, but there is a 180-degree difference in phase shift at a low-to-high index boundary, compared to a high-to-low index boundary, which means that these reflections are also in phase. In the case of a mirror at normal incidence, the layers have a thickness of a quarter wavelength.