Learning Outcome: Interference of light and Coherence property of light
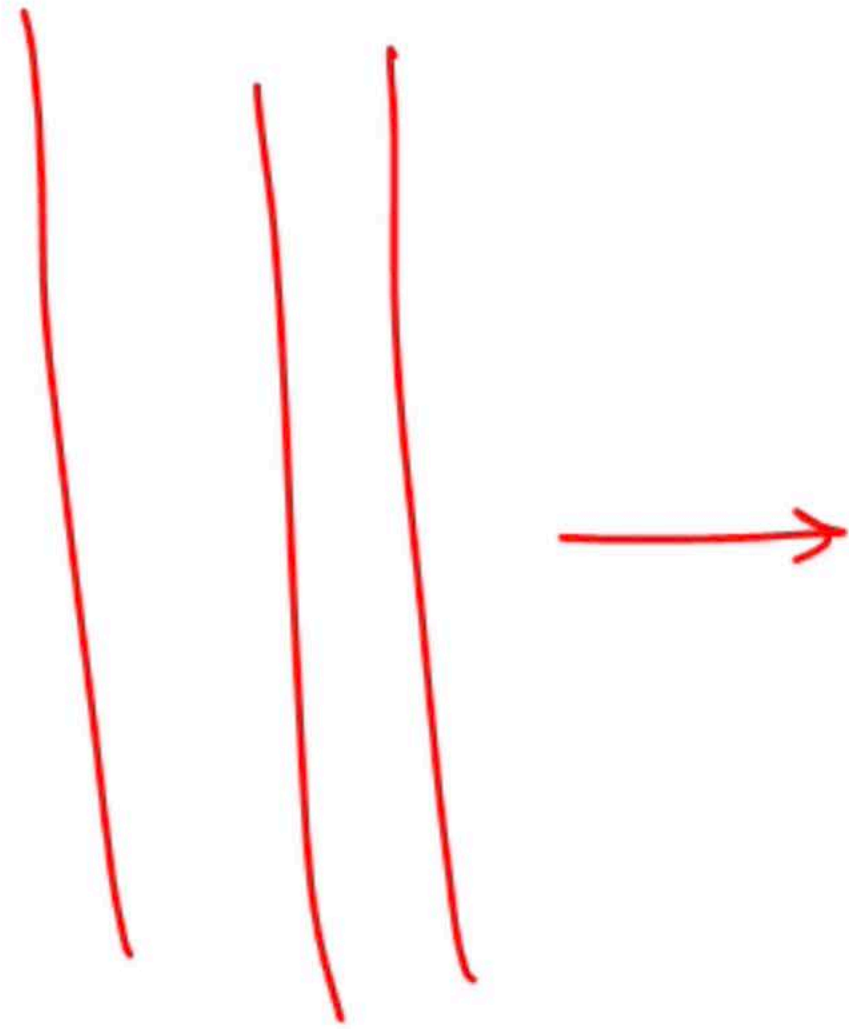
$M \to$ # of slits

$\phi \to$ phase difference from successive slits

Wave component

$$U_m = \sqrt{I_0} \exp[j(m-1)\phi]$$
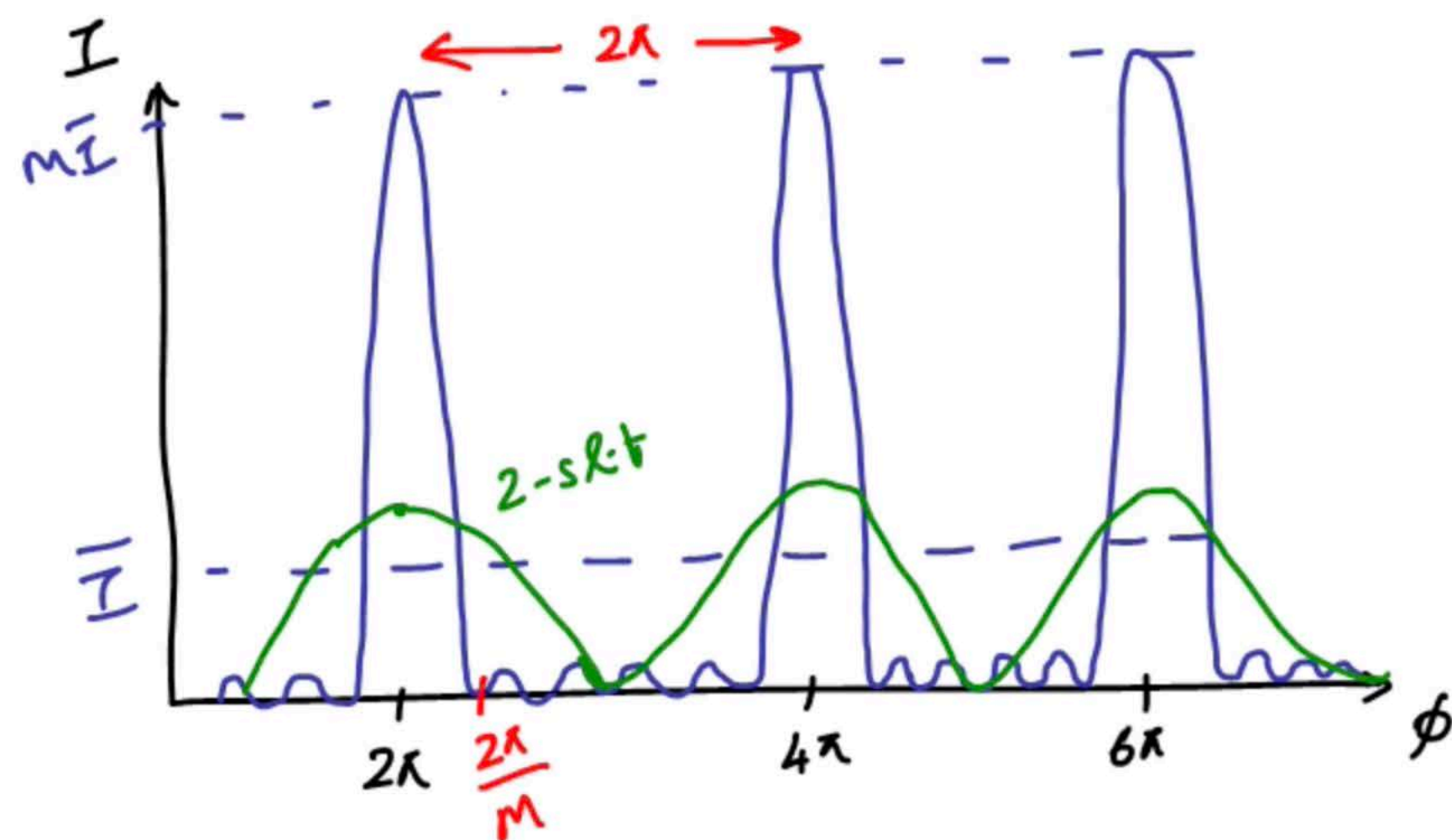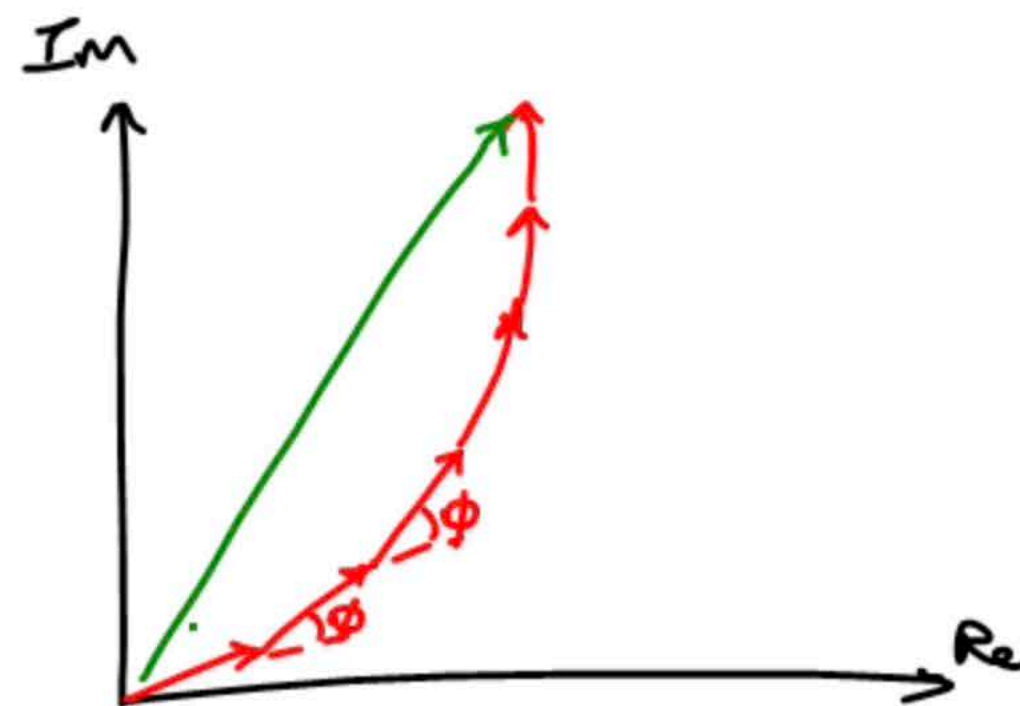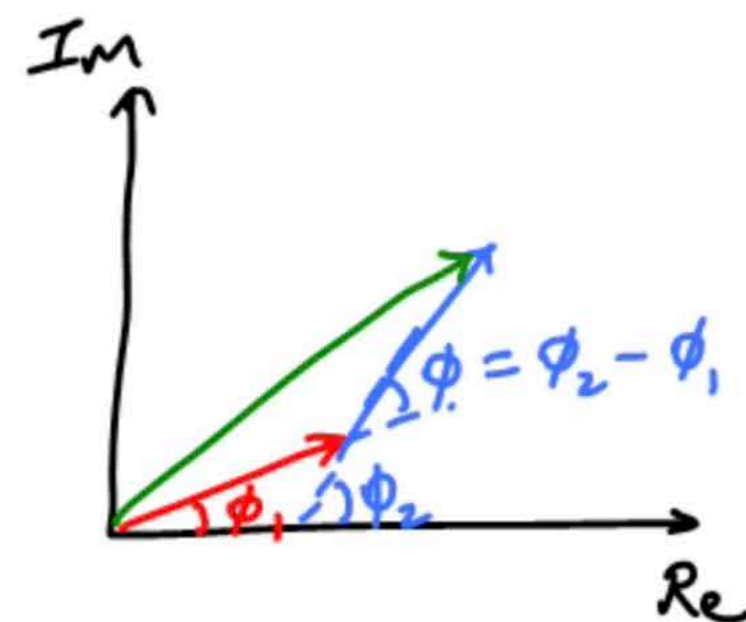
where $m = 1, 2, \dots M$

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Observation plane

Total Wave amplitude $U = \sqrt{I_0}\left(1 + h + h^2 + \cdots + h^{M-1}\right)$ where $h = e^{j\phi}$

$$= \sqrt{I_0} \cdot \frac{1 - h^M}{1 - h} = \sqrt{I_0} \cdot \frac{1 - e^{jM\phi}}{1 - e^{j\phi}}$$

Total Intensity $I = |U|^2 = I_0 \cdot \left|\frac{1 - e^{jM\phi}}{1 - e^{j\phi}}\right|^2 = I_0 \left|\frac{e^{-jM\phi/2} - e^{jM\phi/2}}{e^{-j\phi/2} - e^{j\phi/2}}\right|^2$

$$\boxed{I = I_0 \cdot \frac{\sin^2(M\phi/2)}{\sin^2(\phi/2)}}$$

More # of interfering sources

$\Rightarrow$ Narrower spectral selectivity

$\frac{M\phi_{min}}{2} = \pi$

$\phi_{min} = \frac{2\pi}{M}$

I

$M\bar{I}$

$\bar{I}$

2π

2-slit

2π $\frac{2\pi}{M}$ 4π 6π ϕ

Im $\phi = \phi_2 - \phi_1$ ϕ_1 ϕ_2 Re

Im ϕ ϕ Re

Interferometers

Common Path   Differential Path
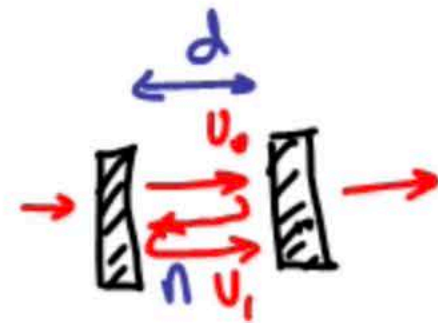
Gyroscope $\phi_{cw} - \phi_{ccw}$

Sagnac   Fabry-Perot   Mach-Zehnder   Michelson

50/50

d

U_0 ϕ_1

n U_1 $\phi_2 - \phi_1$

ϕ_2

d_1 ϕ_1

n_1 ϕ_2 n_2

d_2

$\Delta\phi = \dfrac{2\pi}{\lambda} n \cdot 2d$

$= 2\pi m$
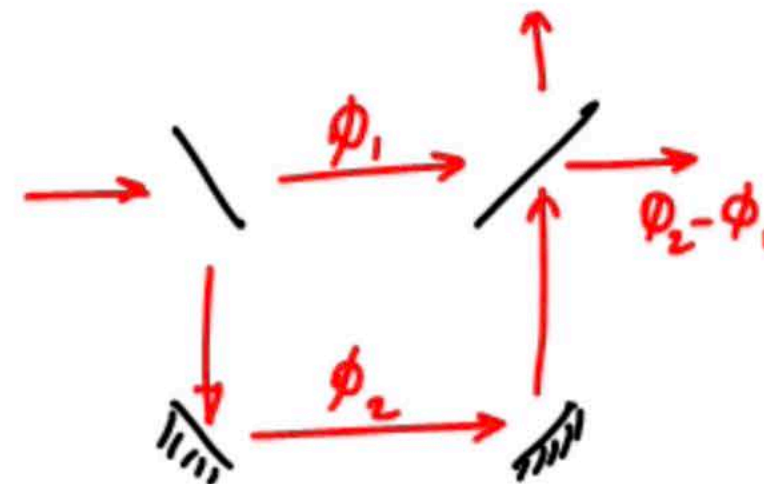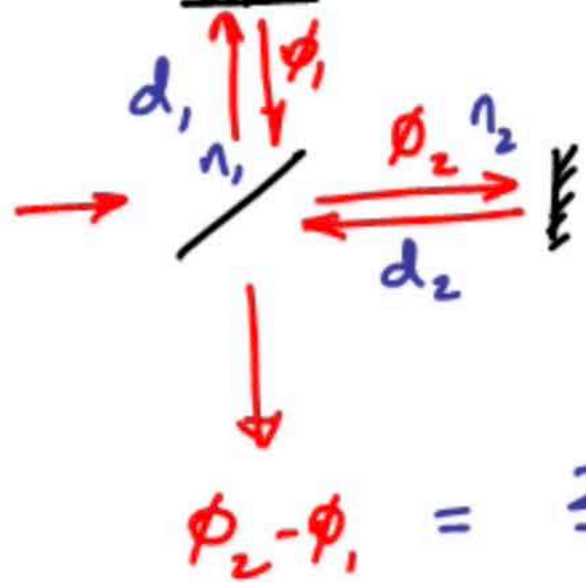
$\phi_2 - \phi_1 = \dfrac{2\pi}{\lambda} 2n_2 d_2 - \dfrac{2\pi}{\lambda} 2n_1 d_1$

$\to$ Optical path length difference

$n=1,$

$$\boxed{d = M \cdot \dfrac{\lambda}{2}}$$

$= \dfrac{2\pi}{\lambda} 2 \left(n_2 d_2 - n_1 d_1 \right)$

Constructive Interference, $\phi_2 - \phi_1 = 2\pi m$

If $n_1 = n_2 = 1,$ $\boxed{d_1 - d_2 = \dfrac{M\lambda}{2}}$

Total Wave amplitude
$$U = \sqrt{I_0}\left(1 + h + h^2 + \ldots + h^{M-1}\right) \qquad \text{where} \quad h = e^{j\phi}$$

$$= \sqrt{I_0} \cdot \frac{1 - h^M}{1 - h} = \sqrt{I_0} \cdot \frac{1 - e^{jM\phi}}{1 - e^{j\phi}}$$

Total Intensity
$$I = |U|^2 = I_0 \cdot \left|\frac{1 - e^{jM\phi}}{1 - e^{j\phi}}\right|^2 = I_0 \left|\frac{e^{-jM\phi/2} - e^{jM\phi/2}}{e^{-j\phi/2} - e^{j\phi/2}}\right|^2$$

$$\boxed{I = I_0 \cdot \frac{\sin^2(M\phi/2)}{\sin^2(\phi/2)}}$$

More # of interfering sources

$\Rightarrow$ Narrower spectral selectivity

$\frac{M\phi_{min}}{2} = \pi$

$\phi_{min} = \frac{2\pi}{M}$

$\angle\phi = \phi_2 - \phi_1$