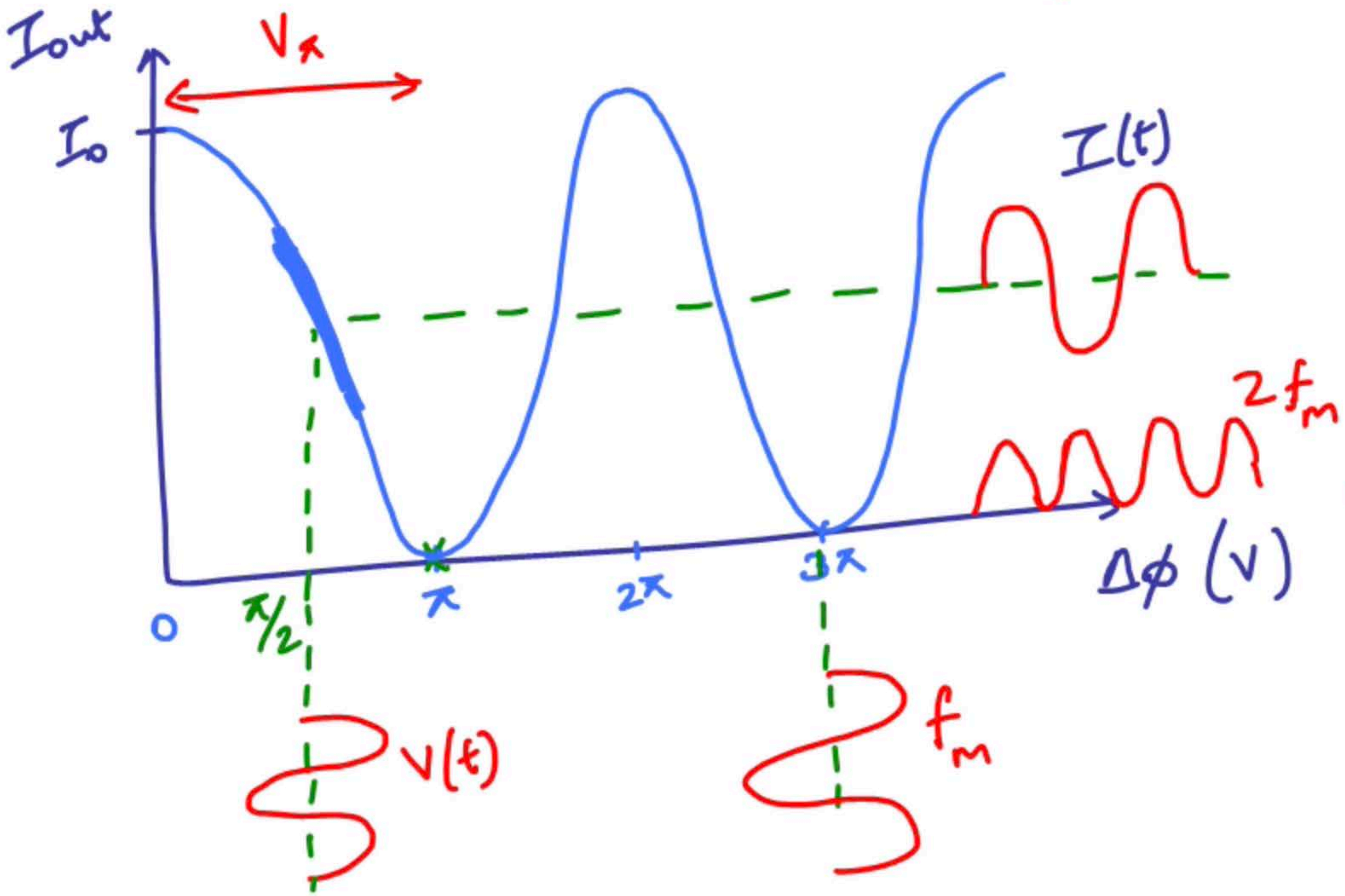
$$\Delta\phi(E) = \cancel{\pi} = \frac{\cancel{2}\pi}{\lambda}\cdot\frac{1}{\cancel{2}}\, r n_0^3 \frac{V_\pi}{d}\cdot L$$

$$\phi(E) = \phi_0 - \pi\cdot\frac{v(t)}{V_\pi} \rightarrow$$ Voltage controlled phase modulation

$$\boxed{V_\pi = \frac{\lambda d}{r n_0^3 L}}$$



Carrier-suppressed modulation

Lets say we need $V_\pi = 5\ V$

We know $n_0 = 2.2$

$r = 30\ pm/V$

$\lambda = 1.5\ \mu m$

$d = 10\ \mu m$

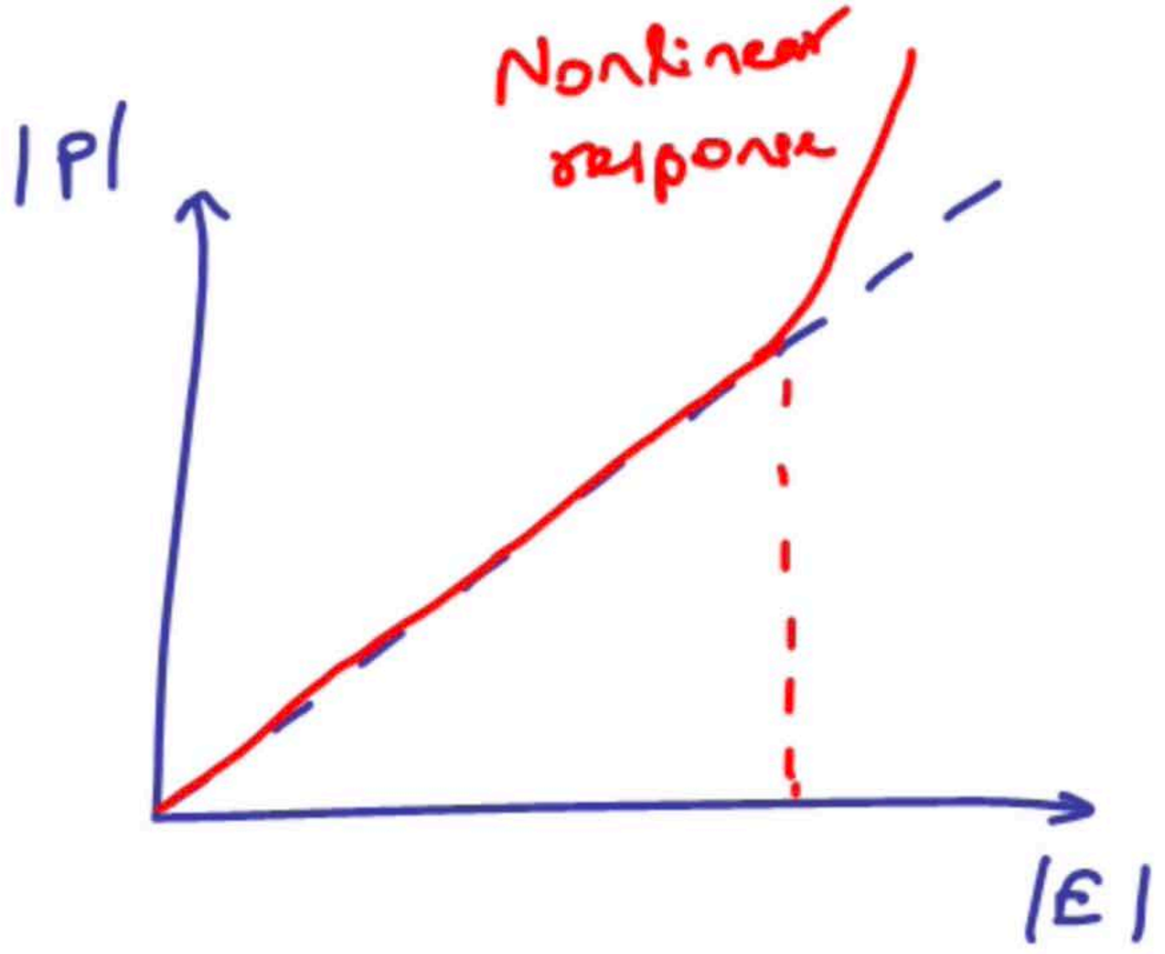
$$\Rightarrow L = \frac{\cancel{1.5\times10^{-6}}\times\cancel{10\times10^{-6}}}{2\ \cancel{30\times10^{-12}}\times(2.2)^3\times 5} \simeq \underline{1\ cm}$$

Nonlinear response of materials to light

$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

$$= \epsilon_0 \vec{E} + \underbrace{\epsilon_0 \chi \vec{E}}_{\vec{P}}$$



$$P = \epsilon_0 \left(\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \cdots \right)$$

Second order susceptibility ($\chi^{(2)} E^2$)

Third order susceptibility ($\chi^{(3)} E^3$)

$E_1 e^{j\omega_1 t} \rightarrow$ $\boxed{\chi^{(2)}}$ (pm/V) $\rightarrow$ $2\omega_1$ → Second Harmonic Generation

$E_2 e^{j\omega_2 t} \rightarrow$ $\omega_1 + \omega_2$

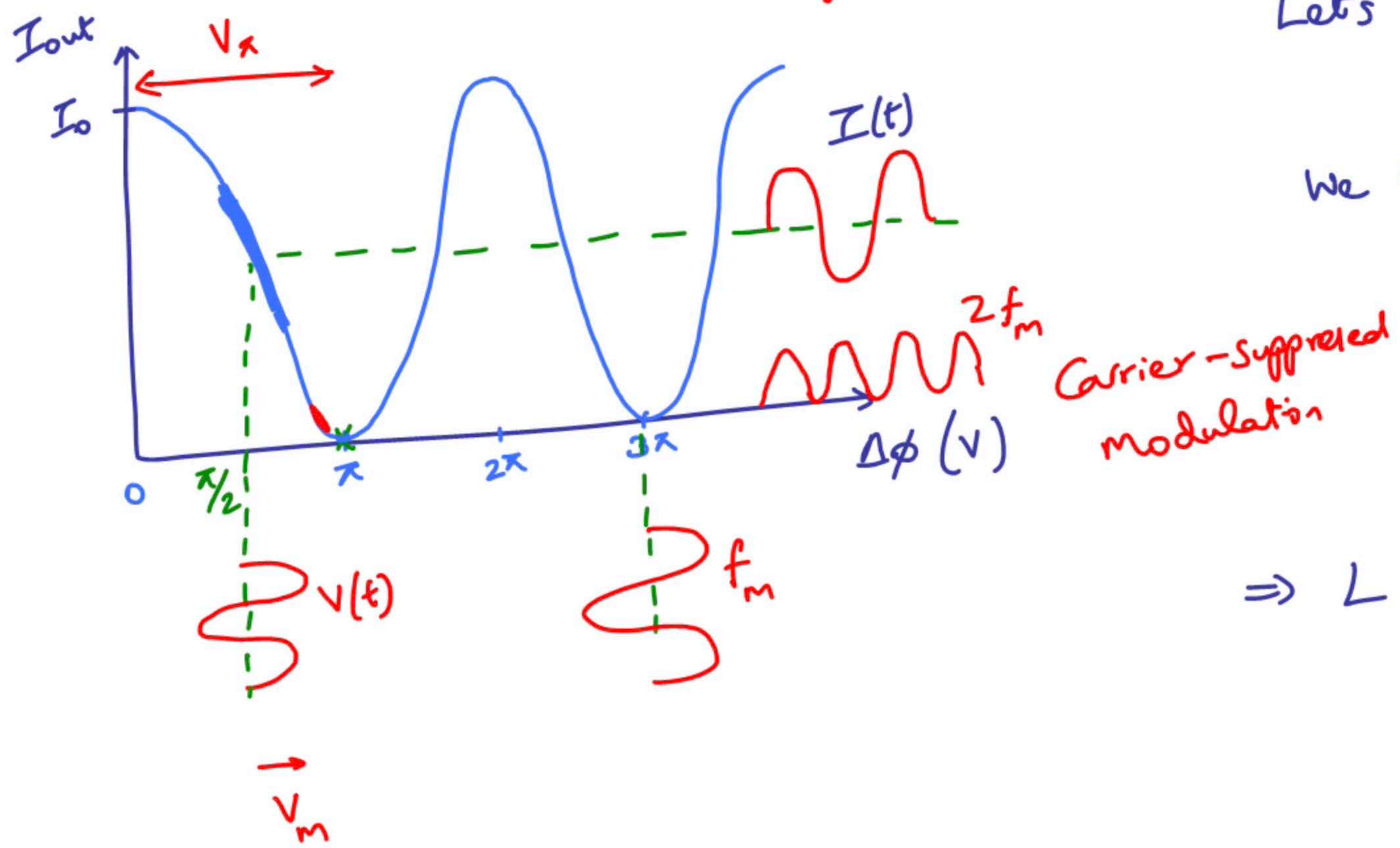
$\omega_1 - \omega_2$ → Difference frequency generation

$2\omega_2$

$$\phi(t) = \phi_0 - \pi \cdot \frac{V(t)}{V_\pi}$$

controlled phase modulation

$$\boxed{V_\pi = \frac{\quad}{r\, n_0^3\, L}}$$



Lets say we need

$V_\pi = 5\ V$

We know $n_0 = 2.2$

$r = 30\ pm/V$

$\lambda = 1.5\ \mu m$

$d = 10\ \mu m$

$$\Rightarrow L = \frac{\cancel{1.5\times10^{-6}} \times \cancel{10\times10^{-6}}}{2\ \cancel{30\times10^{-12}} \times (2.2)^3 \times 5} \approx \underline{1\ cm}$$

Third order susceptibility (χ^3) $\rightarrow$ $\chi^{(3)} E^3$

Energy Conservation

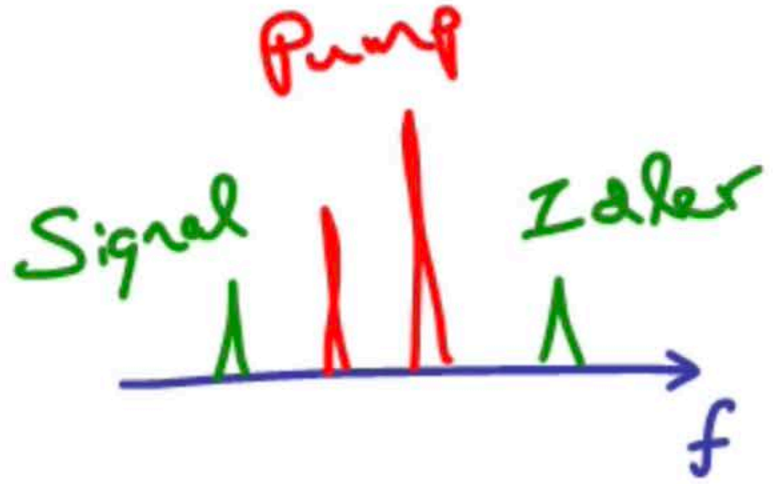
$$\omega_1 + \omega_2 = \omega_3 + \omega_4$$

Momentum Conservation

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4$$

$$\begin{array}{l} E(\omega_1) \rightarrow \\ E(\omega_2) \rightarrow \\ E(\omega_3) \rightarrow \end{array} \boxed{\chi^{(3)}} \rightarrow \begin{array}{l} \omega_1 + \omega_2 + \omega_3 \\ \omega_1 - \omega_2 - \omega_3 \\ \vdots \end{array}$$

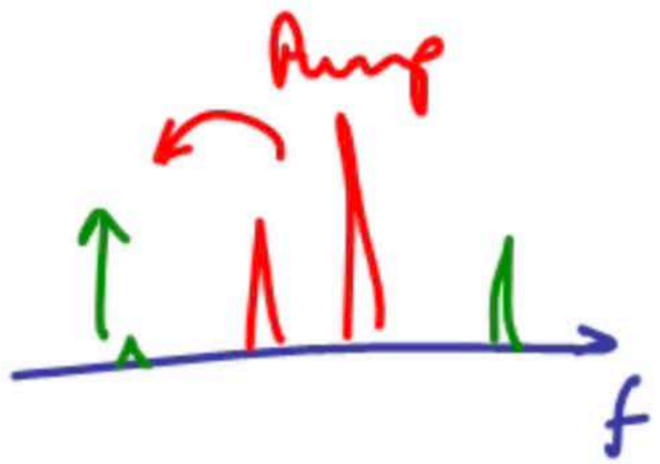
Four-wave mixing



Wavelength Conversion

Self Phase Modulation/

Kerr effect



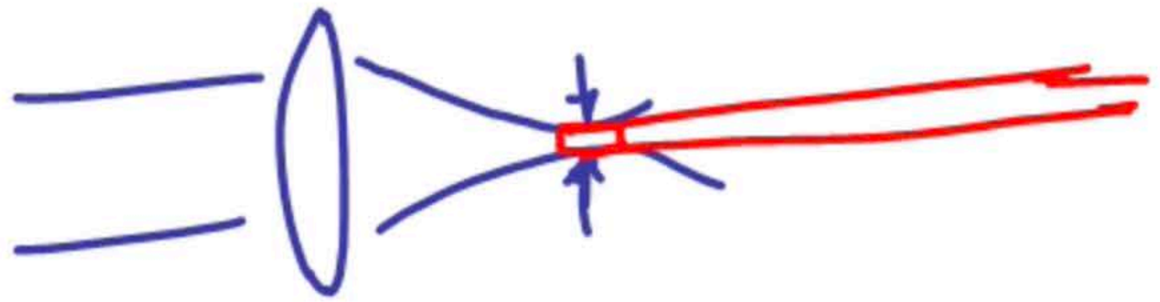
Stimulated Raman Scattering

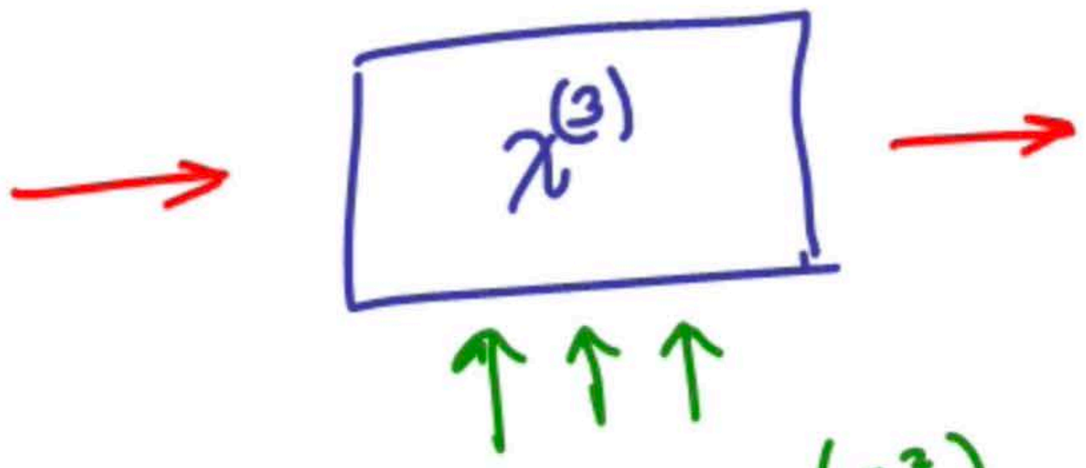
Parametric Amplification

Stimulated Brillouin Scattering

Kerr effect

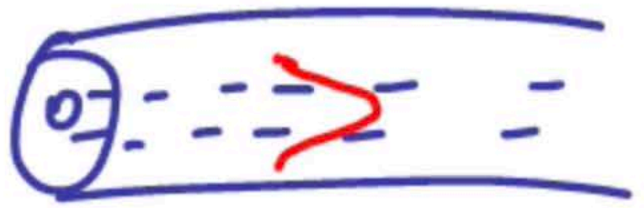
Free Space





Intensity (E^2)

Optical Fiber



$$I = \frac{\text{Power}}{\text{Area}} = \frac{1\,\text{mW}}{\pi w_0^2}$$

$$= \frac{1\,\text{mW}}{\pi (4\times10^{-6})^2}$$

$$= \underline{20}\ \text{MW}/\text{m}^2$$

$$n(E^2) = n_0 - \frac{1}{2} s\, n_0^3 E^2$$

$s \propto \chi^{(3)}$

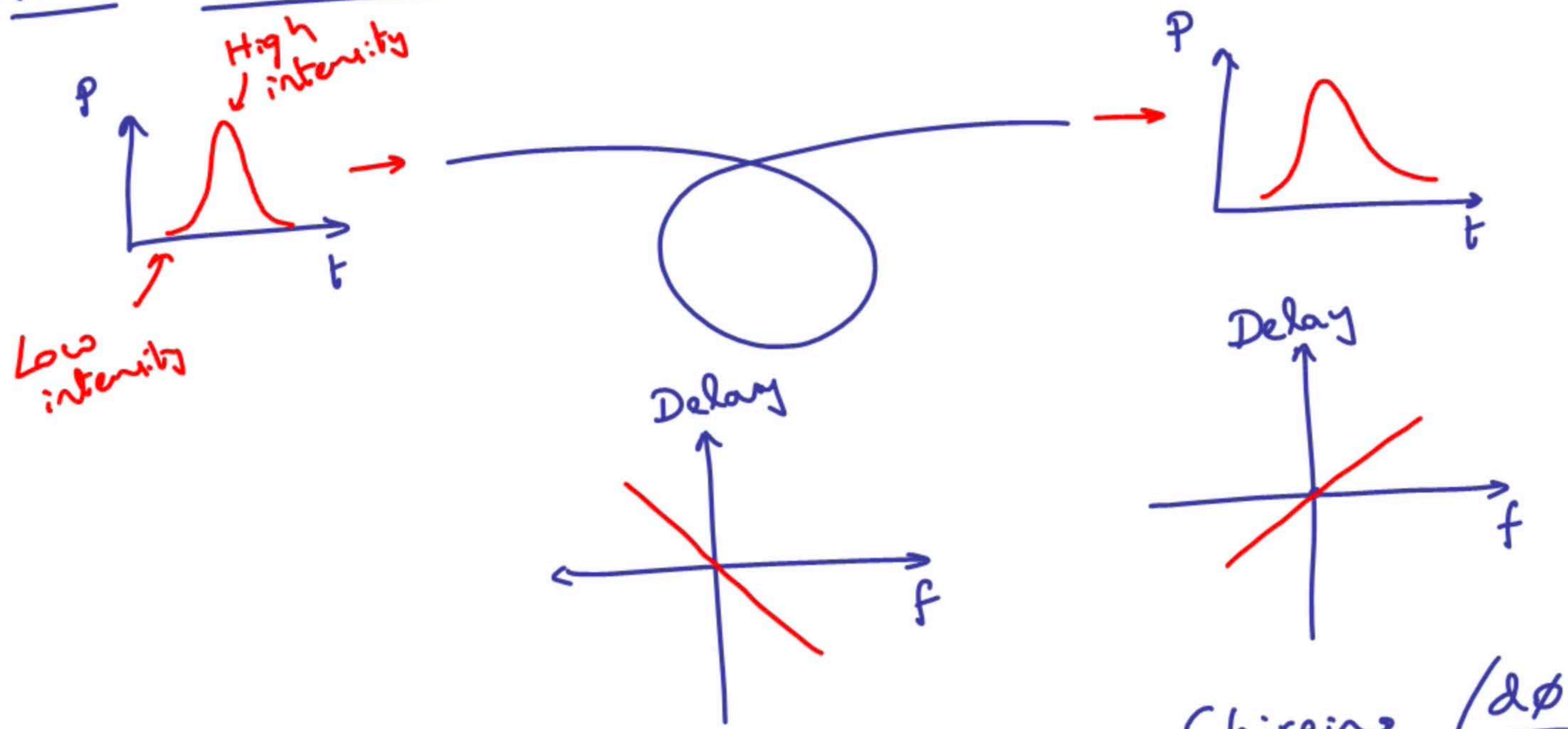
$$n(I) = n_0 + n_2 I$$

where $n_2 \rightarrow$ Kerr index

$n_2 = 10^{-10}\ \text{cm}^2/\text{W}$ in SiO_2

$$\Delta\phi(I) = \frac{2\pi}{\lambda} n_2 \cdot \frac{P}{A} \cdot L$$

Self Phase Modulation



Dispersion + Chirping $\left(\frac{d\phi}{dt}\right)$

$\Rightarrow$ Compensation