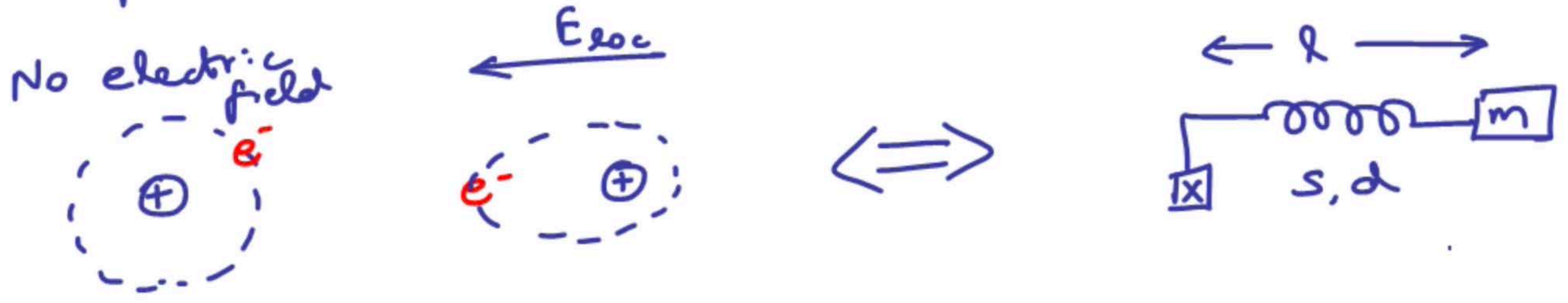
Learning Outcome: Identify the fundamental principles for photon / light manipulation

How does light propagate in a medium?



Displacement is proportional to permittivity (ϵ)

Equation of motion, $$m \cdot \frac{d^2 \ell}{dt^2} + d \cdot \frac{d\ell}{dt} + s \cdot \ell = -e E_{loc}$$

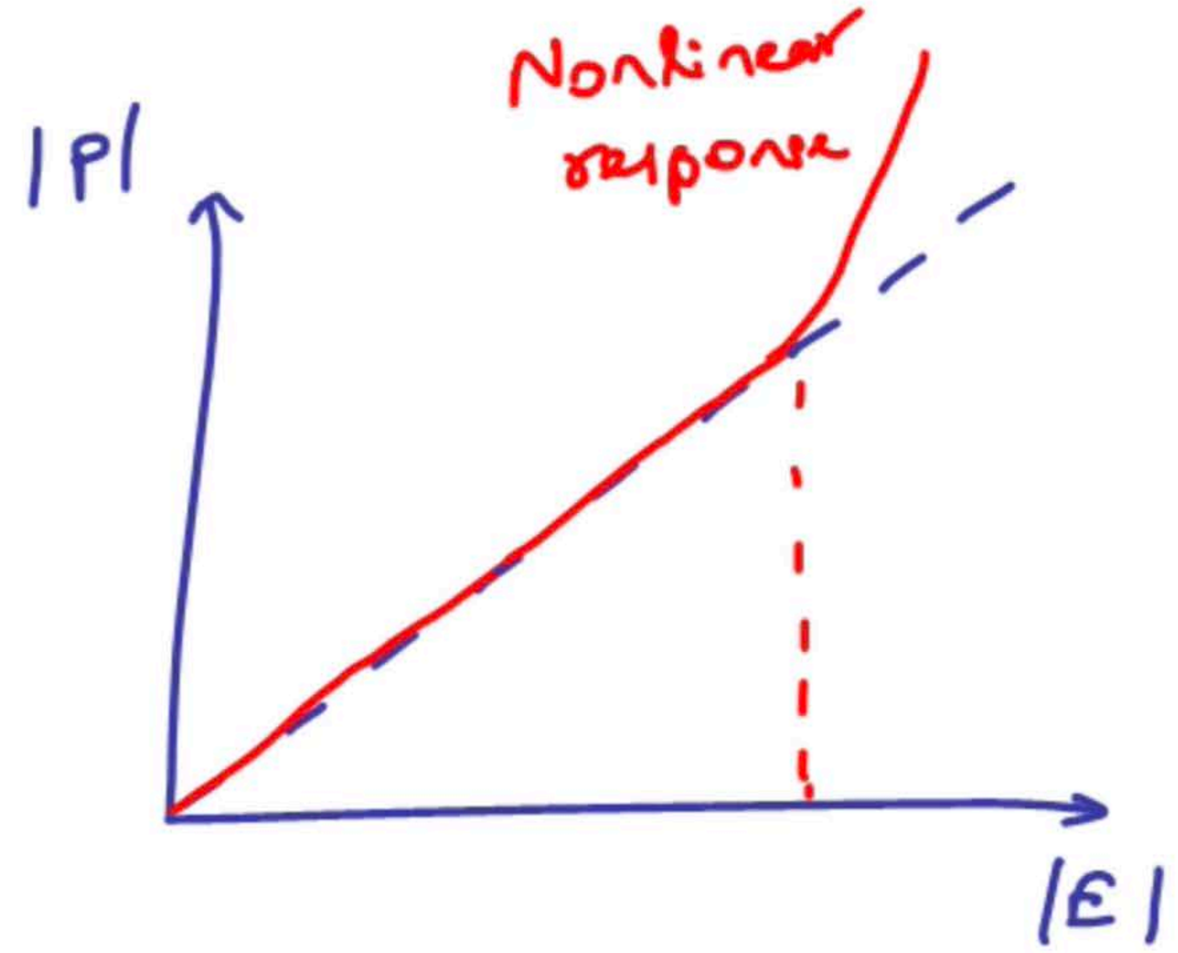
$$\epsilon = \epsilon' - j\epsilon''$$

For time-periodic excitation $\left(e^{j\omega t}\right)$, $$\ell = \frac{-e/m \, E_{loc}}{\omega_0^2 - \omega^2 + j\omega \Gamma}$$

resonance freq. $\omega_0 = \sqrt{s/m}$

damping coeff. d/m

Nonlinear response of materials to light



$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E} \quad \vec{P}$$

$$P = \epsilon_0 \left(\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right)$$

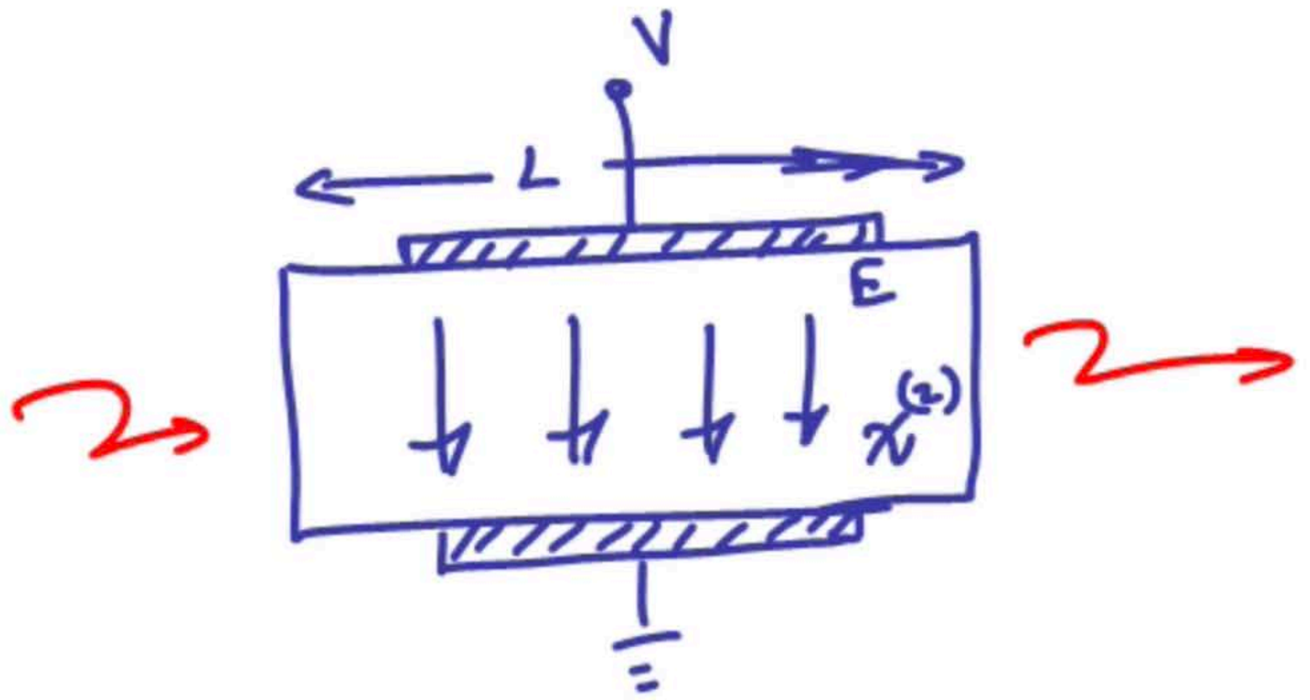
Second order susceptibility

Third order susceptibility

$$\begin{array}{l} E_1 e^{j\omega_1 t} \rightarrow \\ E_2 e^{j\omega_2 t} \rightarrow \end{array} \boxed{\chi^{(2)}} \longrightarrow \begin{array}{l} 2\omega_1 \rightarrow \text{Second Harmonic Generation} \\ \omega_1 + \omega_2 \\ \omega_1 - \omega_2 \rightarrow \text{Difference frequency generation} \\ 2\omega_2 \end{array}$$

pm/V

Pockel's effect



$r \propto \chi^{(2)}$

$$n(E) = n_0 - \frac{1}{2} r n_0^3 E$$

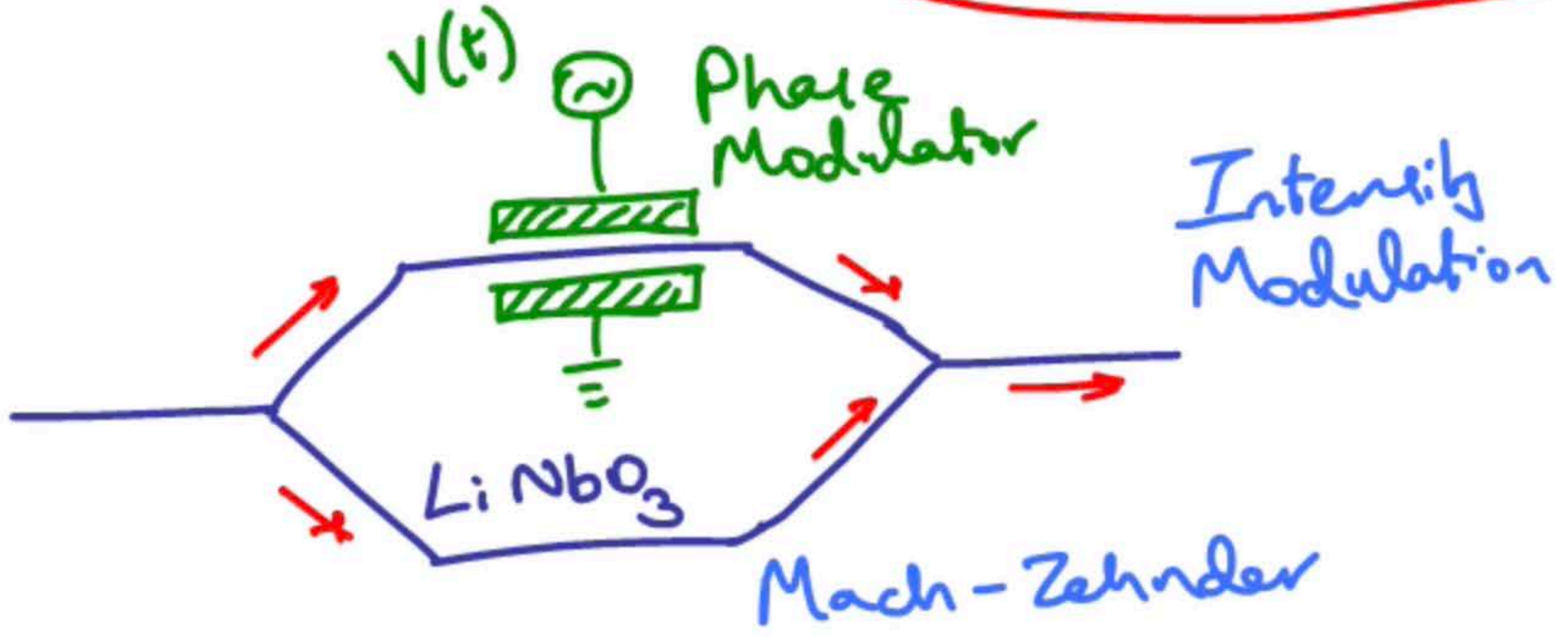
electro-optic coefficient

$LiNbO_3$. $r = 30$ pm/V

SiO_2 . $r = 0.01$ pm/V

$$\phi(E) = \phi_0 - \frac{2\pi}{\lambda} \cdot \frac{1}{2} r n_0^3 E \cdot L$$

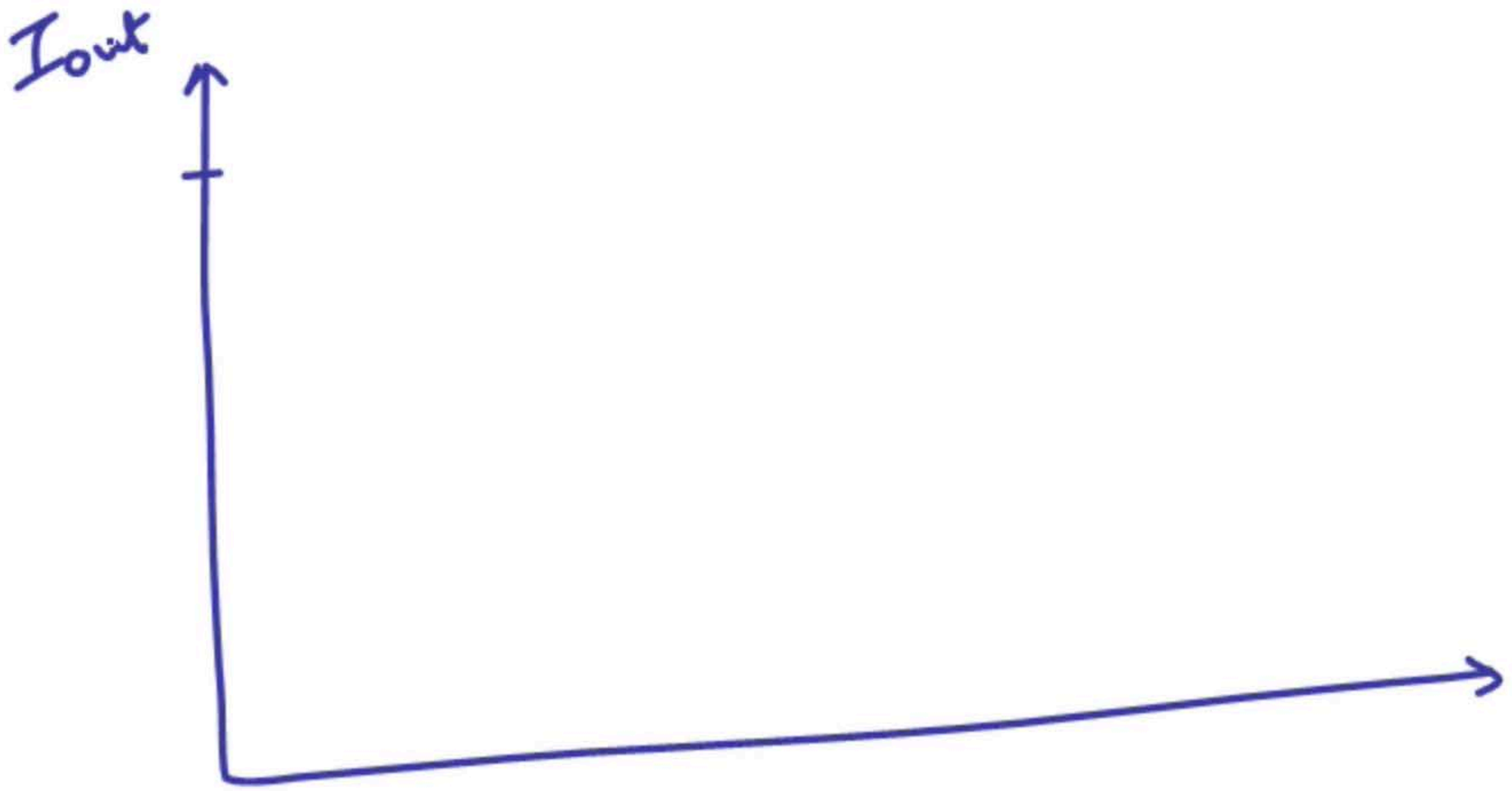
$\Delta\phi(E)$



$$\Delta\phi(t) = \cancel{\pi} = \frac{\cancel{2}\cancel{\pi}}{\lambda}\cdot\frac{1}{\cancel{2}}\, r n_0^3 \,\frac{V_\pi}{d}\cdot L$$

$$\phi(t) = \phi_0 - \pi\cdot\frac{v(t)}{V_\pi}$$

$$\boxed{V_\pi = \frac{\lambda d}{r n_0^3 L}}$$



Lets say we need $V_\pi = 5\ V$

We know $n_0 = 2.2$

$r = 30\ pm/V$

$\lambda = 1.5\ \mu m$

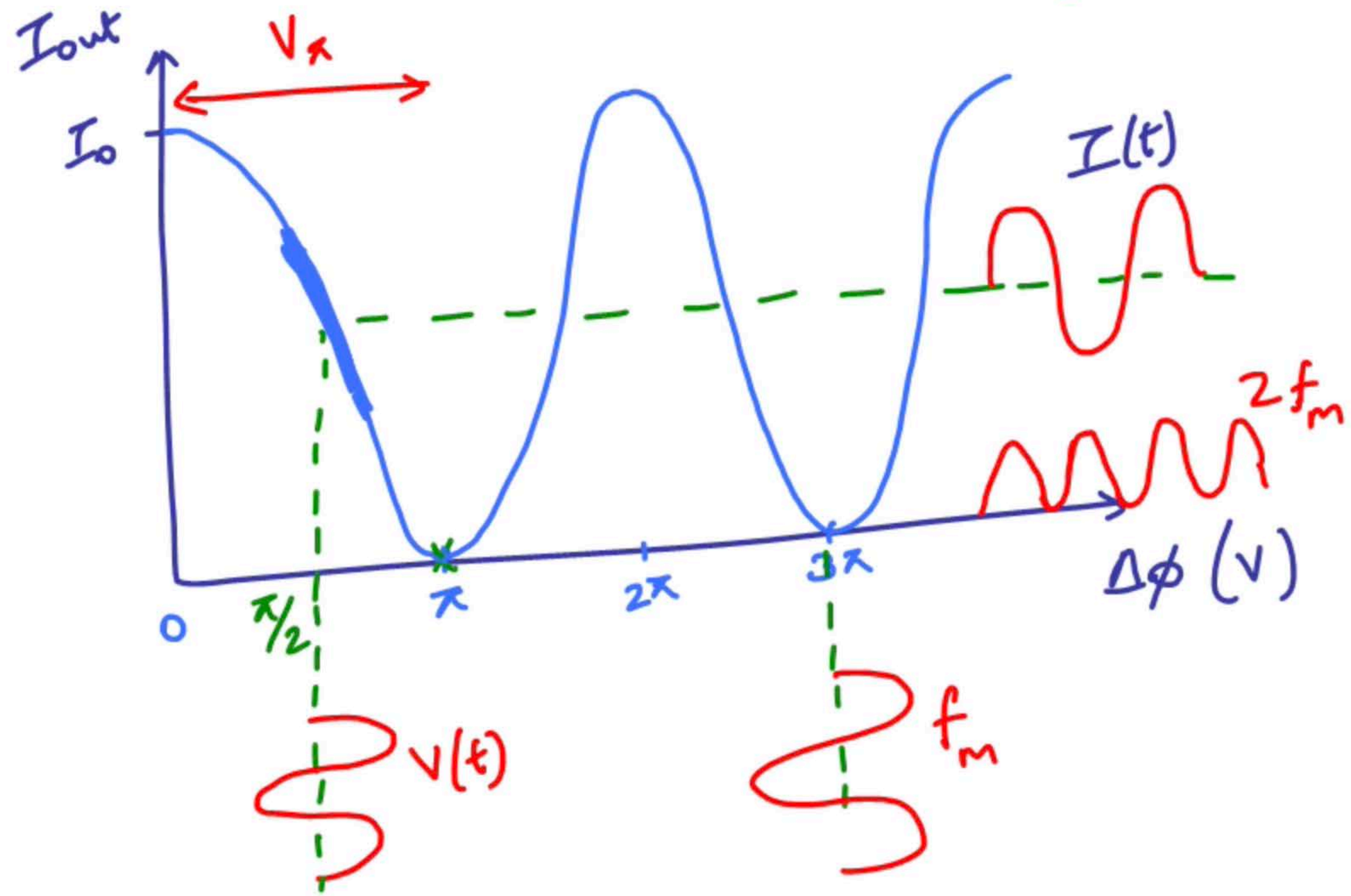
$d = 10\ \mu m$

$$\Rightarrow L = \frac{\cancel{1.5\times10^{-6}}\times\cancel{10\times10^{-6}}}{2\,\cancel{30\times10^{-12}}\times(2.2)^3\times 5} \simeq \underline{1\ cm}$$

$$\Delta\phi(E) = \cancel{\pi} = \frac{\cancel{2\pi}}{\lambda}\cdot\frac{1}{\cancel{2}}\, r n_0^3\, \frac{V_\pi}{d}\cdot L$$

$$\phi(E) = \phi_0 - \pi\cdot\frac{v(t)}{V_\pi}$$ → Voltage controlled phase modulation

$$\boxed{V_\pi = \frac{\lambda d}{r n_0^3 L}}$$



Carrier-suppressed modulation

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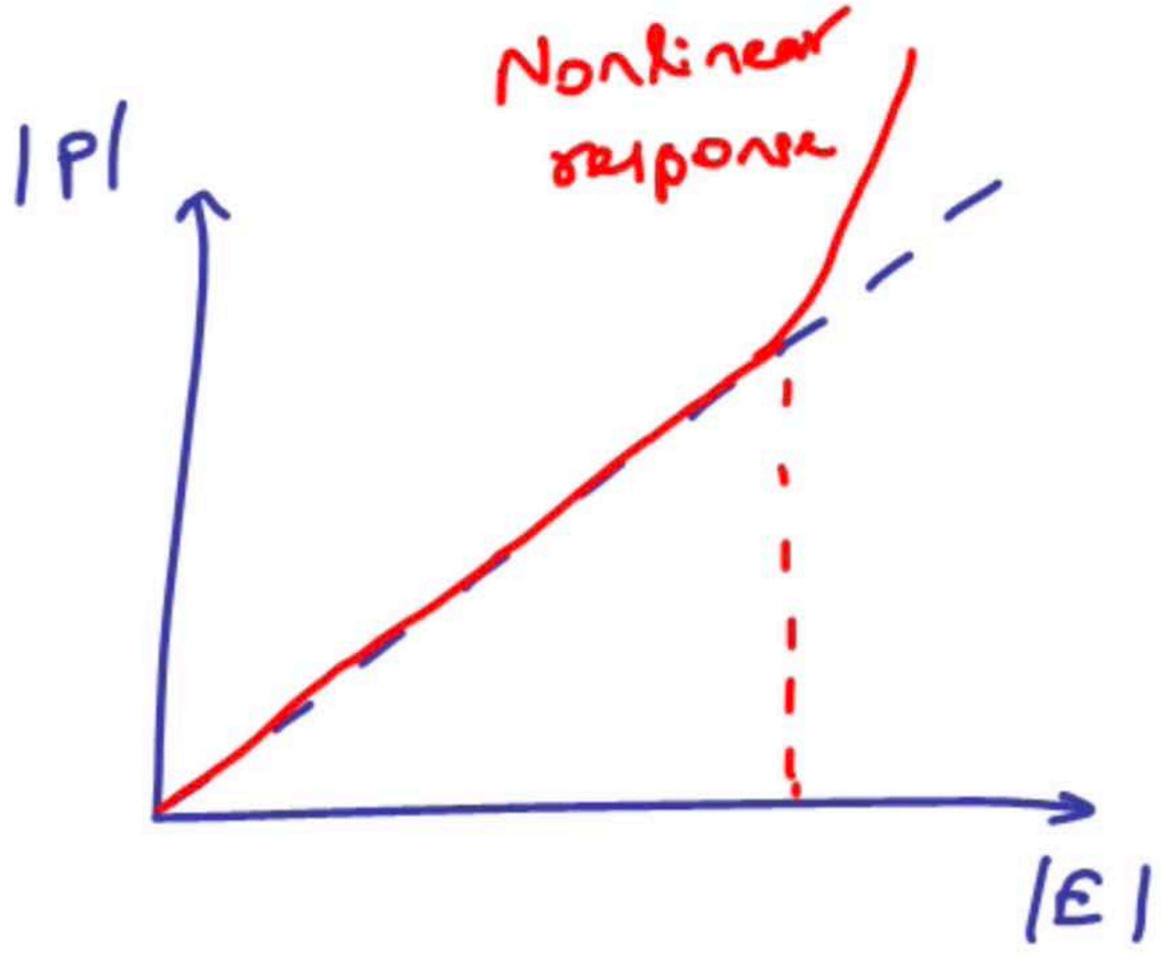
$$= \epsilon_0 \vec{E} + \underbrace{\epsilon_0 \chi \vec{E}}_{\vec{P}}$$



$$P = \epsilon_0 \left(\chi E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots \right)$$

Second order susceptibility

Third order susceptibility

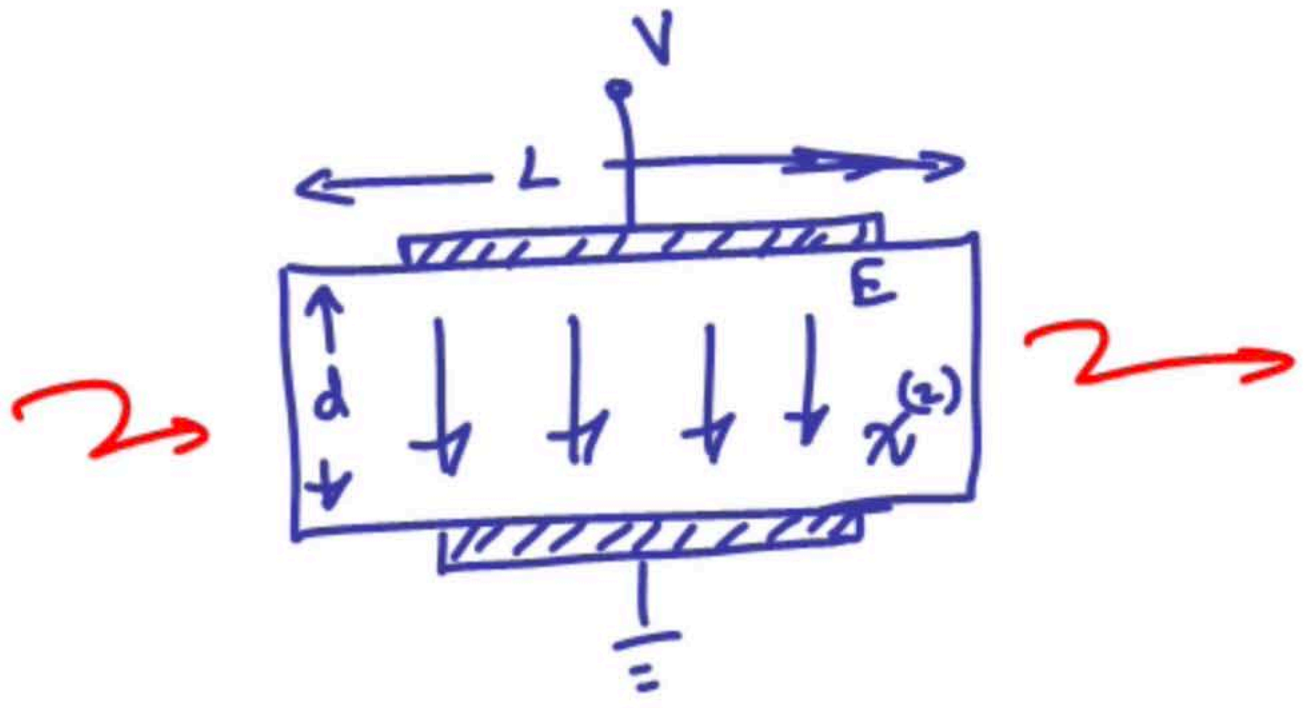
$$E_1 e^{j\omega_1 t},\ E_2 e^{j\omega_2 t} \longrightarrow \boxed{\chi^{(2)}} \longrightarrow 2\omega_1,\ \omega_1 + \omega_2,\ \omega_1 - \omega_2,\ 2\omega_2$$

pm/V

$2\omega_1 \rightarrow$ Second Harmonic Generation

$\omega_1 - \omega_2 \rightarrow$ Difference frequency generation

Pockel's effect



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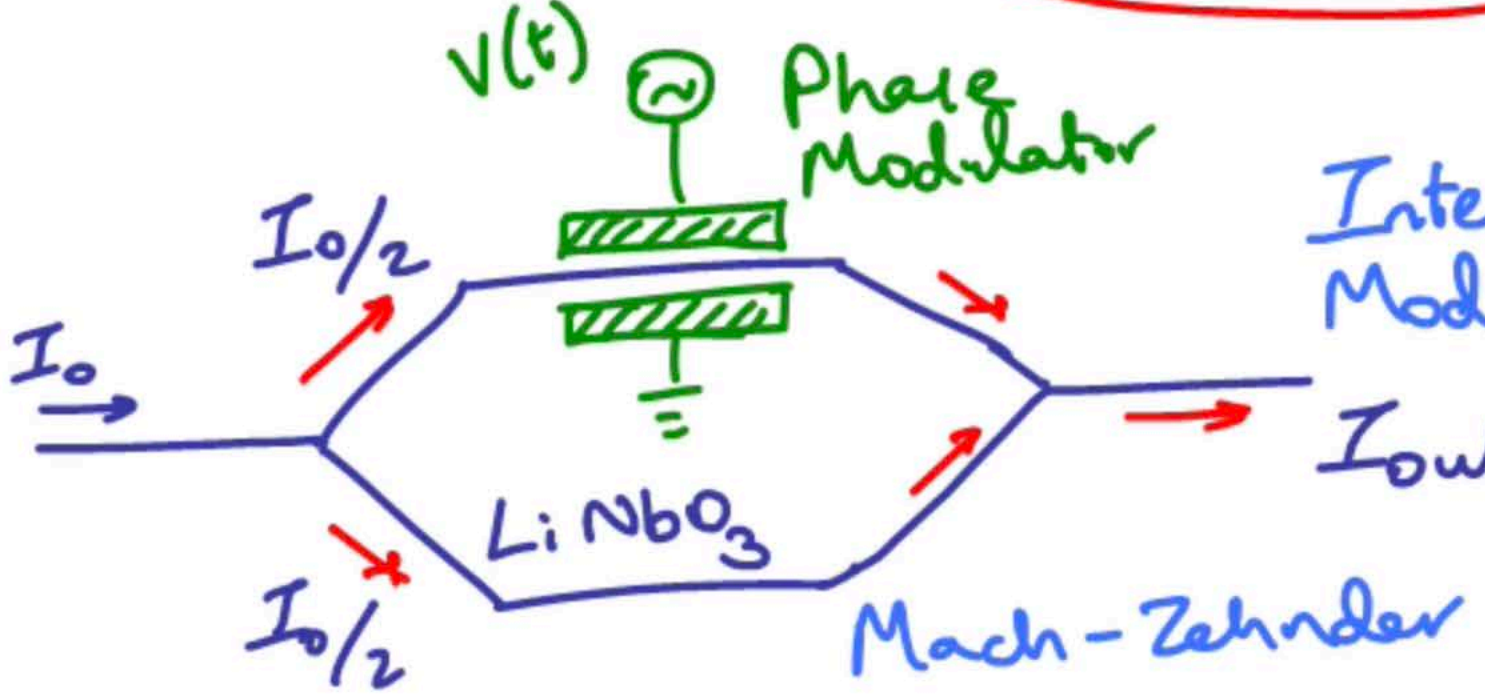
$\Delta\phi(E)$



Intensity Modulation

$$I_{out} = \frac{I_0}{2}(1+\cos\Delta\phi)$$

$$= I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$$