An introduction to coding theory

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Lecture #1A: Introduction to error control coding-I

Books

Textbook:

- "Error Control Coding", by Shu Lin and Daniel J. Costello, Jr., 2nd edition, Prentice Hall, 2004.

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- Cary W. Huffman, Vera Pless, "Fundamentals of Error-Correcting Codes", 1st Edition, Cambridge University Press, 2003.

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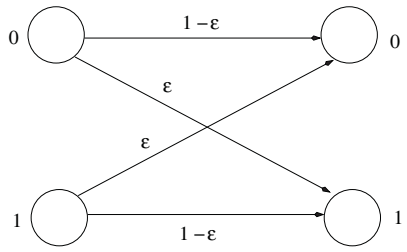
All communications involves three basic steps

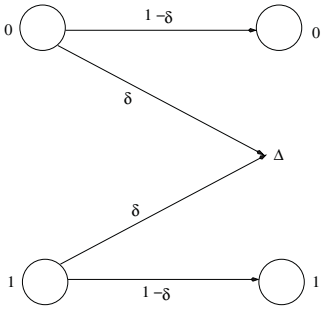
- Encoding a message at its source.
- Transmitting that message through a communication medium.
- Decoding the message at its destination.

Introduction

- The transmission medium in communication is known as channel.



Binary Symmetric Channel



Binary Erasure Channel

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- Example: If the channel capacity of a particular communication link is (say) 2 Gbps. We can communicate over this channel at any desired rate less than 2 Gbps, and achieve arbitrary low error rates.

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- In digital storage systems, channel coding is used to control errors caused by storage medium defects, dust particles, radiation.

Channel Coding

- Example: Repetition codes

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- Rate R=1/2 code

$$0 \rightarrow 00 \qquad 1 \rightarrow 11$$

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- Rate R=1/2 code

$$0 \rightarrow 00 \qquad 1 \rightarrow 11$$

- Rate R=1/3 code

$$0 \rightarrow 000 \qquad 1 \rightarrow 111$$

Channel Coding

- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/2 Repetition codes:	00 00 11 11 00 11
Received coded bits (Single Error):	10 00 11 11 00 11
Received coded bits (Double Error):	11 00 11 11 00 11

Channel Coding

- Example (contd.)

Information bits:	0 0 1 1 0 1
Coded bits using Rate 1/3 Repetition codes:	000 000 111 111 000 111
Received coded bits (Single Error):	100 000 111 111 000 111
Received coded bits (Double Error):	110 000 111 111 000 111

Channel Coding

A message of content and clarity
has gotten to be quite a rarity.
To combat the terror of serious error
use bits of appropriate parity.

– Solomon Golomb