An introduction to coding theory

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Lecture #19: Automatic Repeat reQuest (ARQ) schemes

Outline of the lecture

- Introduction

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- Introduction
- Automatic Repeat reQuest (ARQ) and error detecting codes

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- Protocols for Automatic Repeat Requests

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- Performance Analysis of ARQ systems

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- Hybrid ARQ

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- In *one-way* system, transmission takes place only in one direction, i.e. from transmitter to receiver.
- The error correcting codes used in such a system are referred as *forward error correction (FEC)* codes.

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Automatic Repeat reQuest (ARQ):

- In *two-way* system, there exist a feedback path from the receiver to the transmitter.
- Error correction can be achieved for two-way system using error detection and retransmission, also known as *automatic repeat request (ARQ)*.

ARQ and error detecting codes

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- Let the n bits in a frame be $[c_{n-1}, c_{n-2}, \cdots, c_1, c_0]$ and $n - k$ low order bits (c_0 through c_{n-k-1}) be the parity bits, then the parity bits are chosen such that the polynomial $c(x) = c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_1 x + c_0$ is divisible by g(x).

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- Since c(x) and g(x) are polynomials with binary coefficients, all operations are defined modulo two.

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An introduction to coding theory

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- We can verify that $c(x) = g(x) \cdot (x^{10} + x^2 + x)$ and this choice of the parity bits is unique in the sense that results in a multiple of g(x).
- At the receiver, the received polynomial is checked whether it is divisible by g(x). If it is, the frame is declared error free, else it is declared as erroneous.

CRC codes in practice

- Examples of some popular CRC codes

Code	$g(x)$	n_{max}	d_{min}
CRC-12	$x^{12} + x^{11} + x^3 + x^2 + x + 1$	2,047	4
CRC-ANSI	$x^{16} + x^{15} + x^2 + 1$	32,767	4
CRC-CCITT	$x^{16} + x^{12} + x^5 + 1$	32,767	4

Table: Properties of Three Standard CRC Codes

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- We assume that the frames transmitted on the forward link and the acknowledgments transmitted on the return link are all received in order in which they are sent.
- We assume that there is no undetected error in the feedback channel.

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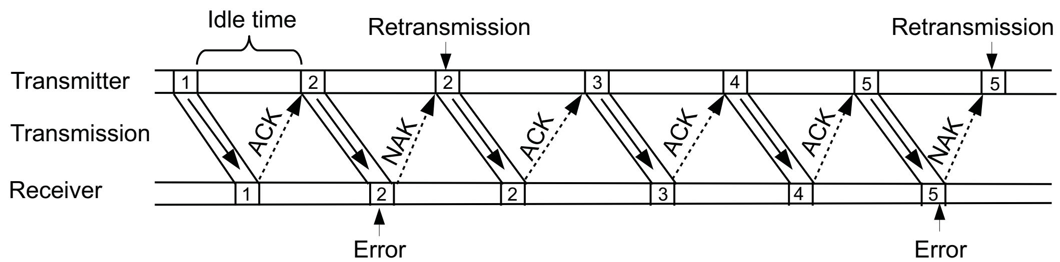
- To avoid potential confusion caused by lost frame, a sequence number is often included in the header of the frame and acknowledgments.



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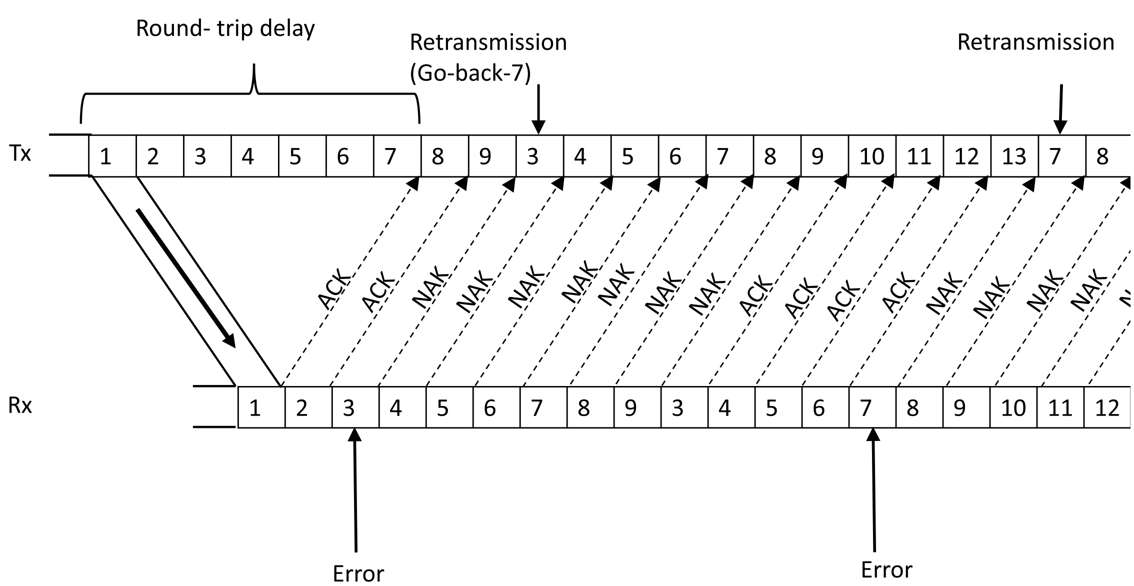
- Appropriate choice of N is governed by the propagation delay of the system and the length of the frames.

Go-Back-N ARQ



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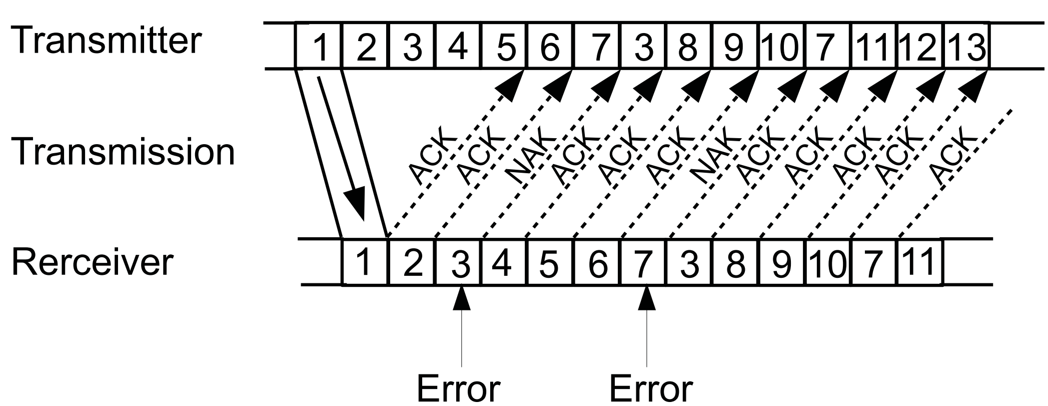
- However, here one has to store the arriving frame in a buffer and reorder them in the correct order.



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  - Frame is received in error and it is detected by the error detecting code.
  - Frame is received in error, however the error detecting code is unable to detect it.

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- Let P_u be the probability that a frame is received in error and the error detecting code is unable to detect it.
- Assuming there is no limit on retransmissions, the frame error rate is given by

$$\begin{aligned}\text{FER} &= P_u + P_d P_u + P_d^2 P_u + P_d^3 P_u \cdots + \\ &= P_u \sum_{i=0}^{\infty} P_d^i = \frac{P_u}{1 - P_d}\end{aligned}$$

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- This analysis is true for any ARQ protocol.
- Reliability of an ARQ protocol depends only on the error detecting capability of the underlying error detecting code and is independent of how many corrupted frames are subsequently retransmitted.

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- The throughput of an ARQ scheme is defined as the ratio of the average number of information bits accepted by the receiver per unit time to the number of bits that can be transmitted over the channel per unit time.
- In selective-repeat ARQ each time a frame is transmitted, it has a probability $P = P_c + P_u$ of being accepted by the receiver where P_c denotes the probability that the frame is delivered error free to the receiver and P_u denotes the probability that the frame is in error but the error detecting code is unable to detect it.

Efficiency analysis: SR-ARQ

- Let T_{SR} be a random variable equal to the number of frame transmissions required for SR ARQ until a particular frame is accepted, then $Prob(T_{SR} = 1) = P, Prob(T_{SR} = 2) = P(1 - P), Prob(T_{SR} = 3) = P(1 - P)^2$ and so on.

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- If we use a (n,k) error detecting code, the receiver accepts k information bits in the amount of time (on average) it takes to transmit n/P over the channel.
- Thus the throughput for selective-repeat ARQ is given by

$$\eta_{SR} = \frac{k}{n}P$$

Efficiency analysis: Go-back-N ARQ

- Let $P = P_c + P_u$ be the probability of being accepted by the receiver where P_c denotes the probability that the frame is delivered error free to the receiver and P_u denotes the probability that the frame is in error but the error detecting code is unable to detect it.

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- Let T_{GBN} be a random variable equal to the number of frame transmissions required in Go-back-N ARQ until a particular frame is accepted, assuming that a full window of N frames must be retransmitted every time a frame is rejected, we have

$$\begin{aligned} E[T_{GBN}] =& 1 \cdot P + (N+1) \cdot (1-P) \cdot P \\ & + (2N+1) \cdot (1-P)^2 \cdot P + \cdots \\ =& 1 + \frac{N(1-P)}{P} \end{aligned}$$

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Efficiency analysis: Go-back-N ARQ (contd.)

- The receiver accepts k information bits in the amount of time (on average) it takes to transmit $n(1 + N(1 - P)/P)$ bits over the channel, so the throughput is given by

$$\eta_{GBN} = \left(\frac{k}{n}\right)\left(\frac{P}{P + N(1 - P)}\right)$$

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- Let T_{SW} be the random variable that represents the delay in terms of frame units incurred from the time a frame is first sent to the time it is finally accepted, we have

$$\begin{aligned} E[T_{SW}] =& (1+\beta)\cdot P + 2\cdot(1+\beta)\cdot(1-P)\cdot P \\ &+ 3\cdot(1+\beta)\cdot(1-P)^2\cdot P + \cdots \\ =& \frac{1+\beta}{P} \end{aligned}$$

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- In HARQ protocols, the transmitted data is encoded for both error correction and error detection. If the receiver detects an error after decoding, it sends a negative acknowledgment (NACK) to the transmitter, which then retransmits the data.

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- Information sequence is first encoded using an error detection code and then using an error correction code.
- At the receiver, the error correcting code first tries to reconstruct the information sequence and then passes the information to the error detection code.
- If the error detection code observes a parity violation, a retransmission request is initiated. Otherwise the packet is accepted.

Type 1 Hybrid ARQ systems

- A simple implementation of Type 1 Hybrid ARQ uses a high rate error detection code and a lower rate error correction code.
- Information sequence is first encoded using an error detection code and then using an error correction code.
- At the receiver, the error correcting code first tries to reconstruct the information sequence and then passes the information to the error detection code.
- If the error detection code observes a parity violation, a retransmission request is initiated. Otherwise the packet is accepted.
- Usually the same information sequence is retransmitted.

Type 2 Hybrid ARQ systems

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- During retransmission, parity bits based on the original information bits is transmitted.
- Original received packet that was received in error is stored in a buffer and is augmented with the newly received parity bits to form a lower rate more powerful forward error correcting code.