An introduction to coding theory

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Lecture #14A: Decoding of low density parity check codes-I

Outline of the talk

- Decoding on BSC: Bit Flipping Algorithm

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    - Example 1: One transmission error case.

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  - Example 2: Two transmission errors case.

Low-density parity check codes

1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1

- Example of a low density code matrix; n=20, j=3, k=4

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  - The nodes on tier 2 represent the other bits in those parity-check sets.

Low-density parity check codes

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0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1
1	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0
0	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0
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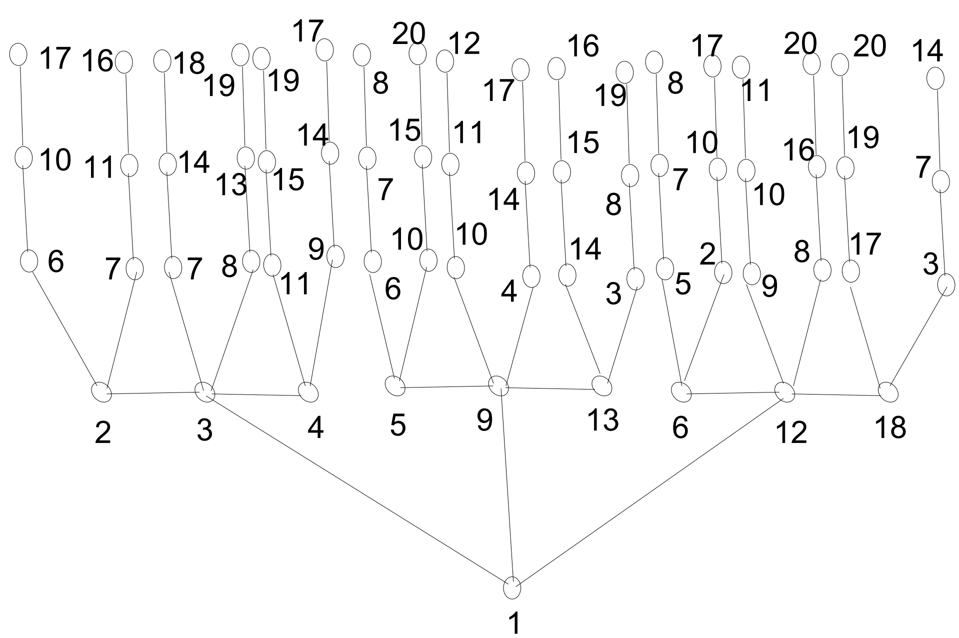
Parity-check set

#	parity-check set
1	{1,2,3,4}
2	{5,6,7,8}
3	{9,10,11,12}
4	{13,14,15,16}
5	{17,18,19,20}
6	{1,5,9,13}
7	{2,6,10,17}
8	{3,7,14,18}
9	{4,11,15,19}
10	{8,12,16,20}
11	{1,6,12,18}
12	{2,7,11,16}
13	{3,8,13,19}
14	{4,9,14,17}
15	{5,10,15,20}

Parity-check set tree



Decoding on BSC: Bit-Flipping Algorithm

Example 1: Single transmission error case

Transmitted bits= {0,0,0,0,0,0,1,1,1,1,1,1,1,0,1,0,1,1,0,0}
Received bits={1,0,0,0,0,0,1,1,1,1,1,1,1,0,1,0,1,1,0,0}

- The first bit is received in error.

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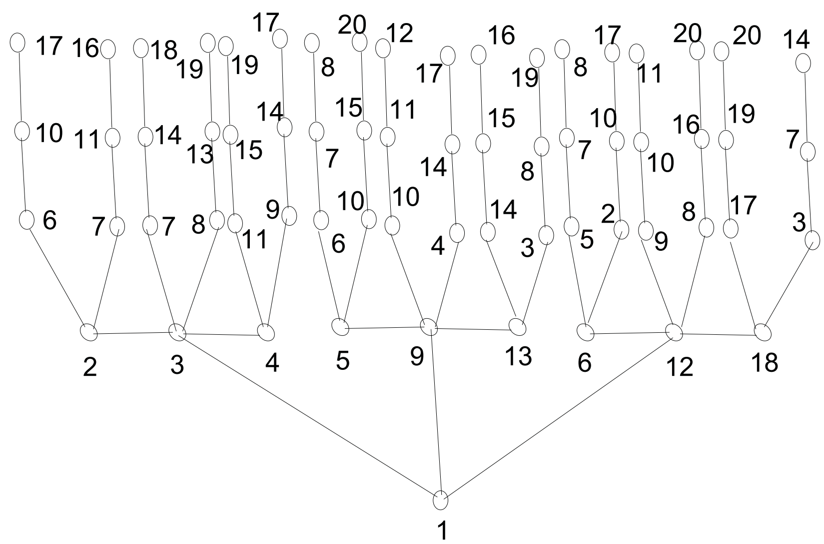
- The first bit is received in error.
- Decoder will try to correct the error.

Decoding on BSC: Bit-Flipping Algorithm

- **Step 1:** Represent the code using parity check set tree.



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10	{8,12,16,20}
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- **Step 2:** First Iteration: Check the parity-check sets containing bit # 1 to see if they are satisfied.

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- All the three parity check set #1, 6, 11 are violated.

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- Flip the first received bit # 1 from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied).
- All the parity equations containing bit 1 are satisfied, hence the first bit is decoded as 0.

Decoding on BSC: Bit-Flipping Algorithm

Example 2: Two transmission errors case

Transmitted bits= {0,0,0,0,0,0,1,1,1,1,1,1,1,0,1,0,1,1,0,0}
Received bits={1,1,0,0,0,0,1,1,1,1,1,1,1,0,1,0,1,1,0,0}

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Decoding on BSC: Bit-Flipping Algorithm

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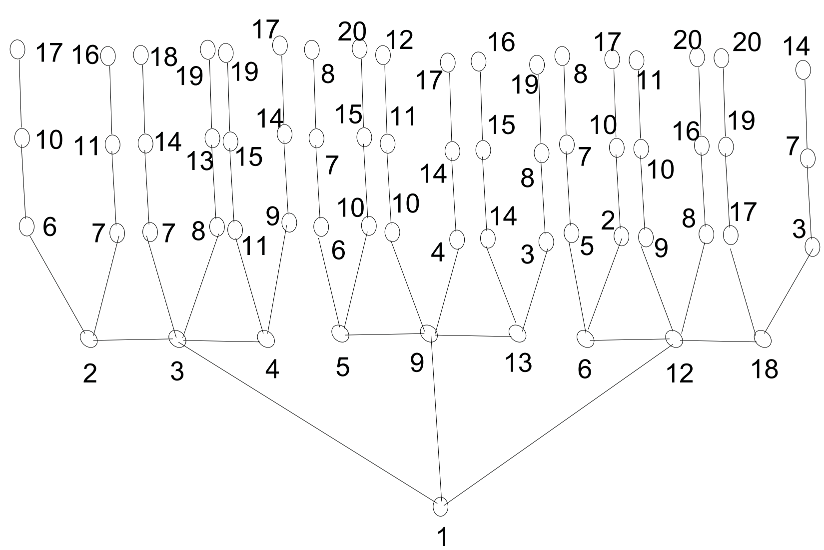
- First two bits are received in error.
- Decoder will try to correct bit #1 and 2.

Decoding on BSC: Bit-Flipping Algorithm

- **Step 1:** Construct parity check-set tree.



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Decoding on BSC: Bit-Flipping Algorithm

- **Step 2:** First Iteration: Check the parity-check set containing bit #1 to see if they are satisfied.
- Two of the three parity check set #6 and 11 are violated. Parity check set #1 is satisfied.

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- Flip the first received bit from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied.
- Parity check-set #6 and 11 are satisfied, but #1 failed.

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- **Step 2:** First Iteration: Check the parity-check set containing bit #1 to see if they are satisfied.
- Two of the three parity check set #6 and 11 are violated. Parity check set #1 is satisfied.
- Since majority of the parity-check set containing bit #1 are violated, there is a strong possibility that first bit is in error.
- Flip the first received bit from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied.
- Parity check-set #6 and 11 are satisfied, but #1 failed.
- Hence the first iteration is not sufficient to correct the errors.

Decoding on BSC: Bit-Flipping Algorithm

- **Step 3:** Second Iteration: Check the parity-check set containing bits in the first tier of the parity check-set tree to see if they are satisfied.

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- Parity check set containing bits 3(#8 and 13), 4(#9 and 14), 5(# 2 and 15), 9(# 3 and 14), 13(# 4 and 13), 12(# 3 and 10), 18(# 5 and 8) are satisfied. One of the parity check set containing bit 6(# 2) is also satisfied.

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- Both the parity check set (# 7 and 12) containing bit 2 and one of the parity check set containing (# 7) containing bit 6 are violated.

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- Both the parity check set (# 7 and 12) containing bit 2 and one of the parity check set containing (# 7) containing bit 6 are violated.
- Bit 2 is common in all of these three parity check set as well as the parity check #1 which was violated after first iteration.
- Hence, there is a strong possibility that second bit is in error.

Decoding on BSC: Bit-Flipping Algorithm

- Flip the second received bit #2 from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied.

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- Flip the second received bit #2 from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied.
- All the parity check sets at first tier are satisfied.
- Now we check the parity-check sets at zero tier(containing first bit) and they are also satisfied.

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- Flip the second received bit #2 from 1 to 0 and recompute the syndrome(check whether the parity constraints are satisfied.
- All the parity check sets at first tier are satisfied.
- Now we check the parity-check sets at zero tier(containing first bit) and they are also satisfied.
- Hence the first and second bits are decoded as 0's.

Decoding on BSC: Bit Flipping Algorithm

- The decoder computes all the parity checks and then changes any bit that is contained in more than some fixed number of unsatisfied parity-check equations.

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- Using these new values, the parity checks are recomputed, and the process is repeated until the parity checks are all satisfied.
- If the parity check sets are small, this decoding procedure is reasonable, since most of the parity-check sets will contain either one transmission error or no transmission errors.
- Thus when most of the parity-check equation checking on a digit are unsatisfied, there is a strong indication that that digit is in error.