An introduction to coding theory

Adrish Banerjee

Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur, Uttar Pradesh
India

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Lecture #18: Convergence of turbo codes

Outline of the lecture

- Introduction

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- Introduction
- Measures for convergence analysis of turbo codes

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- Measures for convergence analysis of turbo codes
  - El Gamal's method

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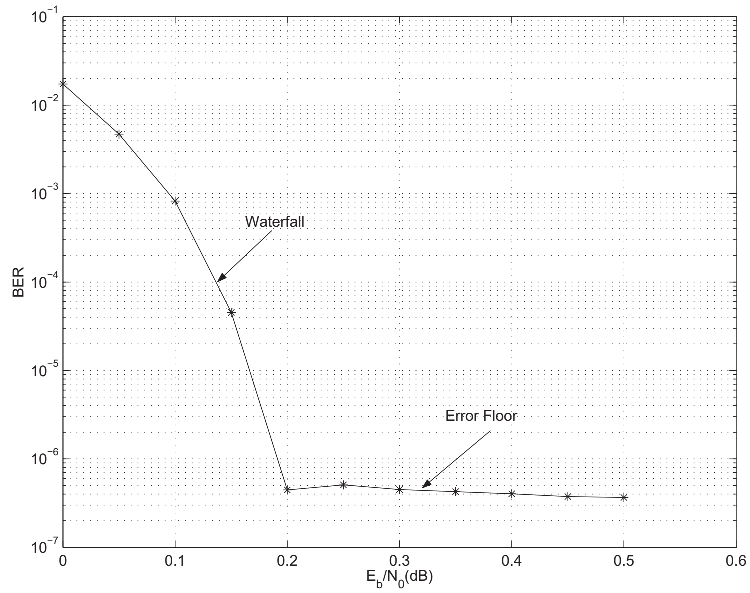
- Limitations of the threshold calculation methods.

Introduction



- Convergence analysis is used to explain the performance of the turbo code in the waterfall region.

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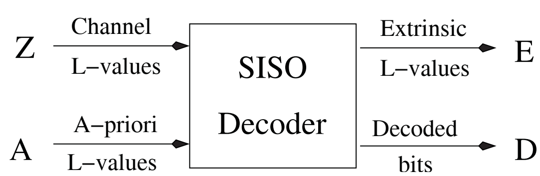

Inputs and Outputs of a soft-input, soft-output (SISO) turbo decoder

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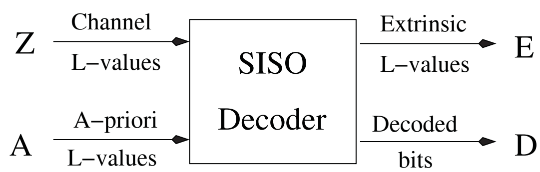

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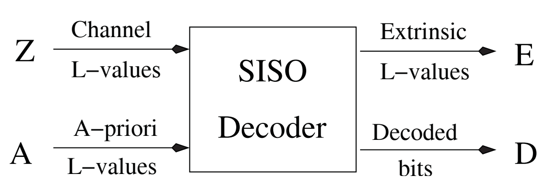

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- Initially, the decoder has no a-priori information about the information bits.
- With increasing iterations, only input to the decoder that is changing is the a-priori information.

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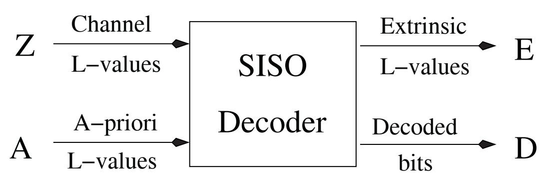
- For asymptotically large block lengths, the smallest channel SNR for which iterative decoding converges is known as the *iterative decoding threshold*.
- Convergence analysis methods provide a tool to compute convergence thresholds for concatenated coding schemes using iterative decoding.
- They also help in the selection of constituent codes and puncturing patterns for turbo coding schemes for good performance in the waterfall region.

Measures for convergence analysis of turbo codes

- SNR [1].
- Density evolution [2].
- EXIT charts [3].

1. H. E. Gamal et. al., "Analyzing the turbo decoder using the Gaussian approximation," IEEE Trans. Inform. Theory, vol. 47, pp. 671-686, Feb 2001.
2. D. Divsalar et. al, "Iterative Turbo Decoder Analysis Based on Density Evolution," IEEE JSAC, vol. 19, pp. 891-907, May 2001
3. S. ten Brink, "Convergence behaviour of iteratively decoded parallel concatenated codes," IEEE Trans. Comm., vol. 49, Oct 2001.

El-Gamal's approach



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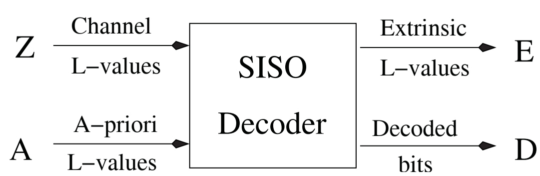

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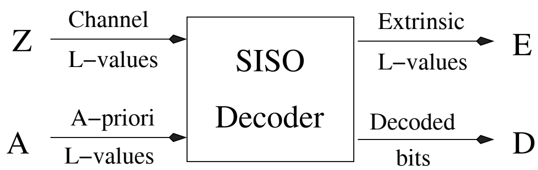

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- For an AWGN channel,

$$z = x + n$$

where z is the received channel value, x is the transmitted bit $(= \pm 1)$, and n is Gaussian distributed with zero mean and variance $N_0/2$.

El-Gamal's approach

- The log-likelihood or L-values are calculated as:

$$Z = \ln \frac{p(z|x=+1)}{p(z|x=-1)} \qquad A = \ln \frac{p(u=+1)}{p(u=-1)},$$

where $u (= \pm 1)$ represents an information bit.

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- For large blocksizes, the probability distribution of the a-priori L-values p_A, are assumed to be Gaussian. In particular, the a-priori L-value A can be modeled as

$$A = \mu_A \cdot u + n_A$$

where the n_A is a zero mean Gaussian random variable with variance σ_A^2 that satisfies the following condition

$$\mu_A = \frac{\sigma_A^2}{2}. \qquad \text{(consistency condition)}$$

El-Gamal's approach

- The a-priori information at the input of the decoder can be characterized by input SNR,

$$\text{SNR}_i = \frac{\mu_A^2}{\sigma_A^2} = \frac{\mu_A}{2}$$

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- Viewing SNR_0 as a function of E_b/N_0, and SNR_i, the transfer characteristics of the decoder can be written as,

$$\mathrm{SNR}_0 = T(\mathrm{SNR}_i, E_b/N_0)$$

Transfer characteristics of a SISO decoder

Step 1 : For a given SNR E_b/N_0, the distribution of a-priori L-value is generated for a particular mean μ_A, and transmitted bits u.

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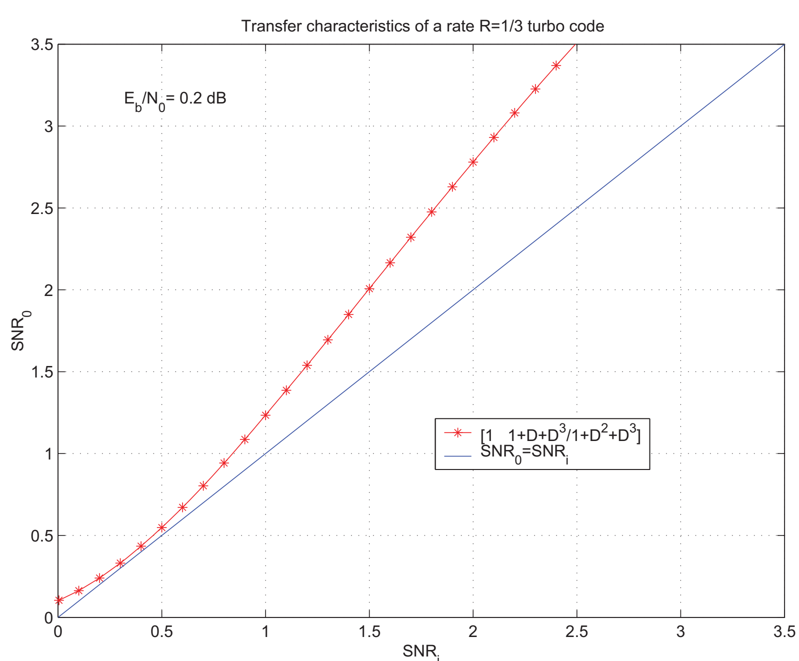
Step 5 : The set of (SNR_i, SNR_0) for different values of μ_a is plotted. This is then used as the transfer characteristics for the SISO module for that particular code, and channel SNR E_b/N_0.

Transfer characteristics of a SISO decoder



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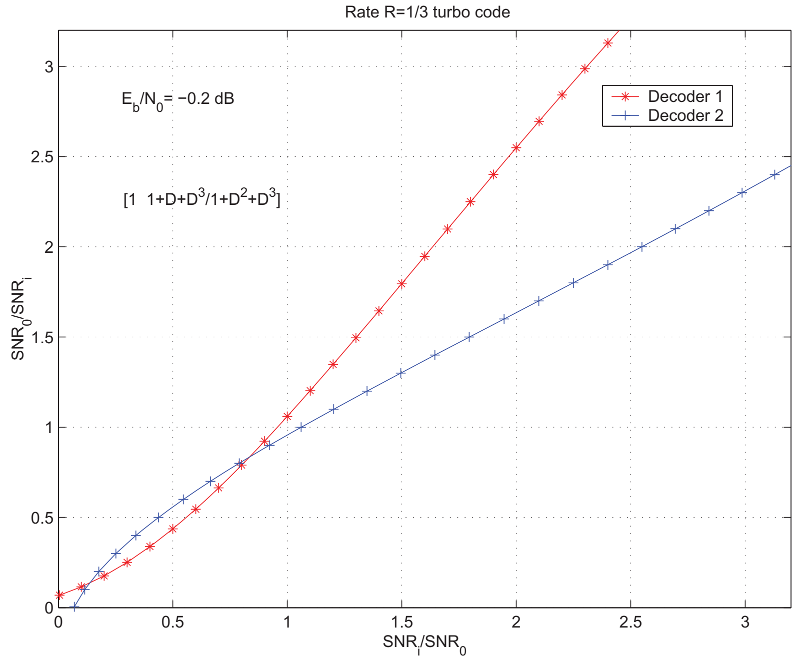
Step 4 : The smallest channel SNR E_b/N_0 for which the transfer characteristics do not touch and a tunnel exists is the convergence threshold for that particular code.

Threshold Calculation



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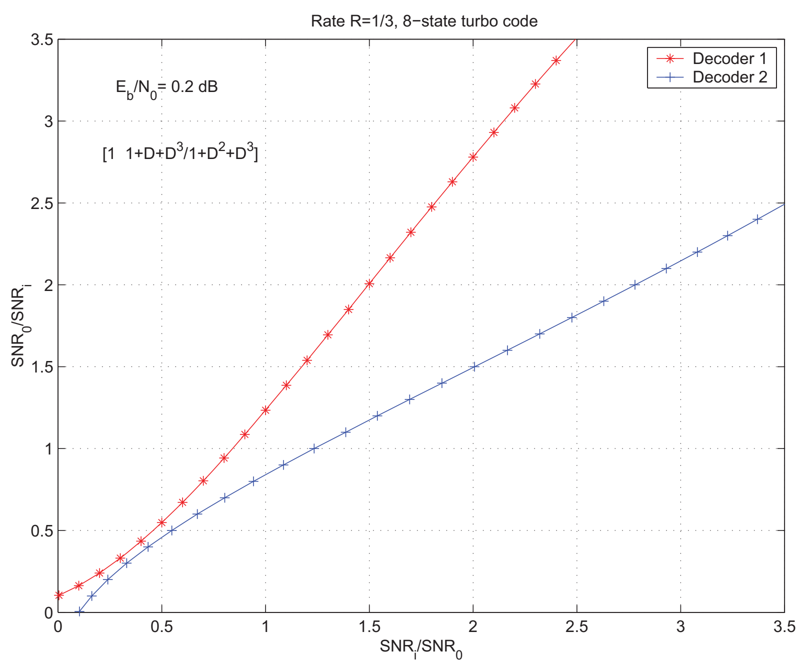

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Step 3 : Since the extrinsic information at the output of decoder 1 becomes the a-priori information at the input of decoder 2, the value of SNR_0 from decoder 1 becomes SNR_i for the first iteration of decoder 2, and the resulting SNR_0 for decoder 2 is determined (horizontally) using the transfer characteristics for decoder 2.

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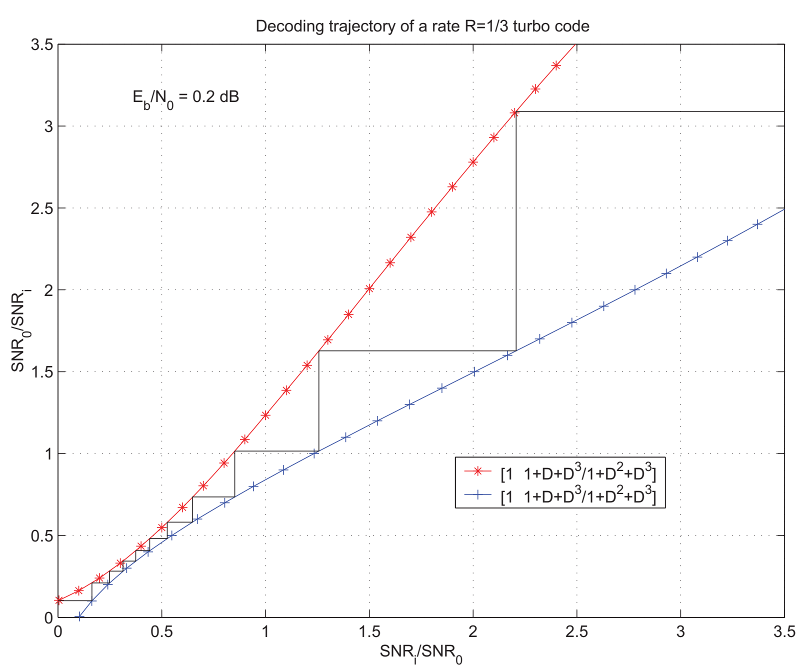

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- Generate input a-priori distribution based on the observed extrinsic information distribution.
- Simulate SISO decoder, and from the generated extrinsic information find the distribution of the extrinsic information.
- The a-priori/extrinsic information can be characterized by the $\text{SNR}_i/\text{SNR}_0$, where mean and variance of the a-priori/extrinsic information is calculated empirically.

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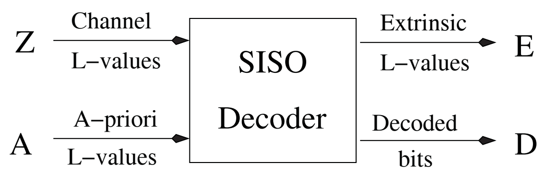
Extrinsic Information Transfer Charts



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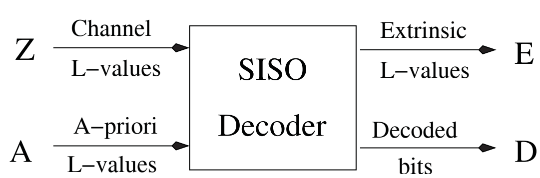

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- The information content of the a-priori probabilities is measured by the mutual information I_A=I(U;A) between the information bits U and the a-priori L-values A.

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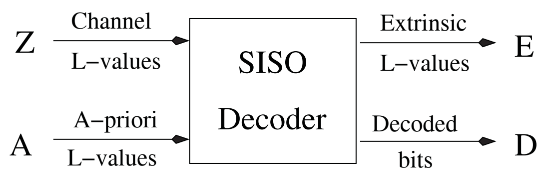

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- The input mutual Information $I(U;A)$ is calculated as:

$$I(U;A) \triangleq \frac{1}{2} \sum_{U=-1,1} \int_{-\infty}^{\infty} p_A(\xi|U=u) \log \frac{p_A(\xi|U=u)}{p_A(\xi)} d\xi$$

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- The probability distribution of the extrinsic L-values p_E, is computed experimentally from Monte Carlo simulations. p_E is then used to calculate the output mutual information $I(U;E)$.

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- Viewing I_E as a function of I_A and E_b/N_o, the extrinsic information transfer characteristic of an encoder is defined as

$$I_E = T(I_A, E_b/N_o).$$

Extrinsic Information Transfer Charts

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- The EXIT chart can then be used to trace the trajectory of iterative decoding and to determine the convergence behavior of the constituent decoders.
- The existence of a “tunnel” implies convergence of iterative decoding.
- As the channel SNR E_b/N_0 is lowered, the two transfer characteristics come closer together (the “tunnel” narrows) until the two curves meet.

Limitations of the convergence analysis methods

- Convergence analysis is based on asymptotically large blocksizes. Practical systems use a finite blocksizes.

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- Convergence analysis is based on asymptotically large blocksizes. Practical systems use a finite blocksizes.
- Some of the methods are based on Gaussian approximation of the distribution of the extrinsic information. This approximation may not hold.