An introduction to coding theory

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Lecture #15: Turbo Codes

Introduction

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- However, structure does not always result in the best distance properties for a code.
- Turbo codes have random-like code design with just enough structure to allow for an efficient iterative decoding method.

Turbo Codes

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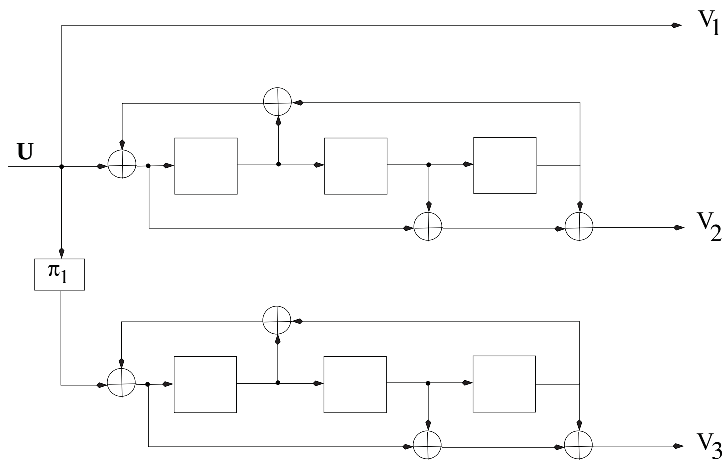
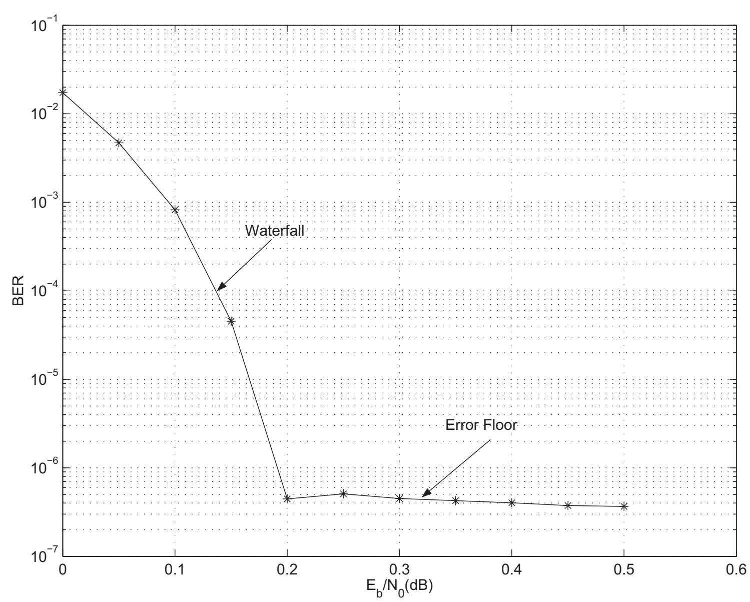
- Example of rate R=1/3 turbo code shown in next slide.

Turbo Codes



Rate 1/3 Turbo Code

Turbo Codes



BER performance of rate R=1/3, turbo code with input information blocklength of 65536 bits.

Turbo Codes

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- Bits can be punctured from the parity sequences in order to produce higher code rates.
- Bits can also be punctured from the information sequence. If some of the information bits are punctured, it is known as partially systematic turbo code. If all the information bits are punctured, it is known as nonsystematic turbo code.

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- Since the MAP decoder uses a forward-backward algorithm, the information is arranged in blocks.

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  - a large decoding delay.
  - significantly weakened performance at BER's below 10^{-6}.

Turbo Codes: Key Developments

- C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo codes," in *Proc. IEEE International Conference on Communications (ICC 93)*, (Geneva, Switzerland), pp. 1064–1070, May 1993.

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- H. El Gamal and A. R. Hammons Jr., "Analyzing the turbo decoder using the Gaussian approximation," in *IEEE Transactions on Information Theory*, vol. IT-47, pp. 671–686, Feb. 2001.