An introduction to coding theory

Adrish Banerjee

Department of Electrical Engineering
Indian Institute of Technology Kanpur
Kanpur, Uttar Pradesh
India

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Lecture #20: Applications of Linear Codes

Outline of the lecture

- Historical timeline
- Applications for convolutional codes
  - Coding in Space Communication
  - Coding for Modems
  - Coding in Wireless Standards

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- Applications for block codes
  - ISBN numbers
  - BookLand Bar Code
  - Universal Product Code (UPC)
  - Coding for Compact Disc
  - Coding for Satellite Communication
  - Coding for Wireless Communications

Historical Timeline

1948	Shannon founded information theory, discovery of Golay codes
1950	Single error correcting Hamming codes
1955	Invention of convolutional codes by Elias.
1960	Reed-Solomon Codes
1960	LDPC codes (Gallager)
1961-63	Sequential decoding of convolutional codes (Wozencraft, Fano)
1967	ML decoding of convolutional codes (Viterbi algorithm)
1968-69	Berlekemp-Massey algorithm to decode BCH and RS codes
1968	Convolutional codes for the Pioneer space vehicle
1974	Convolutional codes for the Helios space vehicle
1976-82	Coded modulation (Ungerboeck codes)
1977	Concatenated codes for Voyager space vehicle
1993	Turbo codes
1995	Graph based codes

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- During the 1970s the NASA standard became a concatenated coding scheme based on a rate-$1/2$, 64-state inner convolutional code and a (255, 223, 33) Reed-Solomon outer code over F_{256} .
  - The overall rate of this code is 0.437, and it achieves an impressive 7.3 dB real coding gain at $P_b(E) \approx 10^{-5}$, i.e., its gap to capacity is only about 2.5 dB.

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- Finally, within the last decade, turbo codes and LDPC codes for deep-space communications have been developed to get within 1 dB of the Shannon limit, and these are now becoming industry standards.

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  - A 16-D shell mapping constellation shaping scheme provided an additional gain of about 0.8 dB.

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- Capacity-approaching coding schemes are now normally included in new wireless standards. In other words, bandwidth-limited coding has moved to these newer, higher bandwidth settings.

Coding in Wireless Standards

- Digital mobile telephony, e.g. Global System for Mobile Communication Standard (GSM)
  - interleaved convolutional code of memory 4 and rate $R = 1/2$.
- Applications of turbo codes are summarized in the next frame.

summarized

Application	turbo code	termination	polynomials	rates
CCSDS (deep space)	binary, 16-state	tail bits	23, 35, 25, 37	1/6, 1/4, 1/3, 1/2
UMTS, CDMA2000	binary, 8-state	tail bits	13, 15, 17	1/4, 1/3, 1/2
DVB-RCS	duo-binary, 8-state	circular	15, 13	1/3 upto 6/7
DVB-RCT	duo-binary, 8-state	circular	15, 13	1/2, 3/4
Inmarsat (Aero-H)	binary, 8-state	no	23, 35	1/2
Eutelsat (Skyplex)	duo-binary, 8-state	circular	15, 13	4/5, 6/7
IEEE 802.16 (WiMaX)	duo-binary, 8-state	circular	15,13	1/2 upto 7/8

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- This code can detect transposition error.

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- When all the 10-digit numbers are used, a new prefix (979) will be started.

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- It can be verified if the UPC is valid by observing the sum, s in Step 4. The sum, s, should be $s = 0 \pmod{10}$.

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  - Check digit, a single checksum digit. The check digit is computed modulo 10, where the weights in the checksum calculation alternate 1 and 3.

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  - fingerprints, scratches, or dust particles from handling.

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- Unfortunately, the compact disc was basically a bursterror channel and the convolutional code failed to correct bursts.

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  - If the error burst exceeds the capability of the interpolator, the decoder blanks out, or mutes the system for the duration of the unreliable samples.

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- Undetected errors occur approximately every 750 hours.

Coding for Compact Disc (CD)

The resulting performance of the CIRC scheme.

- Error correction: error burst of length up to 4000 bits (2.5 mm long scratch)
- Error detection: error burst of length up to 12000 bits (7.5 mm long scratch)
- the lost samples are reconstructed using interpolation.
- Error rate upto 10^{-3}
- Undetected errors occur approximately every 750 hours.
- The compact disc can endure 8 mm holes punched through the disc without noticeable effect.

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S. No	Blocksize	Information Word	Code Rate
1	8176	7154	7/8
2	1280	1024	4/5
3	1536	1024	2/3
4	2048	1024	1/2
5	5120	4096	4/5
6	6144	4096	2/3
7	8192	4096	1/2
8	20480	16384	4/5
9	24576	16384	2/3
10	32768	16384	1/2

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- 10 GBASE-T contains a special class of LDPC codes (so-called Reed-Solomon code-based LDPC codes or RS-LDPC codes). A Reed-Solomon code is used to define the generator matrix of the LDPC code. The LDPC code construction method guarantees that no cycles of length four are contained within the Tanner graph.

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