A brief history of electronics

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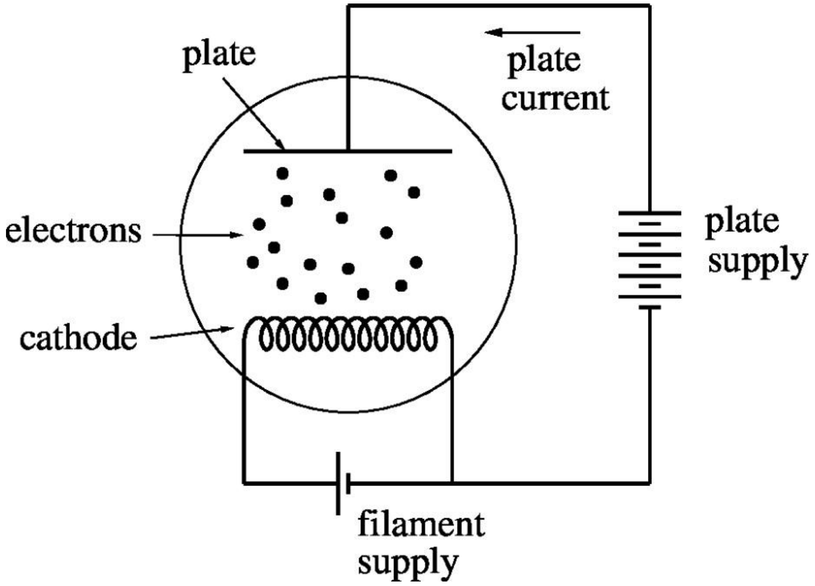
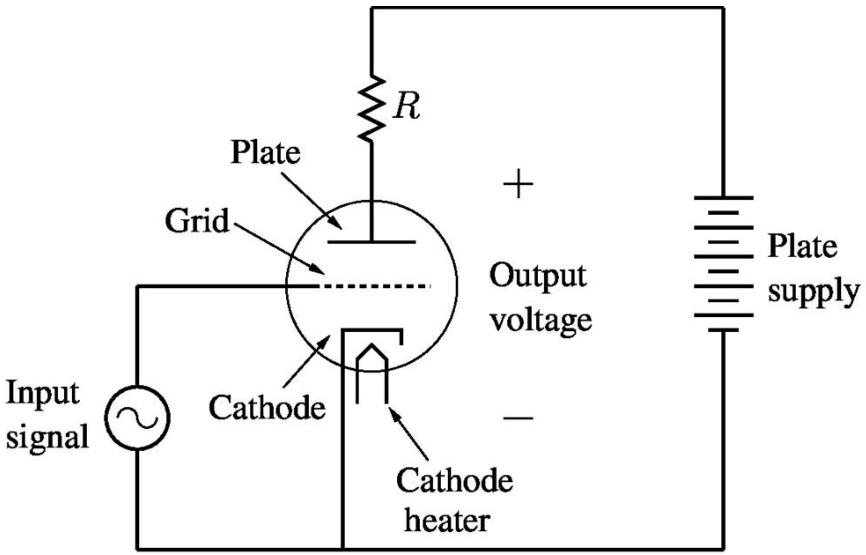
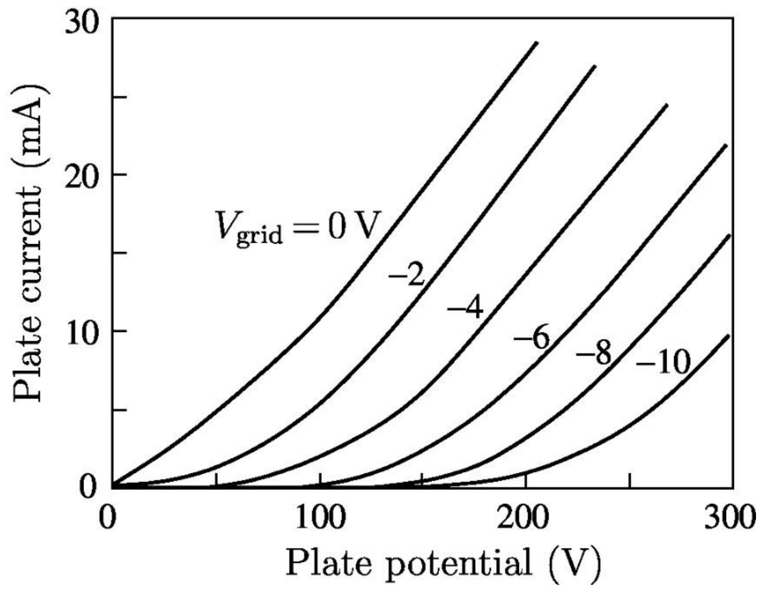
(images taken from internet)

Vacuum tubes

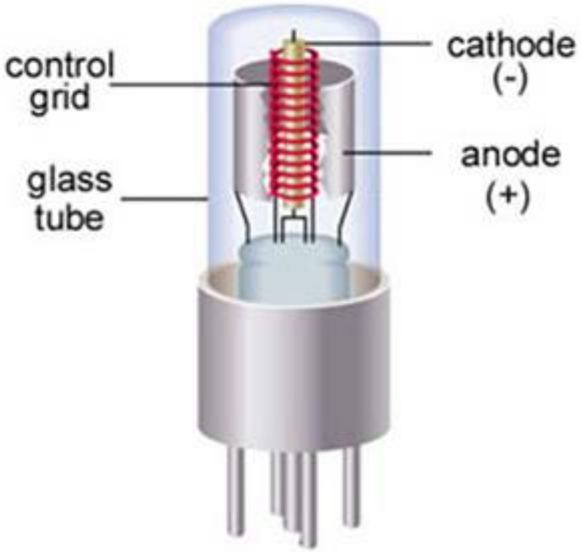
- 1904: the simplest vacuum tube – the diode – was invented by John Fleming.
- 1907: De Forest invented the triode by inserting a third electrode between cathode and anode.



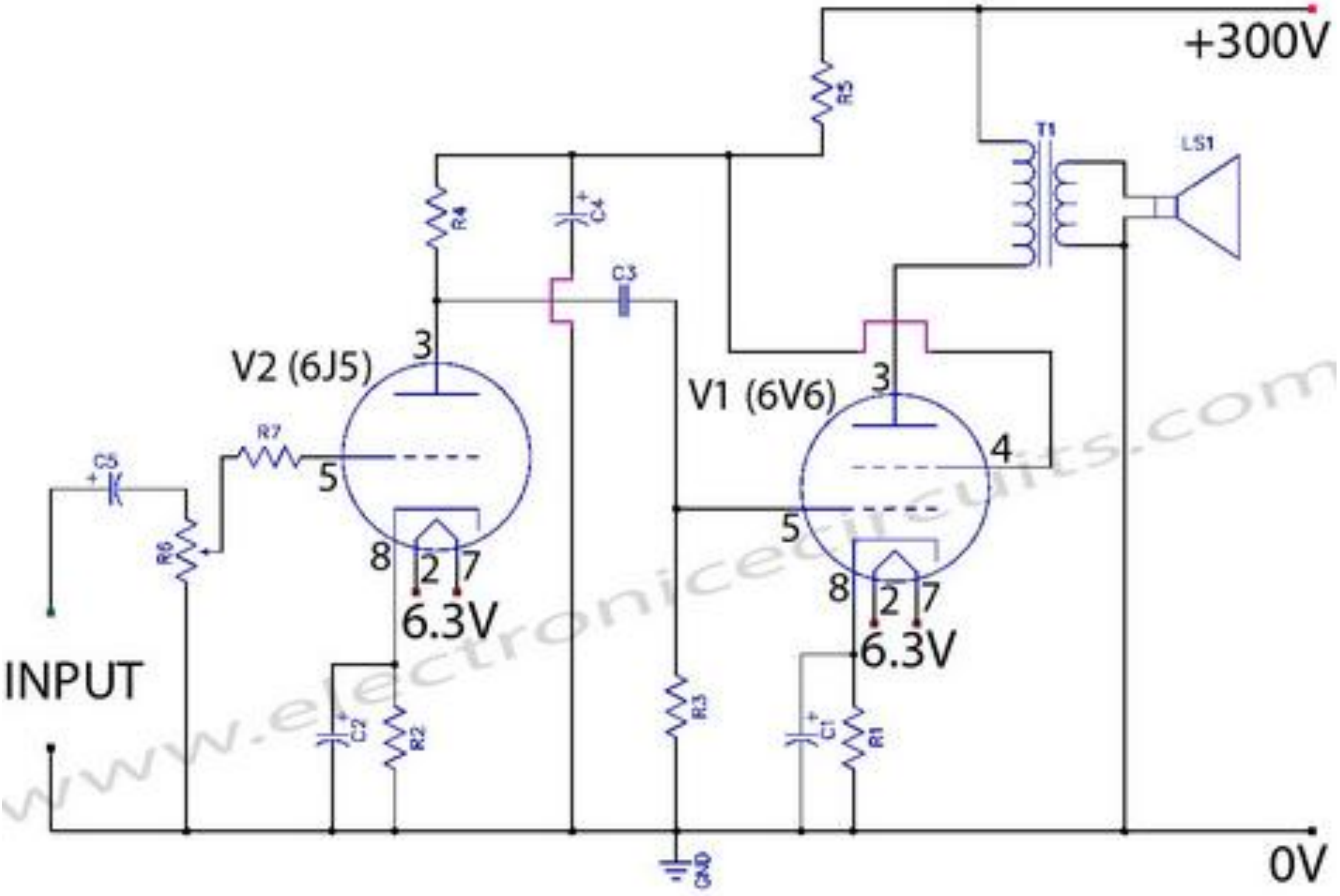




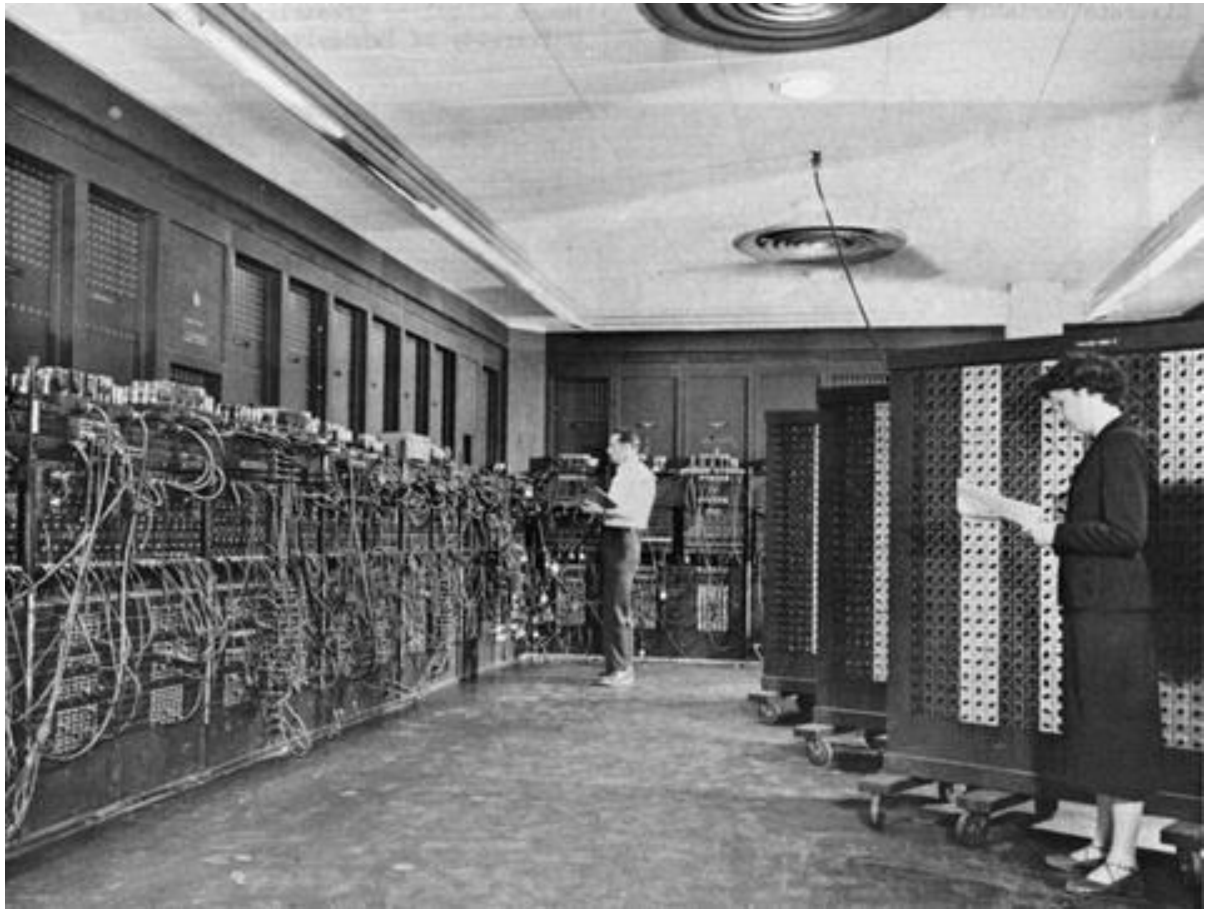
Vacuum Tubes

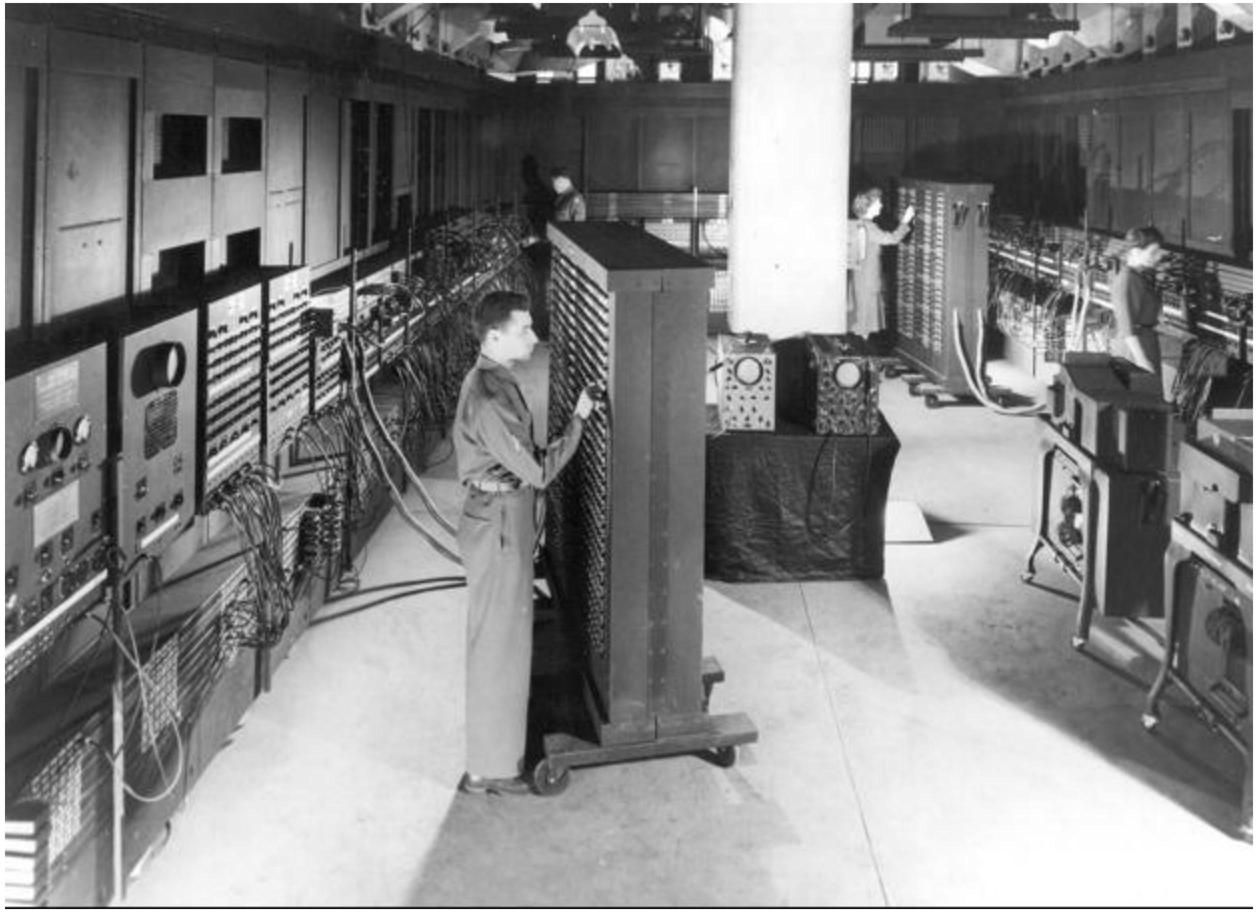


Vacuum tubes: audio amplifier



ENIAC computer (1946, Univ of Pennsylvania)





ENIAC computer

- heralded as a "Giant Brain" by the press
- thousand times faster than electro-mechanical computer
- 17,468 vacuum tubes, 7200 crystal diodes, 1,500 relays, 70,000 resistors, 10,000 capacitors, 6,000 manual switches, and approximately 5,000,000 hand-soldered joints.
- consumed 150 kW
- Input was possible from an IBM card reader
- 100 kHz clock
- Several tubes burned out almost every day, leaving it non-functional about half the time.

ENIAC computer

- could be programmed to perform complex sequences of operations, including loops, branches, and subroutines.
- After the program was figured out on paper, the process of getting the program into ENIAC by manipulating its switches and cables could take days.
- The task of taking a problem and mapping it onto the machine was complex, and usually took weeks.
- The programmers debugged problems by crawling inside the massive structure to find bad joints and bad tubes.
- The first test problem consisted of computations for the hydrogen bomb.

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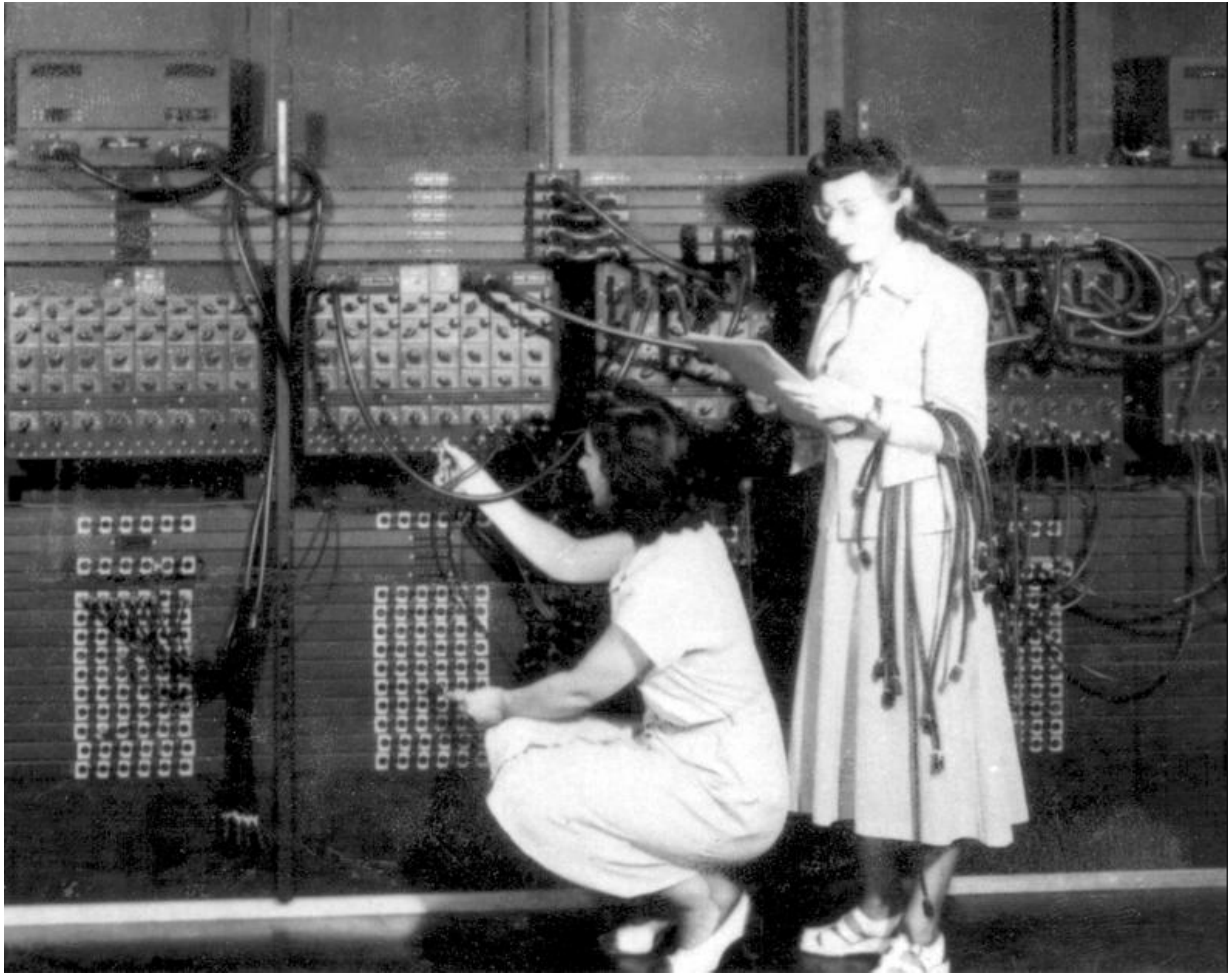
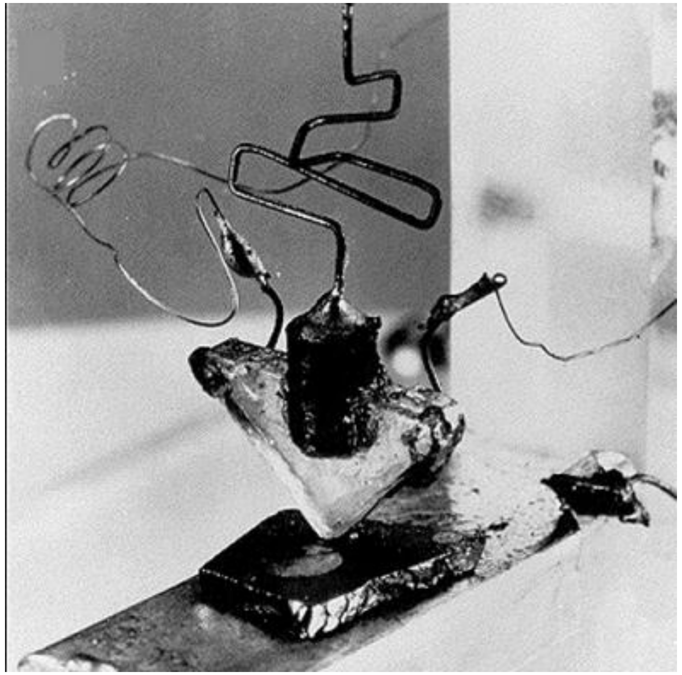
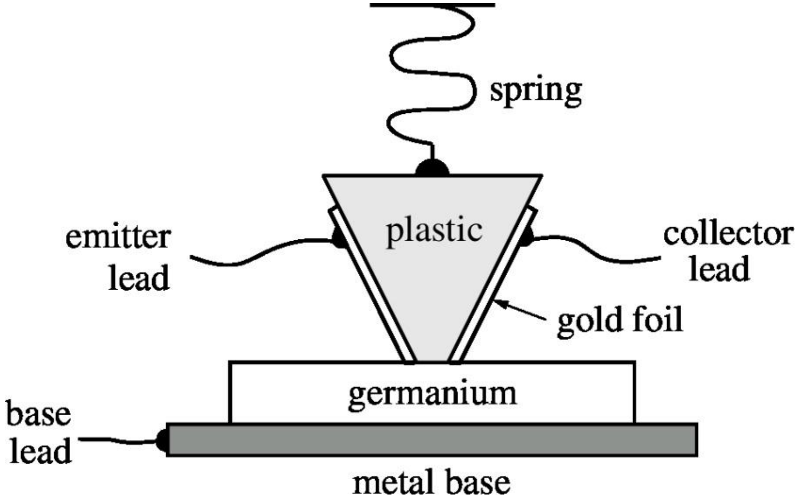
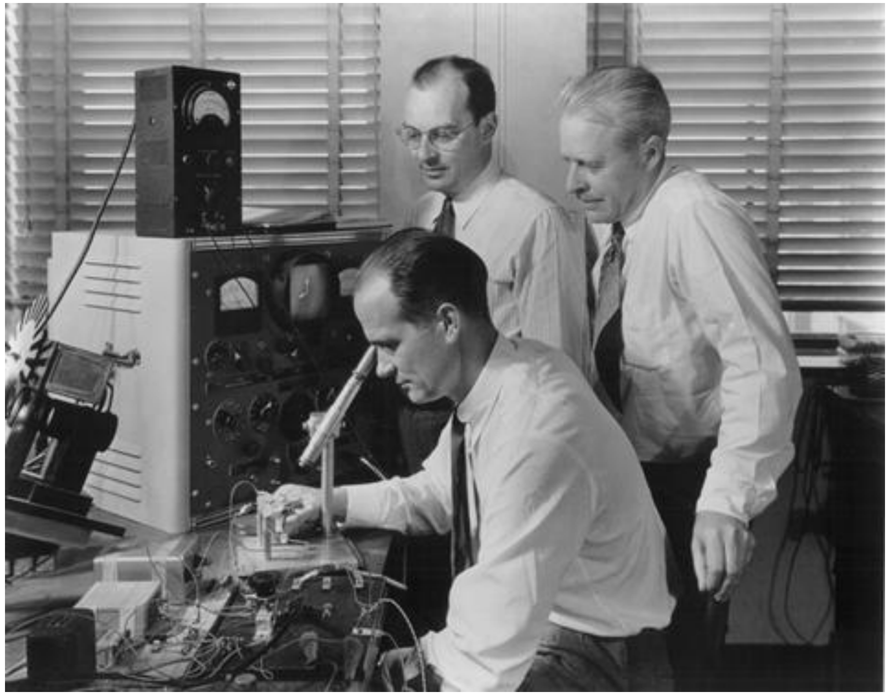


The first transistor

- The vacuum tube was a bulky and fragile device which consumed a significant power.
- 1947: Shockley, Bardeen, and Brattain at Bell Labs invented the first transistor.
- The first transistor was a “point contact transistor.” The modern transistor is a junction transistor, and it is monolithic (in the same semiconductor piece).



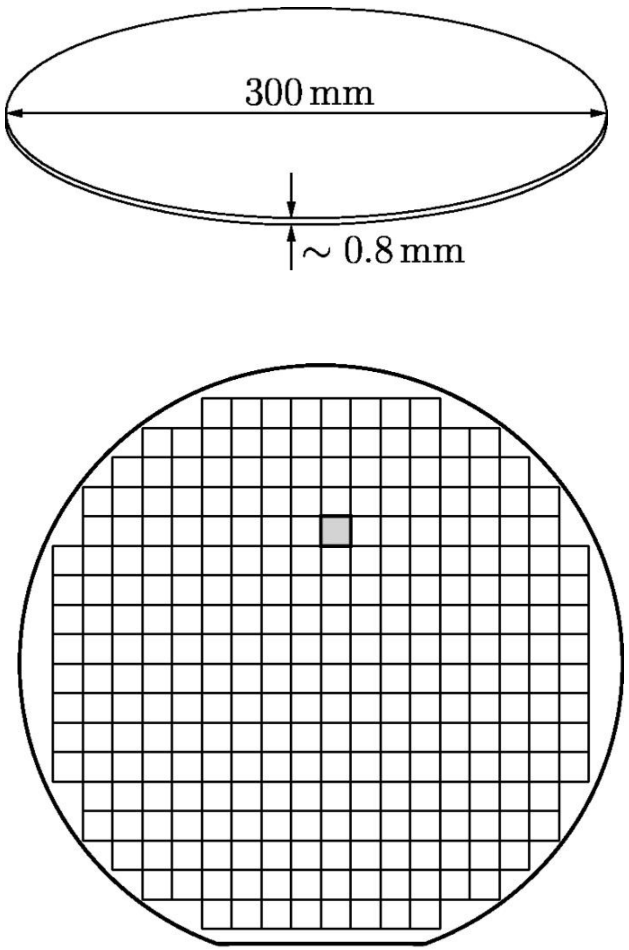


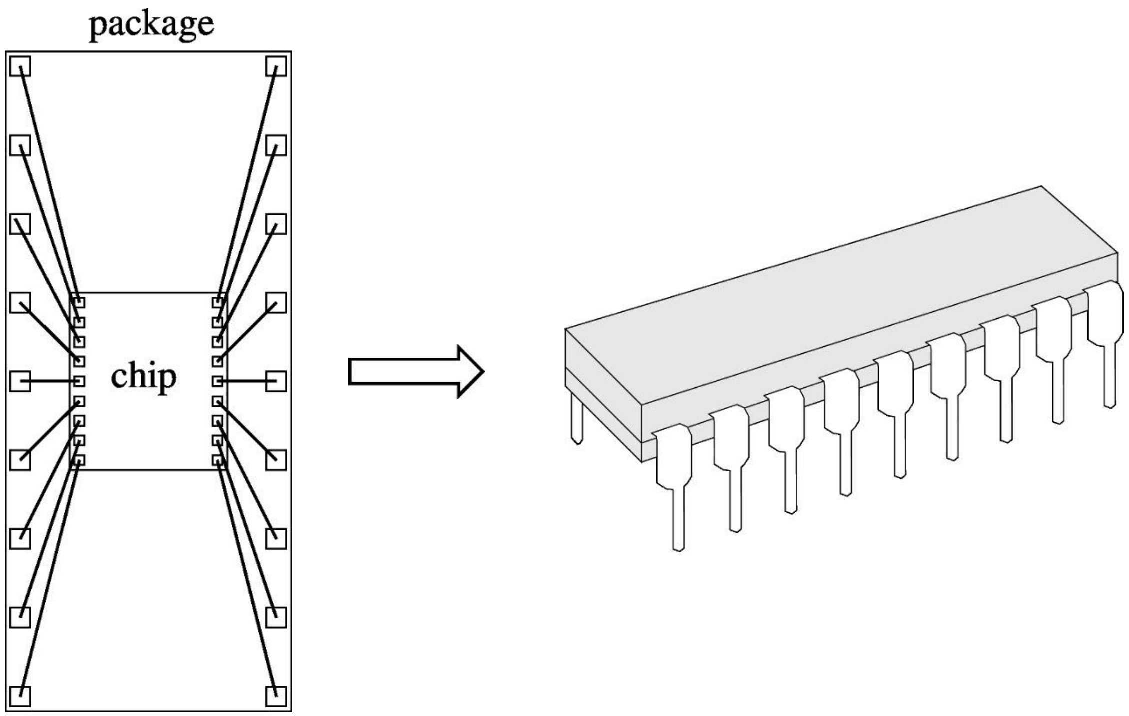


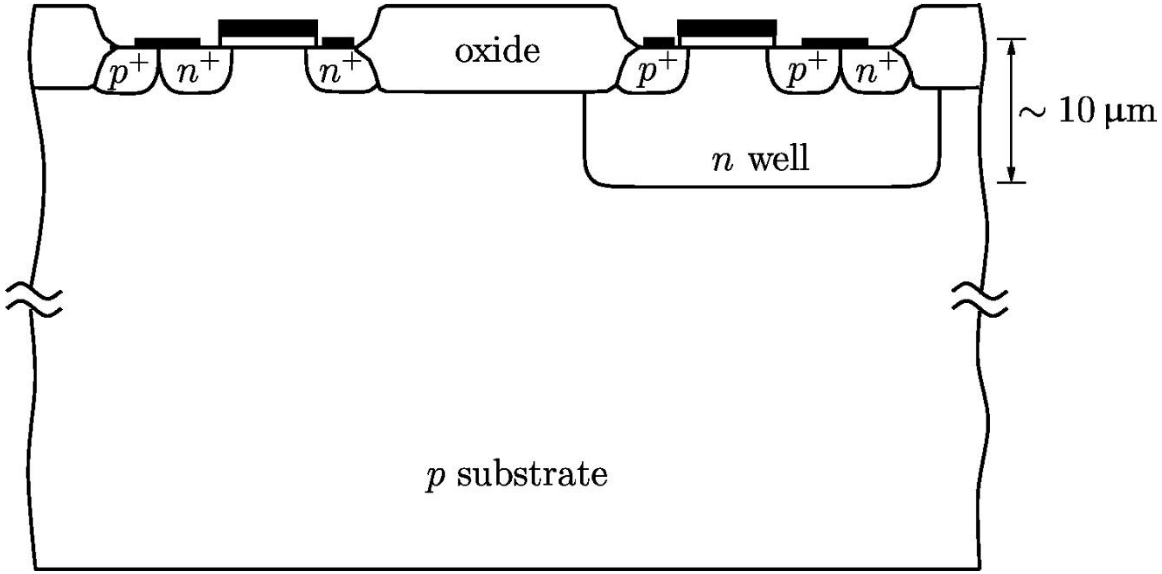
Semiconductor technology

- The bipolar transistor continues to be an important device both as a discrete device and as part of Integrated Circuits (IC).
- However, in digital circuits such as processors and memory, the MOS (Metal Oxide Semiconductor) field-effect transistor has surpassed the bipolar transistor because of the high integration density and low power consumption it offers.
- 1930: patent filed by Lilienfeld for field-effect transistor (FET).
- 1958: Jack Kilby (Texas Instruments) demonstrated the first integrated circuit (bipolar transistor, resistor, capacitor) fabricated on a single piece of germanium.
- The rest is history!

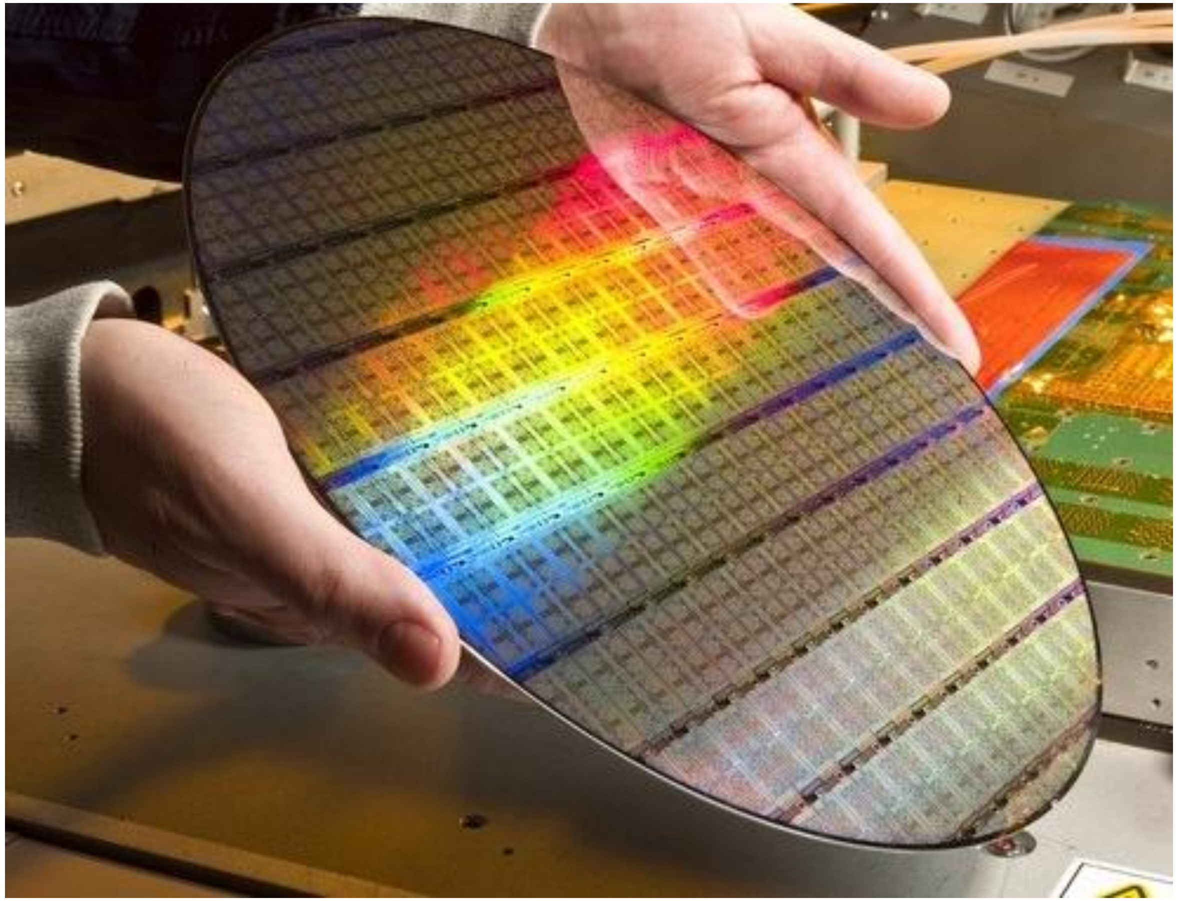
Semiconductor technology





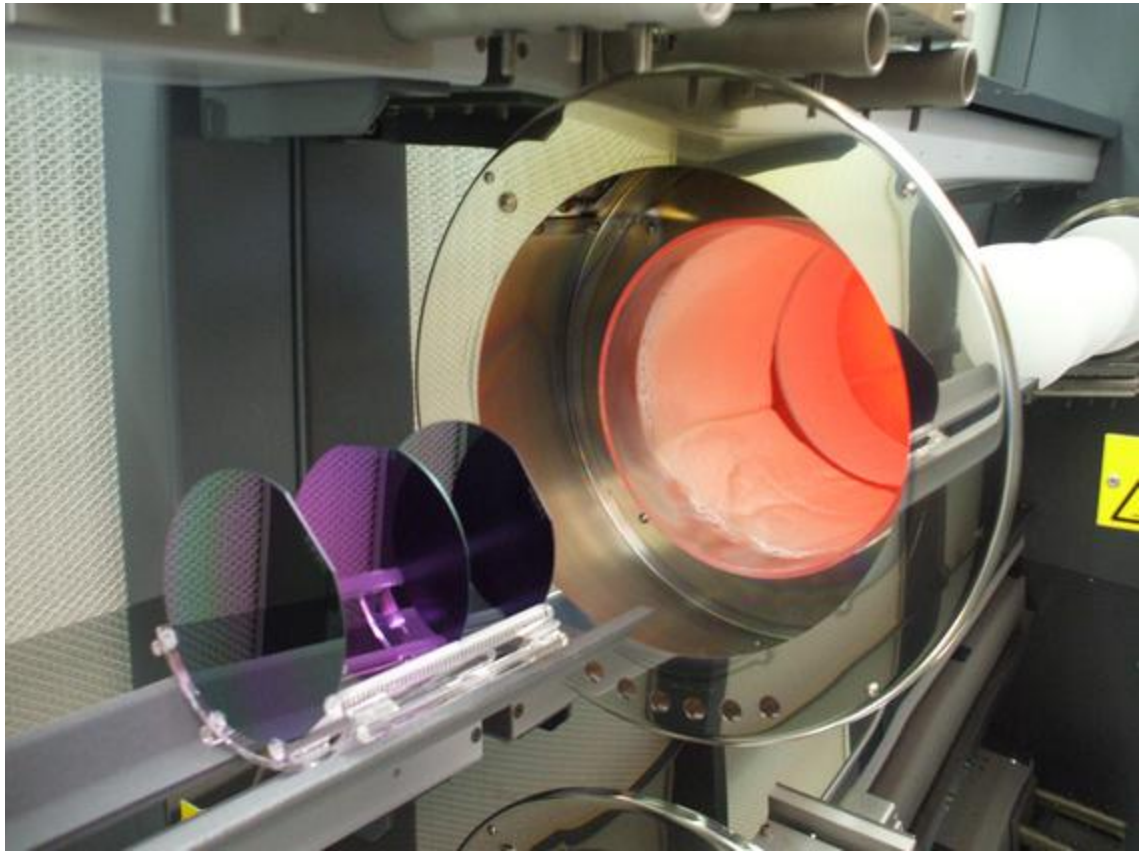


Modern semiconductor technology



silicon wafer

Modern semiconductor technology



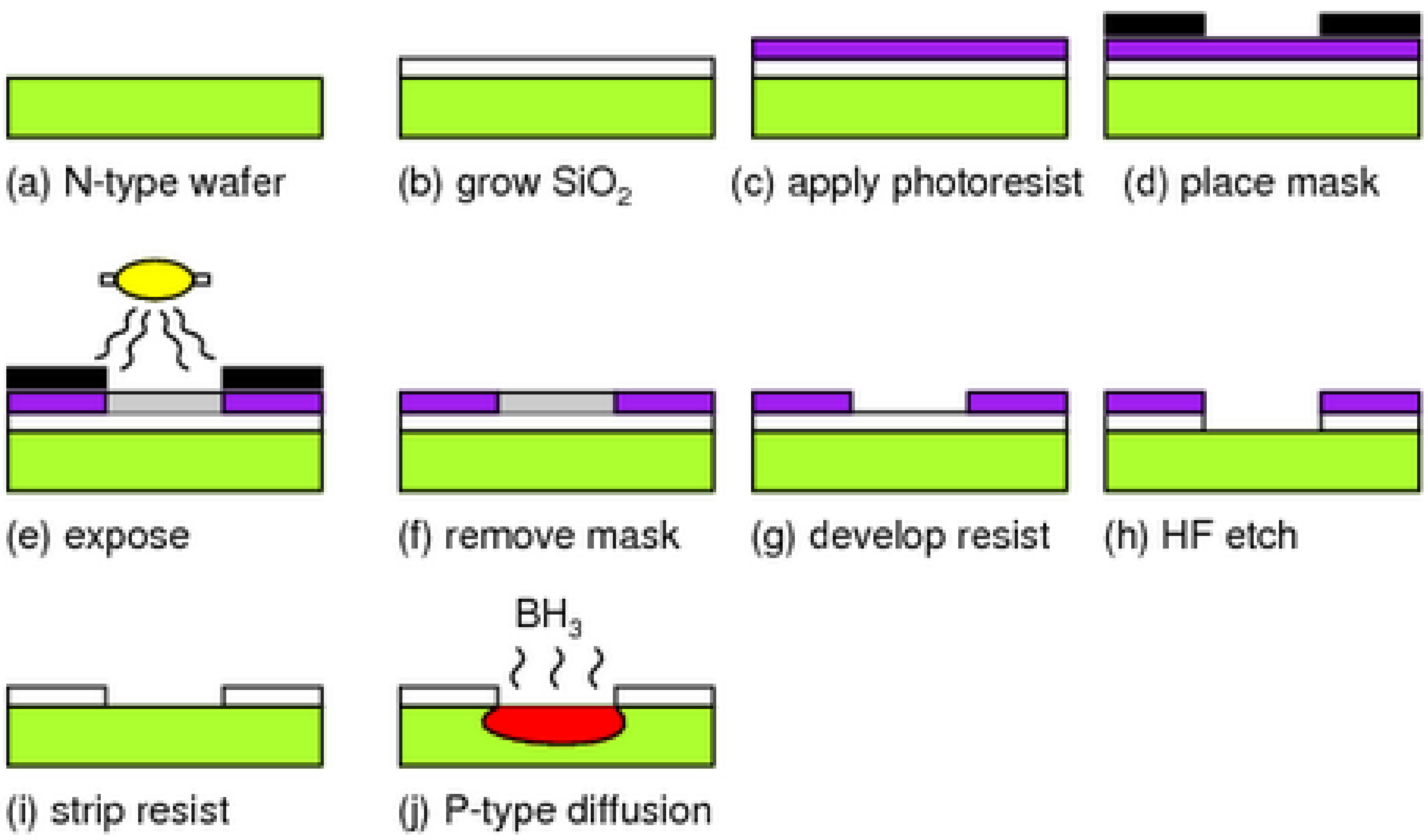
Diffusion furnace

Modern semiconductor technology

Modern semiconductor technology

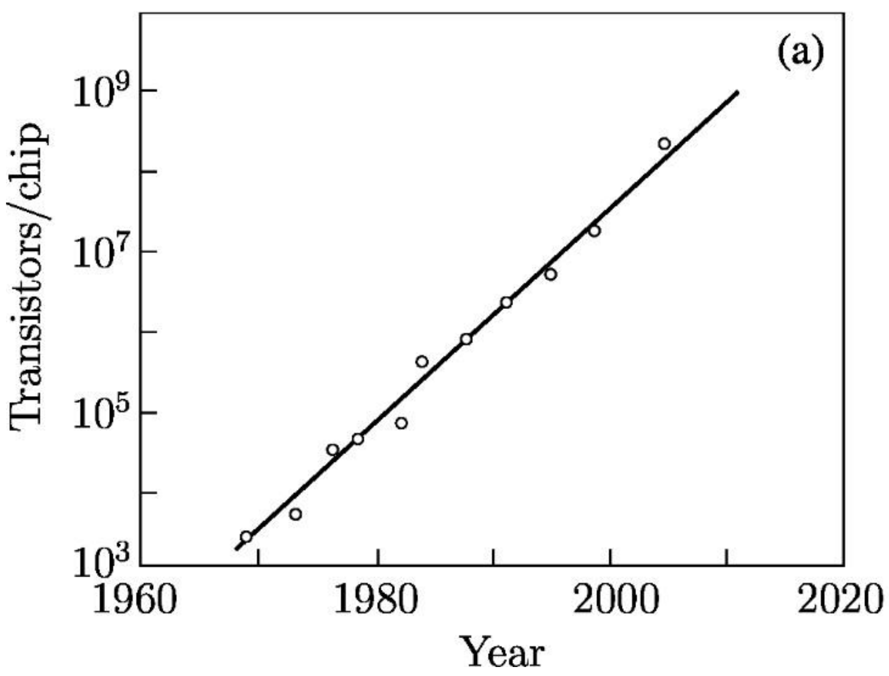
Modern semiconductor technology

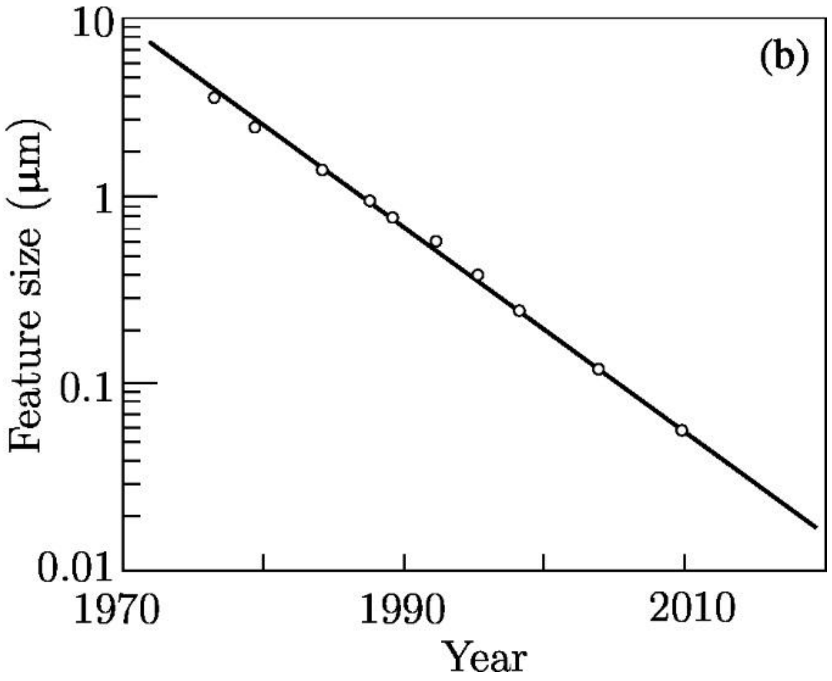
Fabrication of a p-n junction diode



MOS technology: scaling

- Shrinking of the smallest definable dimension ("feature size") on the chip has enabled a huge number of transistors to be integrated on one chip.
- 1970: feature size of 10 μm, 2010: 0.032 μm
- Moore's law: a prediction by Gordon Moore (Intel founder) in 1965: number of transistors will double every two years
- Increased functionality: "system on a chip" is now possible.





Vacuum tube computer with 1 million tubes (not built)

- Each vacuum tube is 5 cm x 5 cm: large area
- Each vacuum tube consumes, say, 1 W to 10 W power: total power in the MW range
- Need to remove the heat dissipated by the tubes
- Poor reliability because of a large number vacuum tubes/soldering joints
- Even if it was actually built, the speed would be much lower than a modern CPU due to parasitic capacitances and inductances of the cables

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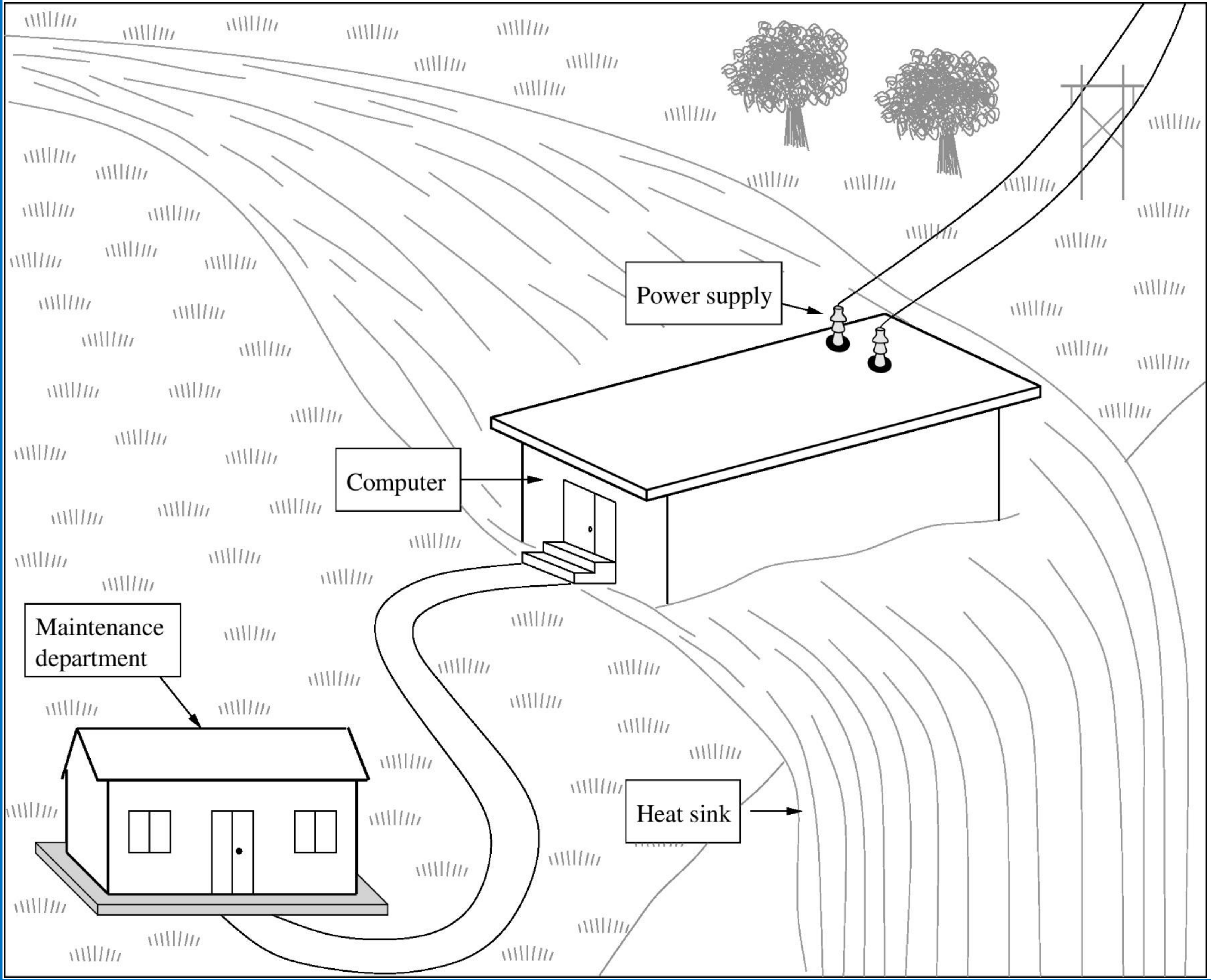

Compare that with your mobile phone!

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* Why does superposition work?

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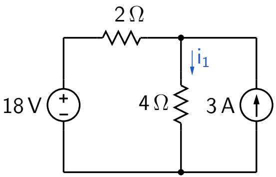
  - Deactivating an independent current source $\Rightarrow$ $I_0 = 0$, i.e., replace the current source with an open circuit.
  - Deactivating an independent voltage source $\Rightarrow$ $V_0 = 0$, i.e., replace the voltage source with a short circuit.

Example 1



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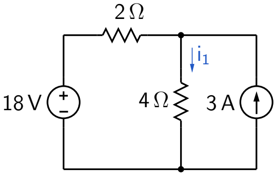

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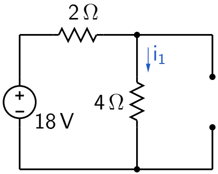

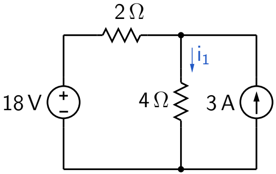

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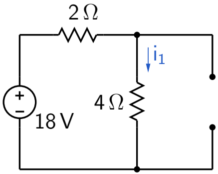

$$i_1^{(1)} = 3\,\mathrm{A}$$

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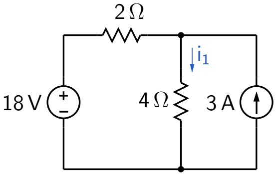

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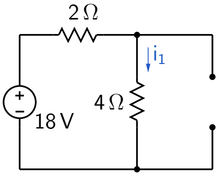

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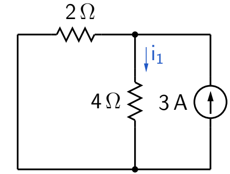

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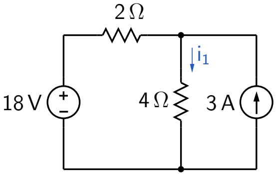

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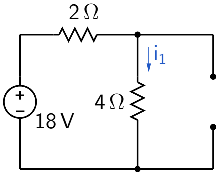

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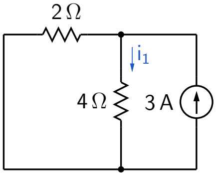

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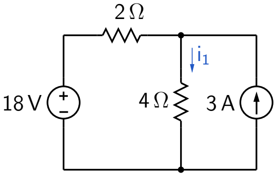

$$i_1^{\text{net}} = i_1^{(1)} + i_1^{(2)} = 3 + 1 = 4\,\text{A}$$

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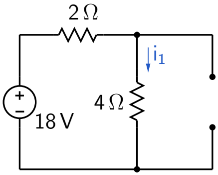

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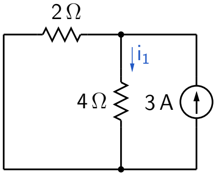

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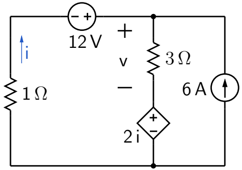

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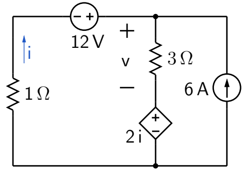

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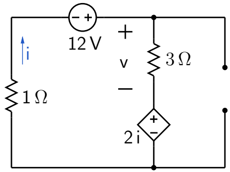

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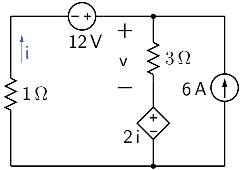

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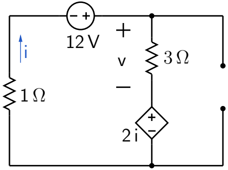

KVL: $-12 + 3\,\mathsf{i} + 2\,\mathsf{i} + \mathsf{i} = 0$

$\Rightarrow \mathsf{i} = 2\,\mathsf{A}\,,\, \mathsf{v}^{(1)} = 6\,\mathsf{V}\,.$

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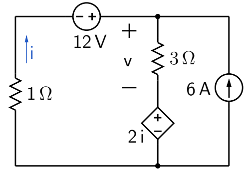

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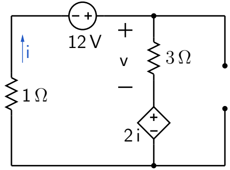

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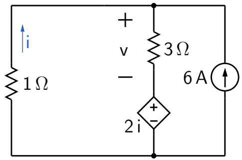

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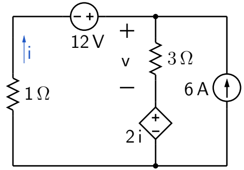

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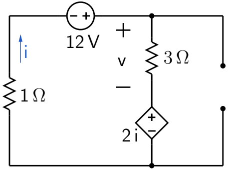

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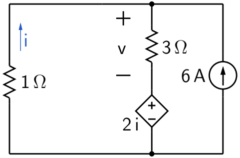
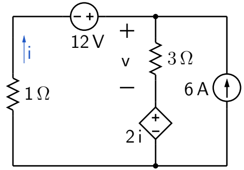


KVL: $i + (6 + i)\,3 + 2\,i = 0$

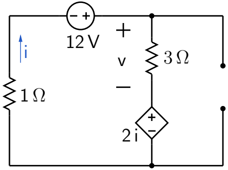
$\Rightarrow i = -3\,A\,,\ v^{(2)} = (-3 + 6) \times 3 = 9\,V\,.$

Example 2



$$v^{\text{net}} = v^{(1)} + v^{(2)} = 6 + 9 = 15\,\text{V}$$

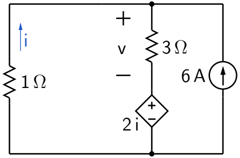
Case 1: Keep V_s, deactivate I_s.



KVL: $-12 + 3\,i + 2\,i + i = 0$

$\Rightarrow i = 2\,\text{A}\,, v^{(1)} = 6\,\text{V}\,.$

Case 2: Keep I_s, deactivate V_s.



KVL: $i + (6 + i)\,3 + 2\,i = 0$

$\Rightarrow i = -3\,\text{A}\,, v^{(2)} = (-3 + 6) \times 3 = 9\,\text{V}\,.$

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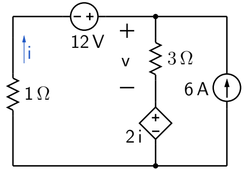
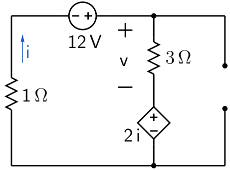


$$\boxed{v^{net} = v^{(1)} + v^{(2)} = 6 + 9 = 15\,\text{V}}$$

Case 1: Keep V_s, deactivate I_s.



KVL: $-12 + 3\,i + 2\,i + i = 0$

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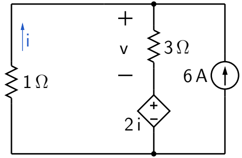

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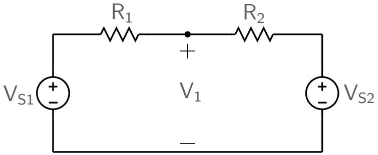
(SEQUEL file: `ee101_superposition_2.sqproj`) www.ee.iitb.ac.in/~sequelnew

Example 3

Find V_1 using superposition.



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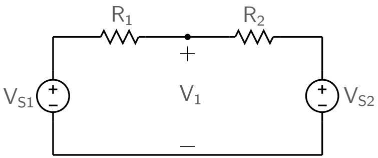
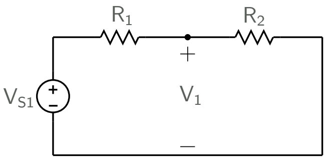


V_{S1} alone:



$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

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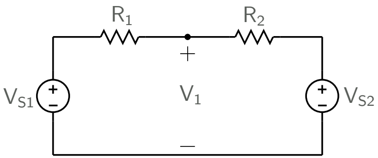

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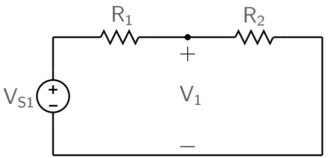
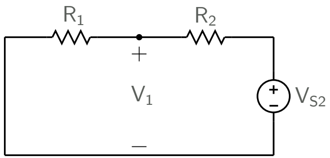


$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

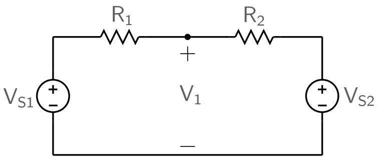
V_{S2} alone:



$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

Example 3

Find V_1 using superposition.



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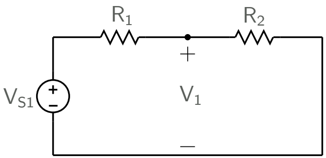

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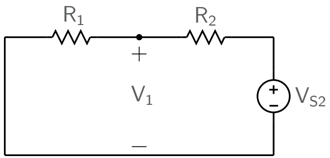
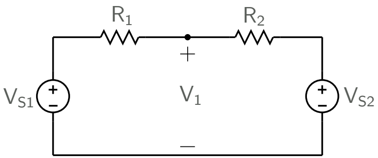


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

$$V_1^{(\text{net})} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{S1} + \frac{R_1}{R_1 + R_2} V_{S2}$$

Example 3 (again)

Find V_1 using superposition.



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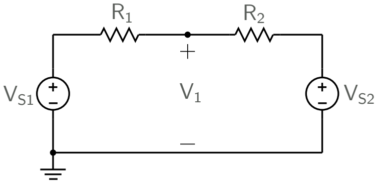
Find V_1 using superposition.



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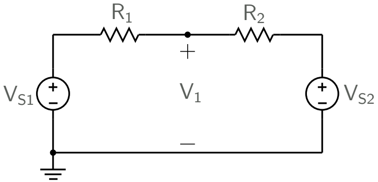

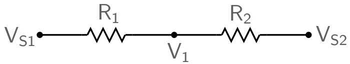

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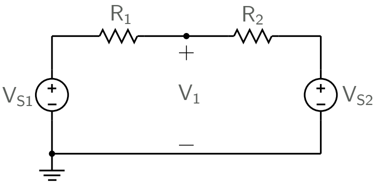
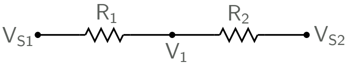
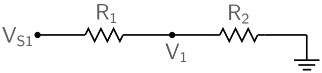
Find V_1 using superposition.





V_{S1} alone:



$$V_1^{(1)} = \frac{R_2}{R_1 + R_2} V_{S1}$$

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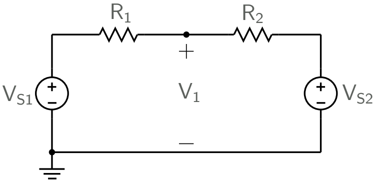
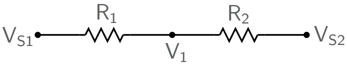
Find V_1 using superposition.





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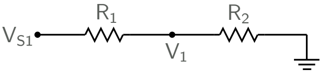

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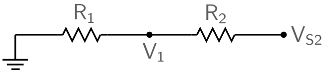
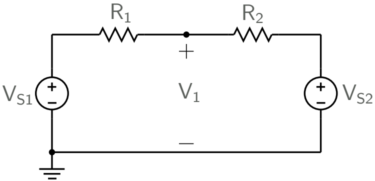
V_{S2} alone:

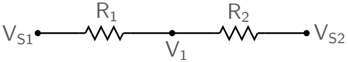


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

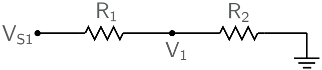
Example 3 (again)

Find V_1 using superposition.





V_{S1} alone:



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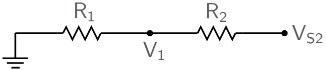
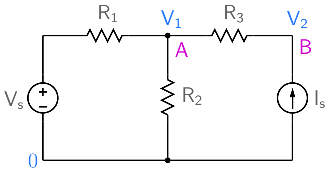


$$V_1^{(2)} = \frac{R_1}{R_1 + R_2} V_{S2}$$

$$V_1^{(\text{net})} = V_1^{(1)} + V_1^{(2)} = \frac{R_2}{R_1 + R_2} V_{S1} + \frac{R_1}{R_1 + R_2} V_{S2}$$

Superposition: Why does it work?



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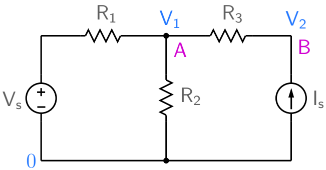
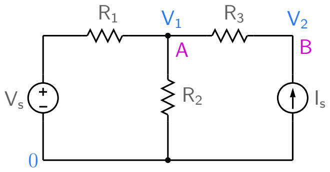


KCL at nodes A and B (taking current leaving a node as positive):

$$\frac{1}{R_1}(V_1 - V_s) + \frac{1}{R_2}V_1 + \frac{1}{R_3}(V_1 - V_2) \;=\; 0\,,$$

$$-I_s + \frac{1}{R_3}(V_2 - V_1) \;=\; 0\,.$$



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Writing in a matrix form, we get (using $G_1 = 1/R_1$, etc.),

$$\begin{bmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}$$

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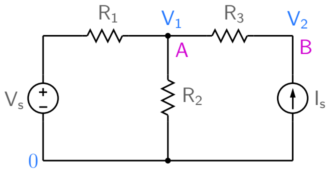

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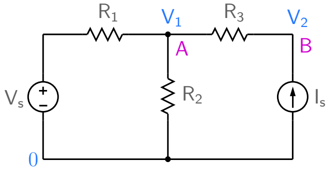
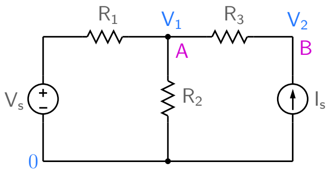
$$\text{i.e.,} \quad \mathbf{A} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix}.$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \begin{bmatrix} G_1 V_s \\ I_s \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}.$$



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We are now in a position to see why superposition works.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} V_s \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} G_1 & m_{12} \\ m_{21} G_1 & m_{22} \end{bmatrix} \begin{bmatrix} 0 \\ I_s \end{bmatrix} \equiv \begin{bmatrix} V_1^{(1)} \\ V_2^{(1)} \end{bmatrix} + \begin{bmatrix} V_1^{(2)} \\ V_2^{(2)} \end{bmatrix}.$$

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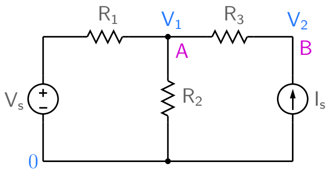

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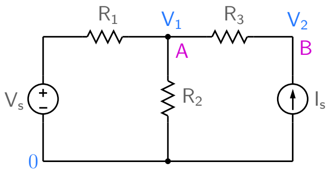
The first vector is the response due to V_s alone (and I_s deactivated).

The second vector is the response due to I_s alone (and V_s deactivated).



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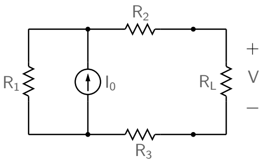
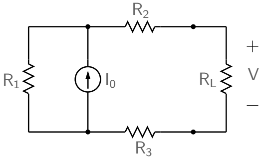
The first vector is the response due to V_s alone (and I_s deactivated).
The second vector is the response due to I_s alone (and V_s deactivated).
All other currents and voltages are linearly related to V_1 and V_2
$\Rightarrow$ Any voltage (node voltage or branch voltage) or current can also be computed using superposition.

Thevenin's theorem



How is V related to the circuit parameters?

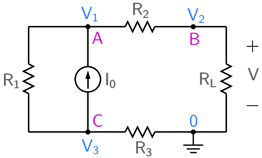
Thevenin's theorem



How is V related to the circuit parameters?

Assign node voltages with respect to a reference node.

Thevenin's theorem



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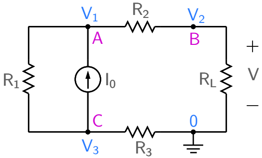

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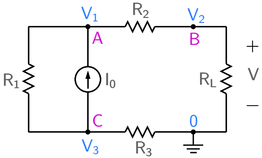
Let $G_1 \equiv 1/R_1$, etc. Write KCL equation at each node, taking current leaving the node as positive.

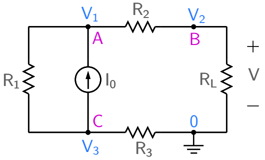


How is V related to the circuit parameters?

Assign node voltages with respect to a reference node.

Let $G_1 \equiv 1/R_1$, etc. Write KCL equation at each node, taking current leaving the node as positive.

$$
\begin{aligned}
\text{KCL at A :} \quad & G_1\,(V_1 - V_3) + G_2\,(V_1 - V_2) - I_0 &= 0\,, \\
\text{KCL at B :} \quad & G_2\,(V_2 - V_1) + G_L\,(V_2 - 0) &= 0\,, \\
\text{KCL at C :} \quad & G_1\,(V_3 - V_1) + G_3\,V_3 + I_0 &= 0\,.
\end{aligned}
$$



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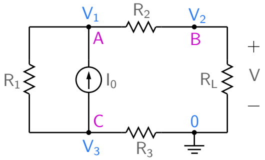
Let $G_1 \equiv 1/R_1$, etc. Write KCL equation at each node, taking current leaving the node as positive.

$$\begin{aligned}
\text{KCL at A :} \quad & G_1\,(V_1 - V_3) + G_2\,(V_1 - V_2) - I_0 &= 0\,,\\
\text{KCL at B :} \quad & G_2\,(V_2 - V_1) + G_L\,(V_2 - 0) &= 0\,,\\
\text{KCL at C :} \quad & G_1\,(V_3 - V_1) + G_3\,V_3 + I_0 &= 0\,.
\end{aligned}$$

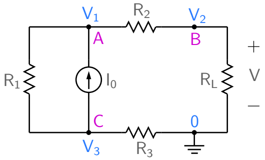
Write in a matrix form:

$$\begin{bmatrix} G_1 + G_2 & -G_2 & -G_1 \\ -G_2 & G_2 + G_L & 0 \\ -G_1 & 0 & G_1 + G_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \\ -I_0 \end{bmatrix},$$

i.e., $\mathbf{G}\,\mathbf{V} = \mathbf{I}_s$. We can solve this matrix equation to get V_2, i.e., the voltage across R_L.



V_2 can be found using Cramer's rule: $$V_2 = \frac{\det \begin{bmatrix} G_1 + G_2 & I_0 & -G_1 \\ -G_2 & 0 & 0 \\ -G_1 & -I_0 & G_1 + G_3 \end{bmatrix}}{\det(\mathbf{G})} \equiv \frac{\Delta_1}{\det(\mathbf{G})}$$



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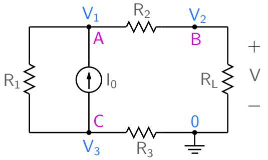
$$\det(\mathbf{G}) = \det \begin{bmatrix} G_1 + G_2 & -G_2 & -G_1 \\ -G_2 & G_2 + G_L & 0 \\ -G_1 & 0 & G_1 + G_3 \end{bmatrix}$$



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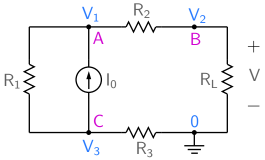
$$\begin{aligned}
\det(\mathbf{G}) &= \det\begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2+G_L & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix} \\
&= \det\begin{bmatrix} G_1+G_2 & -G_2 & -G_1 \\ -G_2 & G_2 & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix} + \det\begin{bmatrix} G_1+G_2 & 0 & -G_1 \\ -G_2 & G_L & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix}
\end{aligned}$$



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&= \Delta + G_L\Delta_2 \quad \text{where} \quad \Delta_2 = \det \begin{bmatrix} G_1+G_2 & 0 & -G_1 \\ -G_2 & 1 & 0 \\ -G_1 & 0 & G_1+G_3 \end{bmatrix}.
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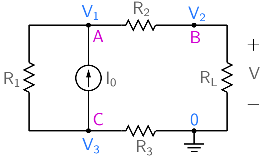

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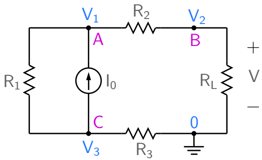
i.e., $V_2 = \dfrac{\Delta_1}{\det(\mathbf{G})} = \dfrac{\Delta_1}{\Delta + G_L\Delta_2}$ (Note: Δ, Δ_1, and Δ_2 are independent of G_L).



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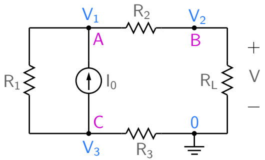

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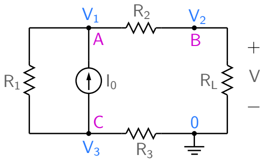

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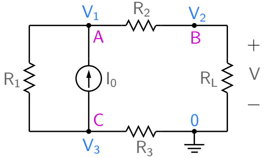

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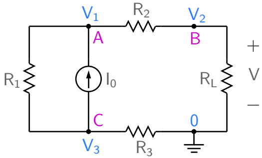
Note that Δ_2/Δ has units of resistance. Define $R_{\mathrm{Th}} = \Delta_2/\Delta$ (Thevenin resistance). Then we have

$$V_2 = \frac{R_L}{R_L + R_{\mathrm{Th}}}\, V_2^{OC}.$$

Thevenin's theorem



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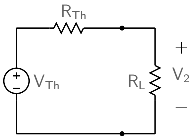
This is simply a voltage division formula, corresponding to the following "Thevenin equivalent circuit" (with $V_{\mathrm{Th}} = V_2^{\mathrm{OC}}$).



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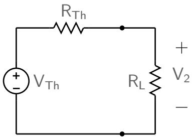
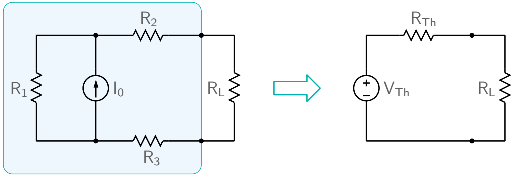
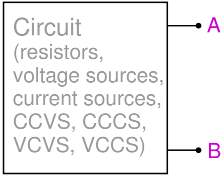
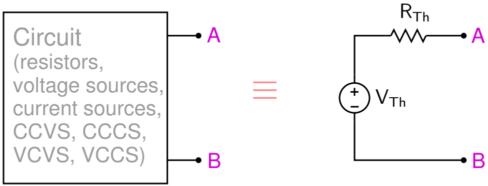


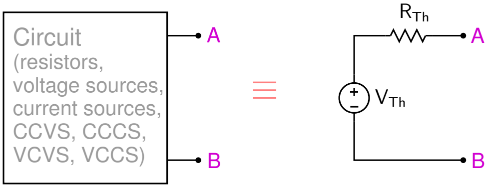
This allows us to replace the original circuit with an equivalent, simpler circuit.



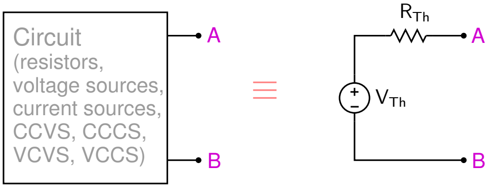
Thevenin's theorem




Circuit
(resistors,
voltage sources,
current sources,
CCVS, CCCS,
VCVS, VCCS)
A
B
R_Th
V_Th
+
−
A
B



* Since the two circuits are equivalent, the open-circuit voltage must be the same in both cases. Let V_{oc} be the open-circuit voltage for the left circuit. For the Thevenin equivalent circuit, the open-circuit voltage is simply V_{Th} since there is no voltage drop across R_{Th} in this case.
  $\rightarrow V_{Th} = V_{oc}$



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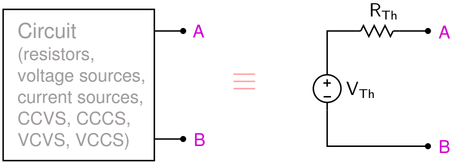
  $\rightarrow V_{Th} = V_{oc}$
* R_{Th} can be found by different methods.

Thevenin's theorem: R_{Th}

Method 1:



* Deactivate all *independent* sources. This amounts to making $V_{\mathrm{Th}} = 0$ in the Thevenin equivalent circuit.

Thevenin's theorem: R_{Th}

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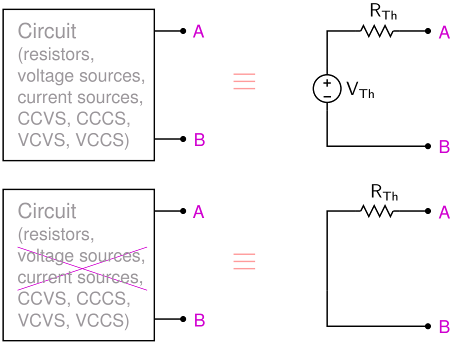

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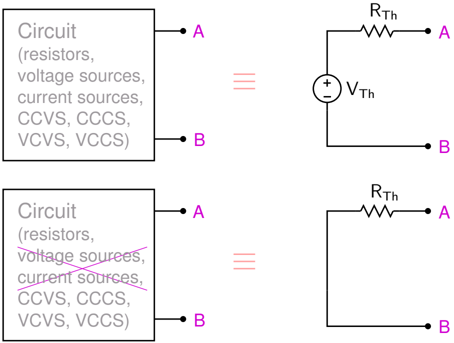

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* Often, R_{Th} can be found by inspection of the original circuit (with independent sources deactivated).

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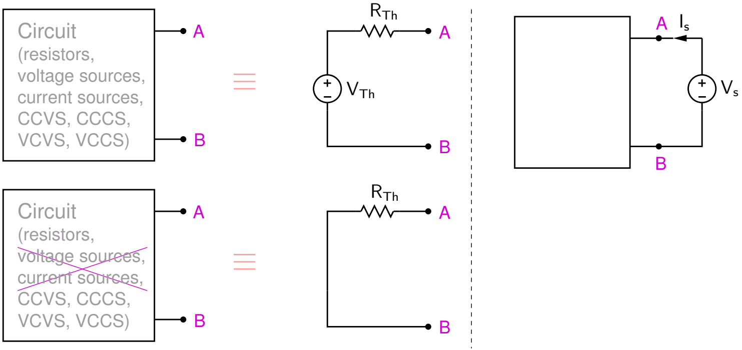

* Deactivate all *independent* sources. This amounts to making $V_{\mathrm{Th}} = 0$ in the Thevenin equivalent circuit.
* Often, R_{Th} can be found by inspection of the original circuit (with independent sources deactivated).
* R_{Th} can also be found by connecting a *test* source to the original circuit (with independent sources deactivated): $R_{\mathrm{Th}} = V_s/I_s$.

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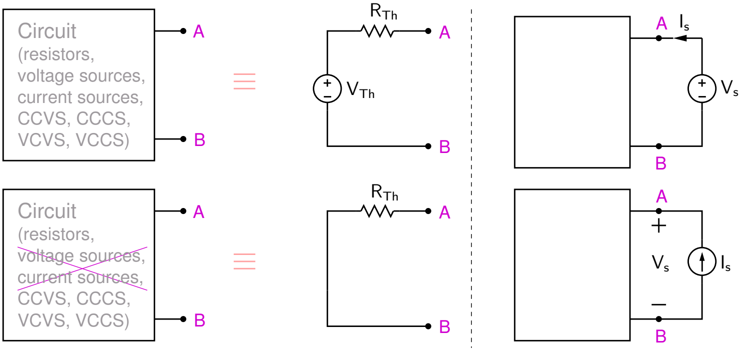

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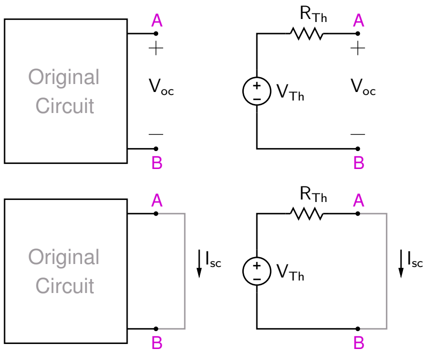
Thevenin's theorem: R_{Th}

Method 2:



* For the Thevenin equivalent circuit, $V_{oc} = V_{Th}$, $I_{sc} = \dfrac{V_{Th}}{R_{Th}} = \dfrac{V_{oc}}{R_{Th}} \rightarrow R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

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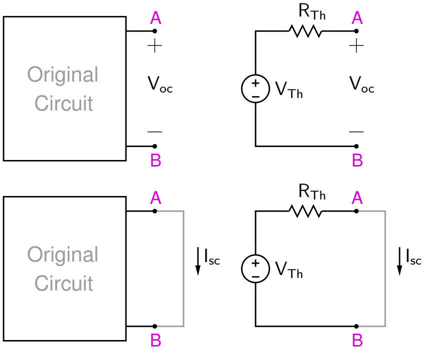

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* In the original circuit, find V_{oc} and I_{sc} $\rightarrow R_{Th} = \dfrac{V_{oc}}{I_{sc}}$.

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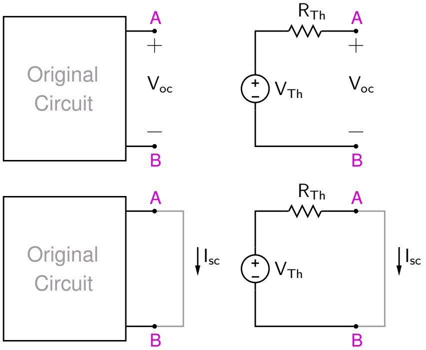
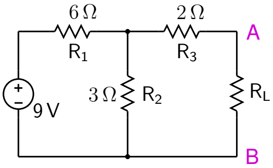
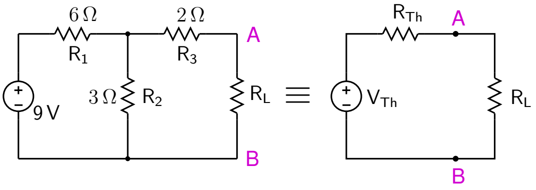


* For the Thevenin equivalent circuit, $V_{\rm oc} = V_{\rm Th}$, $I_{\rm sc} = \dfrac{V_{\rm Th}}{R_{\rm Th}} = \dfrac{V_{\rm oc}}{R_{\rm Th}} \rightarrow R_{\rm Th} = \dfrac{V_{\rm oc}}{I_{\rm sc}}$.
* In the original circuit, find $V_{\rm oc}$ and $I_{\rm sc}$ $\rightarrow R_{\rm Th} = \dfrac{V_{\rm oc}}{I_{\rm sc}}$.
* Note: We do not deactivate any sources in this case.

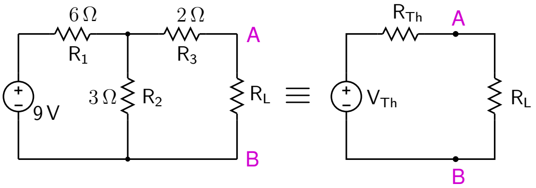
Thevenin's theorem: example

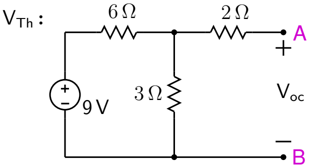


Thevenin's theorem: example



Thevenin's theorem: example





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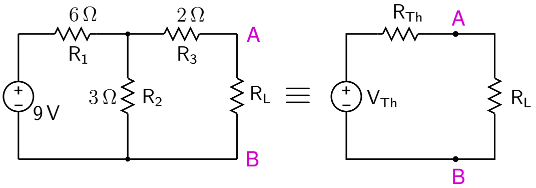
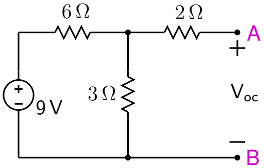


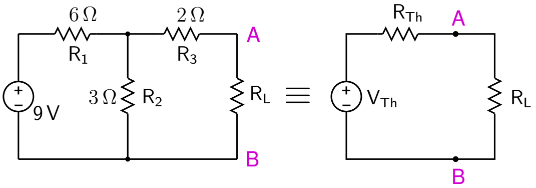
V_{Th} :



$$V_{oc} = 9\,\mathrm{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

Thevenin's theorem: example



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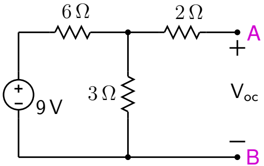
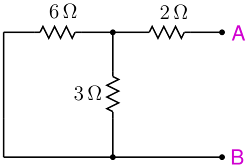


R_{Th} :



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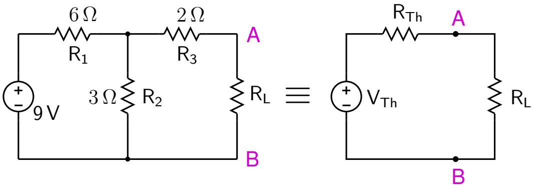

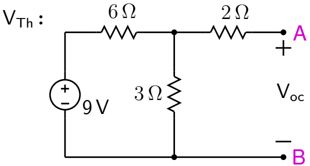

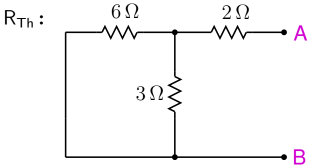

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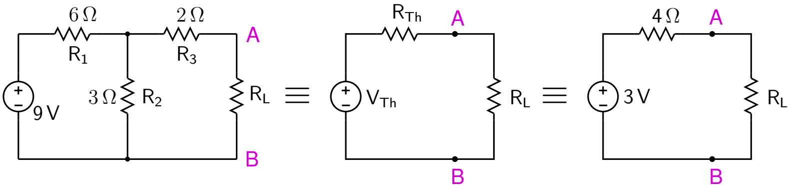
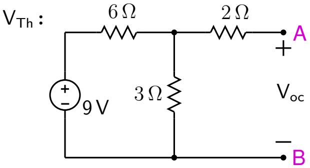
$$= 9\,\mathrm{V} \times \frac{1}{3} = 3\,\mathrm{V}$$

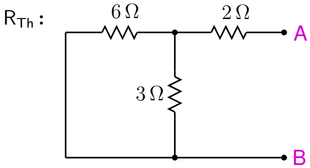
$$\mathrm{R_{Th}} = (\mathrm{R_1} \parallel \mathrm{R_2}) + \mathrm{R_3} = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Thevenin's theorem: example







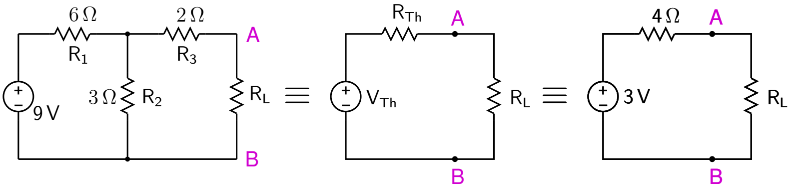
$$V_{oc} = 9\,V \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

$$= 9\,V \times \frac{1}{3} = 3\,V$$

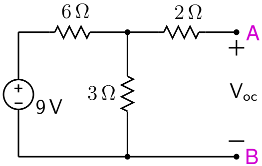
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

Thevenin's theorem: example



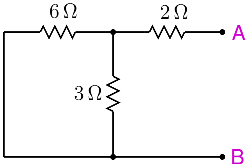
V_{Th} :



$$V_{oc} = 9\,\text{V} \times \frac{3\,\Omega}{6\,\Omega + 3\,\Omega}$$

$$= 9\,\text{V} \times \frac{1}{3} = 3\,\text{V}$$

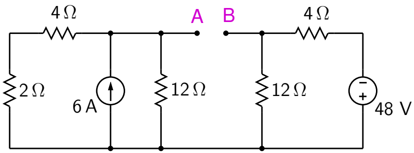
R_{Th} :



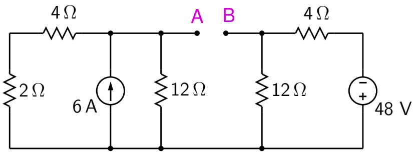
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

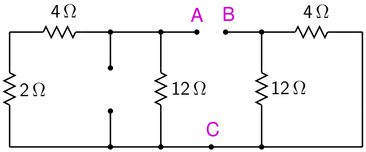
Thevenin's theorem: example



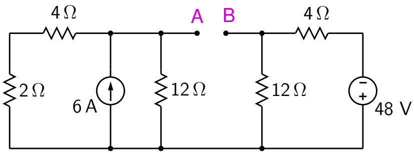
Thevenin's theorem: example



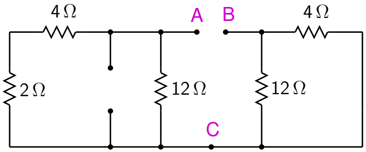
R_{Th}:

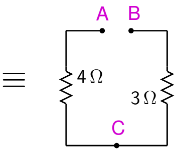


Thevenin's theorem: example

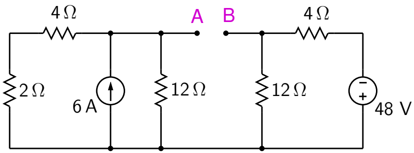


R_{Th}:

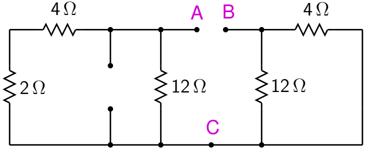




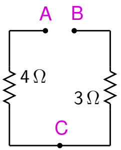
Thevenin's theorem: example



R_{Th}:

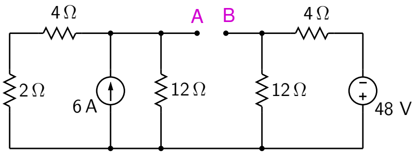


$\equiv$

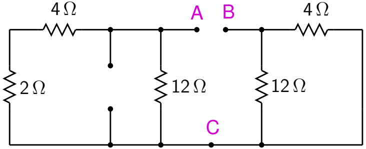


$\Rightarrow \quad R_{Th} = 7\,\Omega$

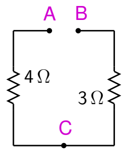
Thevenin's theorem: example



R_{Th}:

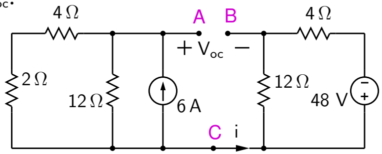


$\equiv$

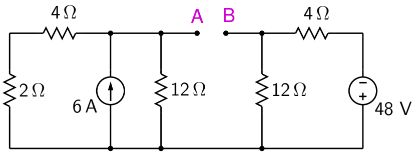


$\Rightarrow \quad R_{Th} = 7\,\Omega$

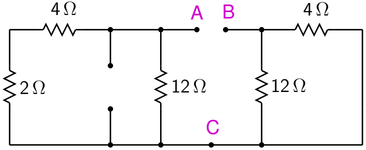
V_{oc}:



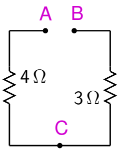
Thevenin's theorem: example



R_{Th}:

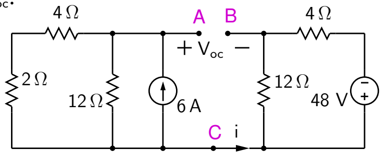


$\equiv$



$\Rightarrow \quad R_{Th} = 7\,\Omega$

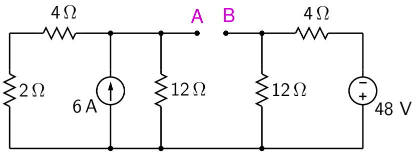
V_{oc}:



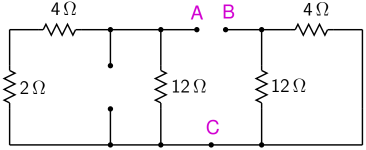
Note: $i = 0$ (since there is no return path).

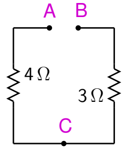
$$
\begin{aligned}
V_{AB} &= V_A - V_B \\
&= (V_A - V_C) + (V_C - V_B) \\
&= V_{AC} + V_{CB} \\
&= 24\,V + 36\,V = 60\,V
\end{aligned}
$$

Thevenin's theorem: example



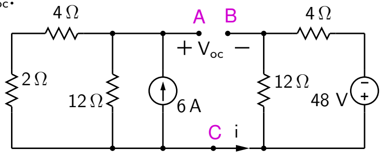
R_{Th}:





$\equiv \quad \Rightarrow \quad R_{Th} = 7\,\Omega$

V_{oc}:



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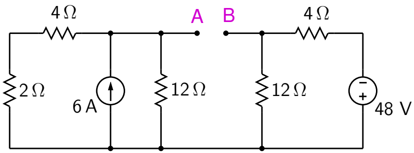
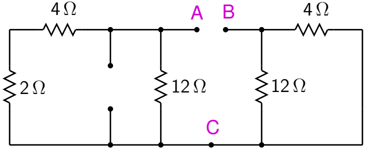
\end{aligned}
$$

$$\boxed{\begin{aligned} V_{Th} &= 60\,\text{V} \\ R_{Th} &= 7\,\Omega \end{aligned}}$$

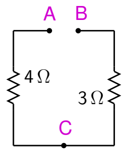
Thevenin's theorem: example



R_{Th}:

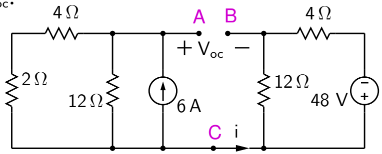


$\equiv$



$\Rightarrow \quad R_{Th} = 7\,\Omega$

V_{oc}:



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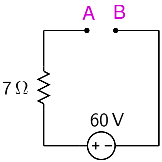
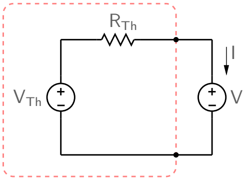
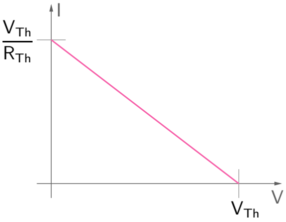
\end{aligned}
$$

$$\boxed{\begin{aligned} V_{Th} &= 60\,V \\ R_{Th} &= 7\,\Omega \end{aligned}}$$

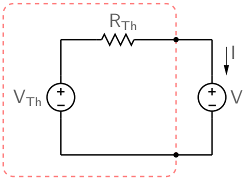


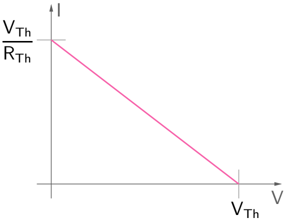




$I = \dfrac{V_{\rm Th} - V}{R_{\rm Th}}$ (Note: negative slope for I versus V plot)

Graphical method for finding V_{Th} and R_{Th}

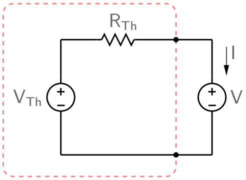


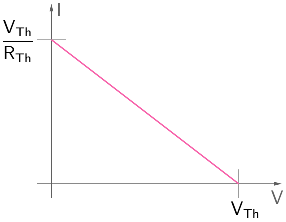


$I = \dfrac{V_{\text{Th}} - V}{R_{\text{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\text{Th}}$ (same as V_{oc})

Graphical method for finding V_{Th} and R_{Th}



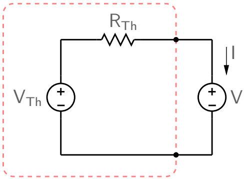


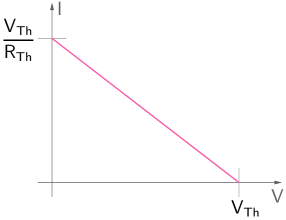
$I = \dfrac{V_{\text{Th}} - V}{R_{\text{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\text{Th}}$ (same as V_{oc})

$V = 0 \rightarrow I = \dfrac{V_{\text{Th}}}{R_{\text{Th}}}$ (same as I_{sc})

Graphical method for finding V_{Th} and R_{Th}



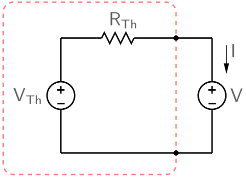


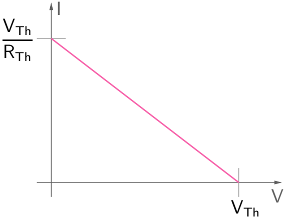
$I = \dfrac{V_{\rm Th} - V}{R_{\rm Th}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\rm Th}$ (same as $V_{\rm oc}$)

$V = 0 \rightarrow I = \dfrac{V_{\rm Th}}{R_{\rm Th}}$ (same as $I_{\rm sc}$)

i.e., a plot of I versus V can be used to find $V_{\rm Th}$ and $R_{\rm Th}$.





$I = \dfrac{V_{\mathrm{Th}} - V}{R_{\mathrm{Th}}}$ (Note: negative slope for I versus V plot)

$I = 0 \rightarrow V = V_{\mathrm{Th}}$ (same as V_{oc})

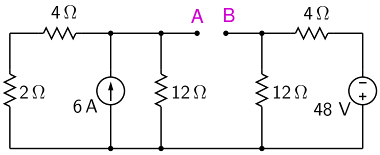
$V = 0 \rightarrow I = \dfrac{V_{\mathrm{Th}}}{R_{\mathrm{Th}}}$ (same as I_{sc})

i.e., a plot of I versus V can be used to find V_{Th} and R_{Th}.

(Instead of a voltage source, we could also connect a resistor load (R), vary R, and then plot I versus V.)

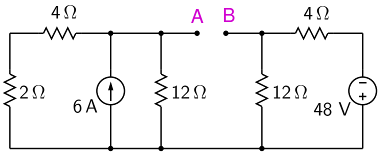
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



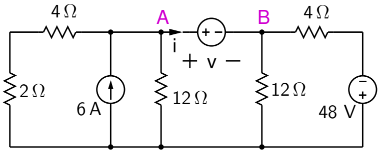
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



Connect a voltage source between A and B.

Plot i versus v.

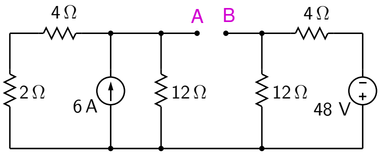


V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

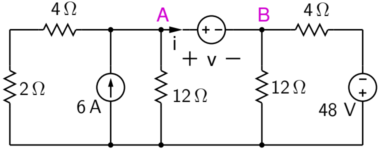
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



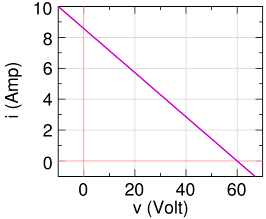
Connect a voltage source between A and B.

Plot i versus v.



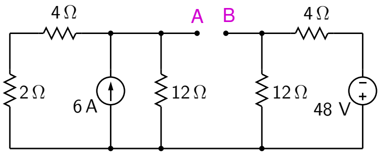
V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.



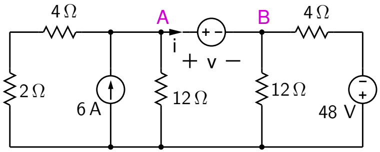
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



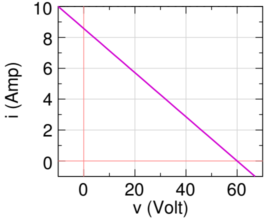
Connect a voltage source between A and B.

Plot i versus v.



V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

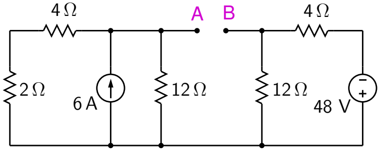


$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

$R_{Th} = V_{oc}/I_{sc} = 7\ \Omega$

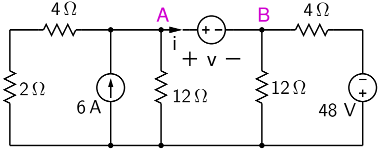
Graphical method for finding V_{Th} and R_{Th}

SEQUEL file: ee101_thevenin_1.sqproj



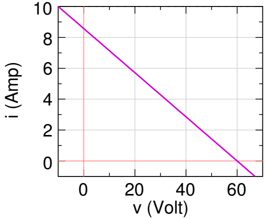
Connect a voltage source between A and B.

Plot i versus v.



V_{oc} = intercept on the v-axis.

I_{sc} = intercept on the i-axis.

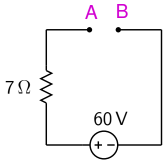


$V_{oc} = 60$ V, $I_{sc} = 8.57$ A

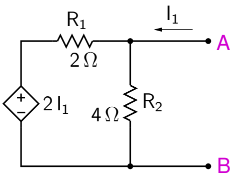
$R_{Th} = V_{oc}/I_{sc} = 7\ \Omega$

$V_{Th} = 60\,\text{V}$

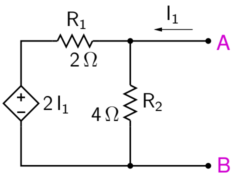
$R_{Th} = 7\,\Omega$



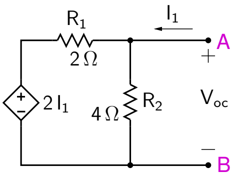
Thevenin's theorem: example



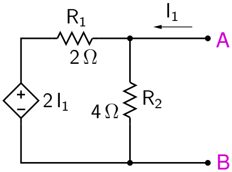
Thevenin's theorem: example



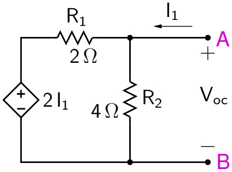
$V_{Th} = V_{oc}$

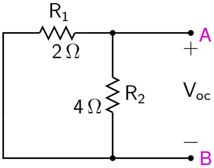


Thevenin's theorem: example



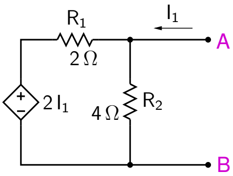
$V_{Th} = V_{oc}$





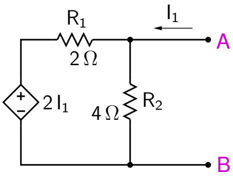
$\boxed{V_{Th} = 0}$

Thevenin's theorem: example



R_{Th}: Deactivate independent sources, connect a test source.

Thevenin's theorem: example



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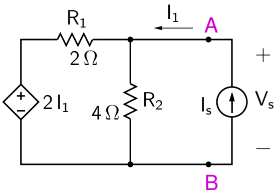
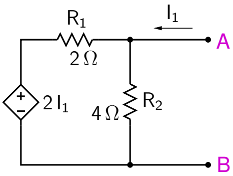


Thevenin's theorem: example



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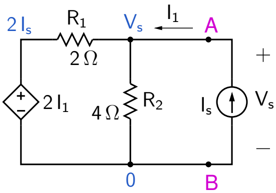
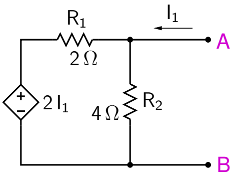


Thevenin's theorem: example



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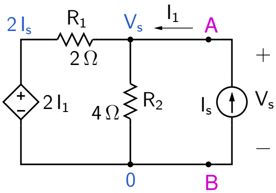
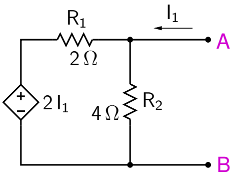


We need to compute $R_{Th} = \dfrac{V_s}{I_s}$.

KCL: $-I_s + \dfrac{V_s}{R_2} + \dfrac{V_s - 2\,I_s}{R_1} = 0$



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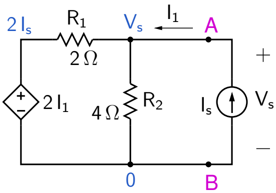

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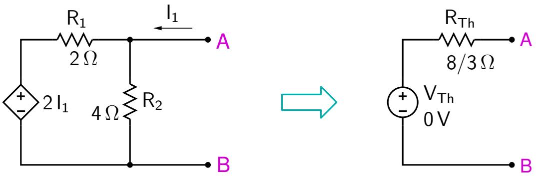
$\rightarrow V_s \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} \right) = I_s \left(1 + \dfrac{2}{R_1} \right)$

$\rightarrow R_{Th} = \dfrac{V_s}{I_s} = \dfrac{8}{3}\,\Omega$

Thevenin's theorem: example



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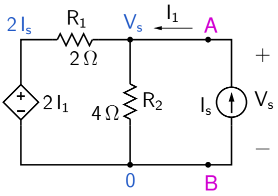

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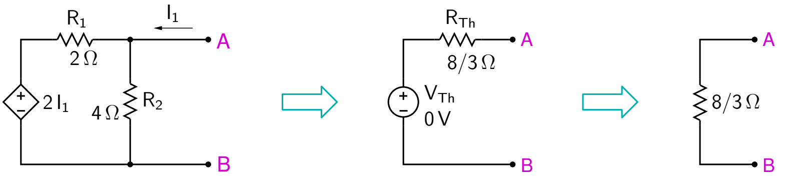
Thevenin's theorem: example



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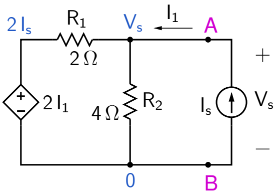

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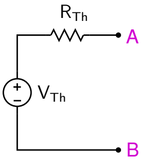
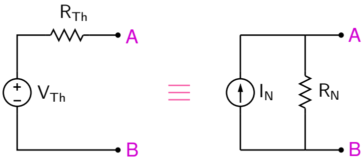
Norton equivalent circuit (source transformation)



Norton equivalent circuit (source transformation)



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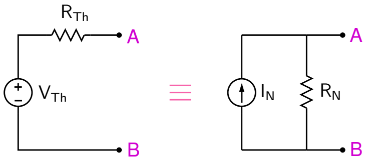
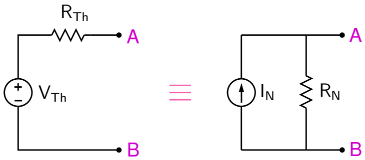


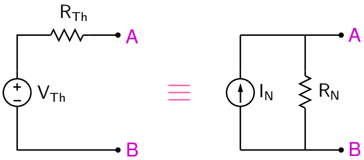
* Consider the open circuit case.

Norton equivalent circuit (source transformation)



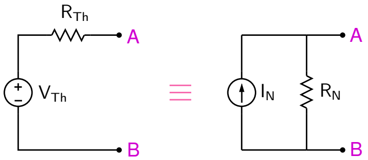
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
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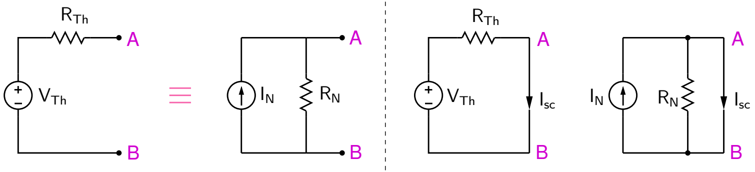
  $\Rightarrow V_{Th} = I_N \, R_N$.

Norton equivalent circuit (source transformation)



- Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
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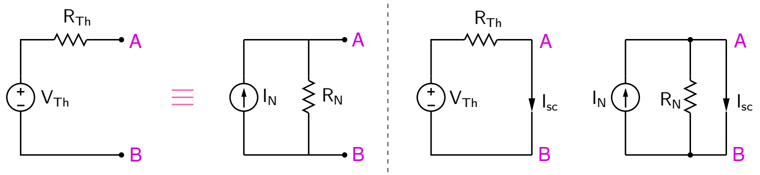
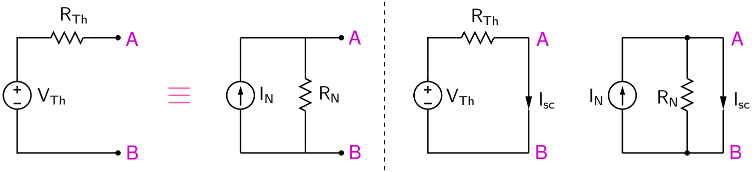
- Consider the short circuit case.

Norton equivalent circuit (source transformation)



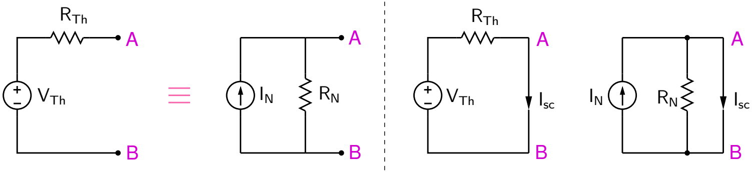
* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.

Norton equivalent circuit (source transformation)

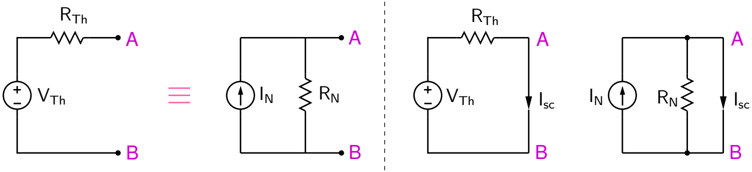


* Consider the open circuit case.
  - Thevenin circuit: $V_{AB} = V_{Th}$.
  - Norton circuit: $V_{AB} = I_N \, R_N$.
  - $\Rightarrow V_{Th} = I_N \, R_N$.
* Consider the short circuit case.
  - Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  - Norton circuit: $I_{sc} = I_N$.

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
  Thevenin circuit: $V_{AB} = V_{Th}$.
  Norton circuit: $V_{AB} = I_N\, R_N$.
  $\Rightarrow V_{Th} = I_N\, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}}\, R_N$



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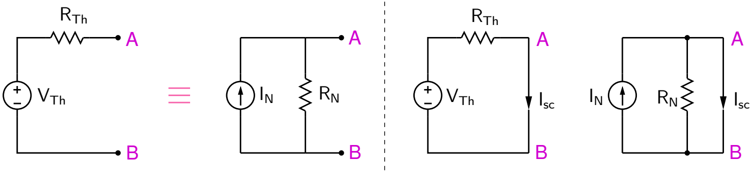

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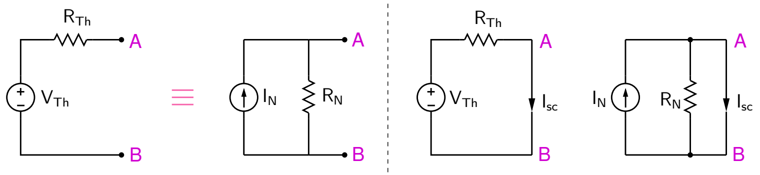
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}} R_N \;\rightarrow\; R_{Th} = R_N$.

$$\boxed{R_N = R_{Th},\ I_N = \frac{V_{Th}}{R_{Th}}}$$

Norton equivalent circuit (source transformation)



* Consider the open circuit case.
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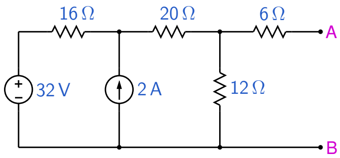
  Norton circuit: $V_{AB} = I_N \, R_N$.
  $\Rightarrow V_{Th} = I_N \, R_N$.
* Consider the short circuit case.
  Thevenin circuit: $I_{sc} = V_{Th}/R_{Th}$.
  Norton circuit: $I_{sc} = I_N$.
  $\Rightarrow V_{Th} = \dfrac{V_{Th}}{R_{Th}} R_N \;\rightarrow\; R_{Th} = R_N$.

$$\boxed{R_N = R_{Th},\; I_N = \frac{V_{Th}}{R_{Th}}} \qquad \boxed{R_{Th} = R_N,\; V_{Th} = I_N R_N}$$

Source transformation: example



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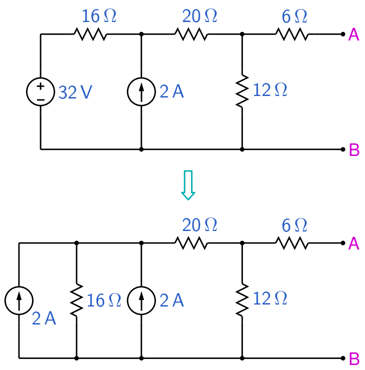

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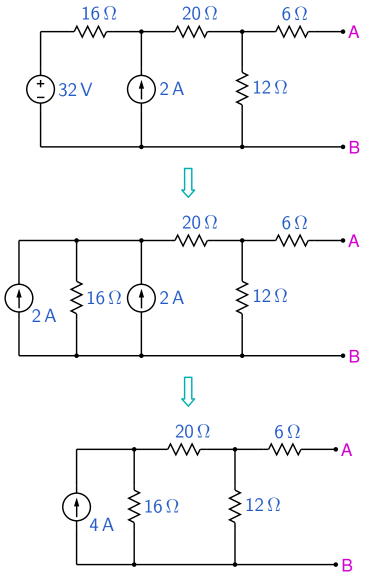

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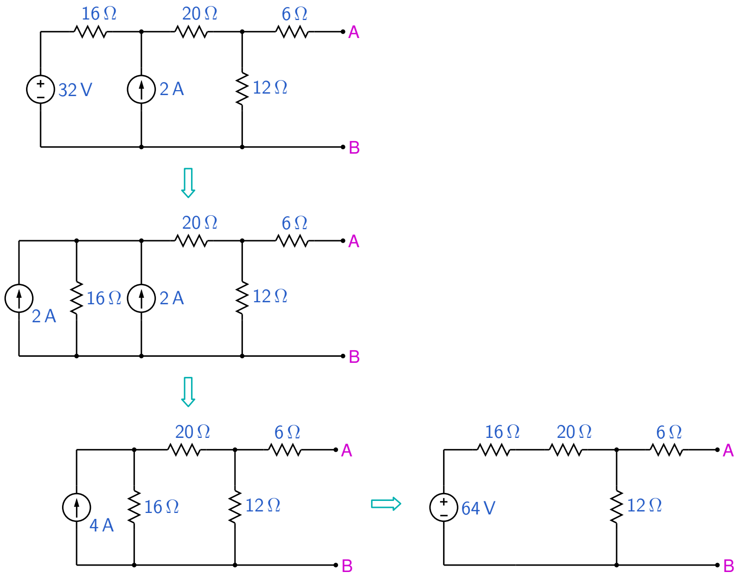

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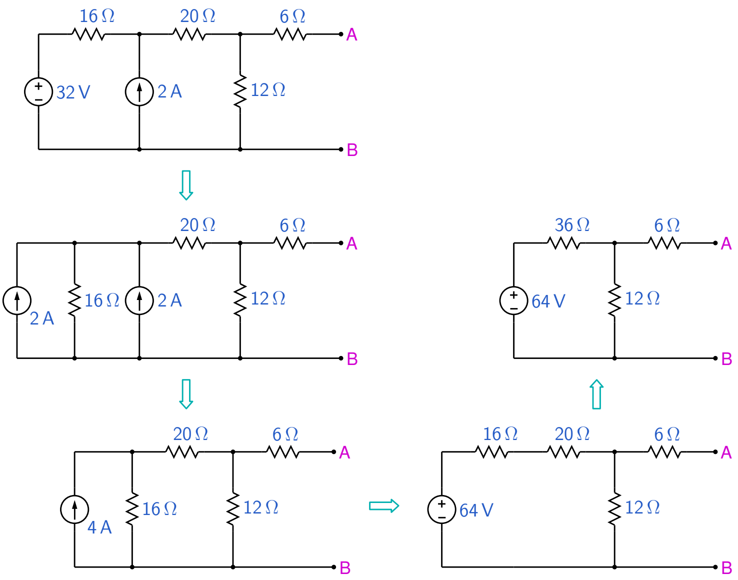
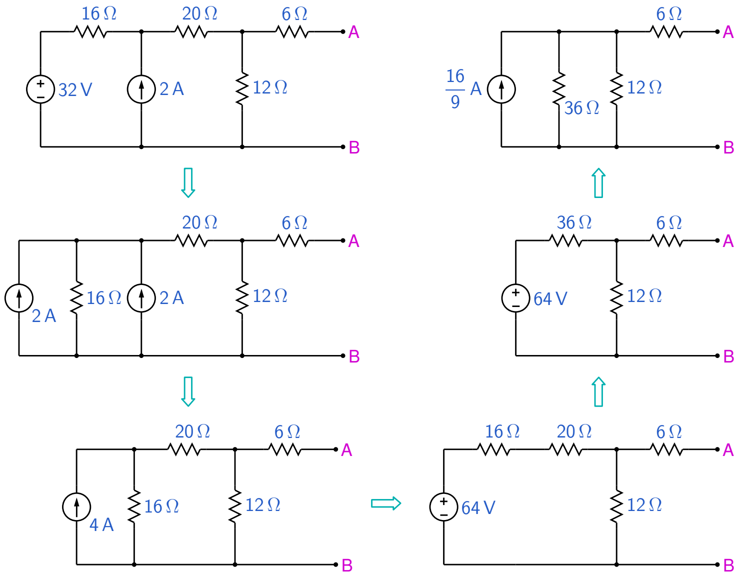
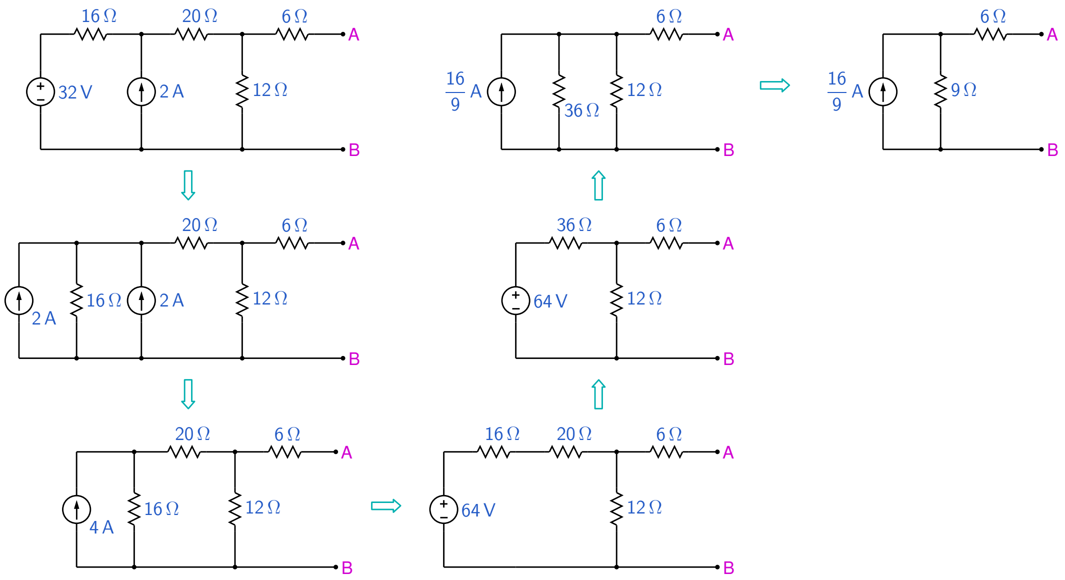


Source transformation: example



Source transformation: example



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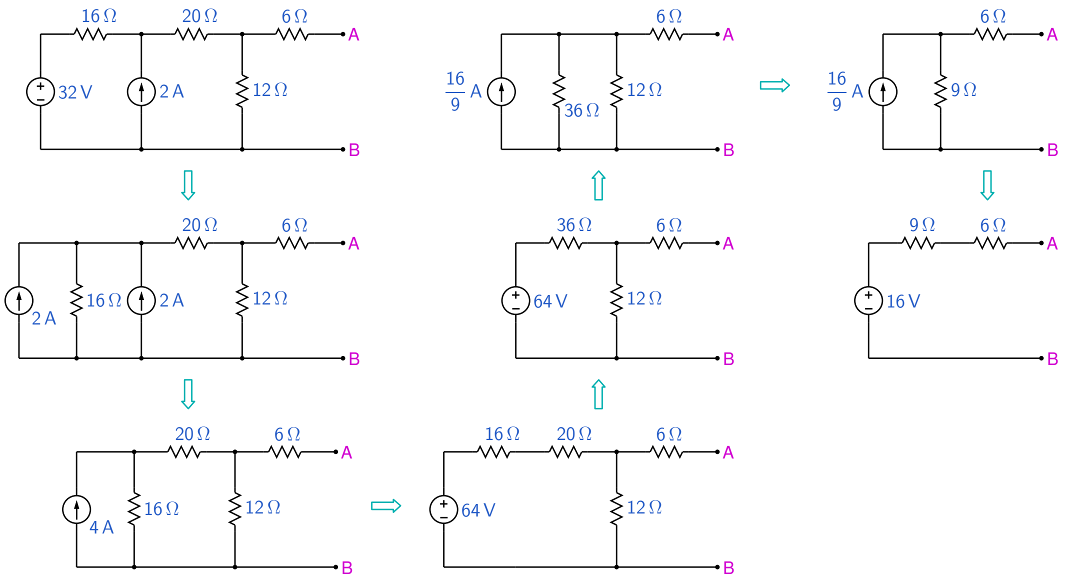

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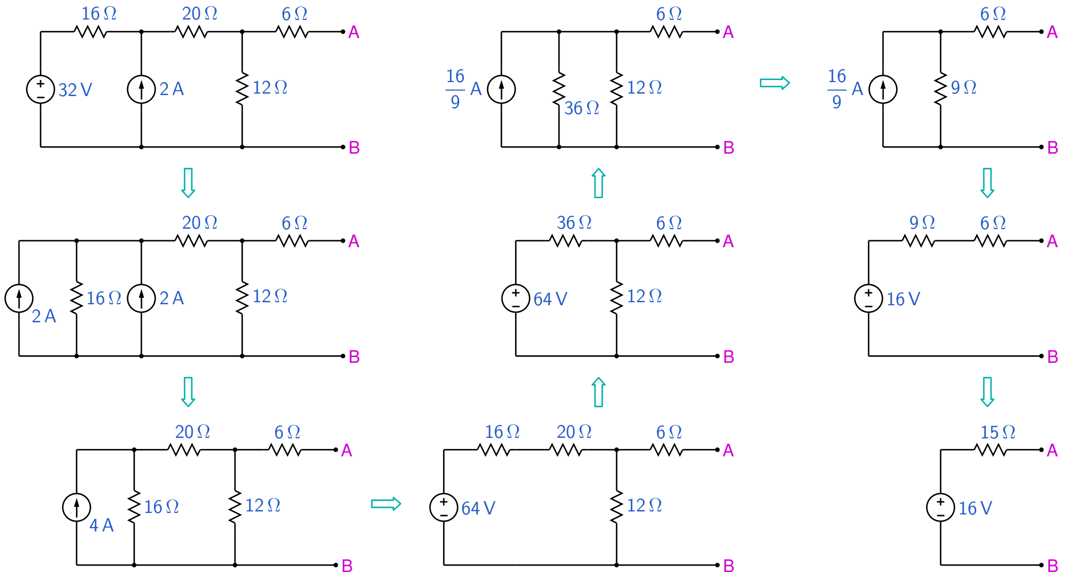
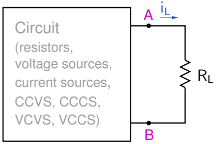


Maximum power transfer



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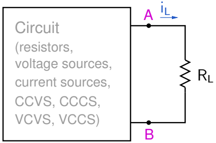
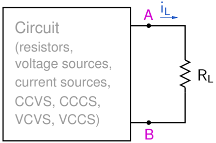


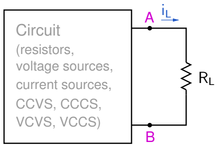
* Power "transferred" to load is, $P_L = i_L^2\, R_L$.

Maximum power transfer



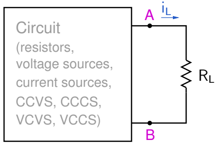
* Power "transferred" to load is, $P_L = i_L^2 \, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?

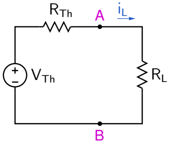
Maximum power transfer



* Power "transferred" to load is, $P_L = i_L^2\, R_L$.
* For a given black box, what is the value of R_L for which P_L is maximum?
* Replace the black box with its Thevenin equivalent.

Maximum power transfer





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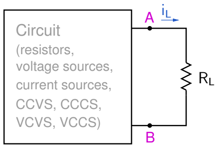

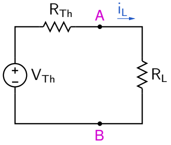

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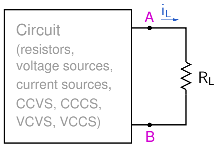
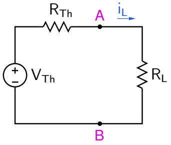
* Replace the black box with its Thevenin equivalent.
* $i_L = \dfrac{V_{Th}}{R_{Th} + R_L}$, $P_L = V_{Th}^2 \times \dfrac{R_L}{(R_{Th} + R_L)^2}$.

Maximum power transfer





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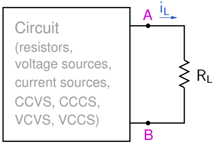
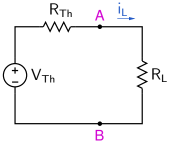
* $i_L = \dfrac{V_{Th}}{R_{Th} + R_L}$, $P_L = V_{Th}^2 \times \dfrac{R_L}{(R_{Th} + R_L)^2}$.
* For $\dfrac{dP_L}{dR_L} = 0$, we need

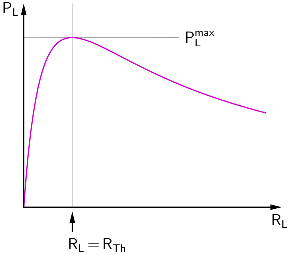
$$\frac{(R_{Th} + R_L)^2 - R_L \times 2\,(R_{Th} + R_L)}{(R_{Th} + R_L)^4} = 0\,,$$

i.e., $R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}$.

Maximum power transfer







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* For $\dfrac{dP_L}{dR_L} = 0$, we need

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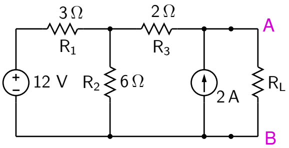
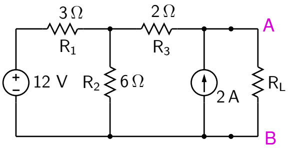
i.e., $R_{Th} + R_L = 2\,R_L \Rightarrow R_L = R_{Th}$.

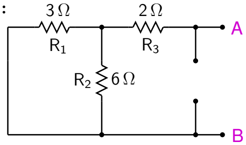
Find R_L for which P_L is maximum.



Find R_L for which P_L is maximum.



R_{Th}:



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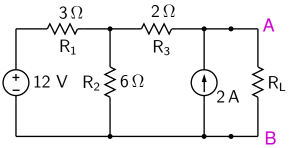
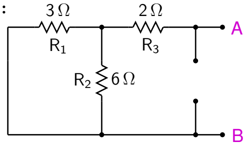


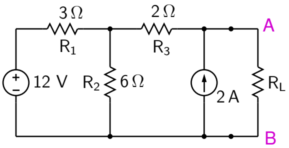
R_{Th}:



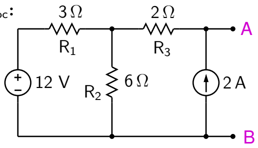
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1 + 2}\right) + 2 = 4\,\Omega$$

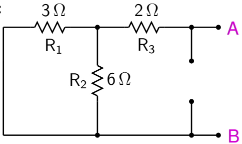
Find R_L for which P_L is maximum.



V_{oc}:



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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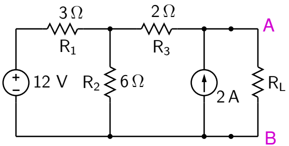
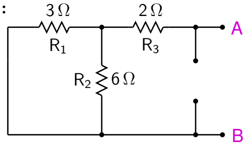
Find R_L for which P_L is maximum.



R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

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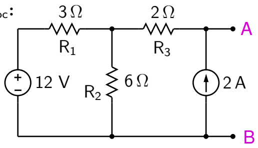
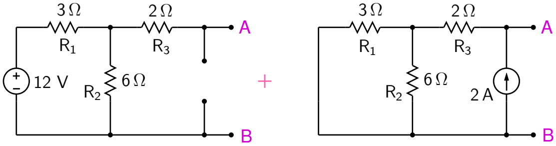
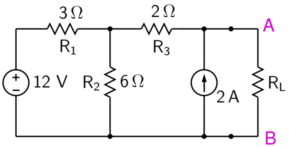
V_{oc}:



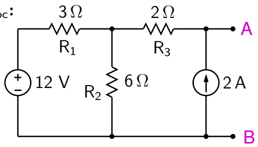
Use superposition to find V_{oc}:



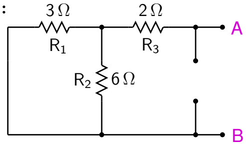
Find R_L for which P_L is maximum.



V_{oc}:



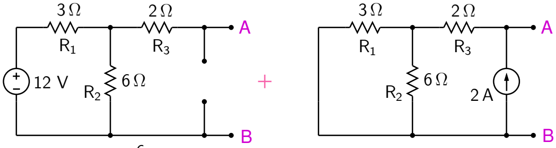
R_{Th}:



$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

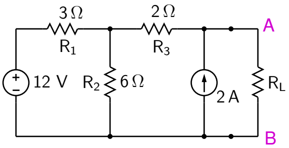
$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

Use superposition to find V_{oc}:

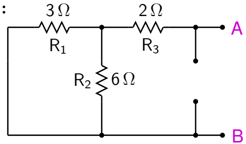


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$

Find R_L for which P_L is maximum.



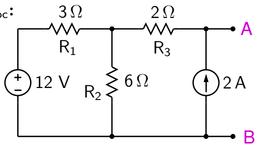
R_{Th}:



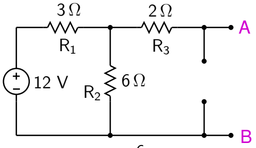
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

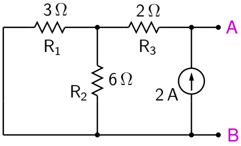
V_{oc}:



Use superposition to find V_{oc}:



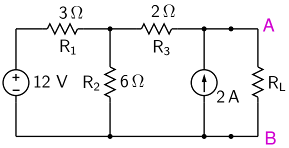
$+$



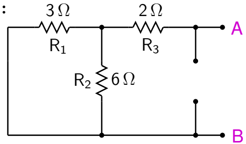
$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\ \mathrm{V}$$

$$V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\mathrm{V}$$

Find R_L for which P_L is maximum.



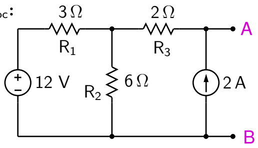
R_{Th}:



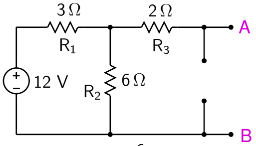
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

V_{oc}:



Use superposition to find V_{oc}:



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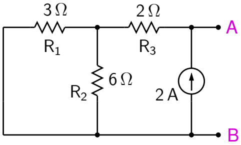
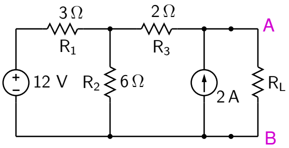


$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8 \text{ V}$$

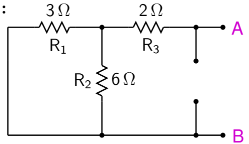
$$V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$

Find R_L for which P_L is maximum.



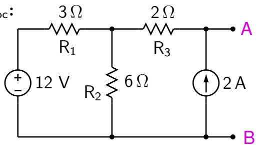
R_{Th}:



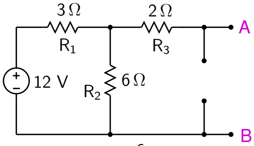
$$R_{Th} = (R_1 \parallel R_2) + R_3 = (3 \parallel 6) + 2$$

$$= 3 \times \left(\frac{1 \times 2}{1+2}\right) + 2 = 4\,\Omega$$

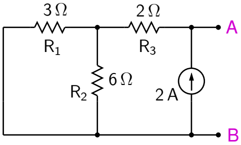
V_{oc}:



Use superposition to find V_{oc}:

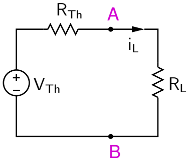


+



$$V_{oc}^{(1)} = 12 \times \frac{6}{9} = 8\ \text{V} \qquad V_{oc}^{(2)} = 4\,\Omega \times 2\,A = 8\,\text{V}$$

$$V_{oc} = V_{oc}^{(1)} + V_{oc}^{(2)} = 8 + 8 = 16\,\text{V}$$



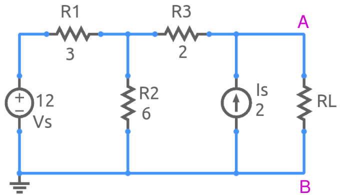
P_L is maximum when $R_L = R_{Th} = 4\,\Omega$

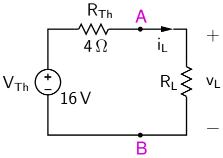
$\Rightarrow i_L = V_{Th}/(2\,R_{Th}) = 2\,\text{A}$

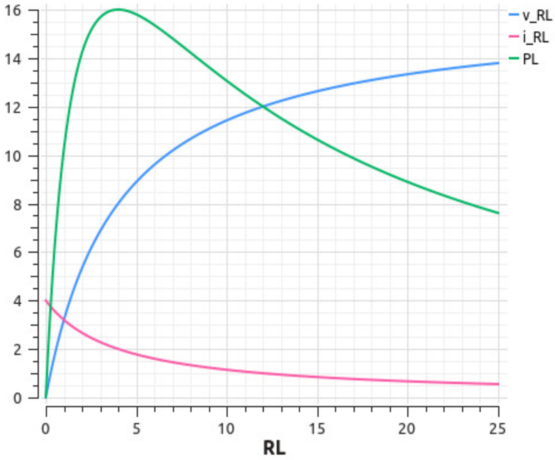
$P_L^{max} = 2^2 \times 4 = 16\,\text{W}$.

Maximum power transfer: simulation results

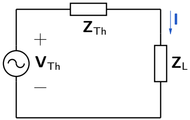
SEQUEL file: ee101_maxpwr_1.sqproj





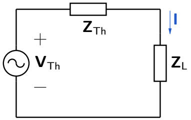


Maximum power transfer (sinusoidal steady state)



Maximum power transfer (sinusoidal steady state)

Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.



Maximum power transfer (sinusoidal steady state)

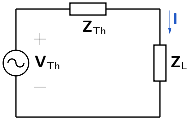
Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

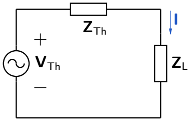
$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L\,.$$



Maximum power transfer (sinusoidal steady state)



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

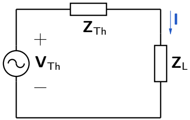
$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} R_L\,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

Maximum power transfer (sinusoidal steady state)



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2}\, I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

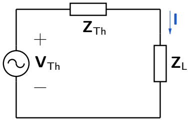
$$= \frac{1}{2}\, \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}\, R_L\,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

With $X_L = -X_{Th}$, we have,

$$P = \frac{1}{2}\, \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2}\, R_L\,,$$

which is maximum for $R_L = R_{Th}$.



Let $\mathbf{Z}_L = R_L + jX_L$, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, and $\mathbf{I} = I_m \angle \phi$.

The power absorbed by $\mathbf{Z}_L$ is,

$$P = \frac{1}{2} I_m^2 R_L$$

$$= \frac{1}{2} \left| \frac{\mathbf{V}_{Th}}{\mathbf{Z}_{Th} + \mathbf{Z}_L} \right|^2 R_L$$

$$= \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2} R_L \,.$$

For P to be maximum, $(X_{Th} + X_L)$ must be zero. $\Rightarrow X_L = -X_{Th}$.

With $X_L = -X_{Th}$, we have,

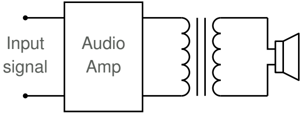
$$P = \frac{1}{2} \frac{|\mathbf{V}_{Th}|^2}{(R_{Th} + R_L)^2} R_L \,,$$

which is maximum for $R_L = R_{Th}$.

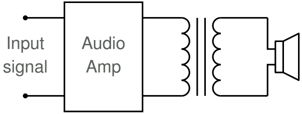
Therefore, for maximum power transfer to the load $\mathbf{Z}_L$, we need,

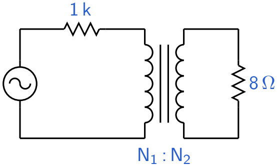
$R_L = R_{Th}$, $X_L = -X_{Th}$, i.e., $\boxed{\mathbf{Z}_L = \mathbf{Z}_{Th}^*.}$

Impedance matching

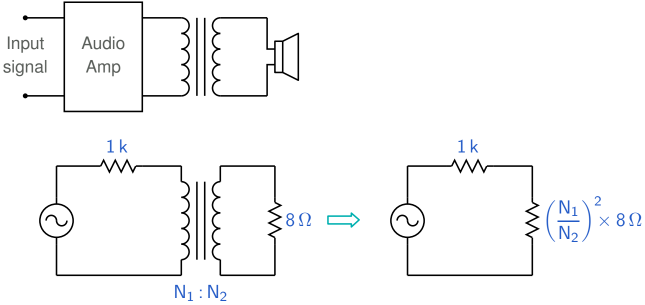


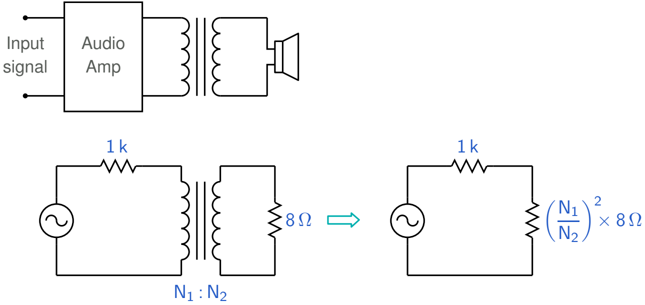
Impedance matching



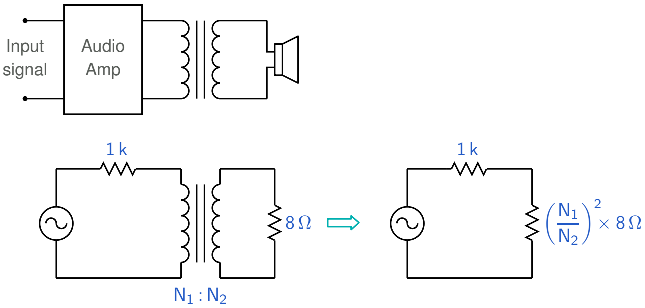


Impedance matching





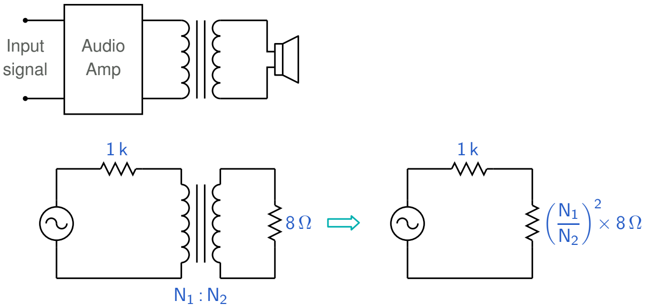
Calculate the turns ratio to provide maximum power transfer of the audio signal.



Calculate the turns ratio to provide maximum power transfer of the audio signal.

$Z_L = Z_{Th}^*$

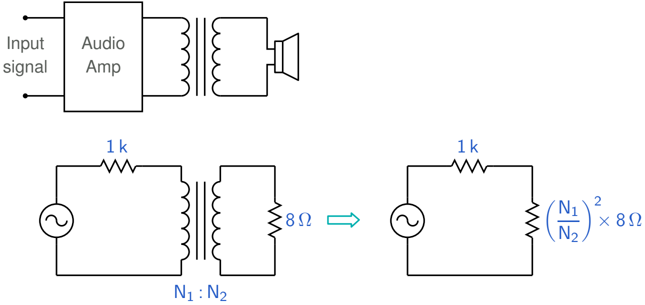
Impedance matching



Calculate the turns ratio to provide maximum power transfer of the audio signal.

$$Z_L = Z^*_{Th} \;\rightarrow\; \left(\frac{N_1}{N_2}\right)^2 \times 8\,\Omega = 1\,\text{k}\Omega$$

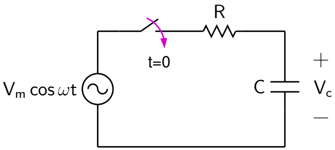
Impedance matching



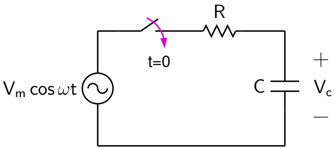
Calculate the turns ratio to provide maximum power transfer of the audio signal.

$$Z_L = Z^*_{Th} \;\rightarrow\; \left(\frac{N_1}{N_2}\right)^2 \times 8\,\Omega = 1\,\text{k}\Omega \;\rightarrow\; \frac{N_1}{N_2} = \sqrt{\frac{1000}{8}} = 11.2$$

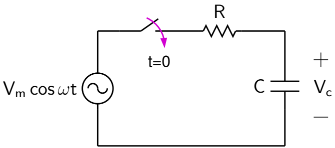
Sinusoidal steady state



Sinusoidal steady state

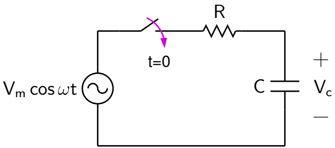


$$R\,(C\,V_c') + V_c = V_m\,\cos\,\omega t\,, \qquad t > 0\,. \tag{1}$$



$$R\,(C\,V_c') + V_c = V_m \cos \omega t\,, \qquad t > 0\,. \tag{1}$$

The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)$.



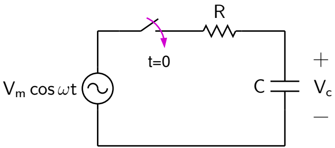
$$R\,(C\,V_c') + V_c = V_m\,\cos\,\omega t\,, \qquad t > 0\,. \tag{1}$$

The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)\,.$
$V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

$$R\,C\,V_c' + V_c = 0\,, \tag{2}$$

from which, $V_c^{(h)}(t) = A\,\exp(-t/\tau)\,,$ with $\tau = RC\,.$

Sinusoidal steady state

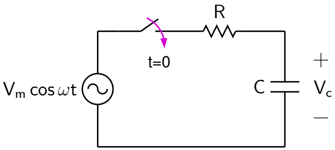


$$R\,(C\,V_c') + V_c = V_m\,\cos\omega t\,,\qquad t > 0\,. \tag{1}$$

The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)\,.$
$V_c^{(h)}(t)$ satisfies the homogeneous differential equation,

$$R\,C\,V_c' + V_c = 0\,, \tag{2}$$

from which, $V_c^{(h)}(t) = A\,\exp(-t/\tau)\,,$ with $\tau = RC\,.$
$V_c^{(p)}(t)$ is a particular solution of (1). Since the forcing function is $V_m\,\cos\omega t$, we try
$V_c^{(p)}(t) = C_1\,\cos\omega t + C_2\,\sin\omega t\,.$



$$R\,(C\,V_c') + V_c = V_m\,\cos\,\omega t\,, \qquad t > 0\,. \tag{1}$$

The solution $V_c(t)$ is made up of two components, $V_c(t) = V_c^{(h)}(t) + V_c^{(p)}(t)\,.$
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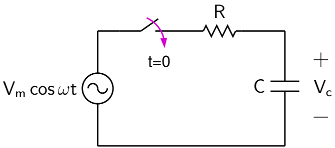

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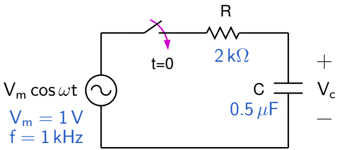
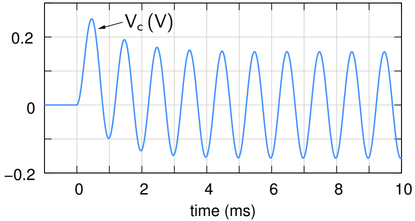
C_1 and C_2 can be found by equating the coefficients of sin ωt and cos ωt on the left and right sides.





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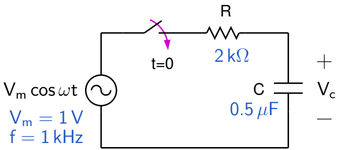

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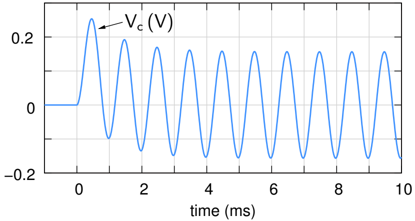

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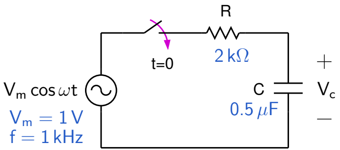

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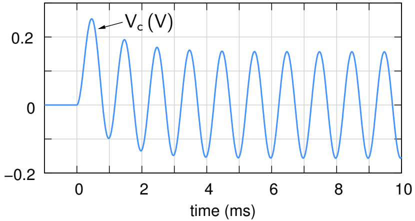

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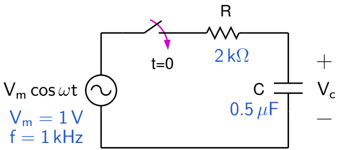

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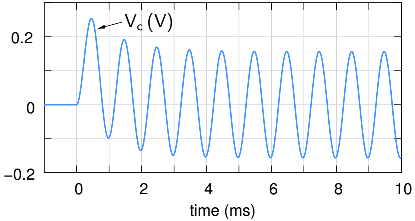

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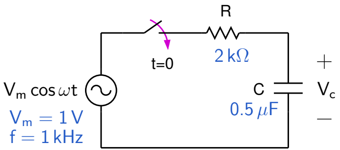
Sinusoidal steady state



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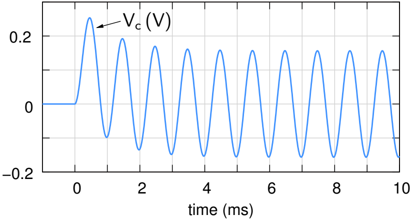

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* This is known as the "sinusoidal steady state" response since all quantities (currents and voltages) in the circuit are sinusoidal in nature.
* Any circuit containing resistors, capacitors, inductors, sinusoidal voltage and current sources (of the same frequency), dependent (linear) sources behaves in a similar manner, viz., each current and voltage in the circuit becomes purely sinusoidal as $t \rightarrow \infty$.

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  &= Re\left[X_m\, e^{j(\omega t+\theta)}\right]
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Sinusoidal steady state: phasors

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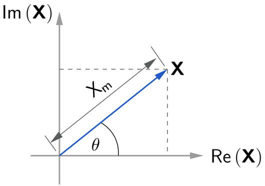
* Use of phasors substantially simplifies analysis of circuits in the sinusoidal steady state.
* Note that a phasor can be written in the polar form or rectangular form,
  $\mathbf{X} = X_m \angle\theta = X_m \, \exp(j\theta) = X_m \, \cos\,\theta + j\, X_m \, \sin\,\theta$.
  The term ωt is always *implicit*.



Phasors: examples

Time domain	Frequency domain
$v_1(t) = 3.2 \cos(\omega t + 30^\circ)$ V	

Phasors: examples

Time domain	Frequency domain
$v_1(t)=3.2\cos(\omega t+30^\circ)$ V	$V_1 = 3.2\angle 30^\circ = 3.2\exp(j\pi/6)$ V

Phasors: examples

Time domain	Frequency domain
$v_1(t)=3.2\cos(\omega t+30^\circ)$ V	$V_1 = 3.2\,\angle 30^\circ = 3.2\exp(j\pi/6)$ V
$i(t) = -1.5\cos(\omega t + 60^\circ)$ A	

Phasors: examples

Time domain	Frequency domain
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Phasors: examples

Time domain	Frequency domain
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Phasors: examples

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Phasors: examples

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Phasors: examples

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$i(t) = -1.5\cos(\omega t + 60^\circ)\ \mathrm{A}$ $= 1.5\cos(\omega t + \pi/3 - \pi)\ \mathrm{A}$ $= 1.5\cos(\omega t - 2\pi/3)\ \mathrm{A}$	$I = 1.5\angle(-2\pi/3)\ \mathrm{A}$
$v_2(t) = -0.1\cos(\omega t)\ \mathrm{V}$ $= 0.1\cos(\omega t + \pi)\ \mathrm{V}$	$V_2 = 0.1\angle\pi\ \mathrm{V}$
$i_2(t) = 0.18\sin(\omega t)\ \mathrm{A}$ $= 0.18\cos(\omega t - \pi/2)\ \mathrm{A}$	$I_2 = 0.18\angle(-\pi/2)\ \mathrm{A}$
	$I_3 = 1 + j\,1\ \mathrm{A}$ $= \sqrt{2}\angle 45^\circ\ \mathrm{A}$

Phasors: examples

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$i_3(t) = \sqrt{2}\cos(\omega t + 45^\circ)$ A	$I_3 = 1 + j\,1$ A $= \sqrt{2}\,\angle\,45^\circ$ A

Addition of phasors

Consider addition of two sinusoidal quantities:

$$\begin{aligned} v(t) &= v_1(t) + v_2(t) \\ &= V_{m1}\,\cos(\omega t + \theta_1) + V_{m2}\,\cos(\omega t + \theta_2) \end{aligned}$$

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Now consider addition of the phasors corresponding to $v_1(t)$ and $v_2(t)$.

$$\begin{aligned} \mathbf{V} &= \mathbf{V_1} + \mathbf{V_2} \\ &= V_{m1} e^{j\theta_1} + V_{m2} e^{j\theta_2} \end{aligned}$$

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$$\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$$
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In the time domain, $\mathbf{V}$ corresponds to $\tilde{v}(t)$, with

$$\tilde{v}(t) = Re\left[\mathbf{V} e^{j\omega t}\right]$$

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$$= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)$$

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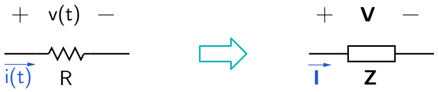
&= Re\left[\left(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2}\right)e^{j\omega t}\right] \\
&= Re\left[V_{m1}e^{j(\omega t+\theta_1)} + V_{m2}e^{j(\omega t+\theta_2)}\right] \\
&= V_{m1}\,\cos\left(\omega t + \theta_1\right) + V_{m2}\,\cos\left(\omega t + \theta_2\right)
\end{aligned}
$$

which is the same as $v(t)$.

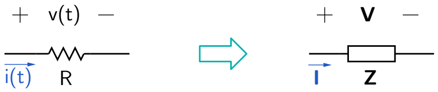
* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.

* Addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state.
* The KCL and KVL equations,
  $\sum i_k(t) = 0$ at a node, and
  $\sum v_k(t) = 0$ in a loop,
  amount to addition of sinusoidal quantities and can therefore be replaced by the corresponding phasor equations,
  $\sum \mathbf{I}_k = \mathbf{0}$ at a node, and
  $\sum \mathbf{V}_k = \mathbf{0}$ in a loop.

Impedance of a resistor

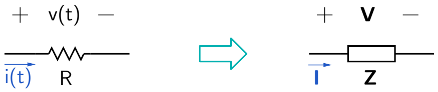


Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

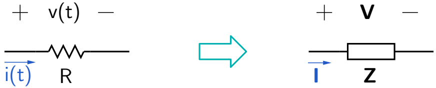
Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

$v(t) = R\, i(t)$

Impedance of a resistor



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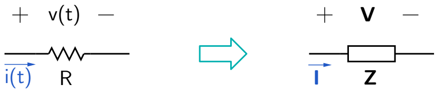
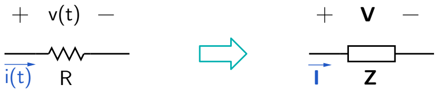
$$v(t) = R\, i(t)$$
$$= R\, I_m \cos(\omega t + \theta)$$



Let $i(t) = I_m \cos(\omega t + \theta)$.

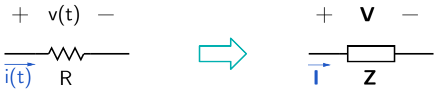
$$
\begin{aligned}
v(t) &= R\, i(t) \\
&= R\, I_m \cos(\omega t + \theta) \\
&\equiv V_m \cos(\omega t + \theta).
\end{aligned}
$$



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = R\,i(t)$$
$$= R\,I_m \cos(\omega t + \theta)$$
$$\equiv V_m \cos(\omega t + \theta).$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,
$\mathbf{I} = I_m \angle\theta$, $\mathbf{V} = R \times I_m \angle\theta$.



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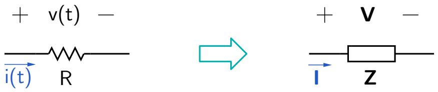
We have therefore the following relationship between $\mathbf{V}$ and $\mathbf{I}$: $\mathbf{V} = R \times \mathbf{I}$.

Impedance of a resistor



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$\begin{aligned} v(t) &= R\, i(t) \\ &= R\, I_m \cos(\omega t + \theta) \\ &\equiv V_m \cos(\omega t + \theta). \end{aligned}$$

The phasors corresponding to $i(t)$ and $v(t)$ are, respectively,

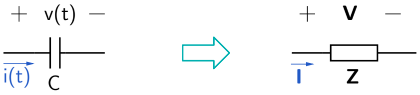
$\mathbf{I} = I_m \angle \theta$, $\mathbf{V} = R \times I_m \angle \theta$.

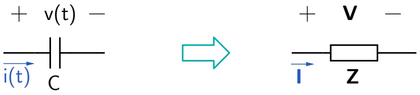
We have therefore the following relationship between $\mathbf{V}$ and $\mathbf{I}$: $\mathbf{V} = R \times \mathbf{I}$.

Thus, the *impedance* of a resistor, defined as, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is

$$\boxed{\mathbf{Z} = R + j\,0}$$

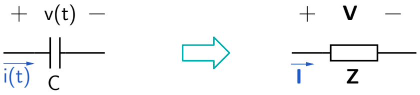
Impedance of a capacitor





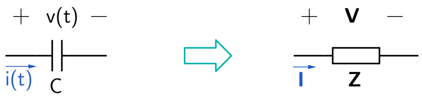
Let $v(t) = V_m \cos(\omega t + \theta)$.

Impedance of a capacitor



Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

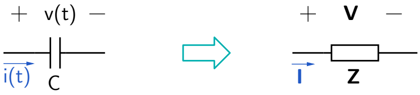


Let $v(t) = V_m \cos(\omega t + \theta)$.

$i(t) = C\,\dfrac{dv}{dt} = -C\,\omega\,V_m \sin(\omega t + \theta)$.

Using the identity, $\cos(\phi + \pi/2) = -\sin\,\phi$, we get

$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.



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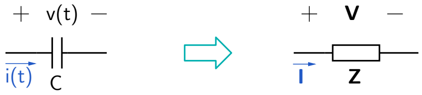
$i(t) = C\,\omega\,V_m\ \cos(\omega t + \theta + \pi/2)$.

In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta + \pi/2)$.



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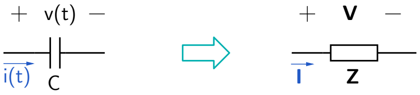
In terms of phasors, $\mathbf{V} = V_m \angle\theta$, $\mathbf{I} = \omega C V_m \angle(\theta + \pi/2)$.

$\mathbf{I}$ can be rewritten as,

$\mathbf{I} = \omega C V_m\, e^{j(\theta+\pi/2)} = \omega C V_m\, e^{j\theta}\, e^{j\pi/2} = j\omega C\, \left(V_m\, e^{j\theta}\right) = j\omega C\,\mathbf{V}$



Let $v(t) = V_m \cos(\omega t + \theta)$.

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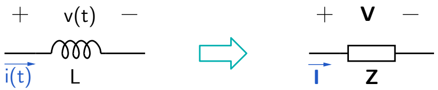
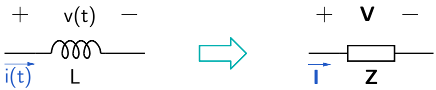
Thus, the *impedance* of a capacitor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = 1/(j\omega C)}$,

and the *admittance* of a capacitor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = j\omega C}$.

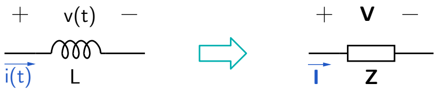
Impedance of an inductor



Impedance of an inductor



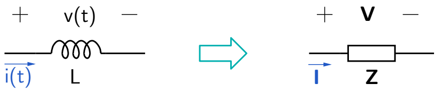
Let $i(t) = I_m \cos(\omega t + \theta)$.



Let $i(t) = I_m \cos(\omega t + \theta)$.

$$v(t) = L\,\frac{di}{dt} = -L\,\omega\,I_m \sin(\omega t + \theta).$$

Impedance of an inductor



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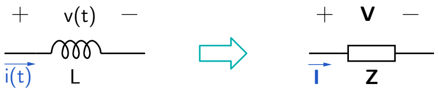
Using the identity, $\cos(\phi + \pi/2) = -\sin\phi$, we get

$v(t) = L\,\omega\,I_m\,\cos(\omega t + \theta + \pi/2)$.

Impedance of an inductor



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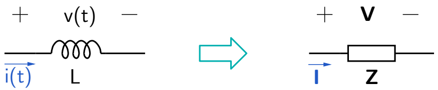
Impedance of an inductor



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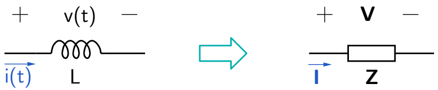
$\mathbf{V} = \omega L I_m\, e^{j(\theta+\pi/2)} = \omega L I_m\, e^{j\theta}\, e^{j\pi/2} = j\omega L\, \left(I_m\, e^{j\theta}\right) = j\omega L\,\mathbf{I}$

Impedance of an inductor



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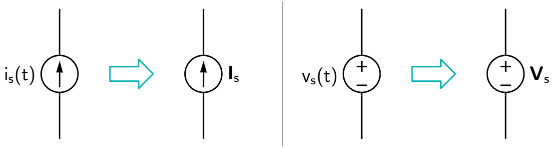
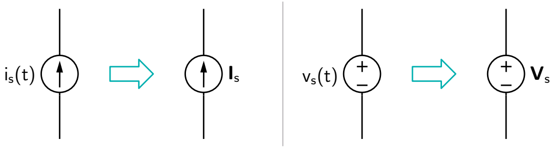
Thus, the *impedance* of an indcutor, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$, is $\boxed{\mathbf{Z} = j\omega L}$,

and the *admittance* of an inductor, $\mathbf{Y} = \mathbf{I}/\mathbf{V}$, is $\boxed{\mathbf{Y} = 1/(j\omega L)}$.

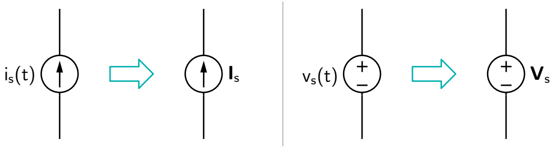


$i_s(t)$
$\mathbf{I}_s$
$v_s(t)$
$\mathbf{V}_s$

Sources



* An independent sinusoidal current source, $i_s(t) = I_m \cos(\omega t + \theta)$, can be represented by the phasor $I_m \angle \theta$ (i.e., a *constant* complex number).



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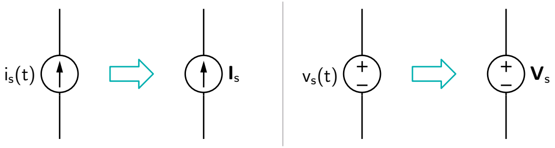
* An independent sinusoidal voltage source, $v_s(t) = V_m \cos(\omega t + \theta)$, can be represented by the phasor $V_m \angle \theta$ (i.e., a *constant* complex number).



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* An independent sinusoidal voltage source, $v_s(t) = V_m \cos(\omega t + \theta)$, can be represented by the phasor $V_m \angle \theta$ (i.e., a *constant* complex number).
* Dependent (linear) sources can be treated in the sinusoidal steady state in the same manner as a resistor, i.e., by the corresponding phasor relationship.
  For example, for a CCVS, we have,
  $v(t) = r\, i_c(t)$ in the time domain.
  $\mathbf{V} = r\, \mathbf{I_c}$ in the frequency domain.

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).

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* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).
* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \textit{constant}$ (a complex number).

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* An independent sinusoidal source in the frequency domain behaves like a DC source, e.g., $\mathbf{V_s} = \textit{constant}$ (a complex number).
* For dependent sources, a time-domain relationship such as $i(t) = \beta\, i_c(t)$ translates to $\mathbf{I} = \beta\, \mathbf{I_c}$ in the frequency domain.

Use of phasors in circuit analysis

* The time-domain KCL and KVL equations $\sum i_k(t) = 0$ and $\sum v_k(t) = 0$ can be written as $\sum \mathbf{I}_k = \mathbf{0}$ and $\sum \mathbf{V}_k = \mathbf{0}$ in the frequency domain.
* Resistors, capacitors, and inductors can be described by $\mathbf{V} = \mathbf{Z}\,\mathbf{I}$ in the frequency domain, which is similar to $V = R\,I$ in DC conditions (except that we are dealing with complex numbers in the frequency domain).
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* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.

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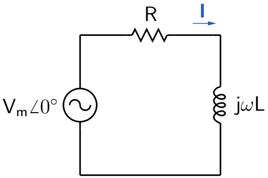
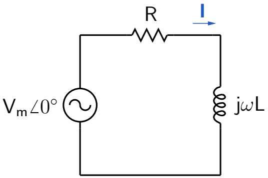
* For dependent sources, a time-domain relationship such as $i(t) = \beta\, i_c(t)$ translates to $\mathbf{I} = \beta\,\mathbf{I_c}$ in the frequency domain.
* Circuit analysis in the sinusoidal steady state using phasors is therefore very similar to DC circuits with independent and dependent sources, and resistors.
* Series/parallel formulas for resistors, nodal analysis, mesh analysis, Thevenin's and Norton's theorems can be directly applied to circuits in the sinusoidal steady state.

RL circuit



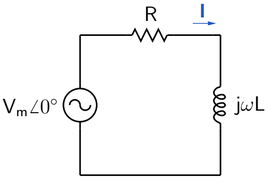
RL circuit



$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L} \equiv I_m\angle(-\theta),$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L/R)$.

RL circuit



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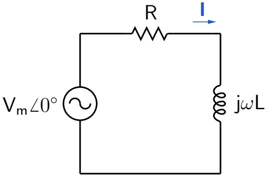
In the time domain, $i(t) = I_m \cos(\omega t - \theta)$, which *lags* the source voltage since the peak (or zero) of $i(t)$ occurs $t = \theta/\omega$ seconds after that of the source voltage.

RL circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle(-\theta),$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L/R)$.

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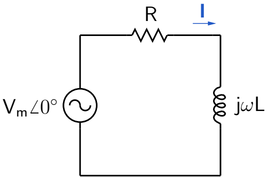
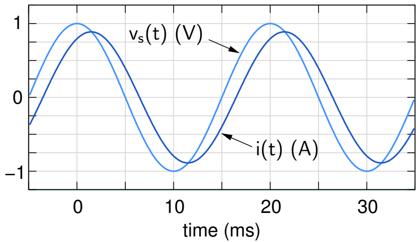
For $R = 1\,\Omega$, $L = 1.6\,\text{mH}$, $f = 50\,\text{Hz}$, $\theta = 26.6°$, $t_{\text{lag}} = 1.48$ ms.

(SEQUEL file: `ee101_rl_ac_1.sqproj`)





R = 1 Ω
L = 1.6 mH

$$\mathbf{I} = \frac{V_m\angle 0}{R + j\omega L} \equiv I_m\angle(-\theta),$$

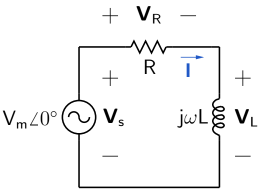
where $I_m = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$, and $\theta = \tan^{-1}(\omega L/R)$.

In the time domain, $i(t) = I_m \cos(\omega t - \theta)$, which *lags* the source voltage since the peak (or zero) of $i(t)$ occurs $t = \theta/\omega$ seconds after that of the source voltage.

For $R = 1\,\Omega$, $L = 1.6\,\text{mH}$, $f = 50\,\text{Hz}$, $\theta = 26.6°$, $t_{\text{lag}} = 1.48$ ms.

(SEQUEL file: `ee101_rl_ac_1.sqproj`)

RL circuit



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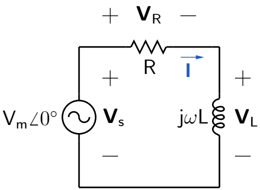

$$\mathbf{I} = \frac{V_m \angle 0}{R + j\omega L} \equiv I_m \angle(-\theta),$$

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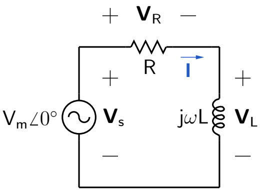
$$\mathbf{V_R} = \mathbf{I} \times R = R\, I_m \angle(-\theta)\,,$$

$$\mathbf{V_L} = \mathbf{I} \times j\omega L = \omega I_m L \angle(-\theta + \pi/2)\,,$$

RL circuit



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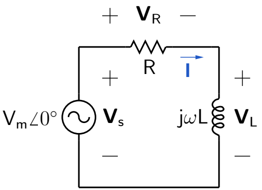
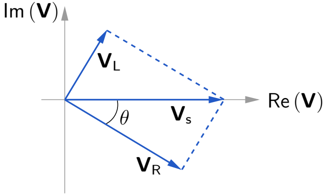
The KVL equation, $\mathbf{V_s} = \mathbf{V_R} + \mathbf{V_L}$, can be represented in the complex plane by a "phasor diagram."





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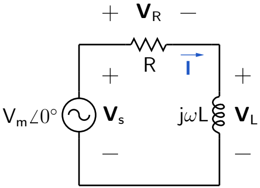

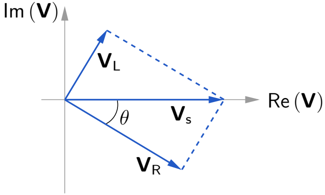

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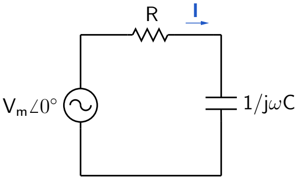
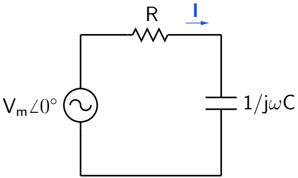
If $R \ll |j\omega L|$, $\theta \to \pi/2$, $|\mathbf{V_L}| \simeq |\mathbf{V_s}| = V_m$.

RC circuit

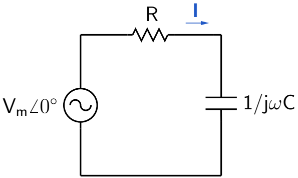


RC circuit



$$\mathbf{I} = \frac{V_m \angle 0}{R + 1/j\omega C} \equiv I_m \angle \theta,$$

where $I_m = \dfrac{\omega C V_m}{\sqrt{1+(\omega RC)^2}}$, and $\theta = \pi/2 - \tan^{-1}(\omega RC)$.



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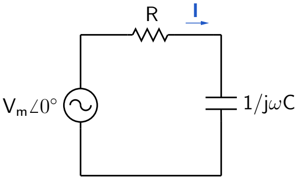
RC circuit



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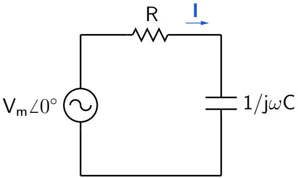
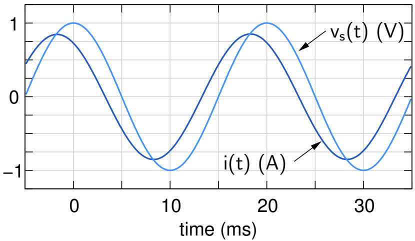
For $R = 1\,\Omega$, $C = 5.3\,\text{mF}$, $f = 50\,\text{Hz}$, $\theta = 31^\circ$, $t_{\text{lead}} = 1.72$ ms.

(SEQUEL file: `ee101_rc_ac_1.sqproj`)

RC circuit





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C = 5.3 mF

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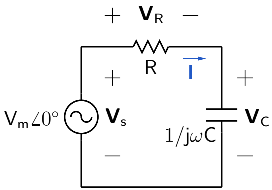

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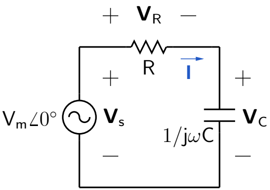

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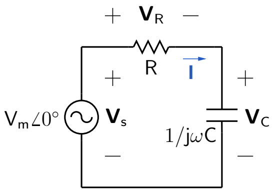
RC circuit



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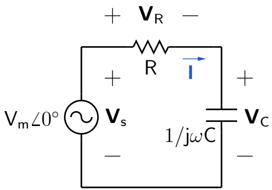

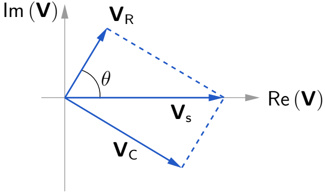

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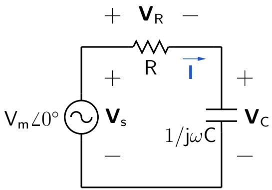

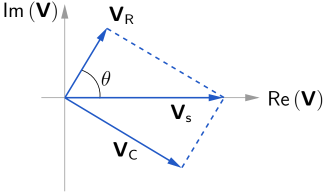

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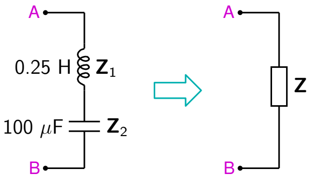
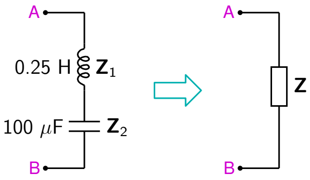
Series/parallel connections



$(\omega = 100 \text{ rad/s})$

Series/parallel connections



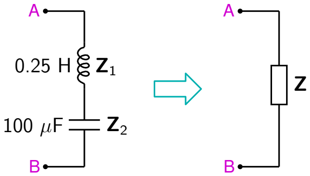
$(\omega = 100$ rad/s$)$

$$\mathbf{Z}_1 = \mathrm{j} \times 100 \times 0.25 = \mathrm{j}\,25\,\Omega$$

$$\mathbf{Z}_2 = -\mathrm{j}/(100 \times 100 \times 10^{-6}) = -\mathrm{j}\,100\,\Omega$$

$$\mathbf{Z} = \mathbf{Z}_1 + \mathbf{Z}_2 = -\mathrm{j}\,75\,\Omega$$

Series/parallel connections



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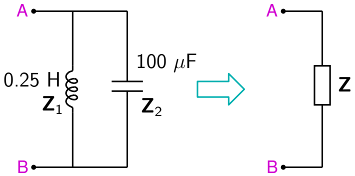

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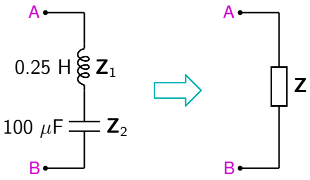

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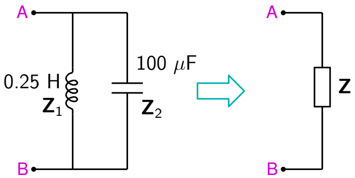


$(\omega = 100 \text{ rad/s})$

$$\mathbf{Z} = \frac{\mathbf{Z}_1 \mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

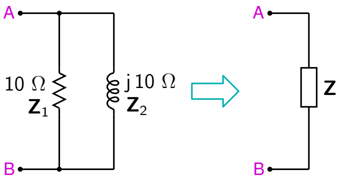
$$= \frac{(\text{j}\,25) \times (-\text{j}\,100)}{\text{j}\,25 - \text{j}\,100}$$

$$= \frac{25 \times 100}{-\text{j}\,75}$$

$$= \text{j}\,33.3\,\Omega$$

Impedance example

Obtain Z in polar form.



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Impedance example

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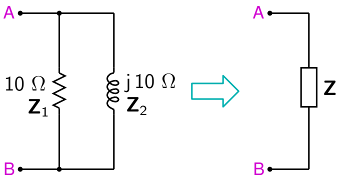


$(\omega = 100$ rad/s$)$

Method 1:

$$\mathbf{Z} = \frac{10 \times \mathrm{j}10}{10 + \mathrm{j}10} = \frac{\mathrm{j}10}{1 + \mathrm{j}}$$

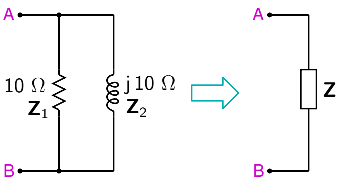
$$= \frac{\mathrm{j}10}{1 + \mathrm{j}} \times \frac{1 - \mathrm{j}}{1 - \mathrm{j}}$$

$$= \frac{10 + \mathrm{j}10}{2} = 5 + \mathrm{j}5\,\Omega$$

Convert to polar form $\rightarrow$ $\mathbf{Z} = 7.07\,\angle\,45^\circ\,\Omega$

Impedance example

Obtain Z in polar form.



$(\omega = 100 \text{ rad/s})$

Method 1:

$$\mathbf{Z} = \frac{10 \times j10}{10 + j10} = \frac{j10}{1 + j}$$

$$= \frac{j10}{1 + j} \times \frac{1 - j}{1 - j}$$

$$= \frac{10 + j10}{2} = 5 + j5\,\Omega$$

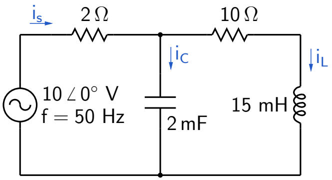
Convert to polar form $\rightarrow$ $\mathbf{Z} = 7.07\,\angle\,45^\circ\,\Omega$

Method 2:

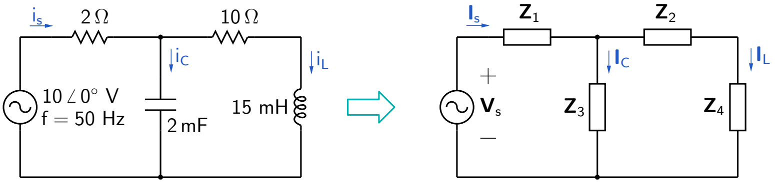
$$\mathbf{Z} = \frac{10 \times j10}{10 + j10} = \frac{100\,\angle\,\pi/2}{10\sqrt{2}\,\angle\,\pi/4}$$

$$= 5\sqrt{2}\,\angle\,(\pi/2 - \pi/4) = 7.07\,\angle\,45^\circ\,\Omega$$

Circuit example



Circuit example



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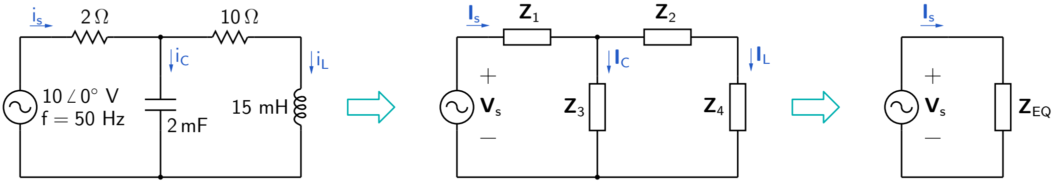
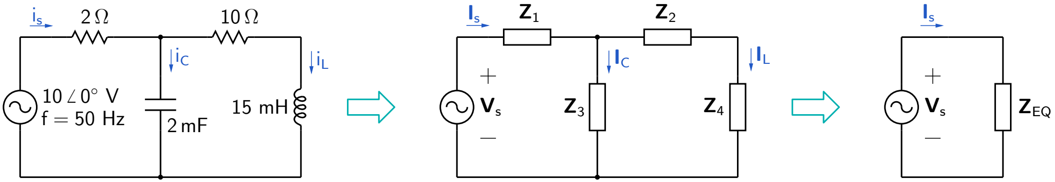
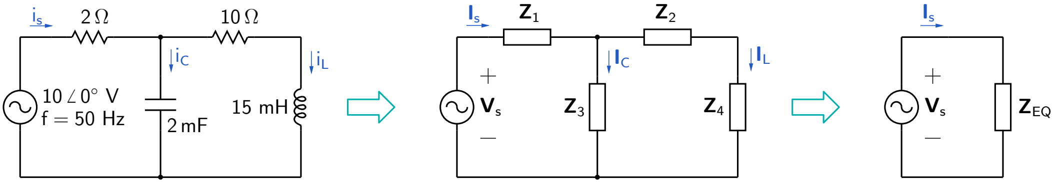


Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

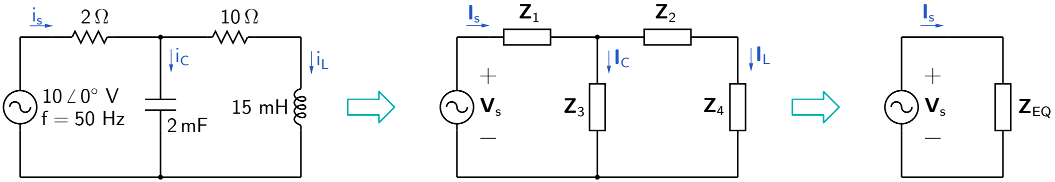
Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

Circuit example



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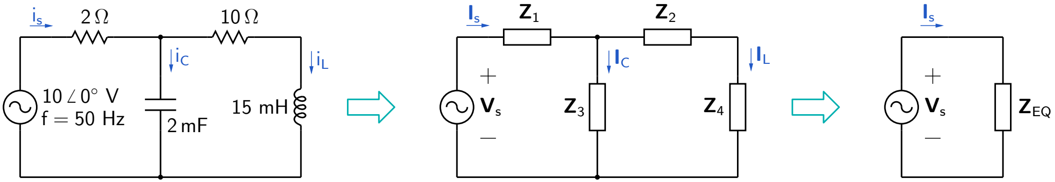
$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$



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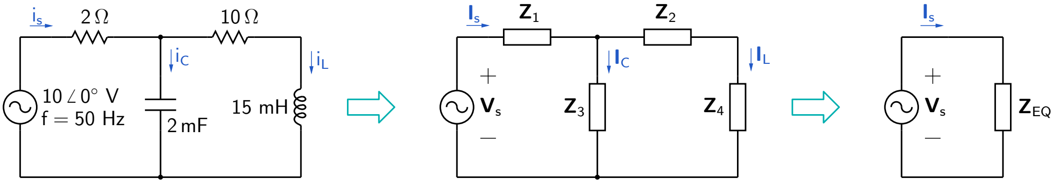
$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

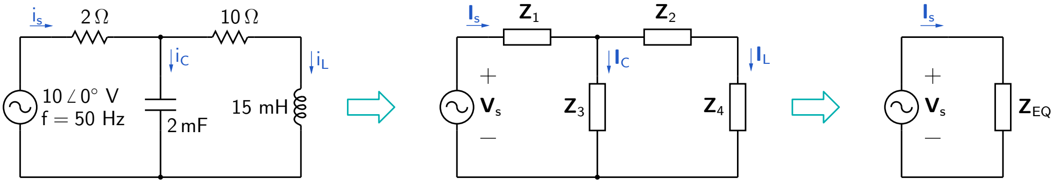
$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

$$= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)}$$

Circuit example



$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

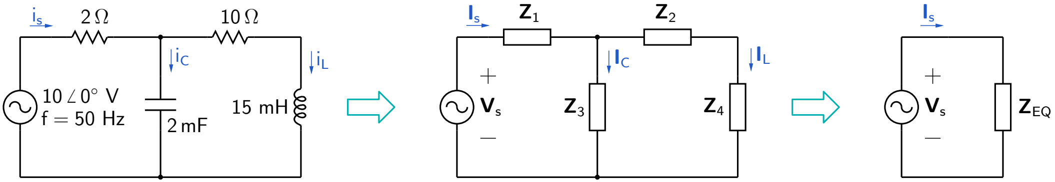
$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\mathbf{Z}_{EQ} = \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4)$$

$$= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7}$$

$$= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)}$$

$$= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67)$$

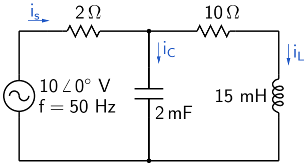


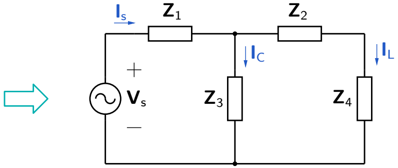
$$\mathbf{Z}_3 = \frac{1}{j \times 2\pi \times 50 \times 2 \times 10^{-3}} = -j\,1.6\ \Omega$$

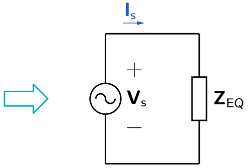
$$\mathbf{Z}_4 = j\,2\pi \times 50 \times 15 \times 10^{-3} = j\,4.7\ \Omega$$

$$\begin{aligned}
\mathbf{Z}_{EQ} &= \mathbf{Z}_1 + \mathbf{Z}_3 \parallel (\mathbf{Z}_2 + \mathbf{Z}_4) \\
&= 2 + (-j\,1.6) \parallel (10 + j\,4.7) = 2 + \frac{(-j\,1.6) \times (10 + j\,4.7)}{-j\,1.6 + 10 + j\,4.7} \\
&= 2 + \frac{1.6\angle(-90^\circ) \times 11.05\angle(25.2^\circ)}{10.47\angle(17.2^\circ)} = 2 + \frac{17.7\angle(-64.8^\circ)}{10.47\angle(17.2^\circ)} \\
&= 2 + 1.69\angle(-82^\circ) = 2 + (0.235 - j\,1.67) \\
&= 2.235 - j\,1.67 = 2.79\angle(-36.8^\circ)\ \Omega
\end{aligned}$$

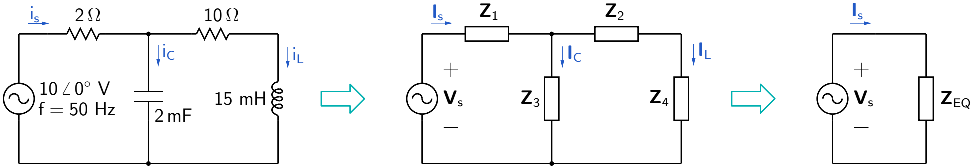
Circuit example (continued)





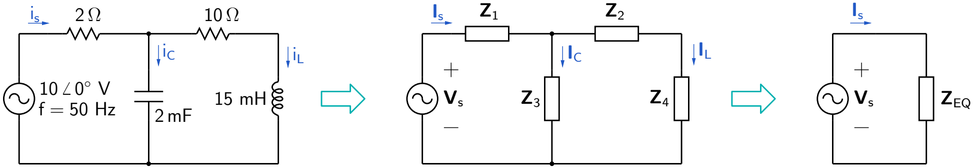


Circuit example (continued)



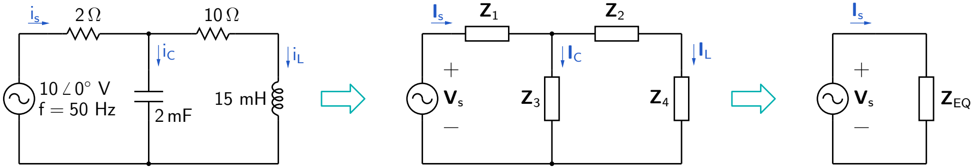
$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

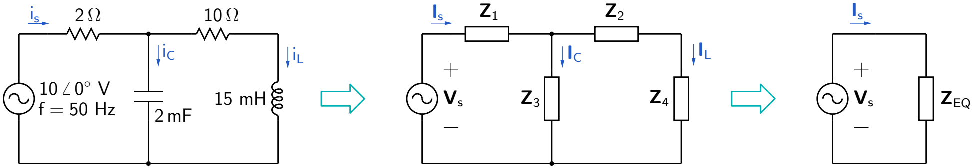


$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$

Circuit example (continued)



$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}_{EQ}} = \frac{10\angle(0^\circ)}{2.79\angle(-36.8^\circ)} = 3.58\angle(36.8^\circ)\ A$$

$$\mathbf{I}_C = \frac{(\mathbf{Z}_2 + \mathbf{Z}_4)}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 3.79\angle(44.6^\circ)\ A$$

$$\mathbf{I}_L = \frac{\mathbf{Z}_3}{\mathbf{Z}_3 + (\mathbf{Z}_2 + \mathbf{Z}_4)} \times \mathbf{I}_s = 0.546\angle(-70.6^\circ)\ A$$