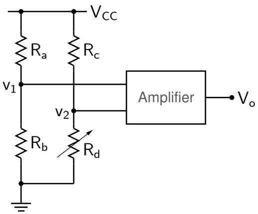
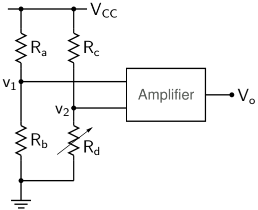
Common-mode and differential-mode voltages





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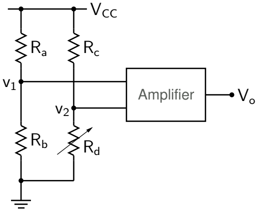
$R_d = R + \Delta R$ varies with the quantity to be measured. Typically, ΔR is a small fraction of R.



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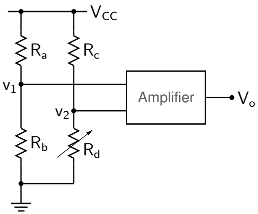

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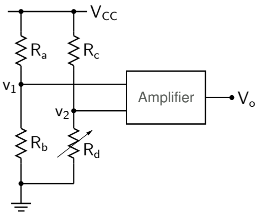

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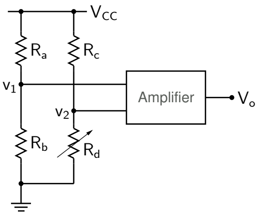

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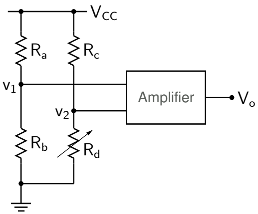
where $x = \Delta R/R$.



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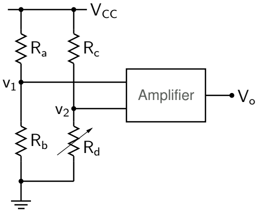
For example, with $V_{CC} = 15\ V$, $R = 1\,\text{k}$, $\Delta R = 0.01\,\text{k}$,

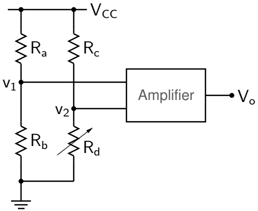
$v_1 = 7.5\ V$,

$v_2 = 7.5 + 0.0375\ V$.

Common-mode and differential-mode voltages

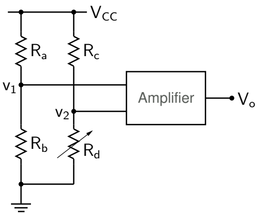


$v_1 = 7.5\ V\ ,\ \ v_2 = 7.5 + 0.0375\ V\ .$



$v_1 = 7.5\ V$, $v_2 = 7.5 + 0.0375\ V$.

The amplifier should only amplify $v_2 - v_1 = 0.0375\ V$ (since that is the signal arising from ΔR).



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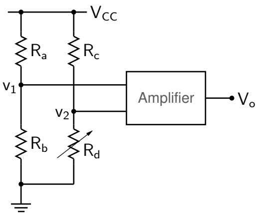

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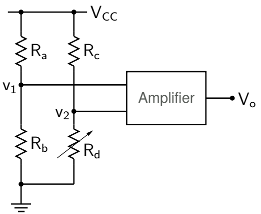

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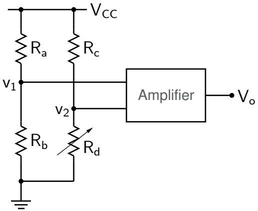
In the above example, $v_c \approx 7.5\ V$, $v_d = 37.5\,\text{m}V$.



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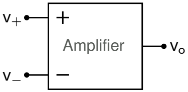
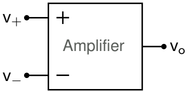
Note that the common-mode voltage is quite large compared to the differential-mode voltage.

This is a common situation in transducer circuits.



$$v_+ = v_c + v_d/2$$
$$v_- = v_c - v_d/2$$

An ideal amplifier would only amplify the difference $(v_+ - v_-)$, giving
$v_o = A_d\,(v_+ - v_-) = A_d\,v_d$,
where A_d is called the "differential gain" or simply the gain (A_V).



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In practice, the output can also have a common-mode component:

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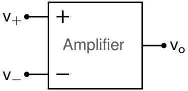

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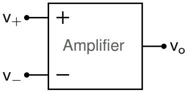
$$\boxed{\mathrm{CMRR} = \frac{A_d}{A_c}}$$



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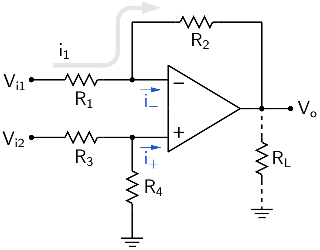
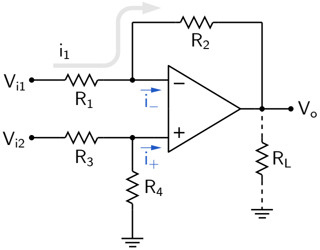
For the 741 op-amp, the CMRR is 90 dB ($\simeq$ 30,000), which may be considered to be infinite in many applications. In such cases, mismatch between circuit components will determine the overall common-mode rejection performance of the circuit.



i_1
R_2
V_{i1}
R_1
i_-
V_o
V_{i2}
R_3
i_+
R_L
R_4



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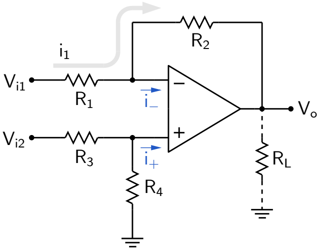
Large input resistance of op-amp $\rightarrow$ $i_+ = 0,\ \ V_+ = \dfrac{R_4}{R_3 + R_4}\, V_{i2}\,.$



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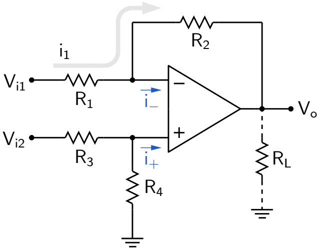

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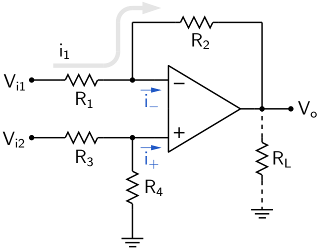
$i_- \approx 0 \rightarrow V_o = V_- - i_1\, R_2 \approx V_+ - \dfrac{R_2}{R_1}\,(V_{i1} - V_+)$.



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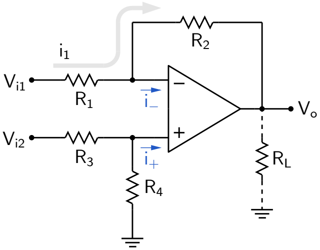
$V_o = \dfrac{R_2}{R_1}\,(V_{i2} - V_{i1})$.



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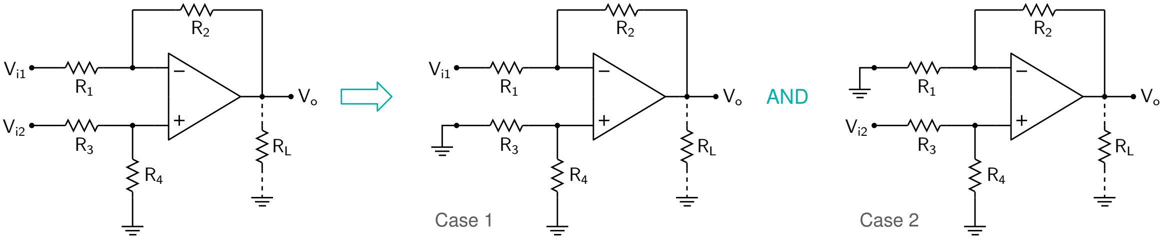
$V_o = \dfrac{R_2}{R_1}(V_{i2} - V_{i1})$.

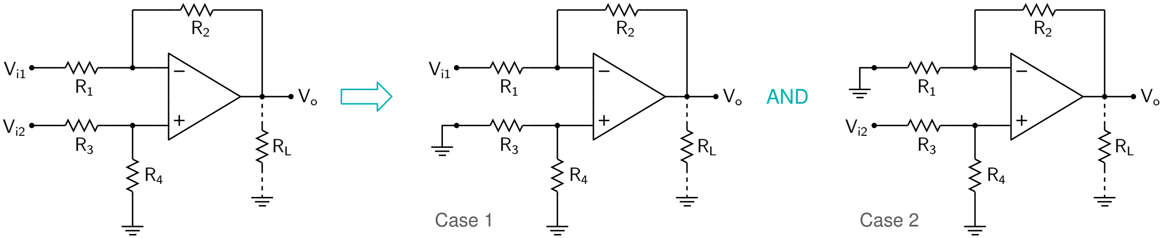
The circuit is a “difference amplifier.”

Difference amplifier



Method 2:

Since the op-amp is operating in the linear region, we can use superposition:



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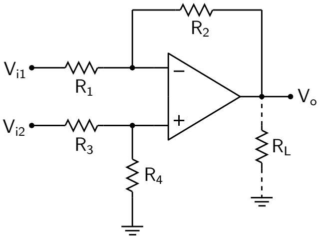

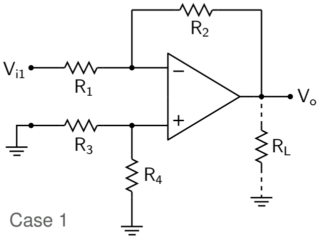

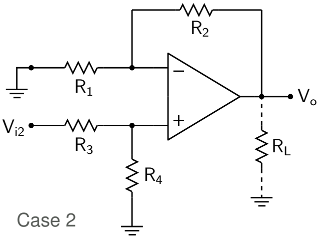

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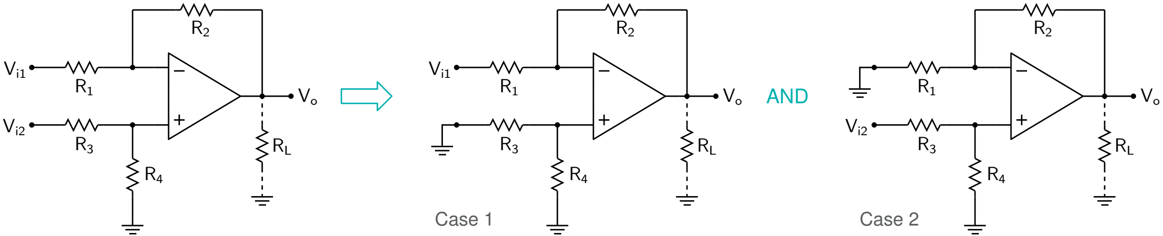
Case 2: Non-inverting amplifier, with $V_i = \dfrac{R_4}{R_3 + R_4}\, V_{i2}\,.$

$$\rightarrow V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2}\,.$$

Difference amplifier



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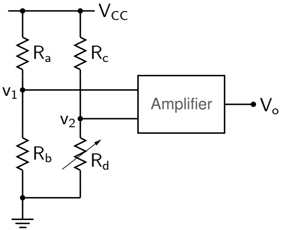
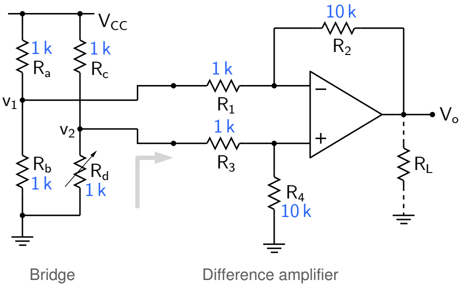
The net result is,

$$V_o = V_{o1} + V_{o2} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1} = \frac{R_2}{R_1}\,(V_{i2} - V_{i1})\,,\ \text{if}\ \frac{R_4}{R_3} = \frac{R_2}{R_1}.$$

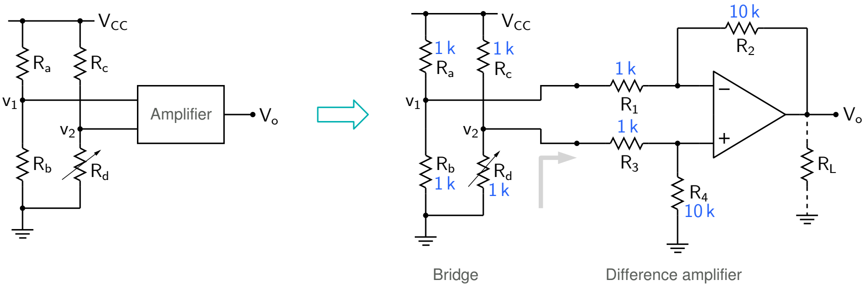


Vcc
Ra
Rc
v1
v2
Rb
Rd
Amplifier
Vo



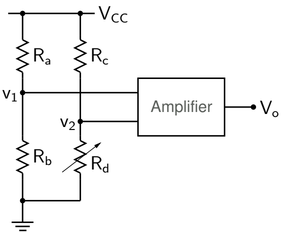
Vcc
1 k
Ra
1 k
Rc
v1
v2
Rb
1 k
Rd
1 k
10 k
R2
1 k
R1
1 k
R3
R4
10 k
Vo
RL
Bridge
Difference amplifier

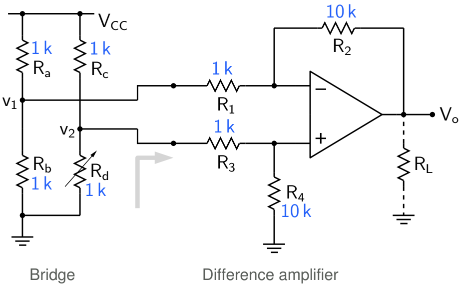
Difference amplifier



The resistance seen from v_2 is $(R_3 + R_4)$ which is small enough to cause v_2 to change.

This is not desirable.





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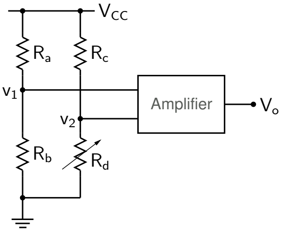
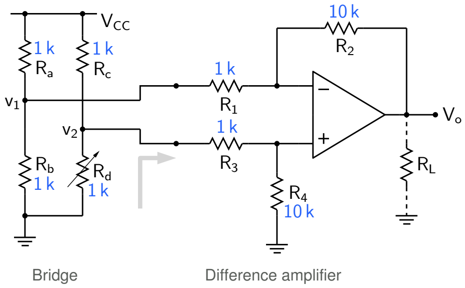
Difference amplifier





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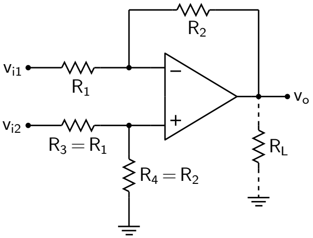
We will discuss an improved difference amplifier later. Before we do that, let us discuss another problem with the above difference amplifier which can be important for some applications (next slide).



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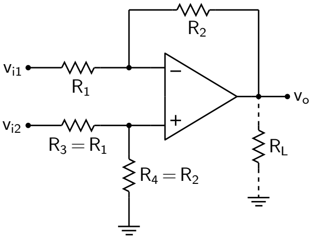
$v_{i2} = v_c + (v_d/2)$



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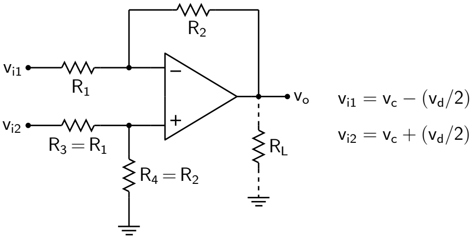

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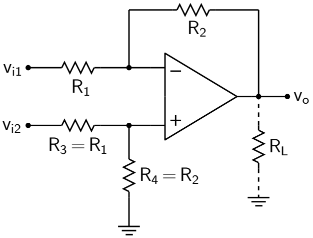

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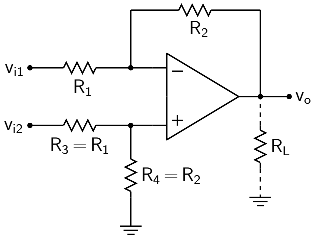
$$\begin{aligned} v_o \ &= \frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)v_{i2} - \frac{R_2}{R_1}\,v_{i1} = \frac{R_2}{R_1+\Delta R+R_2}\left(1+\frac{R_2}{R_1}\right)v_{i2} - \frac{R_2}{R_1}\,v_{i1} \\ &\simeq \frac{R_2}{R_1}(v_d - x\,v_c)\,, \text{with } x = \frac{\Delta R}{R_1+R_2} \qquad \text{(show this)} \end{aligned}$$



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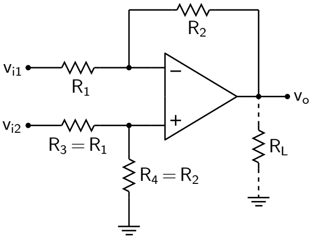

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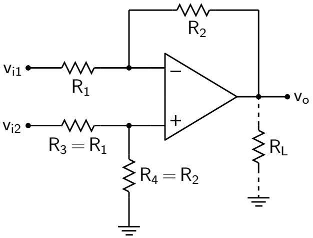
$|A_c| = \dfrac{\Delta R}{R_1+R_2}\,\dfrac{R_2}{R_1} \ll |A_d| = \dfrac{R_2}{R_1}$. However, since v_c can be large compared to v_d, the effect of A_c cannot be ignored.



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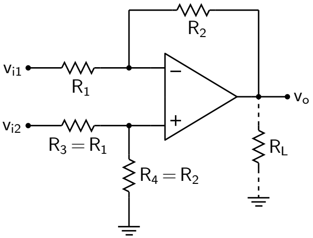

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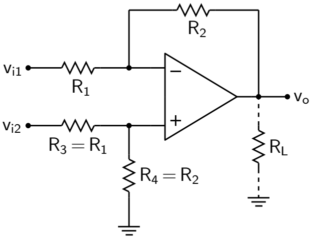

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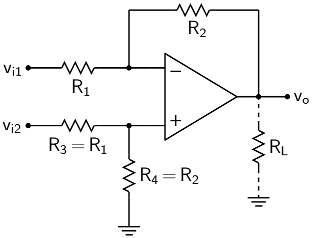
$|v_o^c| = |A_c\,v_c| = 0.0091 \times 7.5 = 0.068\,V$.



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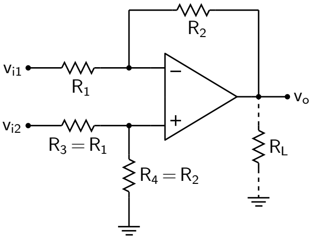

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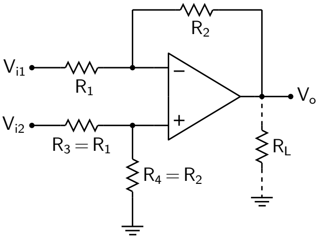
The (spurious) common-mode contribution is substantial.

If we measure v_o, we will conclude that $v_d = \dfrac{v_o}{A_d}$, but in reality, it would be different.

$\rightarrow$ need a circuit which will drastically reduce the common-mode component at the output.

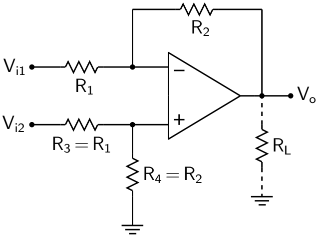
Difference amplifier: resistance mismatch



$$A_d = \frac{R_2}{R_1}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{i2} - \frac{R_2}{R_1}\, V_{i1}$$

Difference amplifier: resistance mismatch



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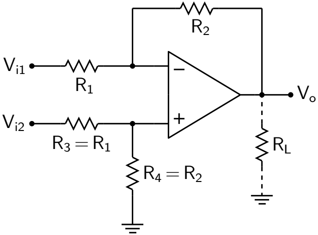

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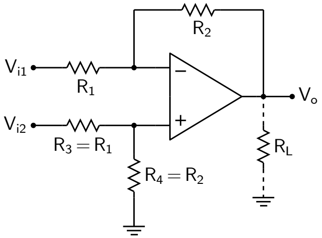
Difference amplifier: resistance mismatch



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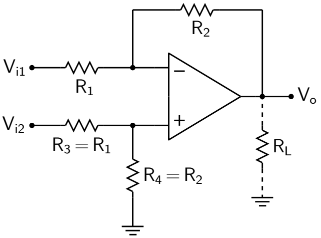

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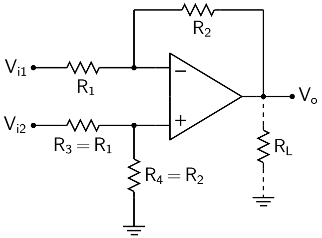

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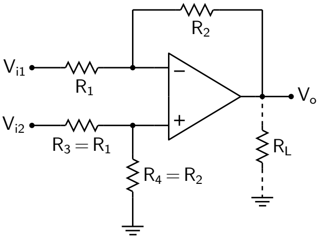
Difference amplifier: resistance mismatch



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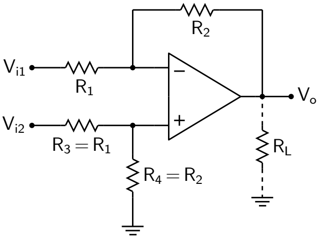

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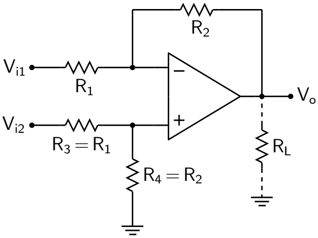
Difference amplifier: resistance mismatch



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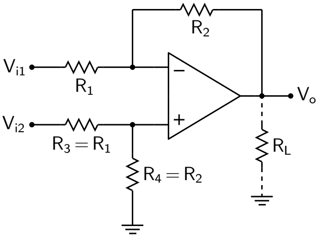
Difference amplifier: resistance mismatch



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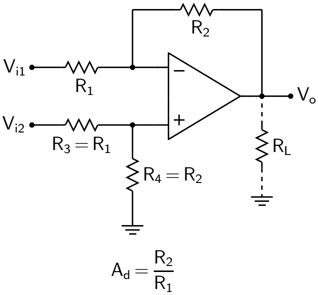

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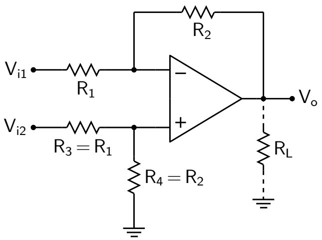

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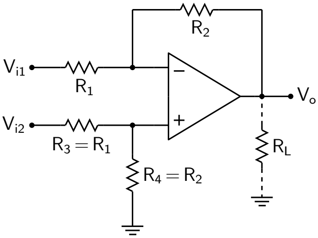
$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$



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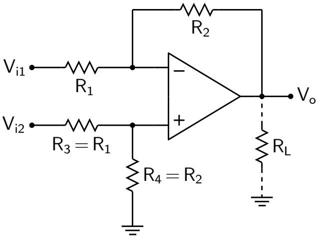
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

$$= \frac{1}{2}\,4\,x = 2\,x \text{ (worst case)}$$

Difference amplifier: resistance mismatch



$$A_d = \frac{R_2}{R_1}$$

$$A_c = \frac{R_4}{R_3 + R_4}\,(x_1 - x_2 - x_3 + x_4).$$

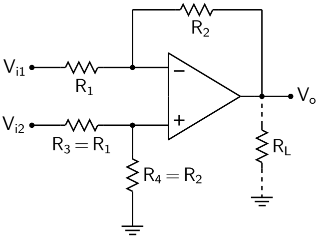
$$\frac{R_4}{R_3 + R_4} \approx \frac{R_4^0}{R_3^0 + R_4^0}.$$

(1) $R_1^0 = R_2^0$ (i.e., $R_3^0 = R_4^0$)

$$A_c = \frac{1}{2}\,(x_1 - x_2 - x_3 + x_4)$$

$$= \frac{1}{2}\,4\,x = 2\,x \text{ (worst case)}$$

(2) $R_1^0 \ll R_2^0$ (i.e., $R_3^0 \ll R_4^0$)



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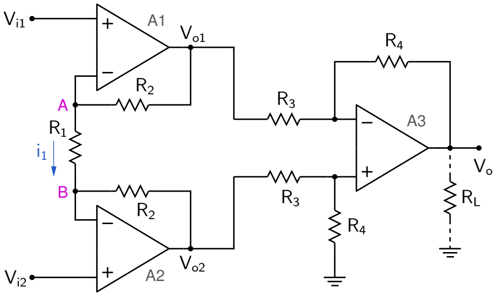
Improved difference amplifier



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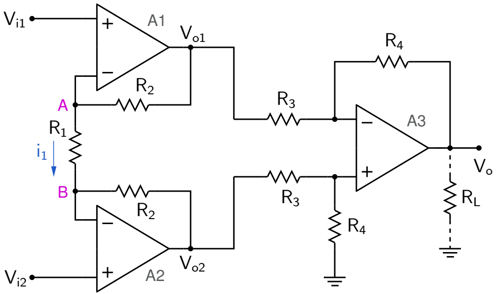

$$V_+ \approx V_- \rightarrow V_A = V_{i1}\,,\;\; V_B = V_{i2}\,, \rightarrow i_1 = \frac{1}{R_1}\,(V_{i1} - V_{i2})\,.$$

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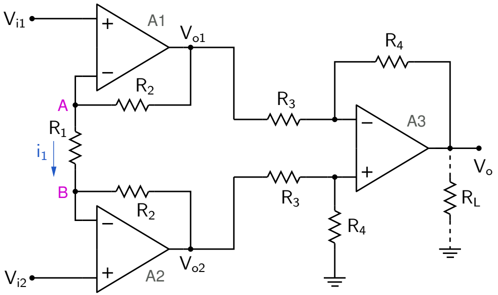

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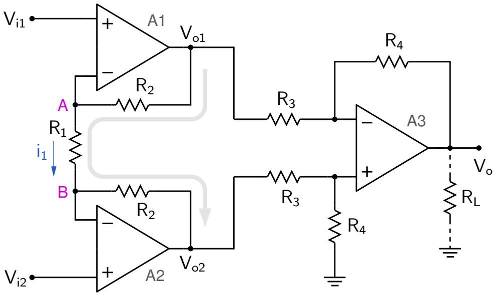

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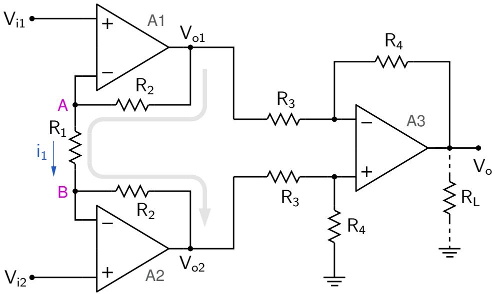

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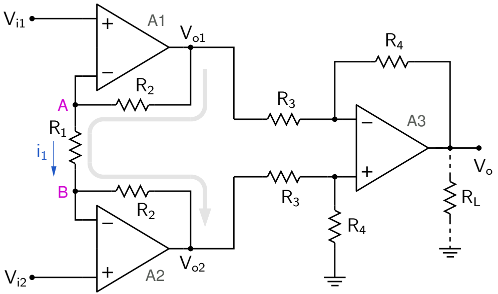

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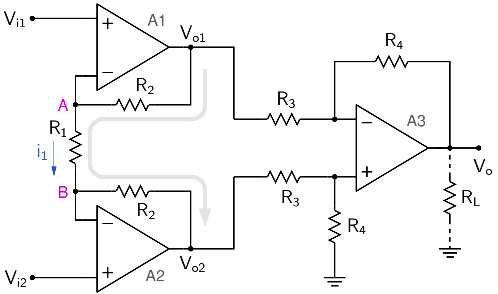

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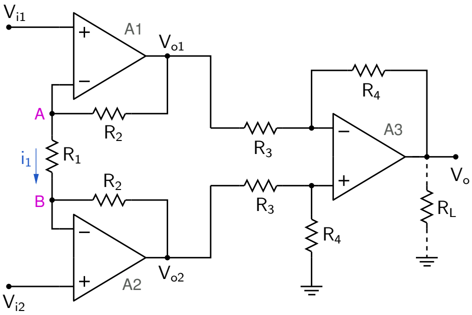
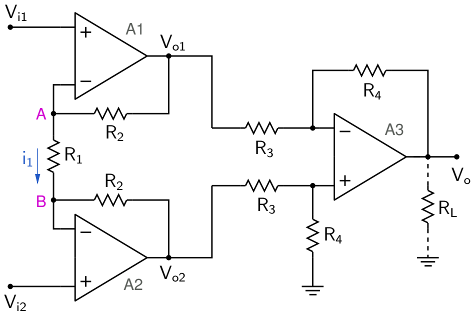
This circuit is known as the “instrumentation amplifier.”

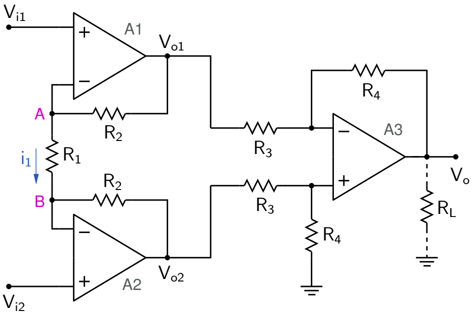
Instrumentation amplifier



Instrumentation amplifier

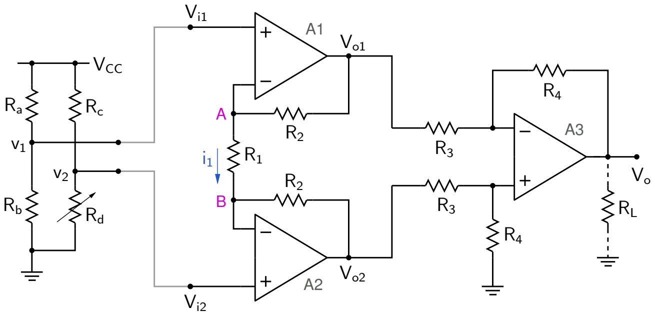


The input resistance seen from V_{i1} or V_{i2} is large (since an op-amp has a large input resistance).



The input resistance seen from V_{i1} or V_{i2} is large (since an op-amp has a large input resistance).
$\rightarrow$ the amplifier will not "load" the preceding stage, a desirable feature.

Instrumentation amplifier



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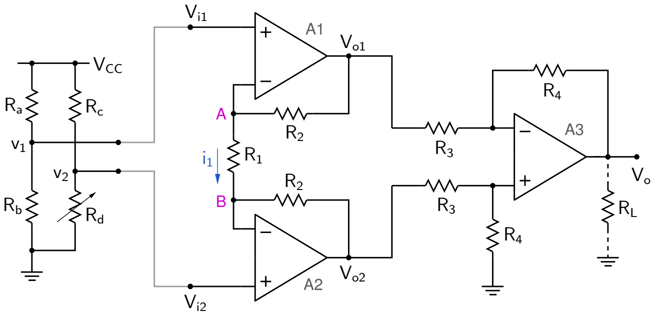
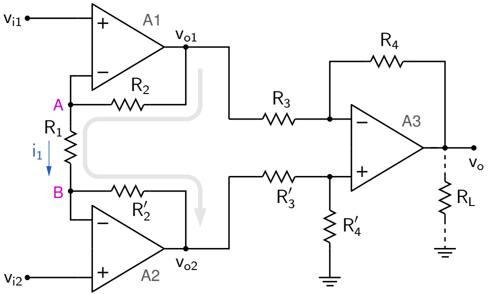


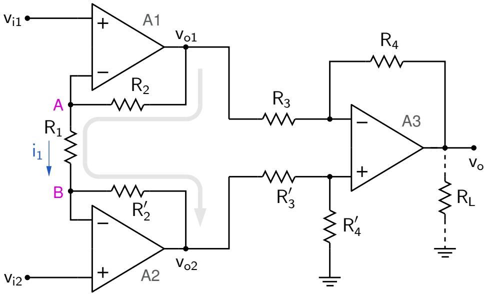
The input resistance seen from V_{i1} or V_{i2} is large (since an op-amp has a large input resistance).
$\rightarrow$ the amplifier will not "load" the preceding stage, a desirable feature.
As a result, the voltages v_1 and v_2 in the bridge circuit will remain essentially the same when the bridge circuit is connected to the instrumentation amplifier.

Instrumentation amplifier: common-mode rejection



$$v_{i1} = v_c - (v_d/2)$$

$$v_{i2} = v_c + (v_d/2)$$



$v_{i1} = v_c - (v_d/2)$

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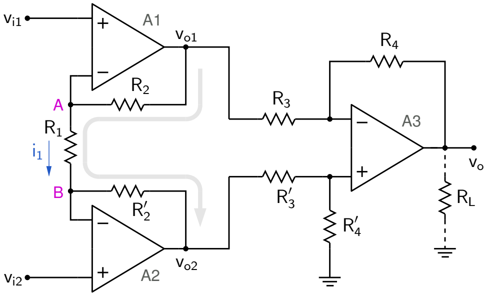
Note that v_{o1} serves as v_{i1} for the difference amplifier, and v_{o2} as v_{i2}. Let us find the differential-mode and common-mode components associated with v_{o1} and v_{o2}.

$$v'_{id} = v_{o2} - v_{o1}, \quad v'_{ic} = \frac{1}{2}\,(v_{o1} + v_{o2})$$



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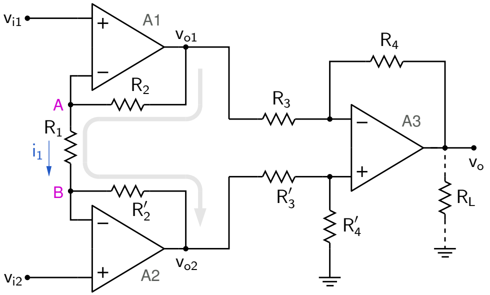
$$v'_{id} = (R_2 + R'_2 + R_1)\,\frac{1}{R_1}\left[\left(v_c + \frac{v_d}{2}\right) - \left(v_c - \frac{v_d}{2}\right)\right] = \left(1 + \frac{R_2 + R'_2}{R_1}\right)\,v_d\,.$$



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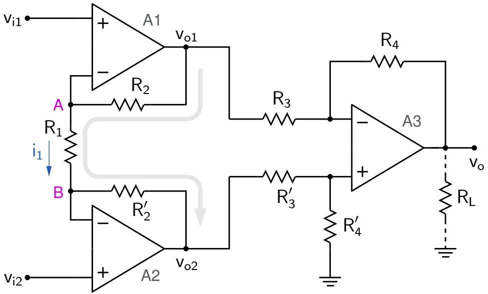
$$v_{id}' = (R_2 + R_2' + R_1)\,\frac{1}{R_1}\left[\left(v_c + \frac{v_d}{2}\right) - \left(v_c - \frac{v_d}{2}\right)\right] = \left(1 + \frac{R_2 + R_2'}{R_1}\right)\,v_d\,.$$

$$v_{ic}' = \frac{1}{2}\left[\left(v_c - \frac{v_d}{2}\right) + i_1 R_2 + \left(v_c + \frac{v_d}{2}\right) - i_1 R_2'\right] \approx v_c\,.$$

Instrumentation amplifier: common-mode rejection



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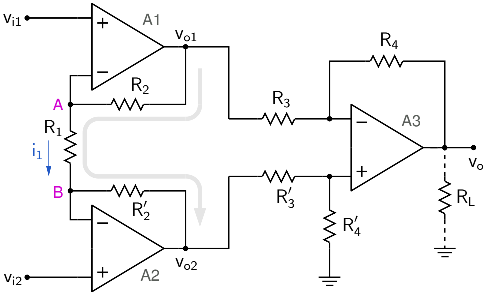
$\rightarrow$ v_d has got amplified but not v_c $\rightarrow$ overall improvement in CMRR.



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(Note that resistor mismatch in the second stage needs to be considered, but it will have a limited effect.)

Current-to-voltage conversion

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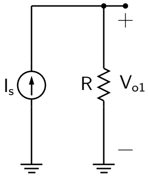

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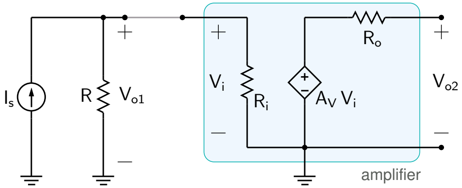
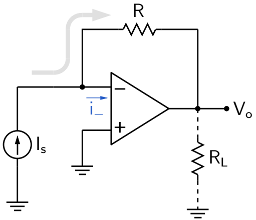
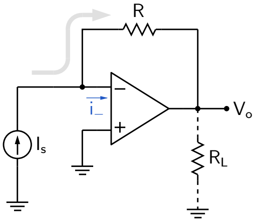


However, this simple approach will not work if the next stage in the circuit (such as an amplifier) has a finite R_i, since it will modify V_{o1} to $V_{o1} = I_s\,(R_i \parallel R)$, which is not desirable.

Current-to-voltage conversion

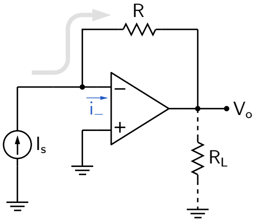


Current-to-voltage conversion



$V_- \approx V_+$, and $i_- \approx 0 \Rightarrow V_o = V_- - I_s R = -I_s R$.

Current-to-voltage conversion



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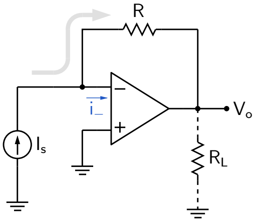
The output voltage is proportional to the source current, *irrespective* of the value of R_L, i.e., irrespective of the next stage.



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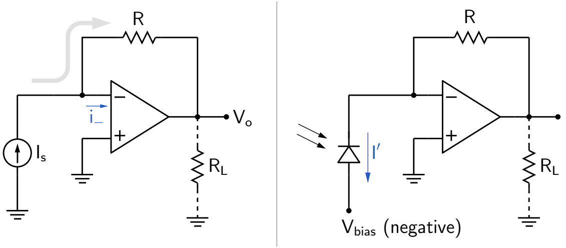
Example: a photocurrent detector.



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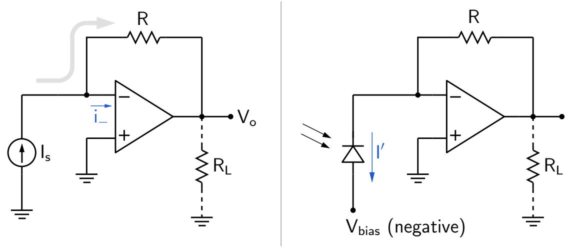

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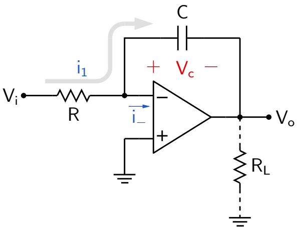
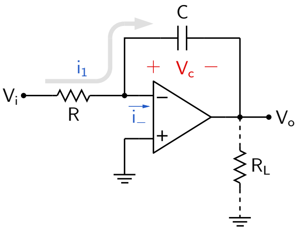
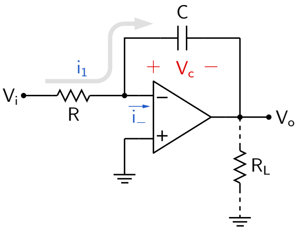
$V_o = I'\, R$. (Note: The diode is under a reverse bias, with $V_n = 0\ V$ and $V_p = V_{\text{bias}}$.)


C
i_1
+ V_c −
V_i
R
i_-
V_o
R_L



$V_- \approx V_+ = 0\,V \rightarrow i_1 = V_i/R\,.$



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Since $i_- \approx 0$, the current through the capacitor is i_1.

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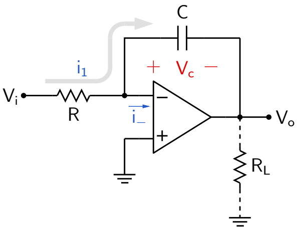

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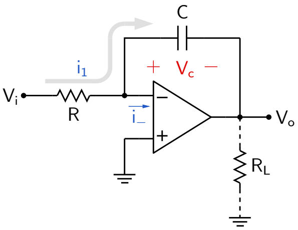
Op-amp circuits (linear region)



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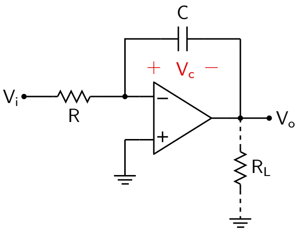
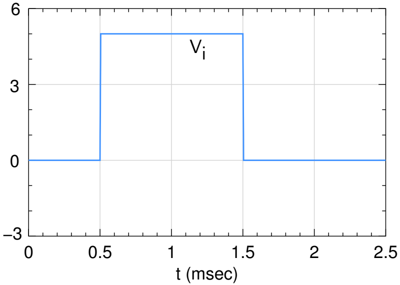
$$V_c = V_- - V_o = 0 - V_o = -V_o \rightarrow C\left(-\frac{dV_o}{dt}\right) = \frac{V_i}{R}$$

$$\boxed{V_o = -\frac{1}{RC}\int V_i\,dt}$$

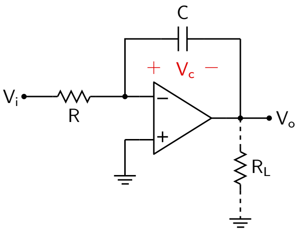
The circuit works as an integrator.

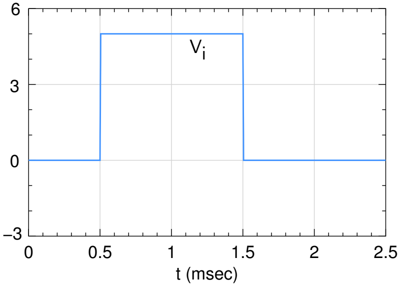




Given: $R = 10\,\mathrm{k}$, $C = 0.2\,\mu\mathrm{F}$.

If $V_o = 0\,\mathrm{V}$ at $t = 0$, find $V_o(t)$ (Let $t_0 = 0.5\,\mathrm{msec}$, $t_1 = 1.5\,\mathrm{msec}$).



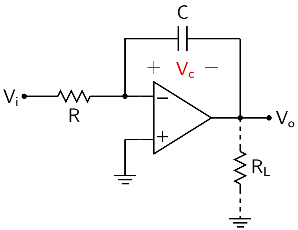


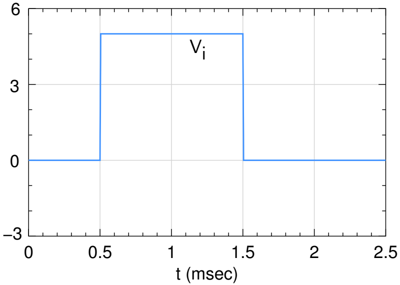
Given: $R = 10\,\mathrm{k}$, $C = 0.2\,\mu\mathrm{F}$.

If $V_o = 0\,\mathrm{V}$ at $t = 0$, find $V_o(t)$ (Let $t_0 = 0.5\,\mathrm{msec}$, $t_1 = 1.5\,\mathrm{msec}$).

$$V_o = -\frac{1}{RC}\int V_i\,dt, \quad \tau \equiv RC = 2\,\mathrm{msec}.$$

Integrator





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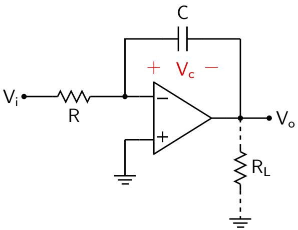
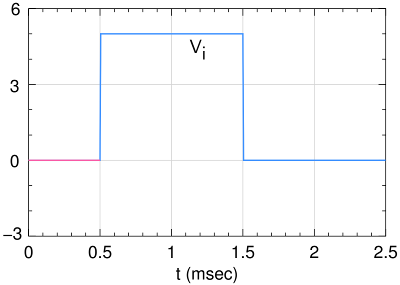
* For $t < t_0$, $V_o(t) - V_o(0) = -\dfrac{1}{\tau}\displaystyle\int_0^t 0\,dt' = 0 \rightarrow V_o(t) = V_o(0) = 0\,\text{V}$





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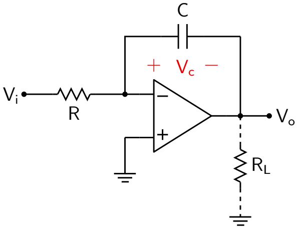
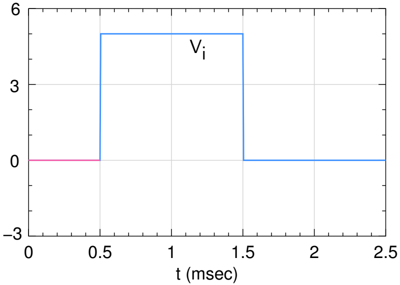
Integrator





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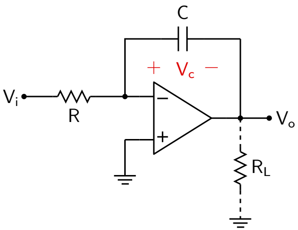
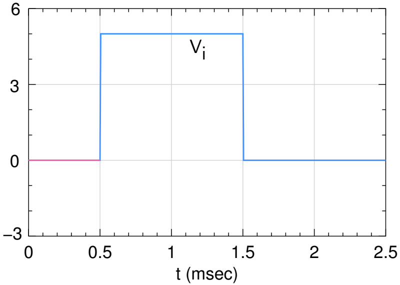
Integrator





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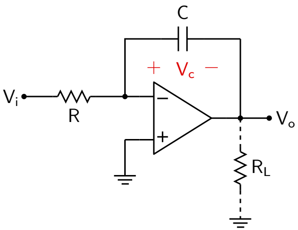
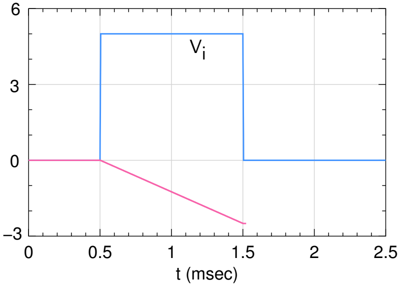
  At $t = t_1$, $V(t_1) - V(t_0) = -\dfrac{1}{2\,\text{msec}}\,5\,\text{V} \times 1\,\text{msec} = -2.5\,\text{V} \rightarrow V_o(t_1) = -2.5\,\text{V}$.

Integrator





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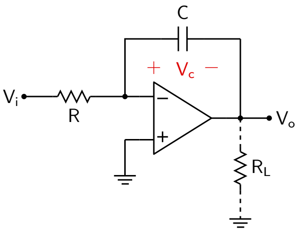
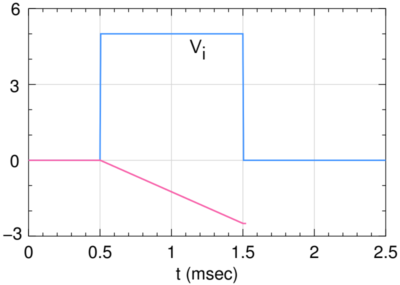
Integrator





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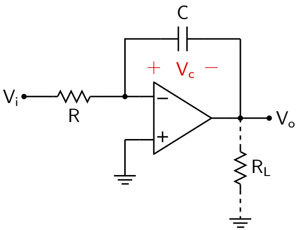

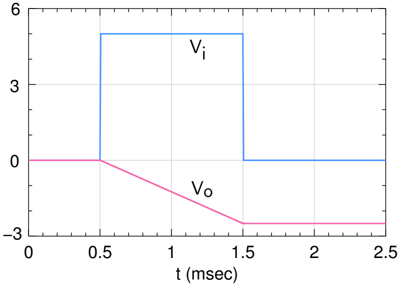

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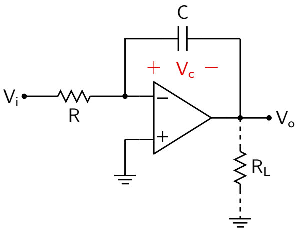

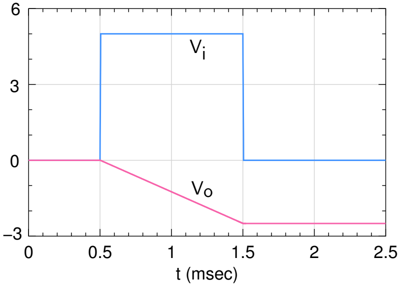

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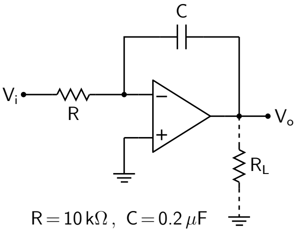
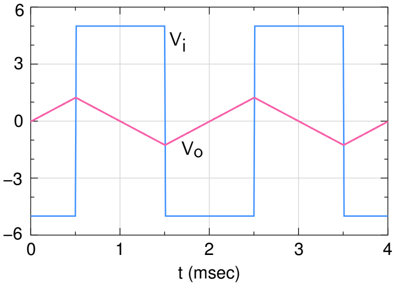
SEQUEL file: `ee101_integrator_1.sqproj`

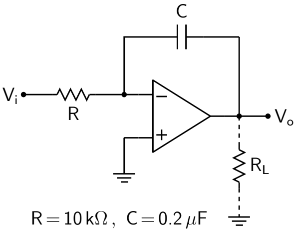


$R = 10\,k\Omega\,,\;\; C = 0.2\,\mu F$

$$V_o = -\frac{1}{RC}\int V_i\,dt$$



* An integrator can be used to convert a square wave to a triangle wave.



$R = 10\,\mathrm{k}\Omega\,,\ \ C = 0.2\,\mu\mathrm{F}$

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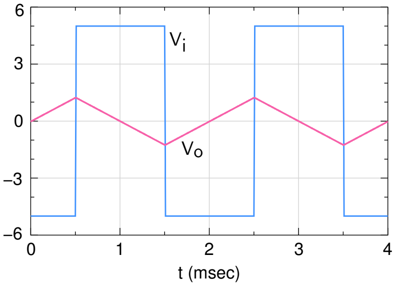
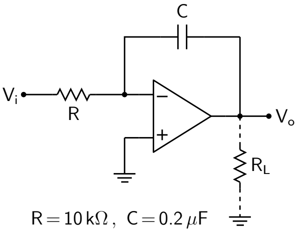


* An integrator can be used to convert a square wave to a triangle wave.
* In practice, the circuit needs a small modification, as discussed in the following.



$$R = 10\,\mathrm{k}\Omega\,,\;\; C = 0.2\,\mu\mathrm{F}$$

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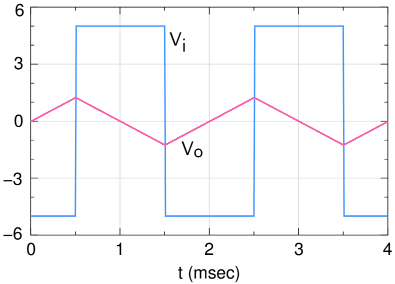
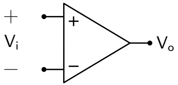
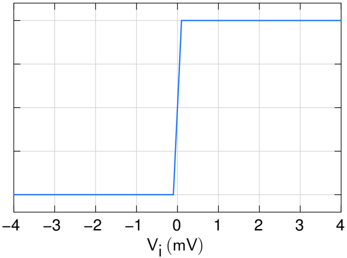


* An integrator can be used to convert a square wave to a triangle wave.
* In practice, the circuit needs a small modification, as discussed in the following.

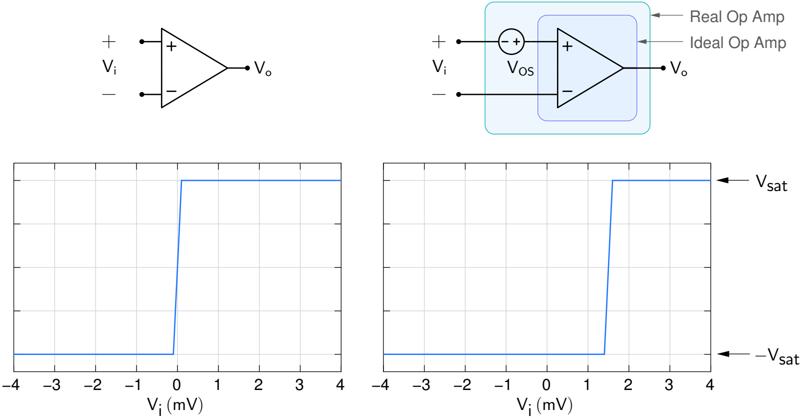
SEQUEL file: `ee101_integrator_2.sqproj`

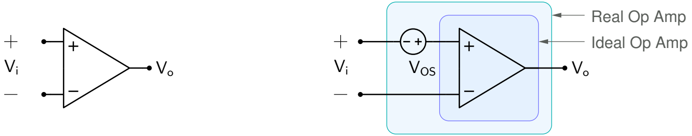
Practical op-amps: Offset voltage

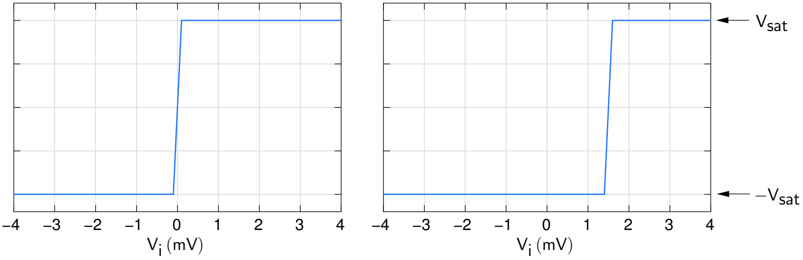




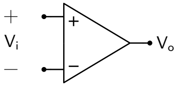
Practical op-amps: Offset voltage

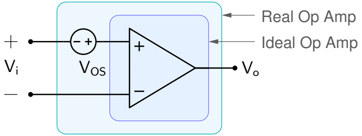


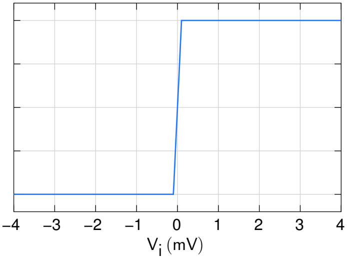


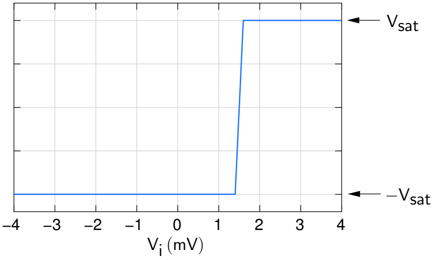


For the real op-amp, $V_o = A_V((V_+ + V_{OS}) - V_-)$.









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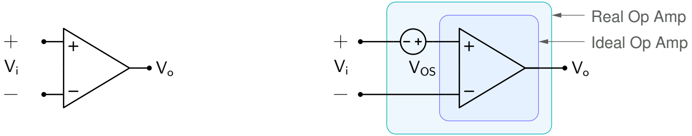
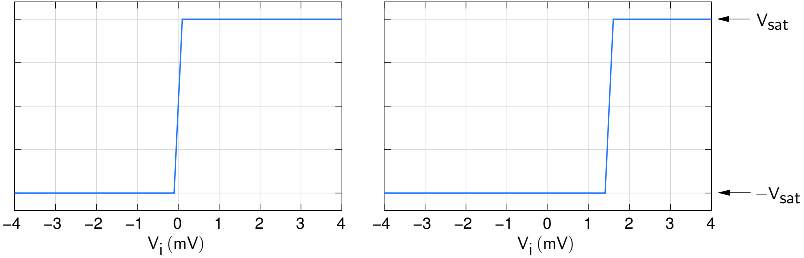
For $V_o = 0\ V$, $V_+ + V_{OS} - V_- = 0 \rightarrow V_i = V_+ - V_- = -V_{OS}$.

Practical op-amps: Offset voltage





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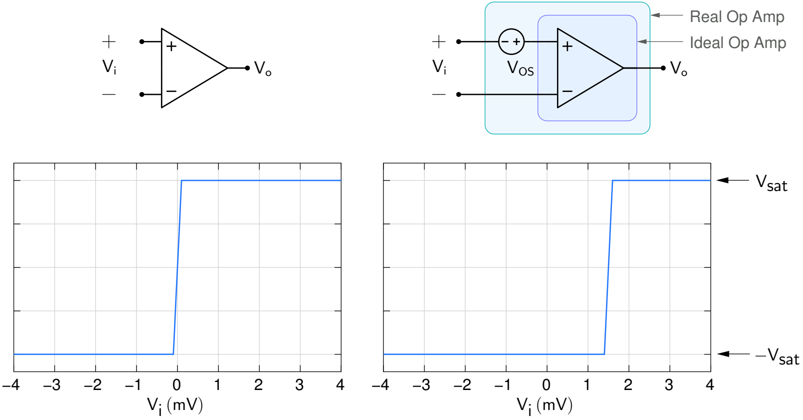
V_o versus V_i curve gets shifted (Note: V_{OS} is negative in the above example).



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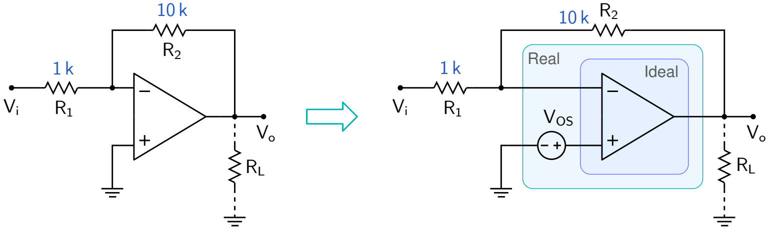
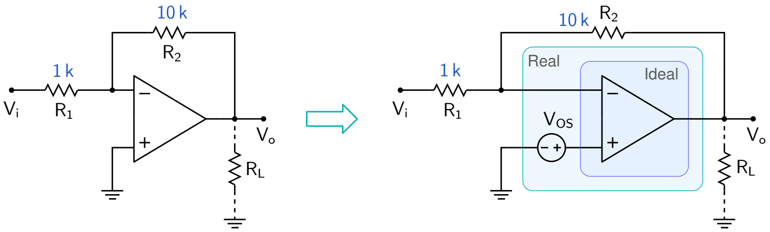
V_o versus V_i curve gets shifted (Note: V_{OS} is negative in the above example).

741: $-6\,\text{m}V < V_{OS} < 6\,\text{m}V$, OP-77: $-50\,\mu V < V_{OS} < 50\,\mu V$.

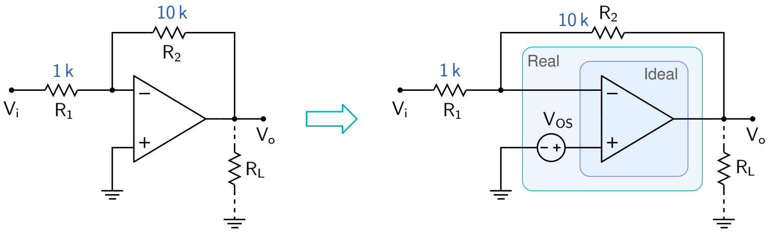
Effect of V_{OS}: inverting amplifier





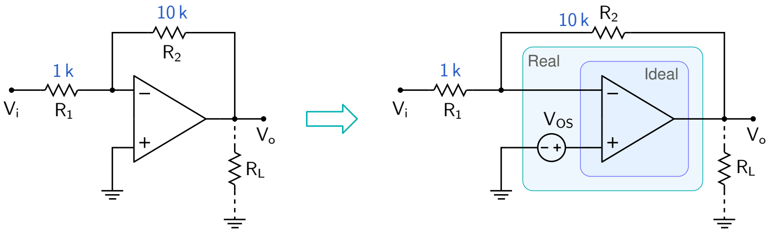
By superposition, $V_o = -\dfrac{R_2}{R_1}\,V_i + V_{OS}\left(1 + \dfrac{R_2}{R_1}\right)$.

Effect of V_{OS}: inverting amplifier



By superposition, $V_o = -\dfrac{R_2}{R_1}\,V_i + V_{OS}\left(1+\dfrac{R_2}{R_1}\right).$

For $V_{OS} = 2\,\mathrm{m}V$, the contribution from V_{OS} to V_o is $22\,\mathrm{m}V$,



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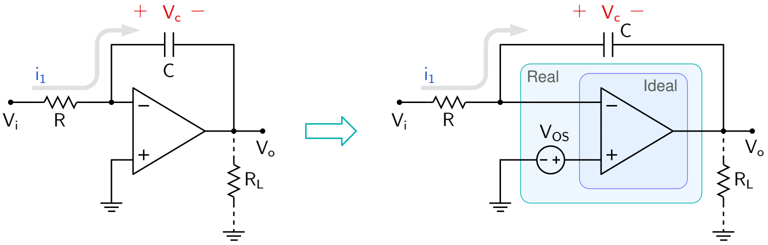
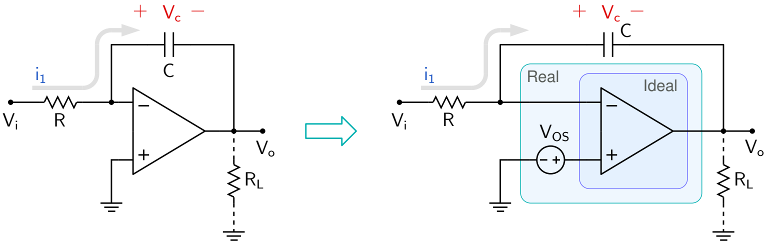
For $V_{OS} = 2\,\mathrm{m}V$, the contribution from V_{OS} to V_o is $22\,\mathrm{m}V$,

i.e., a DC shift of $22\,\mathrm{m}V$.

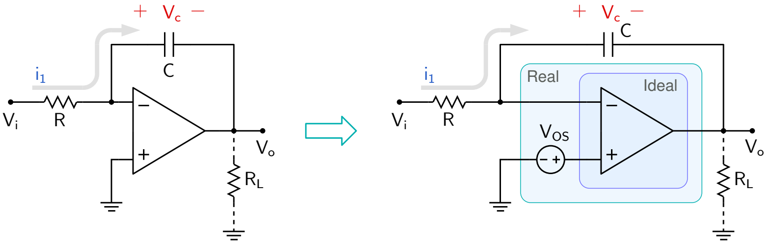
Effect of V_{OS}: integrator





$$V_- \approx V_+ = V_{OS} \rightarrow i_1 = \frac{1}{R}(V_i - V_{OS}) = C\,\frac{dV_c}{dt}\,.$$

Effect of V_{OS}: integrator



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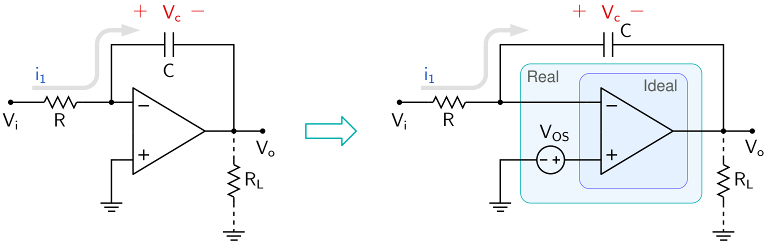

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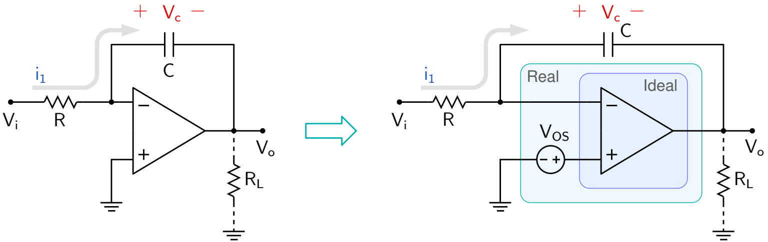
Even with $V_i = 0\,V$, V_c will keep rising or falling (depending on the sign of V_{OS}).

Eventually, the Op Amp will be driven into saturation.



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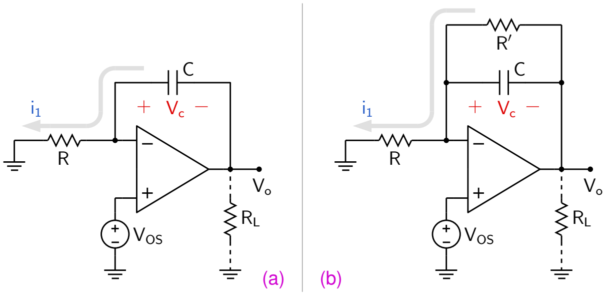
Eventually, the Op Amp will be driven into saturation.

$\rightarrow$ need to address this issue!

Effect of V_{OS}: integrator with $V_i = 0$



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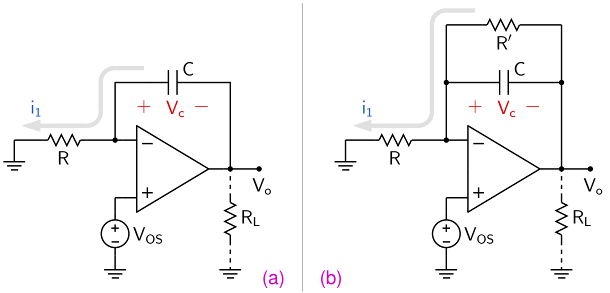
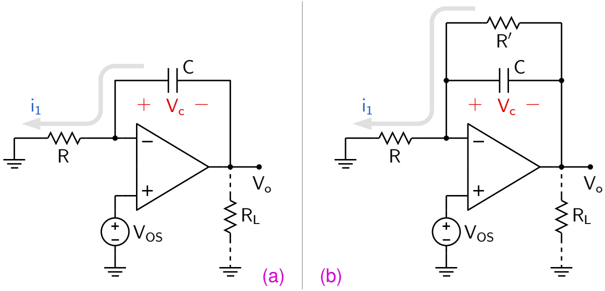


(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

$$V_c = -\frac{1}{RC}\int V_{OS}\,dt \rightarrow \text{op-amp saturates.}$$



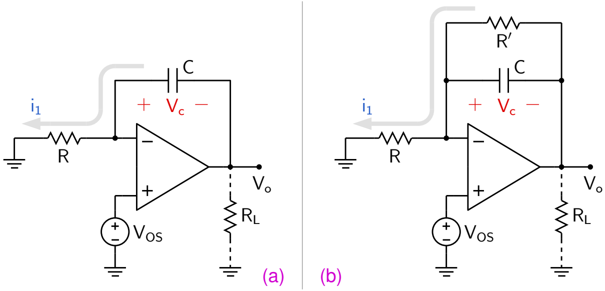
(a) $i_1 = \dfrac{V_{OS}}{R} = -C\,\dfrac{dV_c}{dt}$

$V_c = -\dfrac{1}{RC}\displaystyle\int V_{OS}\,dt \rightarrow$ op-amp saturates.

(b) There is a DC path for the current.

$\rightarrow V_o = \left(1 + \dfrac{R'}{R}\right) V_{OS}.$

Effect of V_{OS}: integrator with $V_i = 0$



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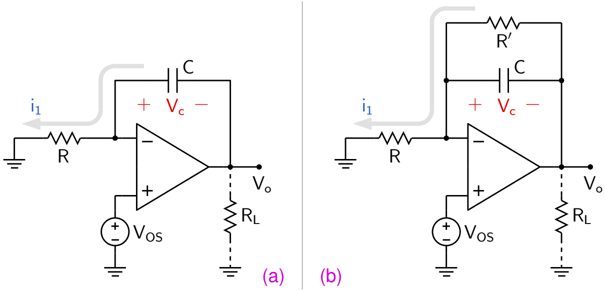

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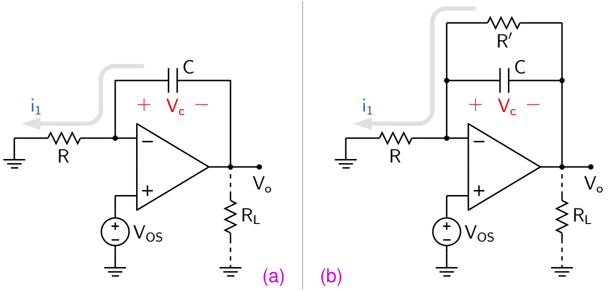

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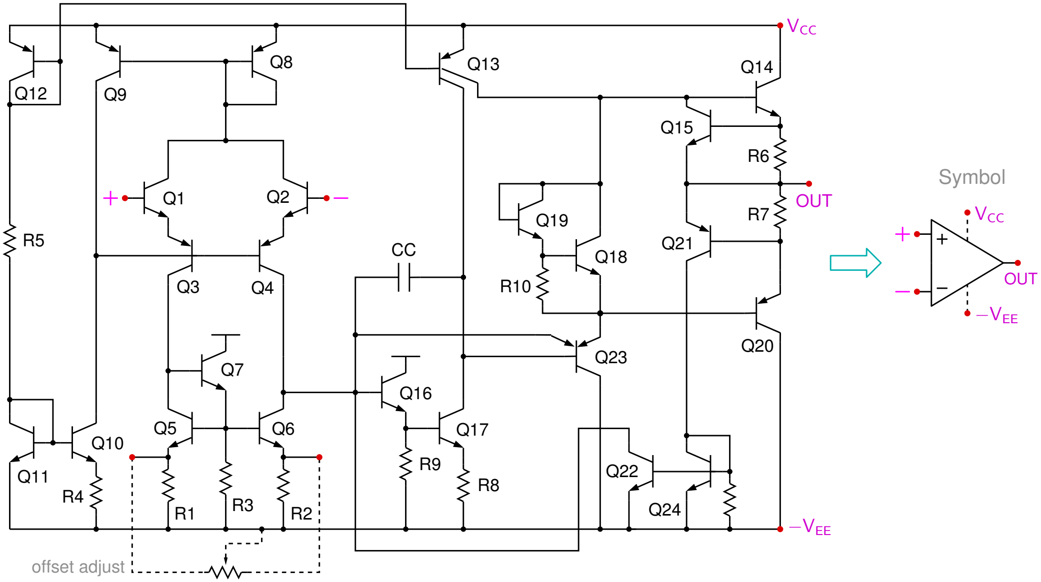
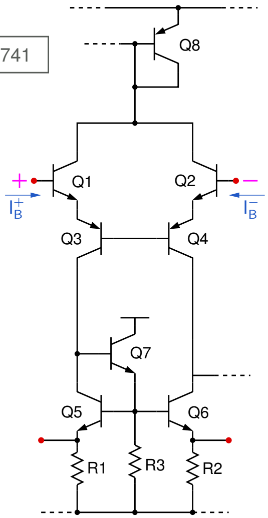
$\rightarrow R' \gg 1/\omega C$ at the frequency of interest.

Op-amp 741: offset null

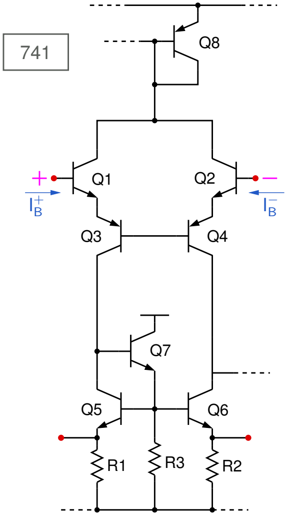


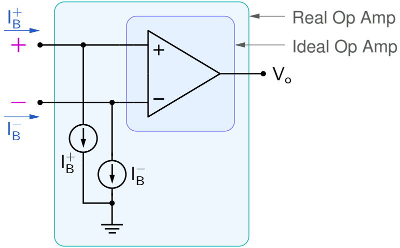
M. B. Patil, IIT Bombay

741



Input bias currents





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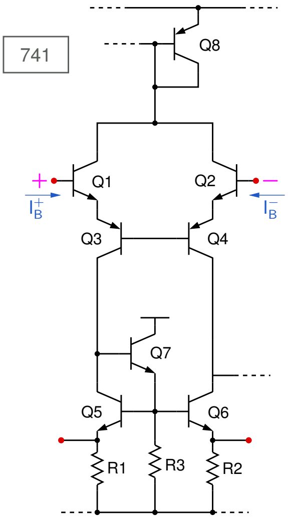
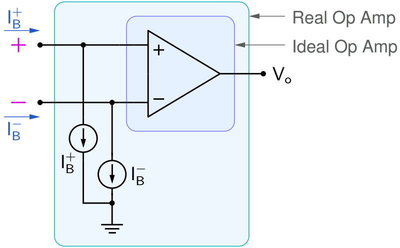


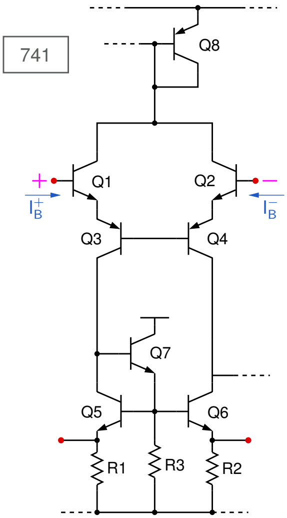


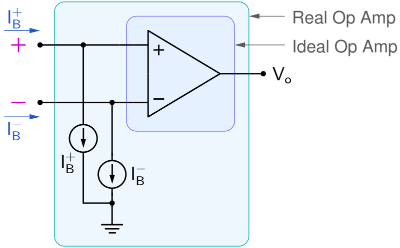
I_B^+ and I_B^- are generally not equal.

$|I_B^+ - I_B^-|$: "offset current" (I_{OS})

$(I_B^+ + I_B^-)/2$: "bias current" (I_B)

Input bias currents





Typical values

Op–Amp	I_B	I_{OS}	V_{OS}	Type
741	80 nA	20 nA	1 mV	BJT input
OP77	1.2 nA	0.3 nA	10 μV	BJT input
411	50 pA	25 pA	0.8 mV	FET input

I_B^+ and I_B^- are generally not equal.

$|I_B^+ - I_B^-|$: "offset current" (I_{OS})

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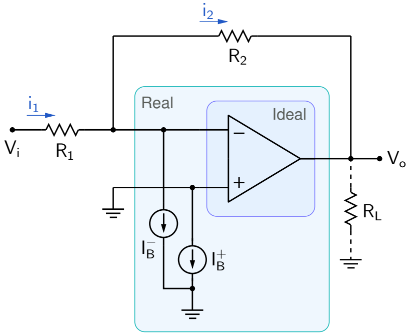
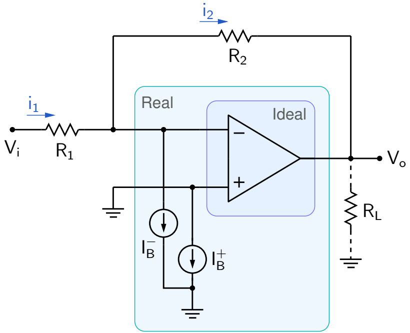

i2
R2
i1
Real
Ideal
Vi
R1
Vo
RL
IB−
IB+

Effect of bias currents: inverting amplifier



Assume that the op-amp is ideal in other respects (including $V_{OS} = 0\ V$).

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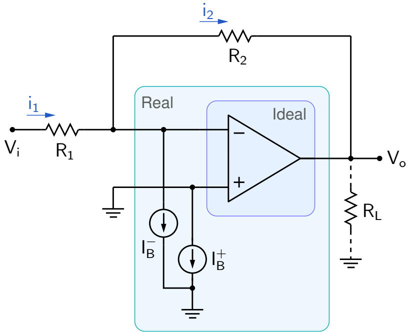

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$V_- \approx V_+ = 0\ V \rightarrow i_1 = V_i / R_1$.

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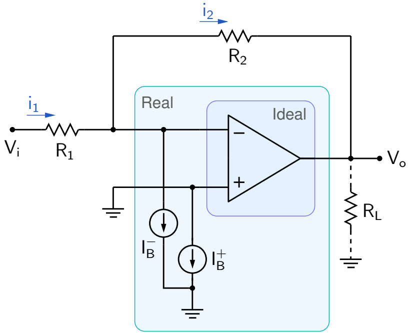

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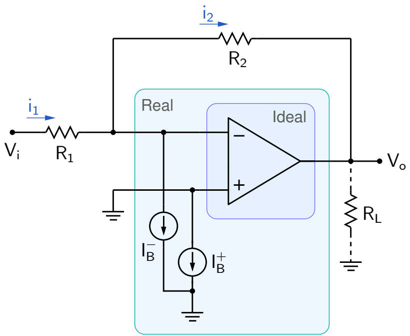
$$i_2 = i_1 - I_B^- \rightarrow V_o = V_- - i_2\, R_2 = 0 - \left(\frac{V_i}{R_1} - I_B^- \right) R_2 = -\frac{R_2}{R_1}\, V_i + I_B^-\, R_2 \,,$$



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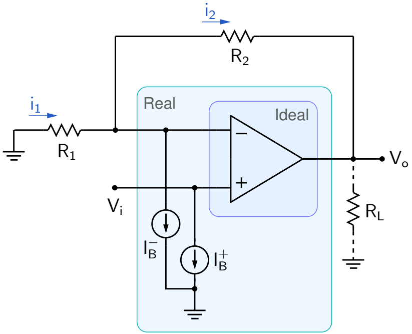
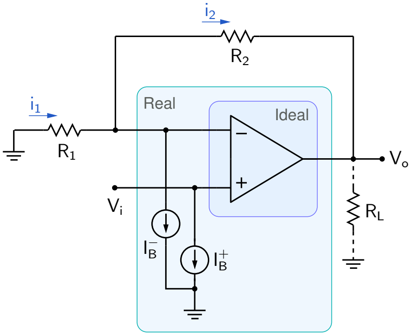
i.e., the bias current causes a DC shift in V_o.

For $I_B^- = 80\,nA$, $R_2 = 10\,k$, $\Delta V_o = 0.8\,mV$.

Effect of bias currents: non-inverting amplifier





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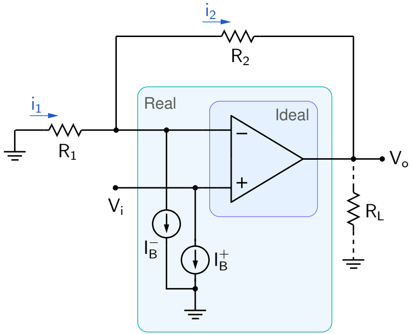

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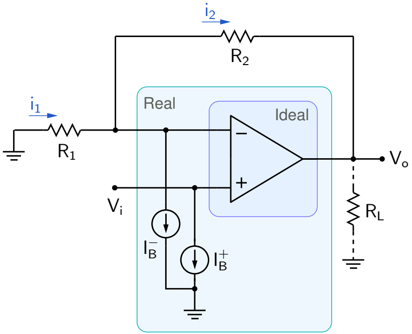

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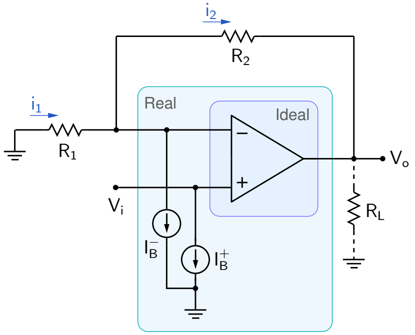

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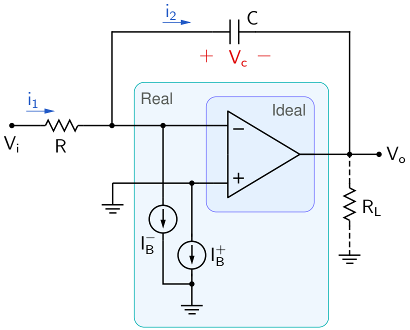
$$V_o = V_- - i_2\,R_2 = V_i - \left(-\frac{V_i}{R_1} - I_B^-\right) R_2 = V_i\left(1 + \frac{R_2}{R_1}\right) + I_B^-\ R_2\ .$$

$\rightarrow$ Again, a DC shift ΔV_o.

Effect of bias currents: integrator



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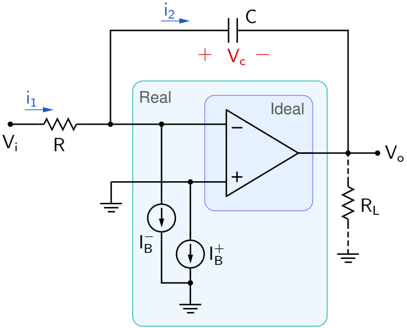
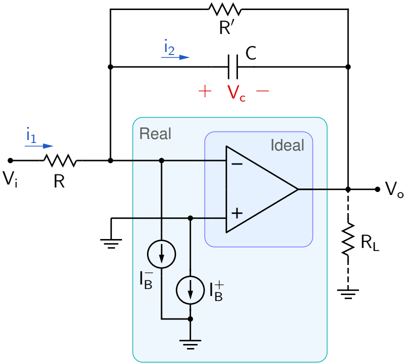


Even with $V_i = 0\,V$, $V_c = \dfrac{1}{C}\displaystyle\int -I_B^-\,dt$ will drive the op-amp into saturation.

Effect of bias currents: integrator



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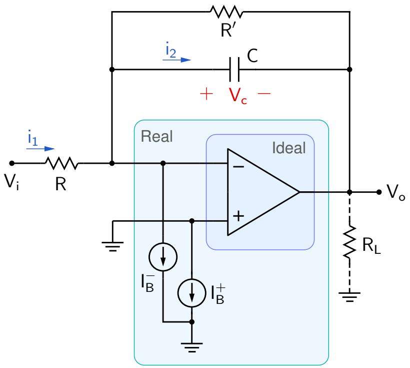
Connecting R' across C provides a DC path for the current, and results in a DC shift $\Delta V_o = I_B^-\,R'$ at the output.



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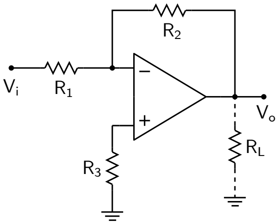
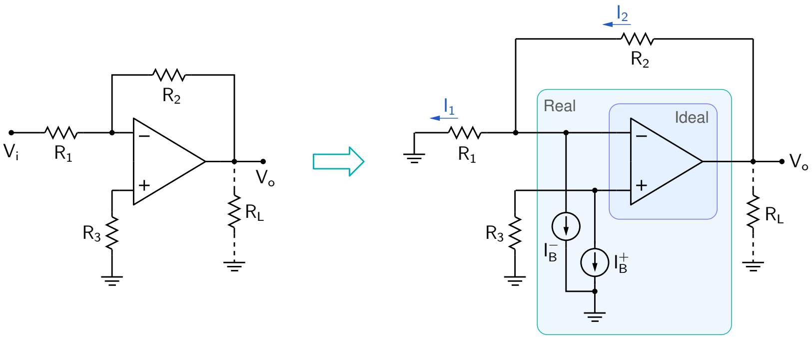
Connecting R' across C provides a DC path for the current, and results in a DC shift $\Delta V_o = I_B^- R'$ at the output.

As we have discussed earlier, R' should be small enough to have a negligible effect on V_o.

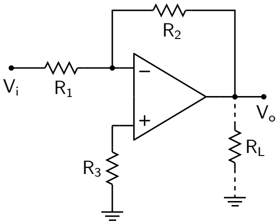
However, R' must be large enough to ensure that the circuit still functions as an integrator.

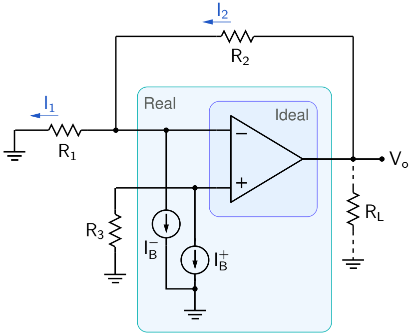
Effect of bias currents: inverting amplifier





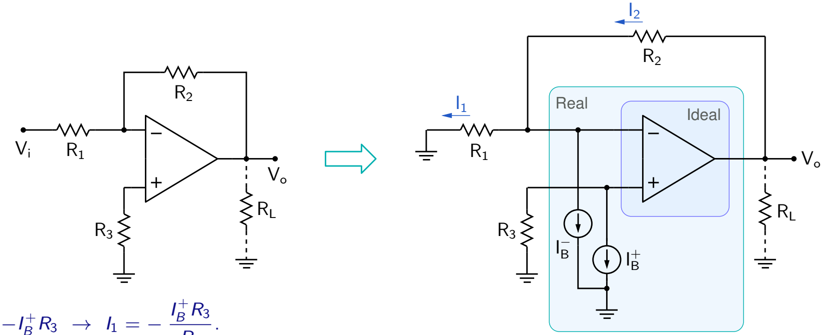
R_2
V_i
R_1
V_o
R_L
R_3
I_2
R_2
I_1
Real
Ideal
R_1
V_o
R_L
R_3
I_B^-
I_B^+





$$V_- \approx V_+ = -I_B^+ R_3 \ \rightarrow \ I_1 = -\frac{I_B^+ R_3}{R_1}.$$

Effect of bias currents: inverting amplifier



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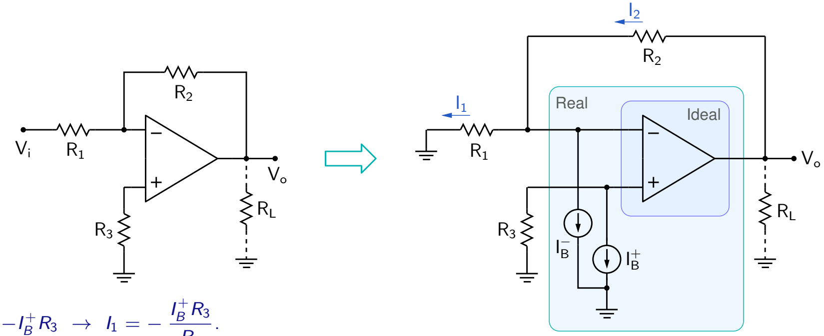

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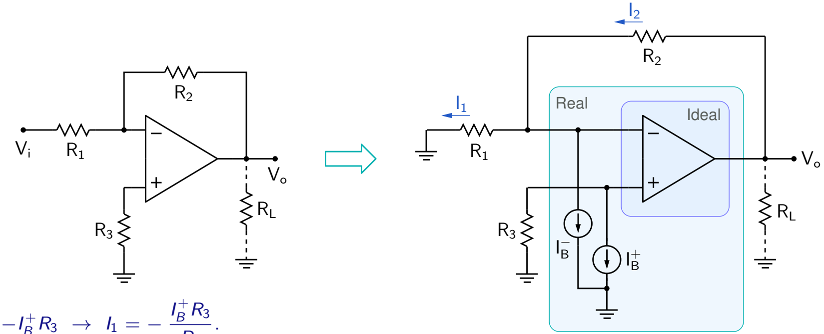
Effect of bias currents: inverting amplifier



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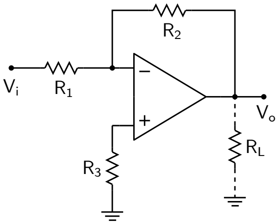
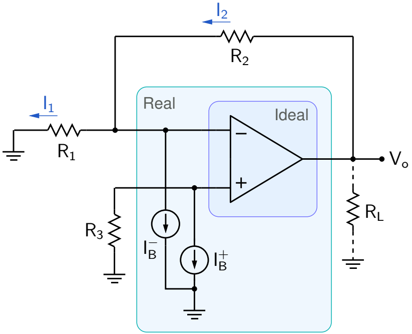
$$V_o = -R_3 \left(1 + \frac{R_2}{R_1}\right)\left(I_B + \frac{I_{OS}}{2}\right) + R_2 \left(I_B - \frac{I_{OS}}{2}\right) = \left(1 + \frac{R_2}{R_1}\right)\left\{ \left[(R_1 \parallel R_2) - R_3 \right] I_B - \left[(R_1 \parallel R_2) + R_3 \right] \frac{I_{OS}}{2} \right\}$$

Effect of bias currents: inverting amplifier





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The first term can be made zero if we select $R_3 = R_1 \,\|\, R_2$.

$\rightarrow V_o = -R_2 I_{OS}$ (Compare with $V_o = R_2 I_B^-$ when R_3 is not connected.)

* For the integrator, V_{OS} and I_B will lead to saturation unless a DC path (a resistor) is provided.

Should we worry about V_{OS} and I_B?

* For the integrator, V_{OS} and I_B will lead to saturation unless a DC path (a resistor) is provided.
* In AC applications (e.g., audio), the DC shift arising due to V_{OS} or I_B is of no consequence since a coupling capacitor will block it anyway.

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* In AC applications (e.g., audio), the DC shift arising due to V_{OS} or I_B is of no consequence since a coupling capacitor will block it anyway.
* A DC shift is a matter of concern when the output is expected to be a DC (or slowly varying) quantity, e.g., a temperature sensor or a strain gauge circuit.

* The unit dB is used to represent quantities on a logarithmic scale.

What is deciBel (dB)?

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  Interesting facts:
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  - Bell considered the telephone an intrusion and refused to put one in his office.
* The unit Bel turned out to be too large in practice $\rightarrow$ deciBel (i.e., one tenth of a Bel).

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  For example, if $P_1 = 20\,W$ and $P_{\mathrm{ref}} = 1\,W$,
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* The gain of a voltage-to-voltage amplifier is often expressed in dB. In that case, the ratio V_o^2/V_i^2 is considered (since $P \propto V^2$ or $P \propto I^2$ for a resistor).

  A_V in dB $= 10\,\log_{10}|V_o/V_i|^2 = 20\,\log_{10}|V_o/V_i|$,

* "dBm" is a related unit used to describe voltages with a reference of 1 mV.

  For example, 2.2 V: $20\,\log_{10}\left(\dfrac{2.2\,\text{V}}{1\,\text{mV}}\right) = 6.85\,\text{dBm}$.

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Let $\widehat{V}_i$ and $\widehat{V}_o$ be the input and output amplitudes.

If $\widehat{V}_i = 2.5\,\text{mV}$ and $A_V = 36.3\,\text{dB}$, compute $\widehat{V}_o$ in dBm and mV.

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dB in audio measurements

* When sound intensity is specified in dB, the reference pressure is $P_{\mathrm{ref}} = 20\,\mu Pa$ (our hearing threshold).
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Industrial area	75 dB

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Industrial area	75 dB
Commercial area	65 dB
Residential area	55 dB

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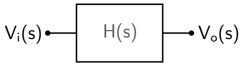
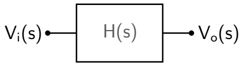
Industrial area	75 dB
Commercial area	65 dB
Residential area	55 dB
Silence zone	50 dB

Bode plots





* The transfer function of a circuit such as an amplifier or a filter is given by,
  $H(s) = V_o(s)/V_i(s), \quad s = j\omega.$
  e.g., $H(s) = \dfrac{K}{1 + s\tau} = \dfrac{K}{1 + j\omega\tau}$

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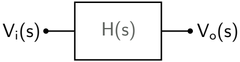

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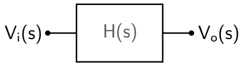

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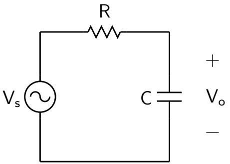
* Bode gave simple rules which allow construction of the above plots in an approximate (asymptotic) manner.

A simple transfer function



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC}\,.$$

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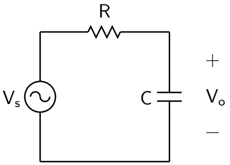

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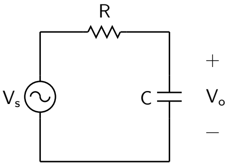

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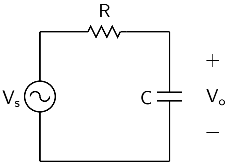

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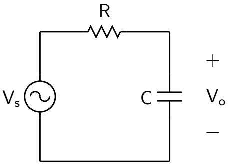

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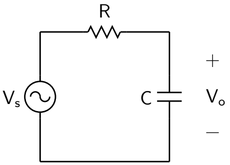
  For $\omega \gg \omega_0$, $\dfrac{\omega}{\omega_0} \gg 1$, $H(j\omega) \approx \dfrac{1}{j\,\dfrac{\omega}{\omega_0}}$, and $|H(j\omega)| \rightarrow \dfrac{1}{\omega}$.

* The magnitude and phase of $H(j\omega)$ are given by,

  $$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}\,, \quad \angle H(j\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right).$$

A simple transfer function



$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s ,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\omega/\omega_0)} ,$$

$$\omega_0 = \frac{1}{RC} .$$

* The circuit behaves like a low-pass filter.
  For $\omega \ll \omega_0$, $\dfrac{\omega}{\omega_0} \ll 1$, $|H(j\omega)| \rightarrow 1$.
  For $\omega \gg \omega_0$, $\dfrac{\omega}{\omega_0} \gg 1$, $H(j\omega) \approx \dfrac{1}{j\,\dfrac{\omega}{\omega_0}}$, and $|H(j\omega)| \rightarrow \dfrac{1}{\omega}$.
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* We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.

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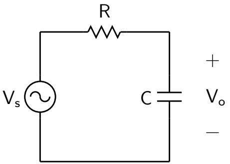

$$V_o = \frac{(1/sC)}{R + (1/sC)}\, V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1{+}sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

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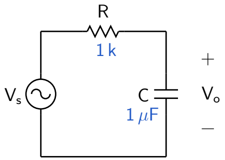
* We are generally interested in a large variation in ω (several orders), and its effect on $|H|$ and $\angle H$.
* The magnitude ($|H|$) varies by orders of magnitude as well.
  The phase ($\angle H$) varies from 0 (for $\omega \ll \omega_0$) to $-\pi/2$ (for $\omega \gg \omega_0$).

A simple transfer function: magnitude



$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC} = 10^3\,\text{rad/s}\,.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

$$\angle(H(j\omega)) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

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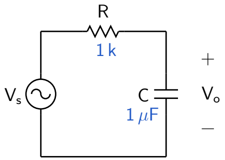

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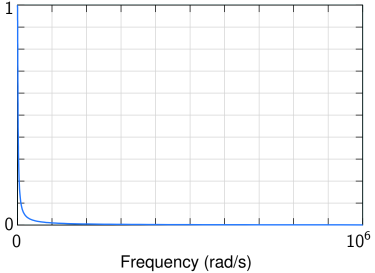
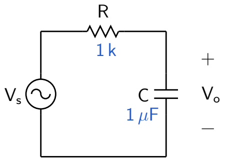


A simple transfer function: magnitude



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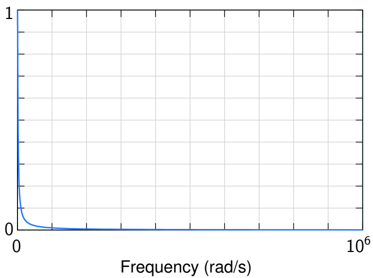
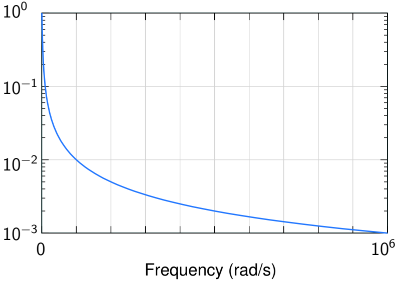
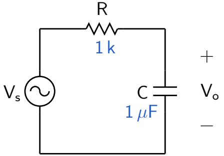




A simple transfer function: magnitude



$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s\,,$$

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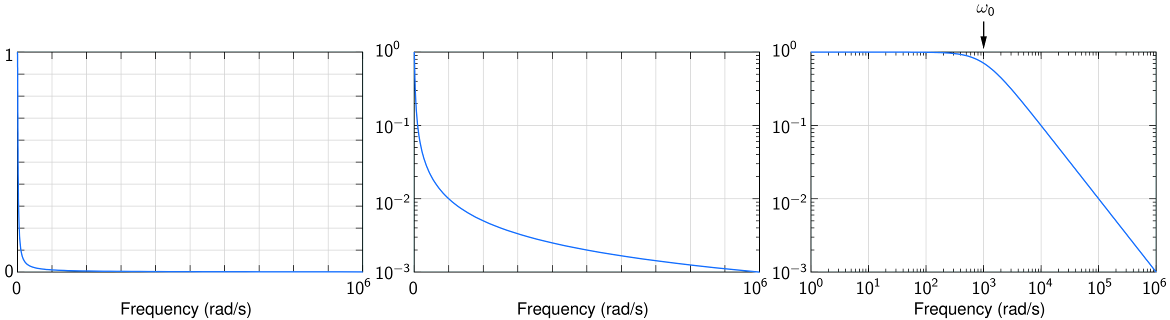

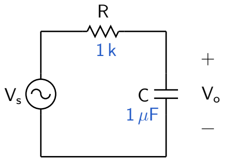

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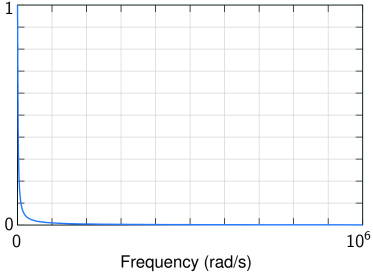
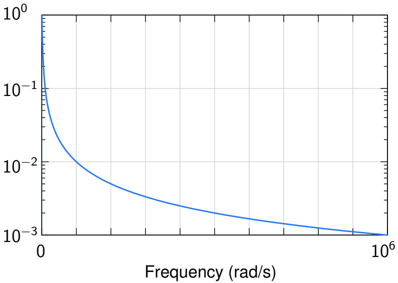
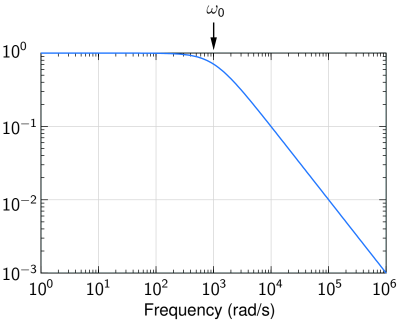
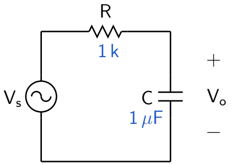






Since ω and $|H(j\,\omega)|$ vary by several orders of magnitude, a linear ω- or $|H|$-axis is not appropriate $\rightarrow$ $\log|H|$ is plotted against $\log\omega$.



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

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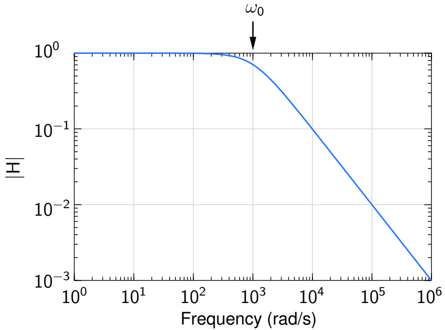
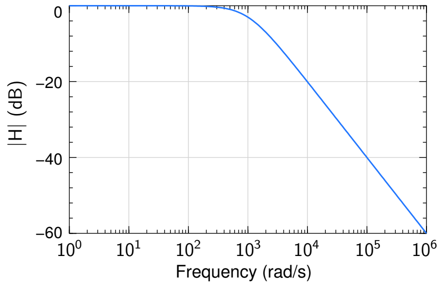
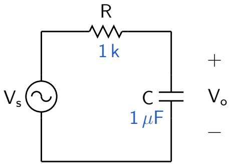




A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1{+}s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

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$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

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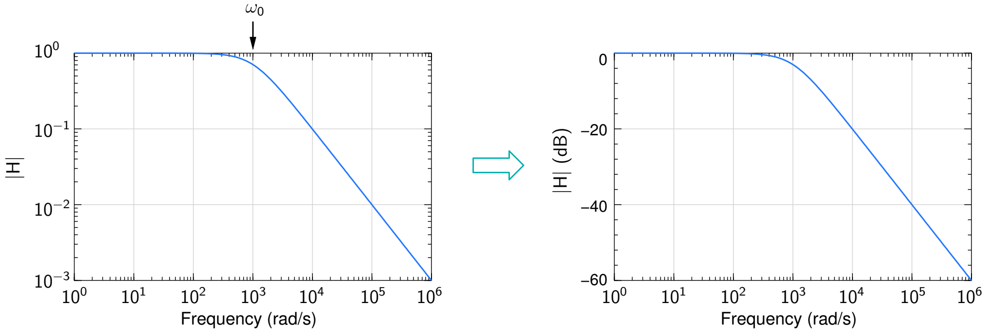
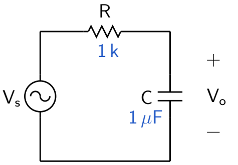


Note that the *shape* of the plot does not change.

$|H|$ (dB) $= 20 \log |H|$ is simply a scaled version of $\log |H|$.

A simple transfer function: magnitude



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+s\,RC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC} = 10^3\,\text{rad/s},\ f_0 = 159\,\text{Hz}\,.$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$$

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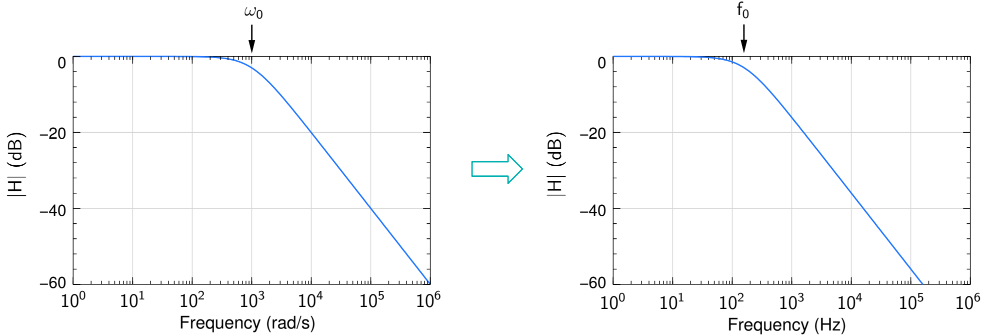
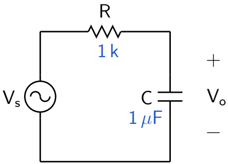


A simple transfer function: magnitude



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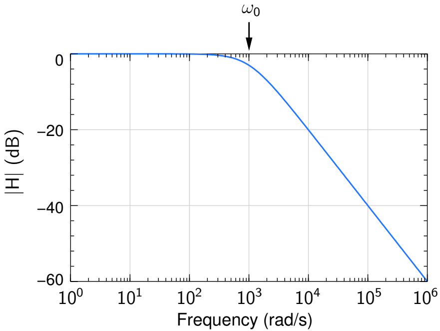
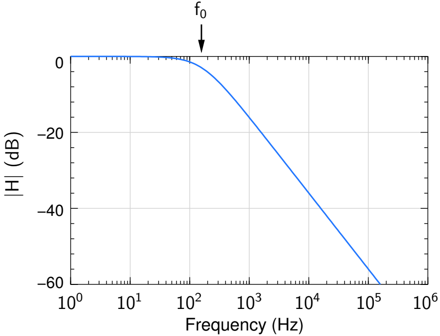
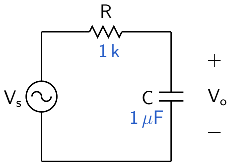




Since $\omega = 2\pi f$, $\log \omega = \log(2\pi) + \log f$ which causes a shift in the x direction, but the *shape* of the plot does not change.

A simple transfer function: phase



$$V_o = \frac{(1/s\,C)}{R + (1/s\,C)}\,V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1+s\,RC} = \frac{1}{1+(j\,\omega/\omega_0)}\,,$$

$$\omega_0 = \frac{1}{RC} = 10^3\,\text{rad/s},\ f_0 = 159\,\text{Hz}\,.$$

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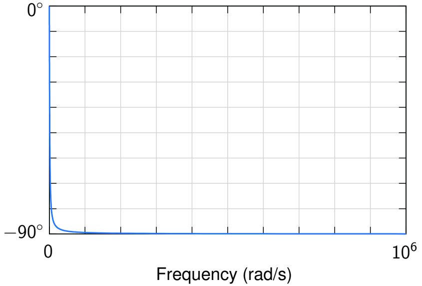
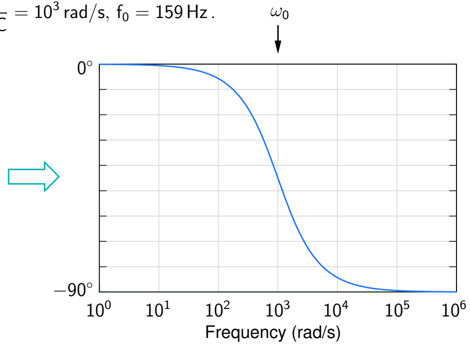
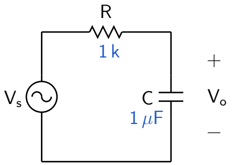
$$\angle(H(j\omega)) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$







$$V_o = \frac{(1/sC)}{R + (1/sC)} V_s\,,$$

$$\rightarrow H(s) = \frac{1}{1 + sRC} = \frac{1}{1 + (j\,\omega/\omega_0)}\,,$$

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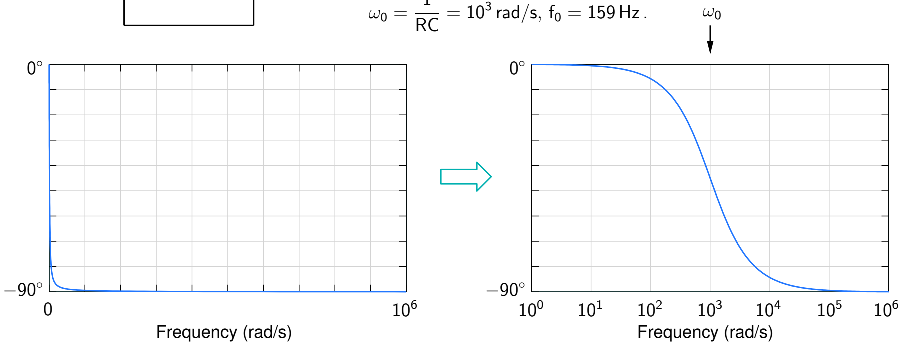
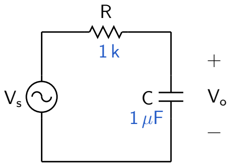


* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to −90° in this example), a linear axis is appropriate for $\angle H$.



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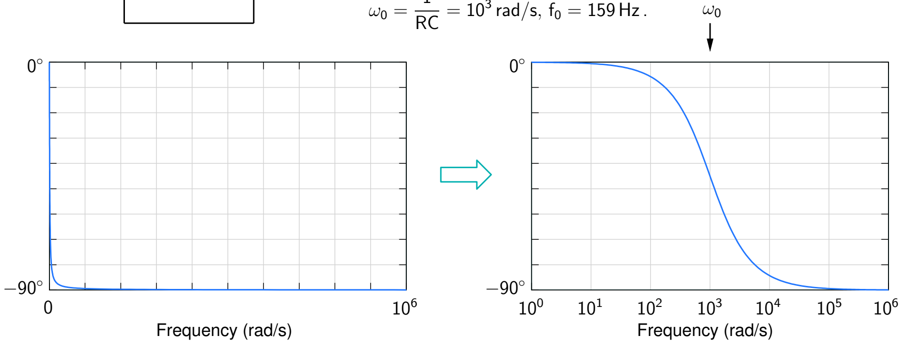

* Since $\angle H = -\tan^{-1}(\omega/\omega_0)$ varies in a limited range (0° to -90° in this example), a linear axis is appropriate for $\angle H$.
* As in the magnitude plot, we use a log axis for ω, since we are interested in a wide range of ω.

Consider $H(s) = \dfrac{K\,(1+s/z_1)(1+s/z_2)\cdots(1+s/z_M)}{(1+s/p_1)(1+s/p_2)\cdots(1+s/p_N)}$.

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$-z_1$, $-z_2, \cdots$ are called the "zeros" of $H(s)$.

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(In addition, there could be terms like s, $s^2, \cdots$ in the numerator.)

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We will assume, for simplicity, that the zeros and poles are real and distinct.

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Construction of Bode plots involves

(a) computing approximate contribution of each pole/zero as a function of ω.

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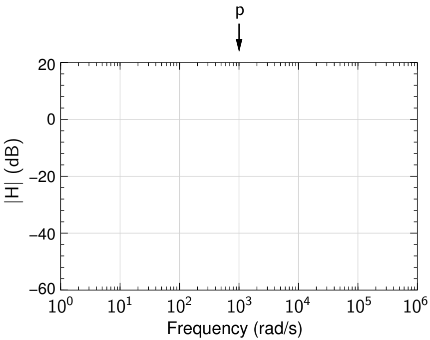
Construction of Bode plots involves

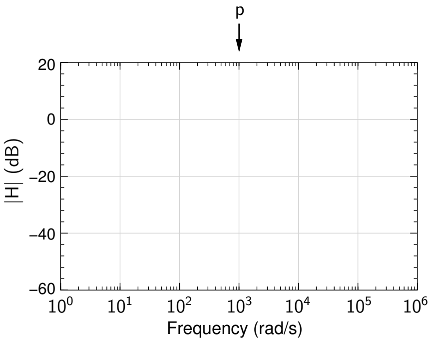
(a) computing approximate contribution of each pole/zero as a function of ω.

(b) combining the various contributions to obtain $|H|$ and $\angle H$ versus ω.



Consider $H(s) = \dfrac{1}{1 + s/p} \rightarrow H(j\omega) = \dfrac{1}{1 + j(\omega/p)}$, $|H(j\omega)| = \dfrac{1}{\sqrt{1 + (\omega/p)^2}}$.

In this example, $p = 10^3$ rad/s.



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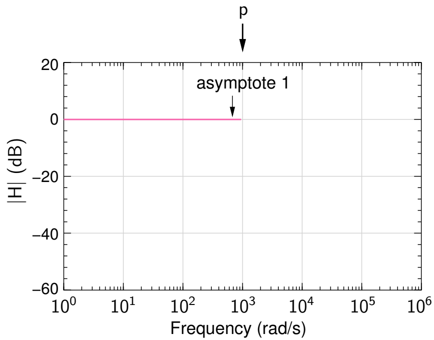
Asymptote 1:

$\omega \ll p$: $|H| \rightarrow 1,\; 20\log|H| = 0\,\text{dB}.$

Contribution of a pole: magnitude



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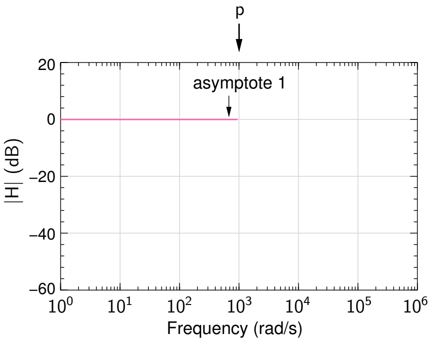

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In this example, $p = 10^3$ rad/s.

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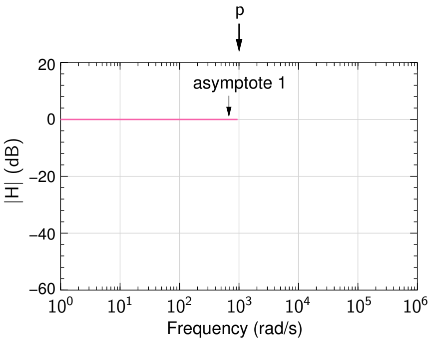
$\omega \ll p$: $|H| \rightarrow 1,\;\; 20\log|H| = 0\,\text{dB}.$

Asymptote 2:

$$\omega \gg p:\; |H| \rightarrow \frac{1}{\omega/p} = \frac{p}{\omega} \rightarrow |H| = 20\,\log\,p - 20\,\log\omega\;(\text{dB})$$

Contribution of a pole: magnitude



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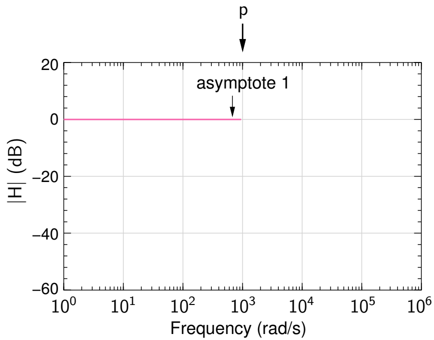

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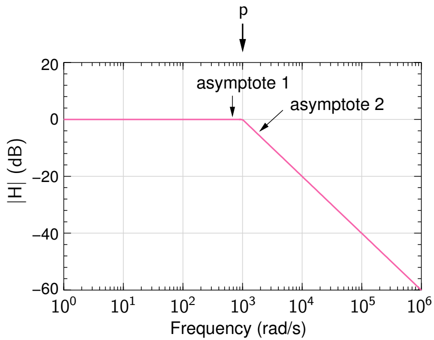

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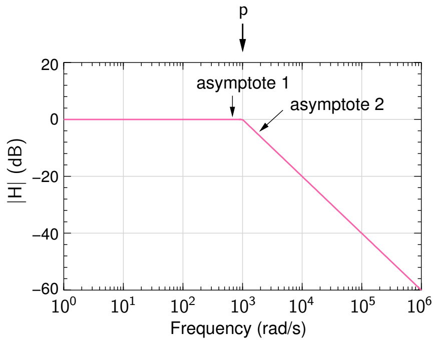

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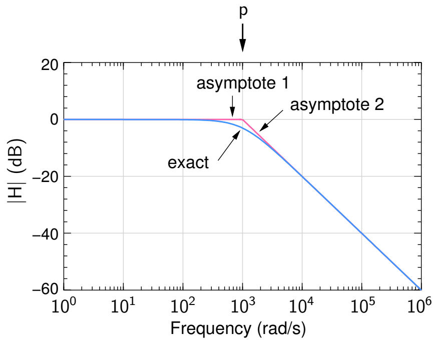

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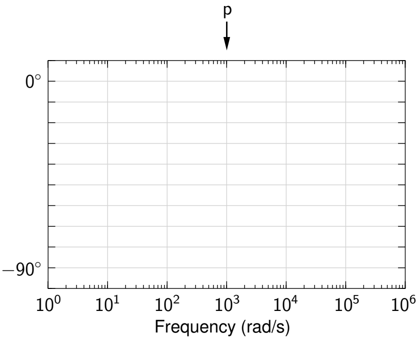

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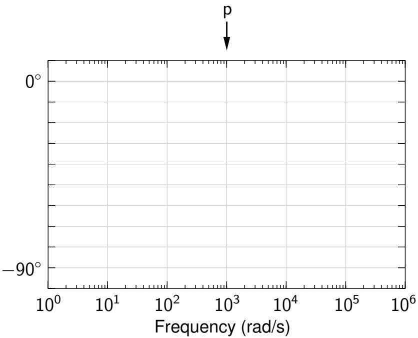

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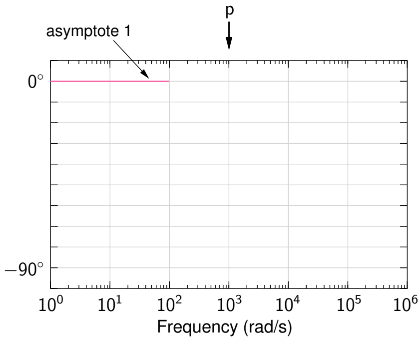

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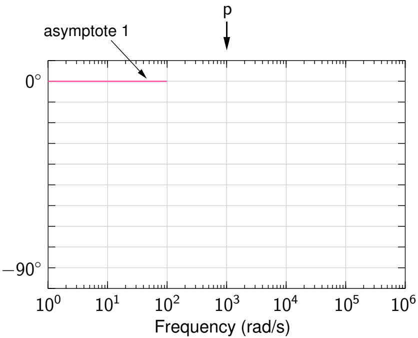

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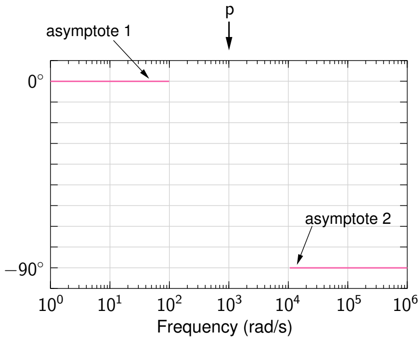

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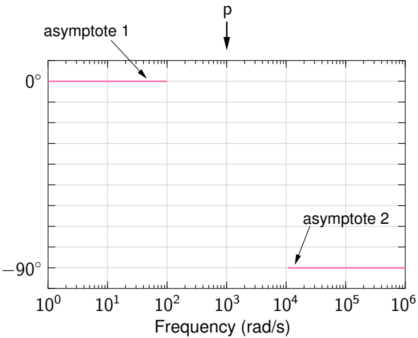

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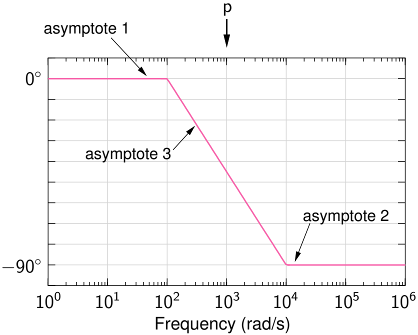
$\rightarrow$ at $\omega = p$, $\angle H = -\pi/4$

(which is also the actual value of $\angle H$).



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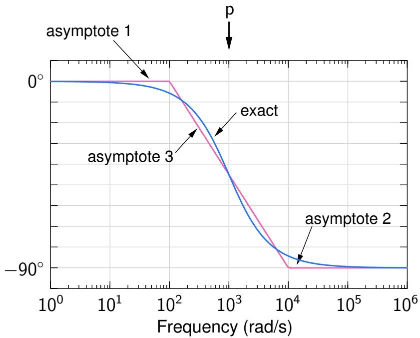

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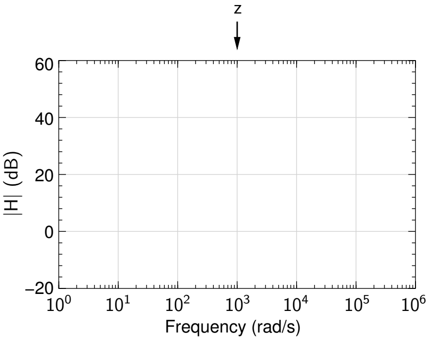
Contribution of a zero: magnitude



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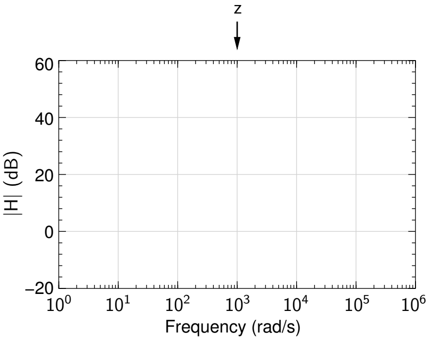

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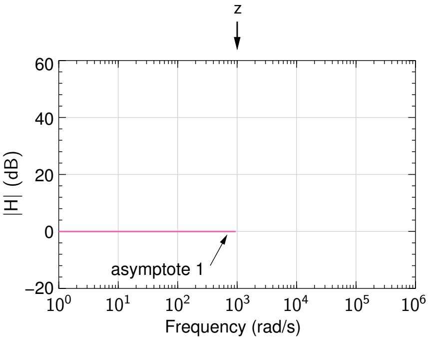

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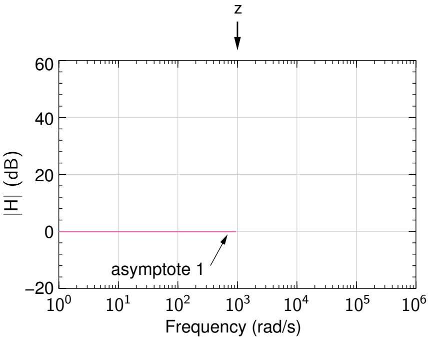

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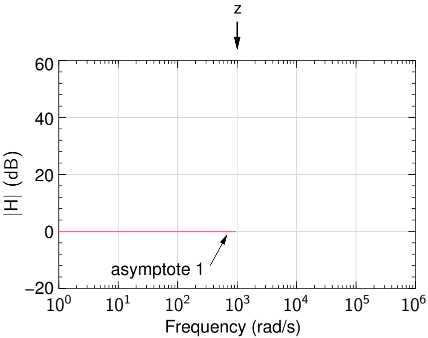

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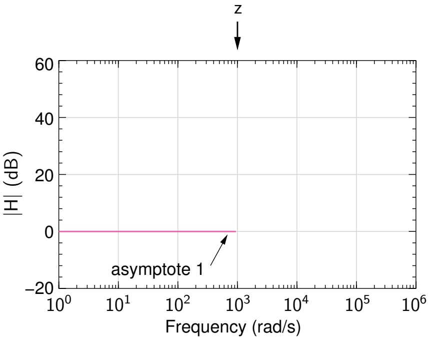

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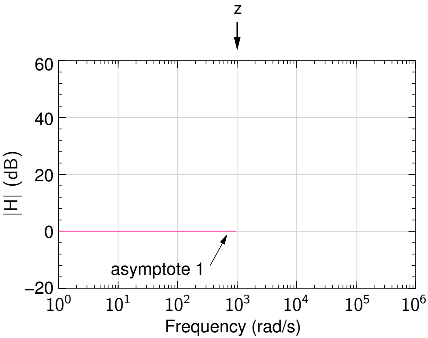

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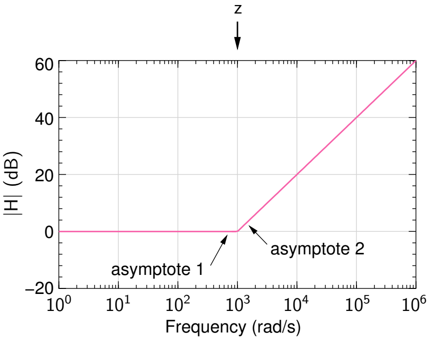

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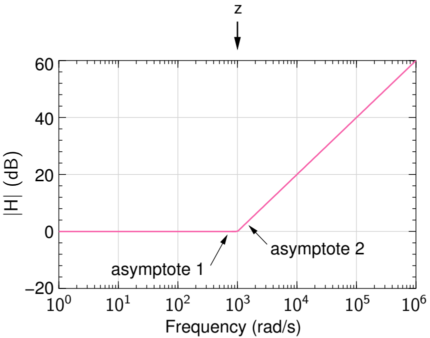

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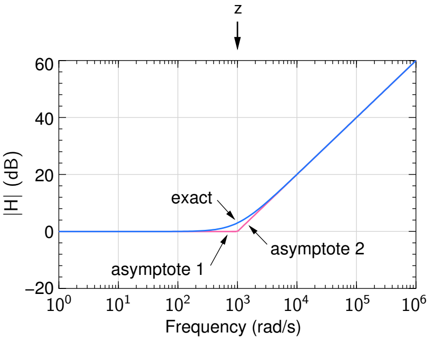
Contribution of a zero: magnitude



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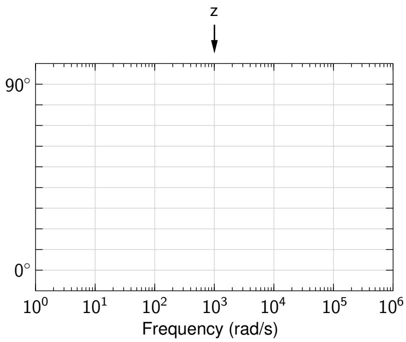

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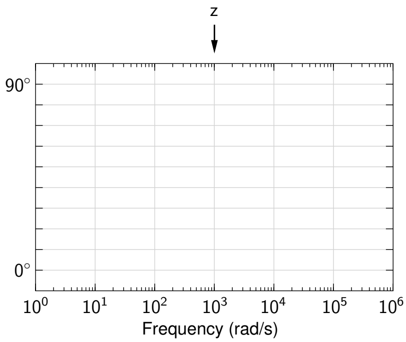

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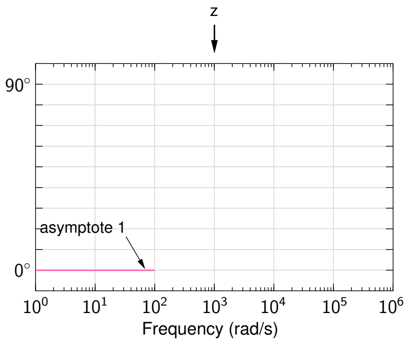

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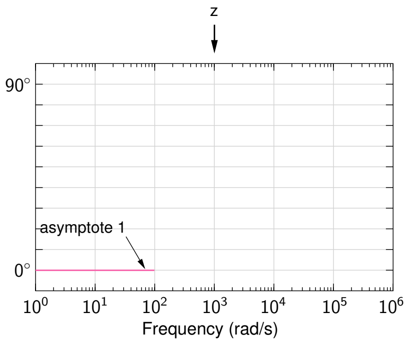

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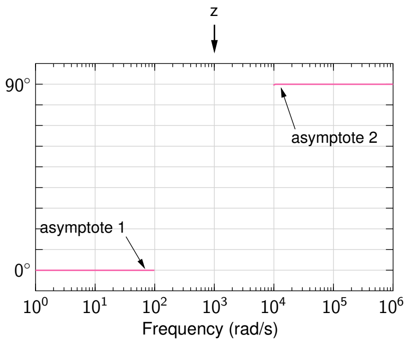

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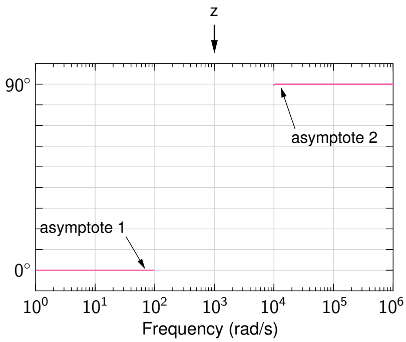

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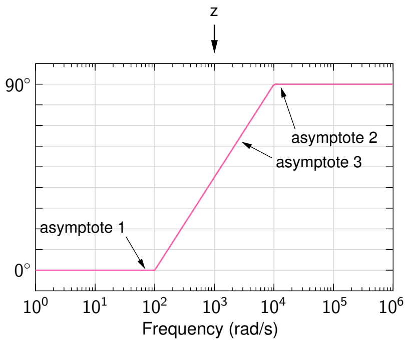

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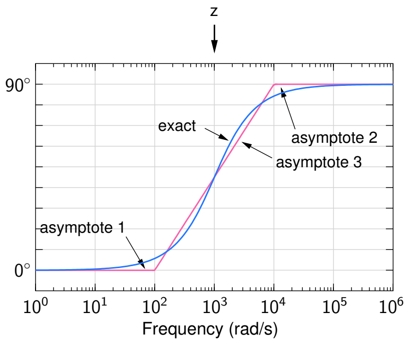

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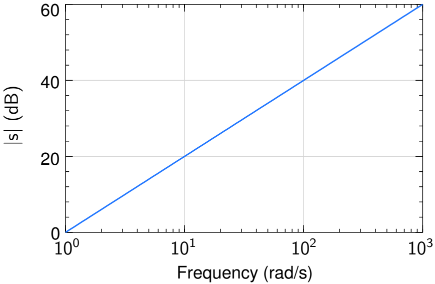

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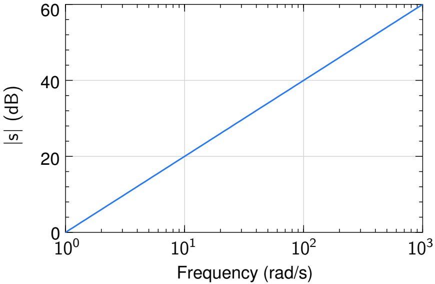

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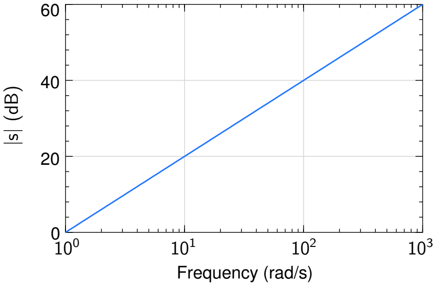

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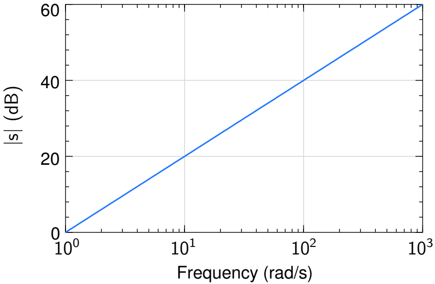

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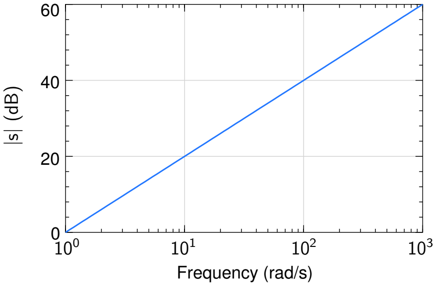

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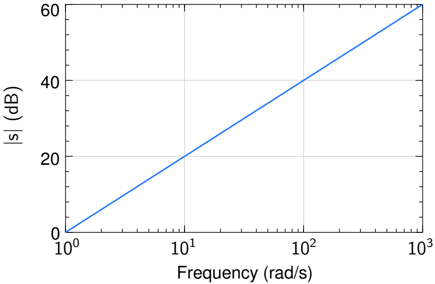

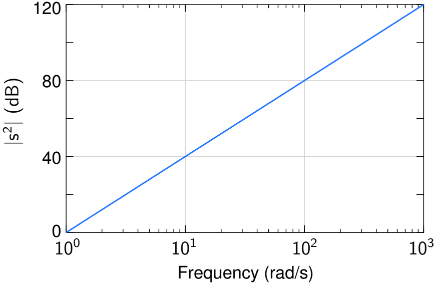

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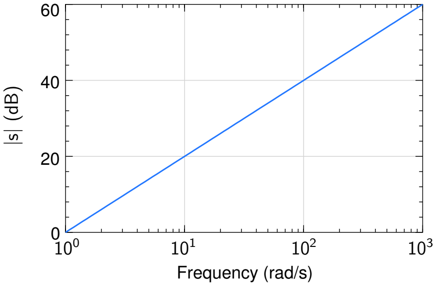

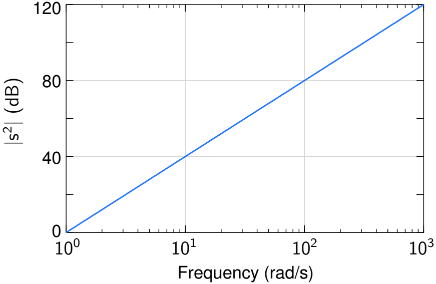

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$H_1(j\omega)$ and $H_2(j\omega)$ are complex numbers.

At a given ω, let $H_1 = K_1\angle\alpha = K_1\,e^{j\alpha}$, and $H_2 = K_2\angle\beta = K_2\,e^{j\beta}$.

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In the Bode phase plot, the contributions due to H_1 and H_2 also get added.

The same reasoning applies to more than two terms as well.

Combining different terms: example

Consider $H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$.

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Let $H(s) = H_1(s)\,H_2(s)\,H_3(s)\,H_4(s)$, where

$H_1(s) = 10$,

$H_2(s) = s$,

$H_3(s) = \dfrac{1}{1+s/p_1}$, $p_1 = 10^2$ rad/s,

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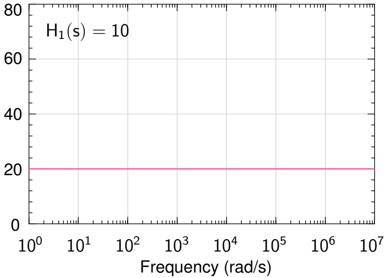
We can now plot the magnitude and phase of H_1, H_2, H_3, H_4 *individually* versus ω and then simply add them to obtain $|H|$ and $\angle H$.

Magnitude plot ($|H|$ in dB)



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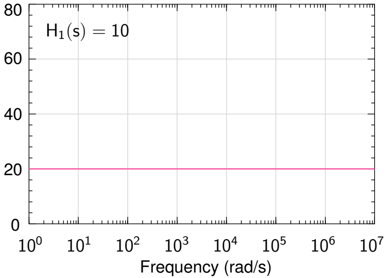

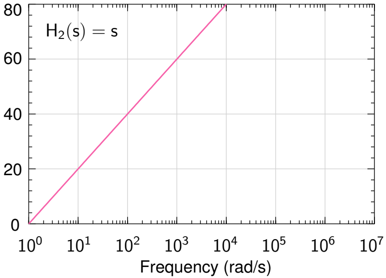

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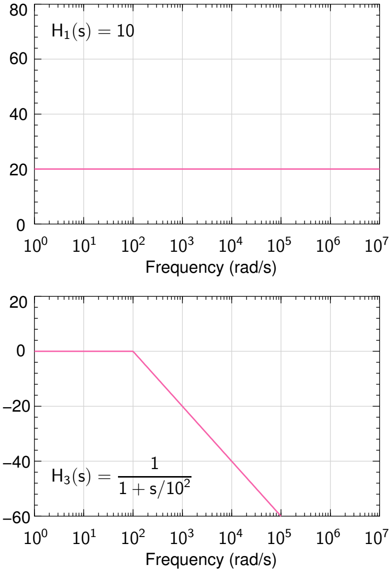

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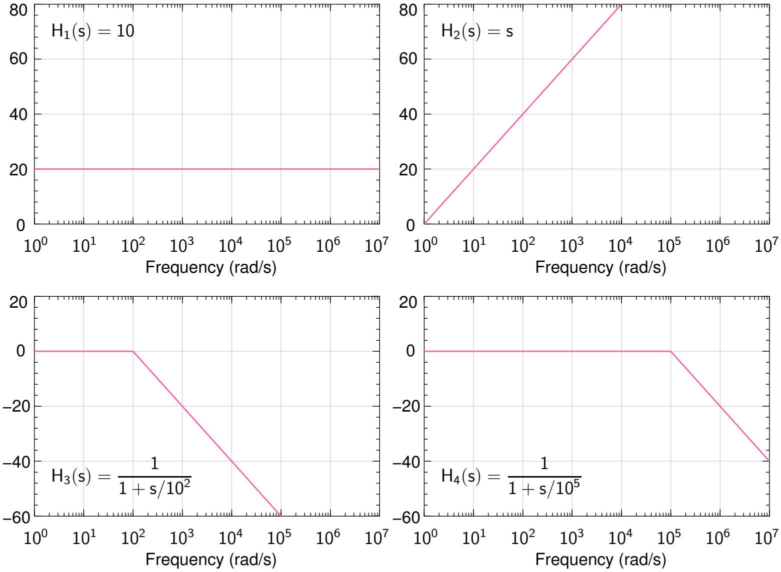

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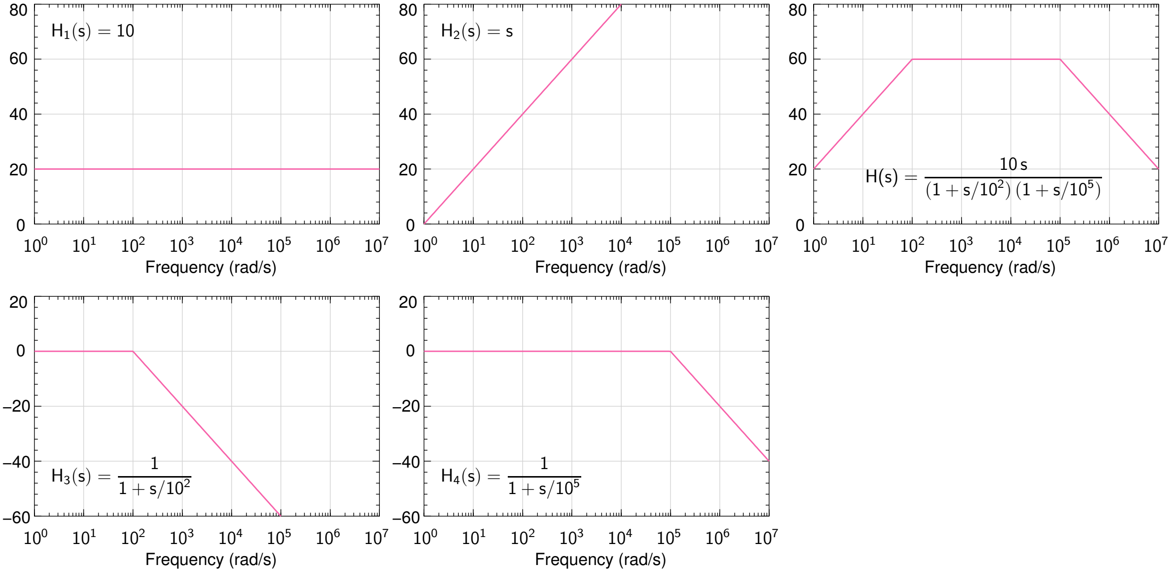

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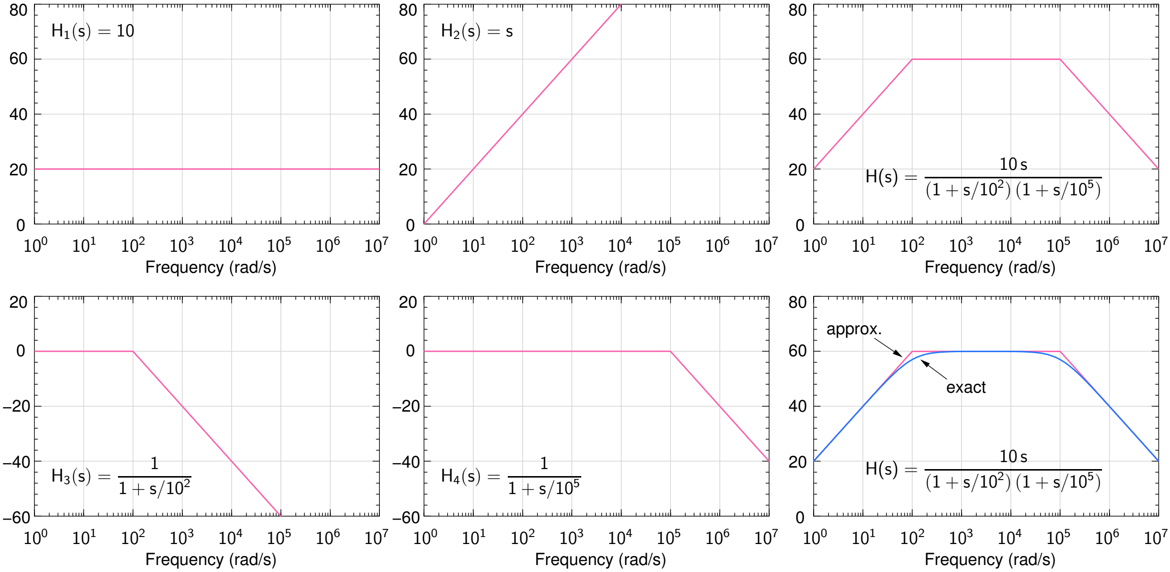
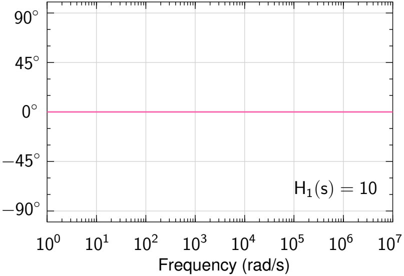
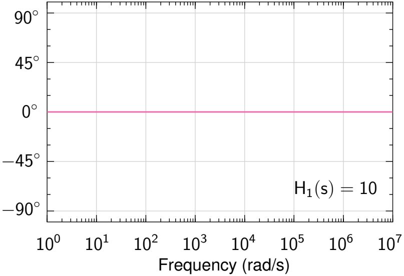


Phase plot



90°
45°
0°
−45°
−90°
$H_1(s) = 10$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



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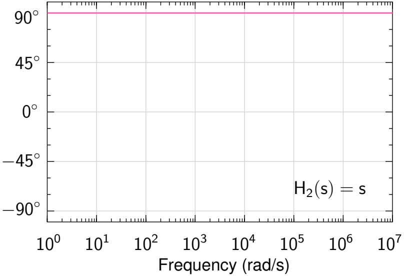
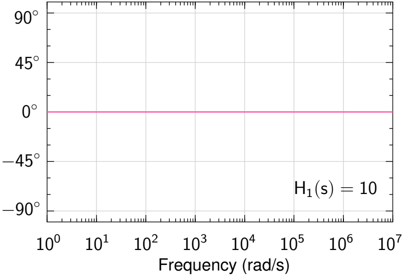


90°
45°
0°
−45°
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot



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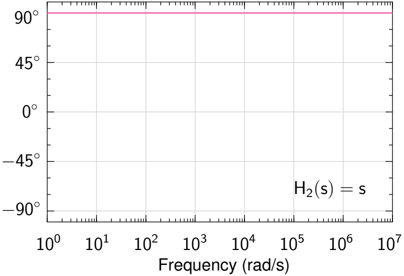

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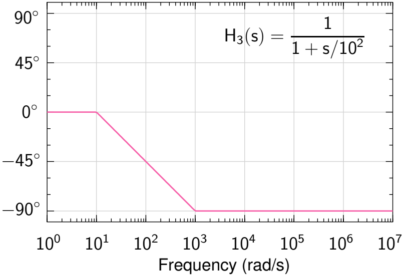
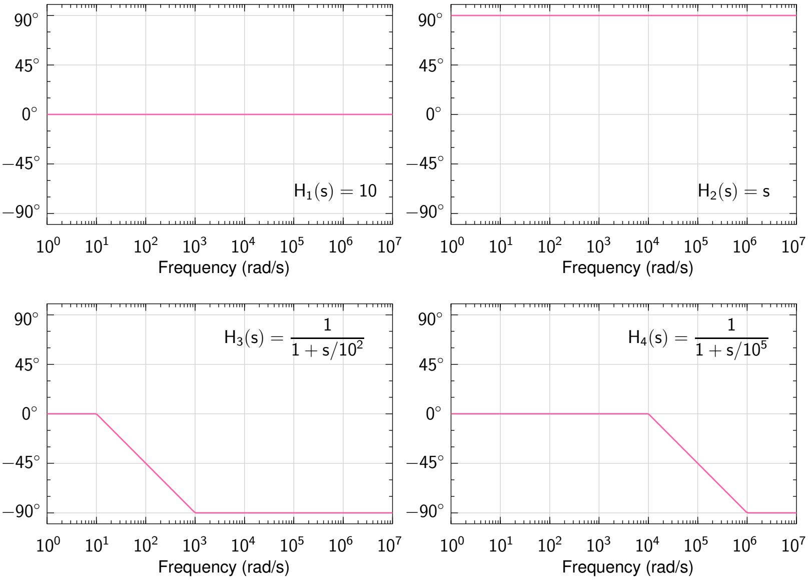
−90°
$H_2(s) = s$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

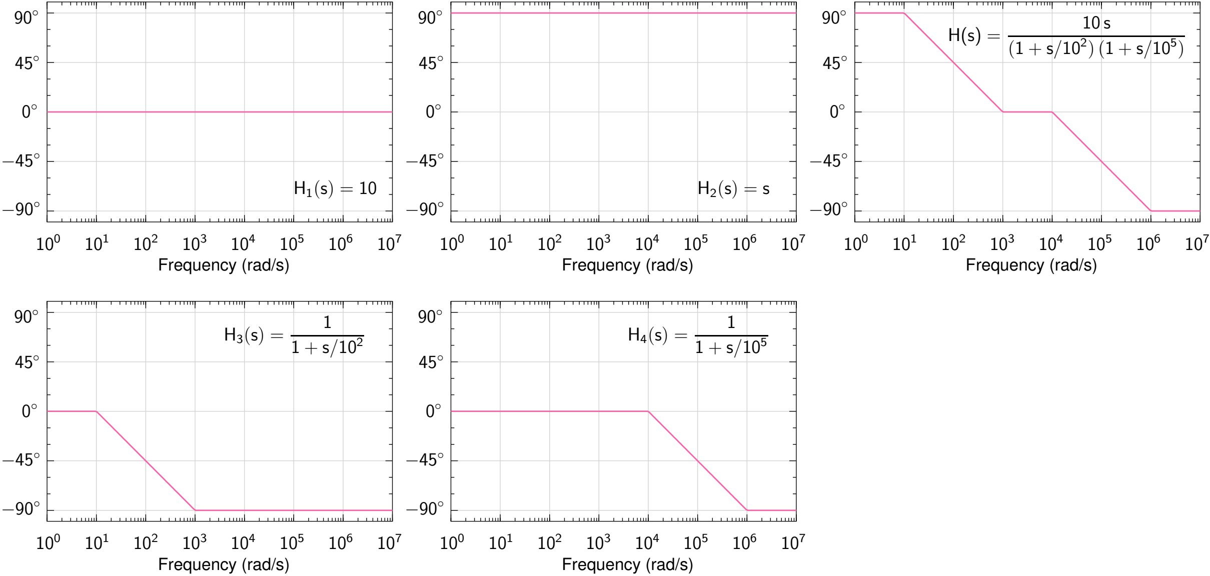


90°
45°
0°
−45°
−90°
$H_3(s) = \frac{1}{1 + s/10^2}$
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

Phase plot

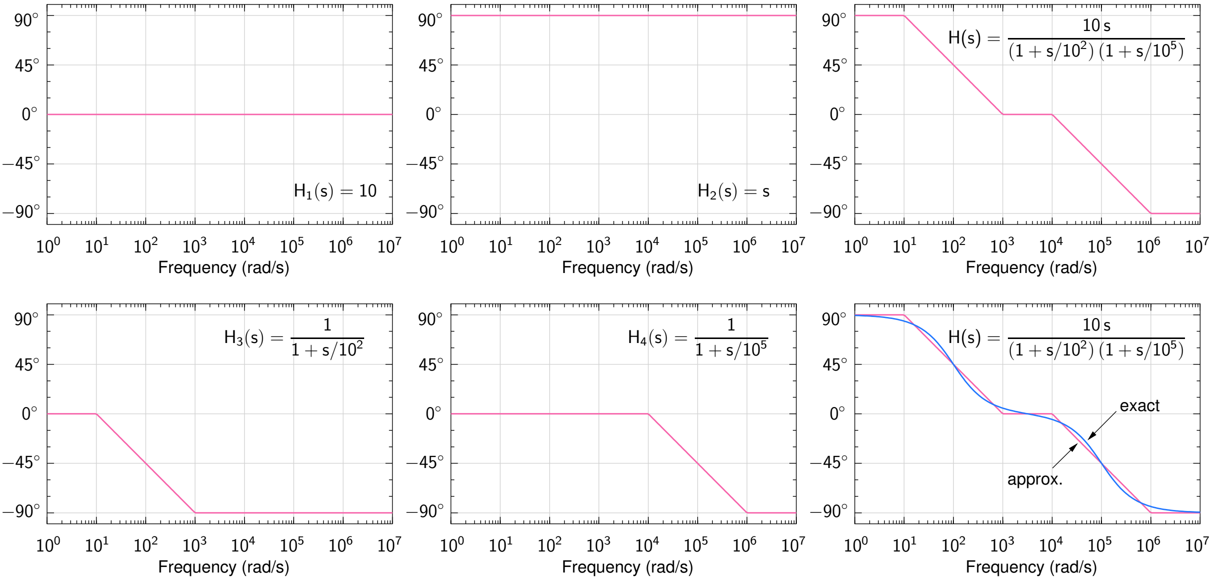

90°
45°
0°
-45°
-90°
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)
$H_1(s) = 10$
$H_2(s) = s$
$H_3(s) = \frac{1}{1 + s/10^2}$
$H_4(s) = \frac{1}{1 + s/10^5}$

Phase plot



90°
45°
0°
−45°
−90°
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)
$H_1(s) = 10$
$H_2(s) = s$
$H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$
$H_3(s) = \dfrac{1}{1+s/10^2}$
$H_4(s) = \dfrac{1}{1+s/10^5}$

Phase plot



$H_1(s) = 10$
$H_2(s) = s$
$H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$
$H_3(s) = \dfrac{1}{1+s/10^2}$
$H_4(s) = \dfrac{1}{1+s/10^5}$
$H(s) = \dfrac{10\,s}{(1+s/10^2)\,(1+s/10^5)}$
exact
approx.
90°
45°
0°
−45°
−90°
10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7
Frequency (rad/s)

How good are the approximations?

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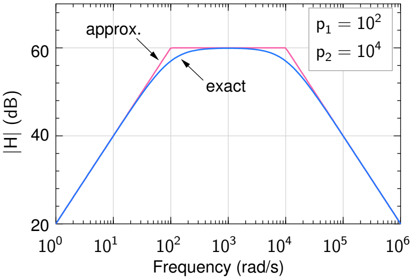
* If two poles p_1 and p_2 are close to each other (say, separated by less than a decade in ω), the error becomes larger (next slide).
* When the poles and zeros are not sufficiently separated, the Bode approximation should be used only for a rough estimate, follwed by a numerical calculation. However, even in such cases, it does give a good idea of the *asymptotic* magnitude and phase plots, which is valuable in amplifier design.

How good are the approximations?

Consider $H(s) = \dfrac{10\,s}{(1 + s/p_1)\,(1 + s/p_2)}$.



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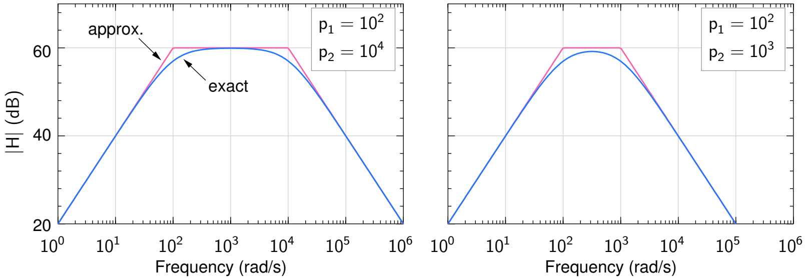

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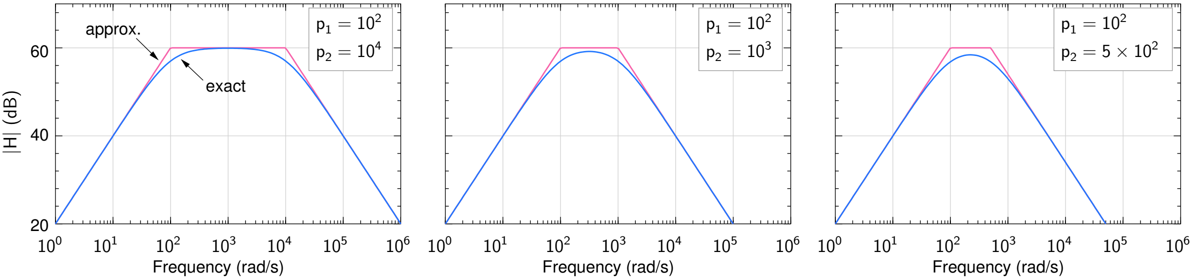

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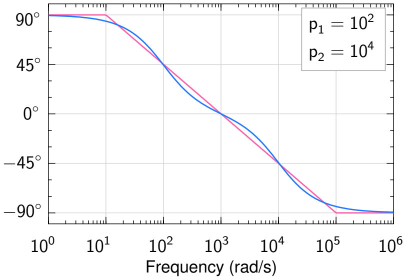

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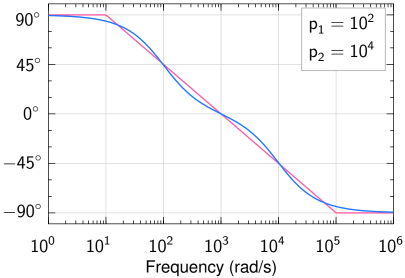

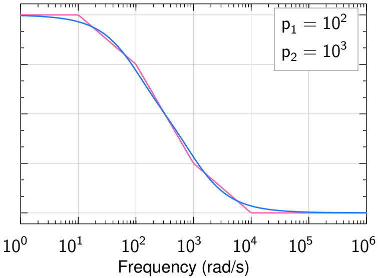

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