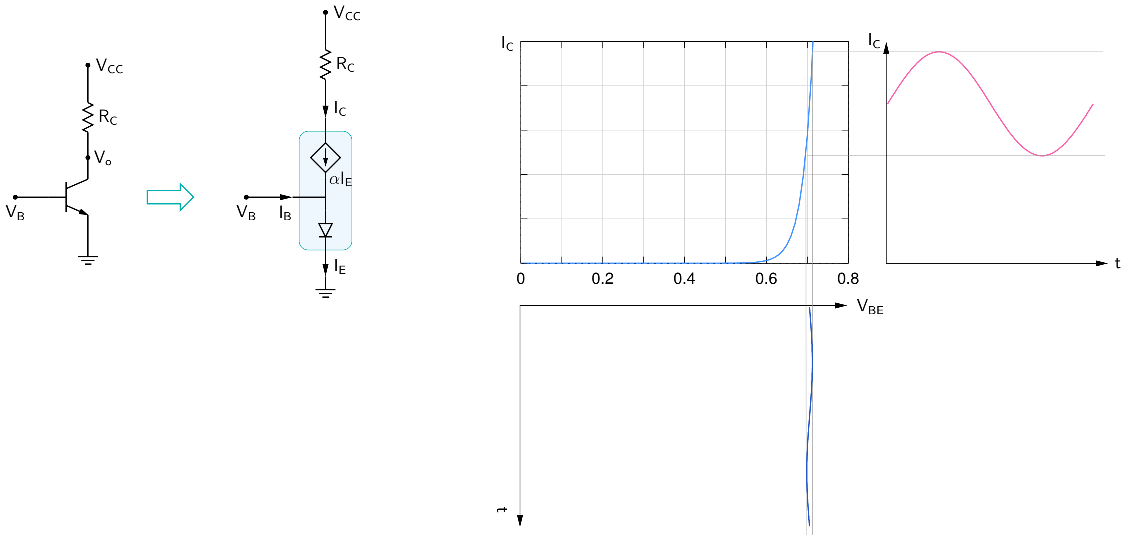
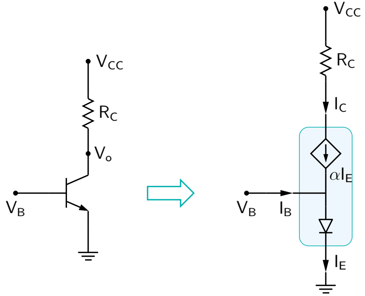
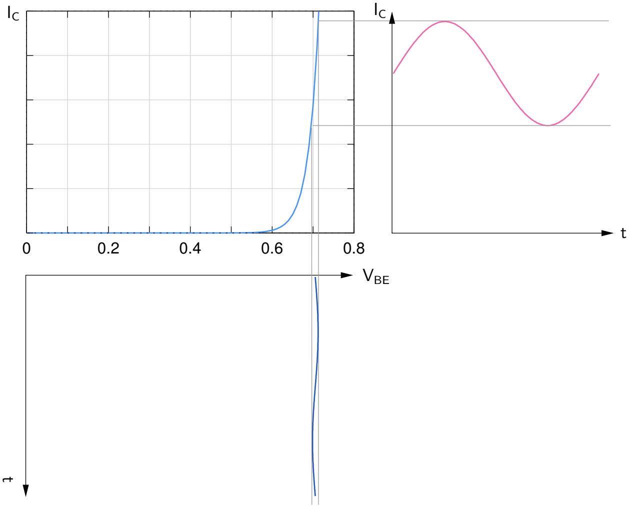
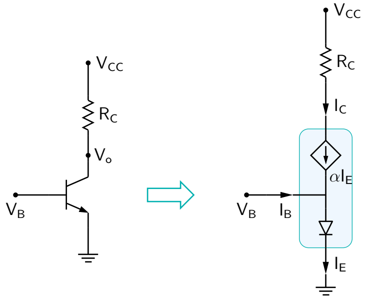
BJT amplifier: basic operation

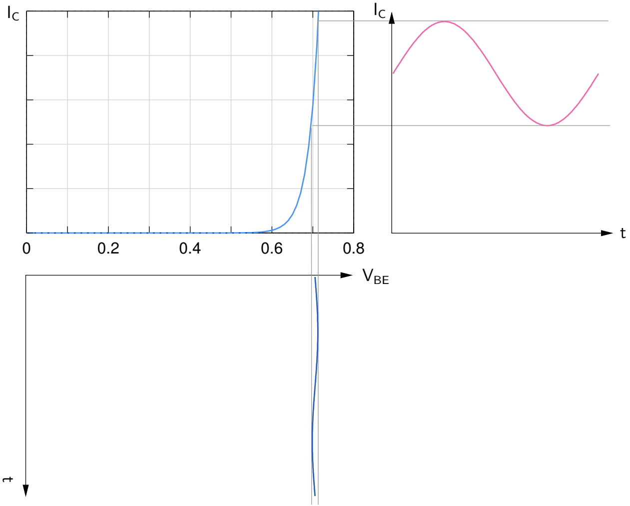




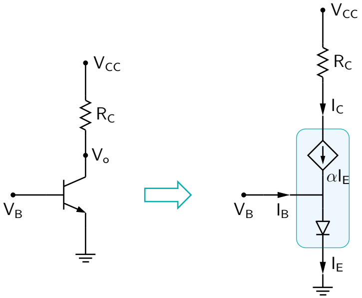


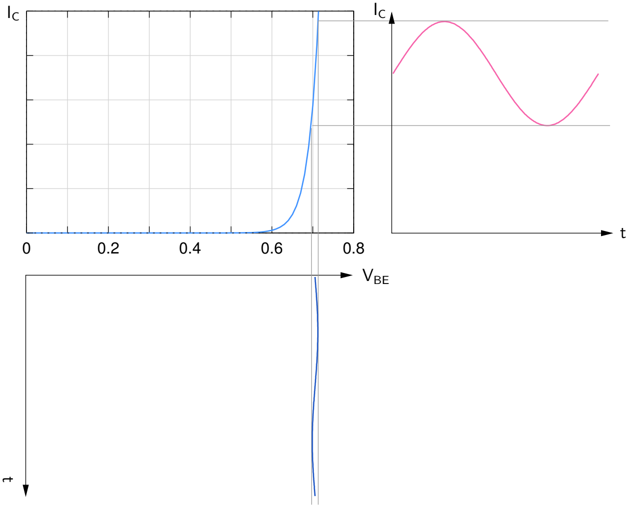
* In the active mode, I_C changes exponentially with V_{BE}: $I_C = \alpha_F I_{ES}\,[\exp(V_{BE}/V_T) - 1]$





* In the active mode, I_C changes exponentially with V_{BE}: $I_C = \alpha_F I_{ES} \left[\exp(V_{BE}/V_T) - 1\right]$
* $V_o(t) = V_{CC} - I_C(t)\,R_C$
  $\Rightarrow$ the amplitude of V_o, i.e., $\widehat{I_C}\,R_C$, can be made much larger than $\widehat{V_B}$.





* In the active mode, I_C changes exponentially with V_{BE}: $I_C = \alpha_F I_{ES} \left[\exp(V_{BE}/V_T) - 1\right]$
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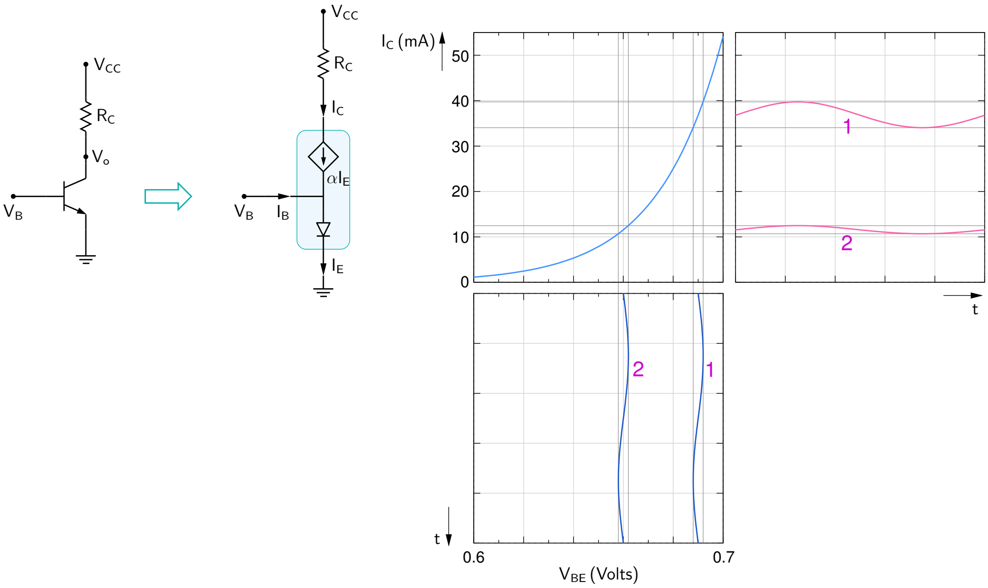
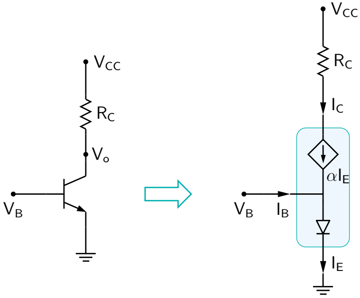
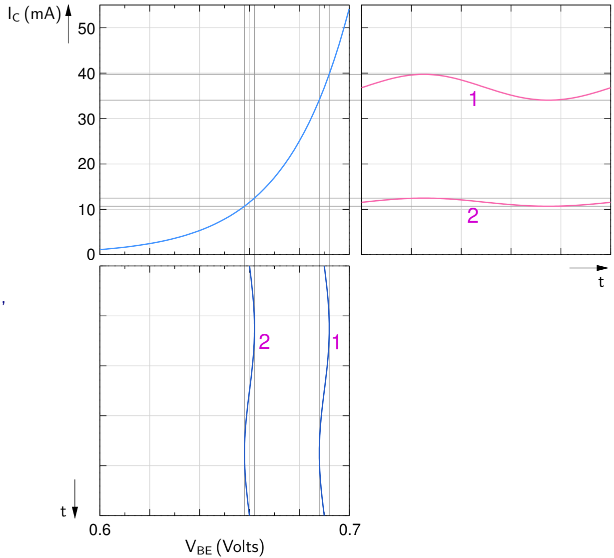
  $\Rightarrow$ the amplitude of V_o, i.e., $\widehat{I_C}\, R_C$, can be made much larger than $\widehat{V_B}$.
* Note that both the input (V_{BE}) and output (V_o) voltages have DC ("bias") components.

BJT amplifier: basic operation

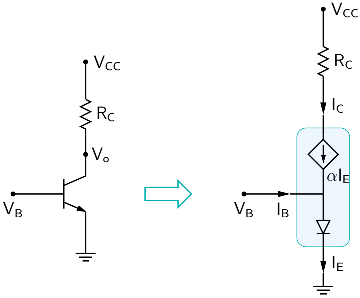


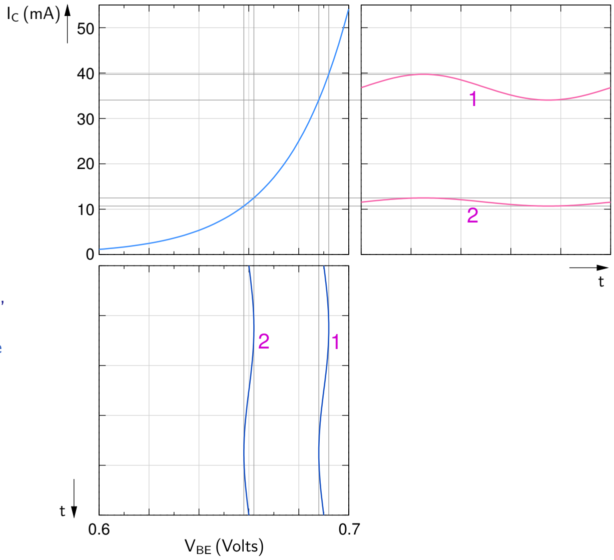




* The gain depends on the DC (bias) value of V_{BE}, the input voltage in this circuit.

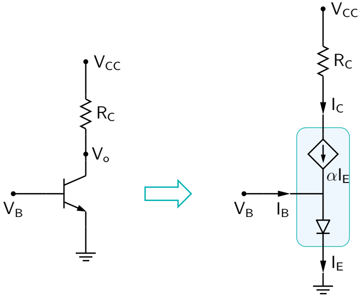
BJT amplifier: basic operation

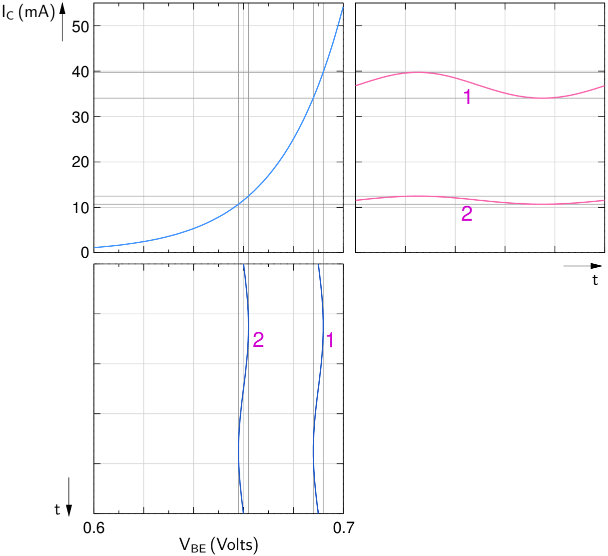




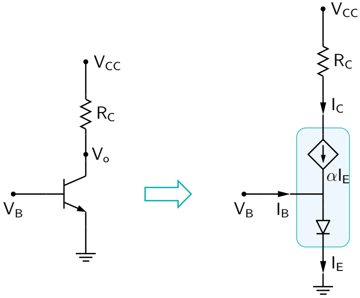
* The gain depends on the DC (bias) value of V_{BE}, the input voltage in this circuit.
* In practice, it is not possible to set the bias value of the input voltage to the desired value (e.g., 0.673 V).

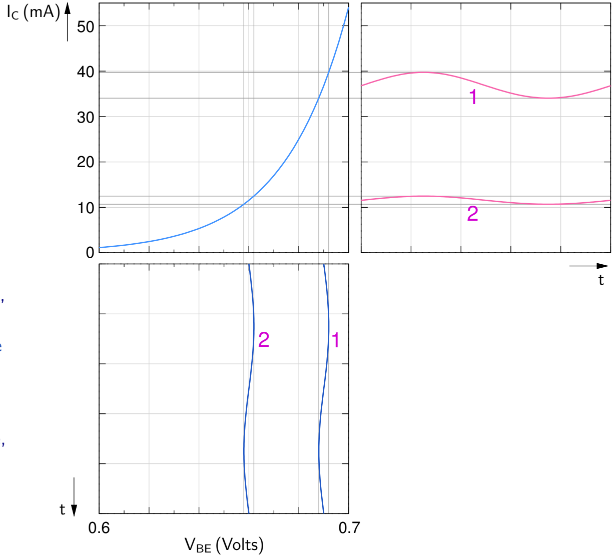
BJT amplifier: basic operation





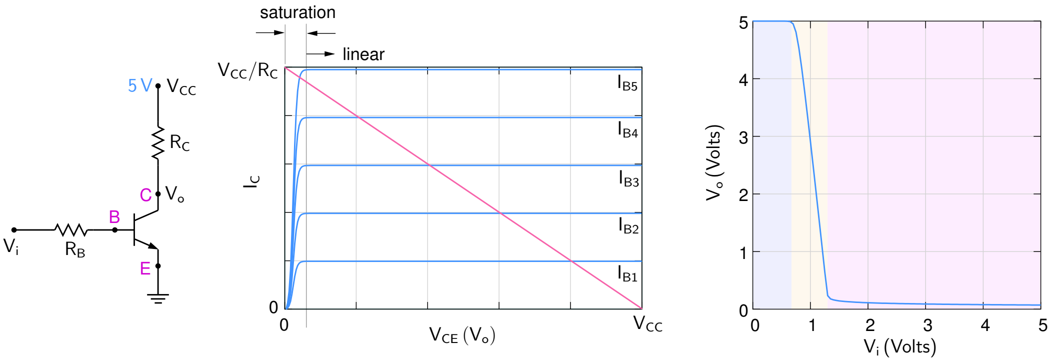
* The gain depends on the DC (bias) value of V_{BE}, the input voltage in this circuit.
* In practice, it is not possible to set the bias value of the input voltage to the desired value (e.g., 0.673 V).
* Even if we could set the input bias as desired, device-to-device variation, change in temperature, etc. would cause the gain to change.
  $\rightarrow$ need a better biasing method.



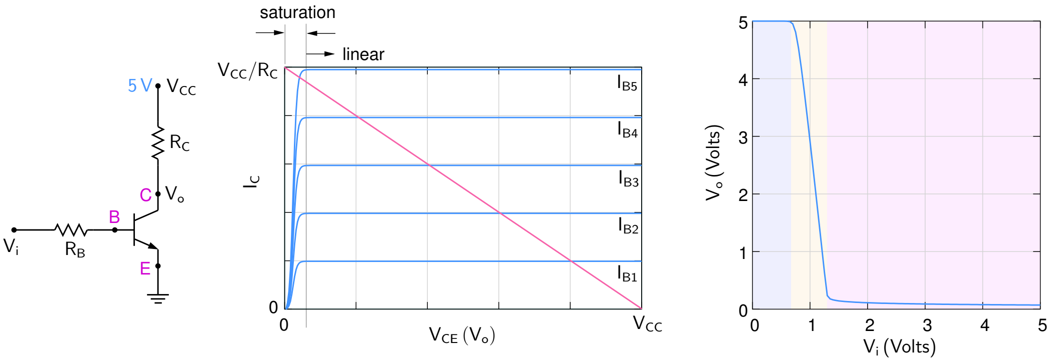


* The gain depends on the DC (bias) value of V_{BE}, the input voltage in this circuit.
* In practice, it is not possible to set the bias value of the input voltage to the desired value (e.g., 0.673 V).
* Even if we could set the input bias as desired, device-to-device variation, change in temperature, etc. would cause the gain to change.
  $\rightarrow$ need a better biasing method.
* Biasing the transistor at a specific V_{BE} is equivalent to biasing it at a specific I_C.

BJT amplifier biasing

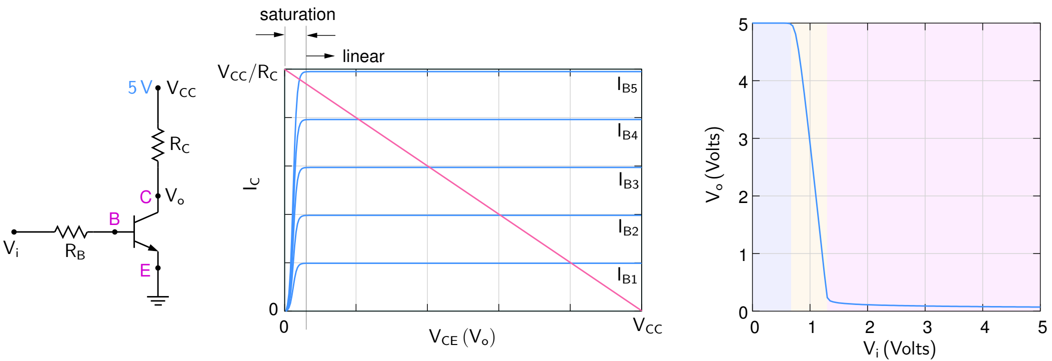


Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).



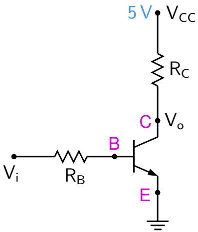
Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).

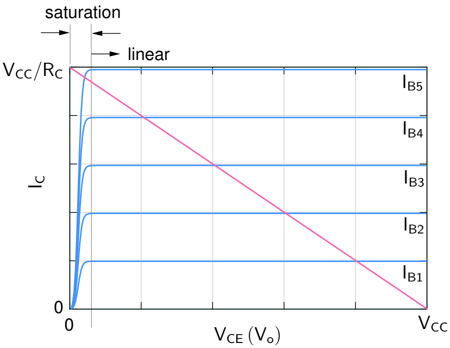
* When $V_i < 0.7\,\text{V}$, the B-E junction is not sufficiently forward biased, and the BJT is in the cut-off mode ($V_{BE} = V_i,\ V_{BC} = V_i - V_{CC}$)

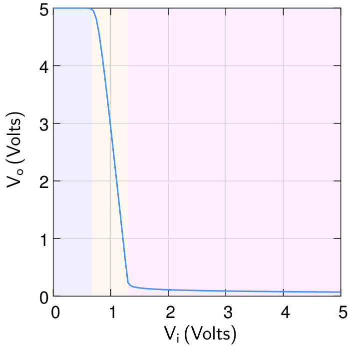


Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).

* When $V_i < 0.7\,\text{V}$, the B-E junction is not sufficiently forward biased, and the BJT is in the cut-off mode ($V_{BE} = V_i$, $V_{BC} = V_i - V_{CC}$)
* When V_i exceeds 0.7 V, the BJT enters the linear region, and $I_B \approx \dfrac{V_i - 0.7}{R_B}$. As V_i increases, I_B and $I_C = \beta I_B$ also increase, and $V_o = V_{CC} - I_C R_C$ falls.

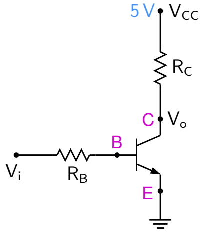


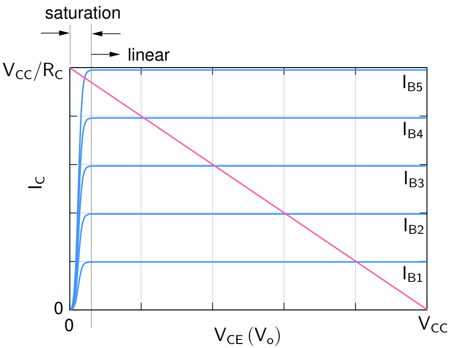


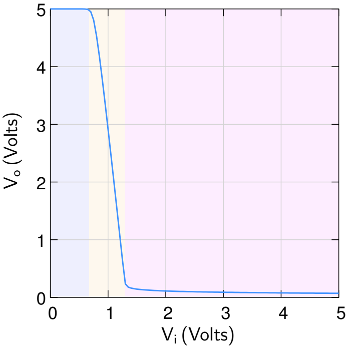


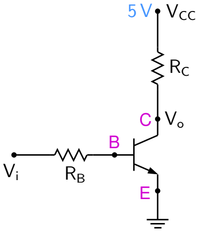
Consider a more realistic BJT amplifier circuit, with R_B added to limit the base current (and thus protect the transistor).

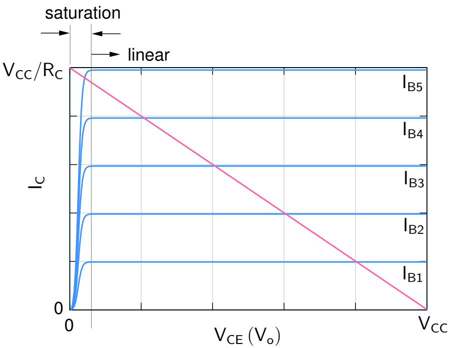
* When $V_i < 0.7\,\text{V}$, the B-E junction is not sufficiently forward biased, and the BJT is in the cut-off mode ($V_{BE} = V_i$, $V_{BC} = V_i - V_{CC}$)
* When V_i exceeds 0.7 V, the BJT enters the linear region, and $I_B \approx \dfrac{V_i - 0.7}{R_B}$. As V_i increases, I_B and $I_C = \beta I_B$ also increase, and $V_o = V_{CC} - I_C R_C$ falls.
* As V_i is increased further, V_o reaches V_{CE}^{sat} (about 0.2 V), and the BJT enters the saturation region (both B-E and B-C junctions are forward biased).

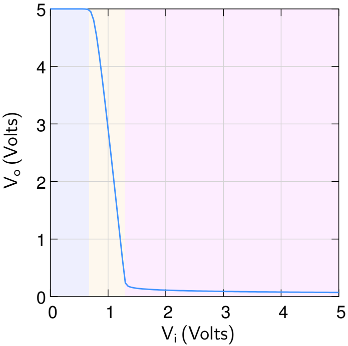

5 V
V_{CC}
R_C
C
V_o
B
V_i
R_B
E


saturation
linear
V_{CC}/R_C
I_C
I_{B5}
I_{B4}
I_{B3}
I_{B2}
I_{B1}
0
0
V_{CE} (V_o)
V_{CC}


V_o (Volts)
0
1
2
3
4
5
V_i (Volts)

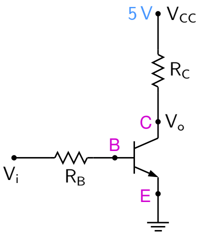


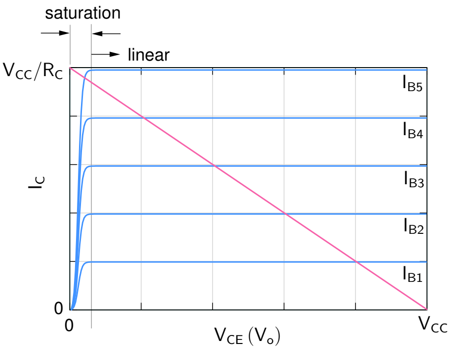


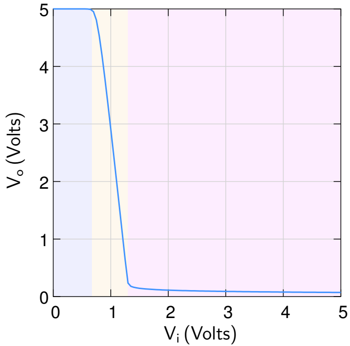


* The gain of the amplifier is given by $\dfrac{dV_o}{dV_i}$.

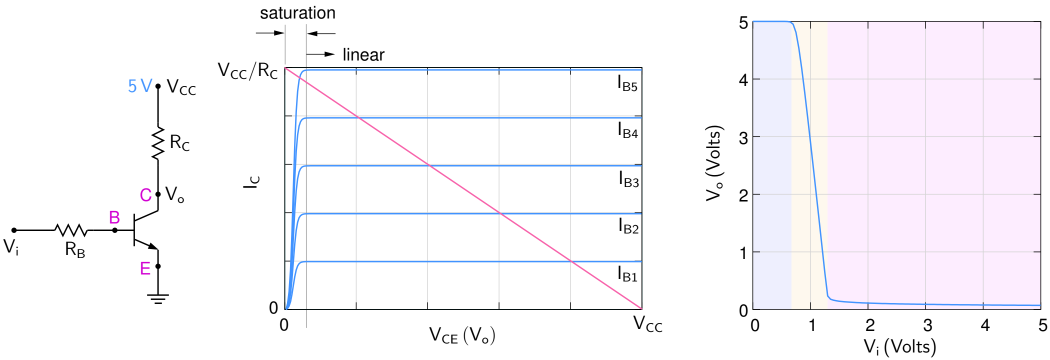
BJT amplifier biasing







* The gain of the amplifier is given by $\dfrac{dV_o}{dV_i}$.
* Since V_o is nearly constant for $V_i < 0.7\ V$ (due to cut-off) and $V_i > 1.3\ V$ (due to saturation), the circuit will not work an an amplifier in this range.



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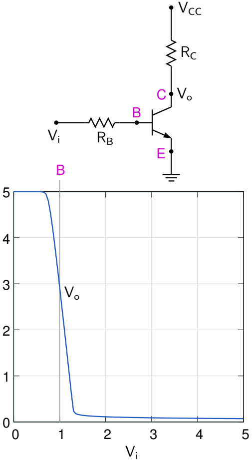
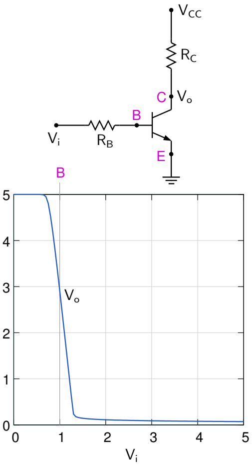
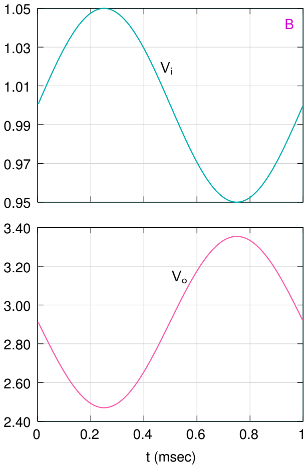
* Since V_o is nearly constant for $V_i < 0.7\ V$ (due to cut-off) and $V_i > 1.3\ V$ (due to saturation), the circuit will not work an an amplifier in this range.
* Further, to get a large swing in V_o without distortion, the DC bias of V_i should be at the centre of the amplifying region, i.e., $V_i \approx 1\ V$.

BJT amplifier biasing

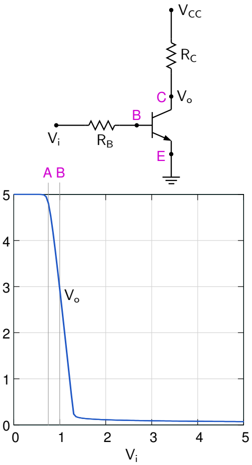


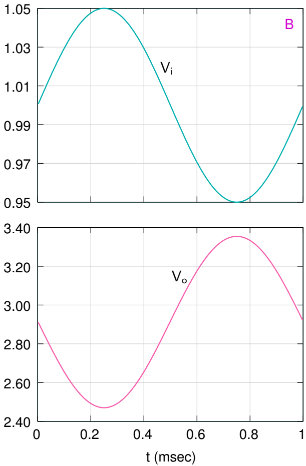
BJT amplifier biasing





BJT amplifier biasing





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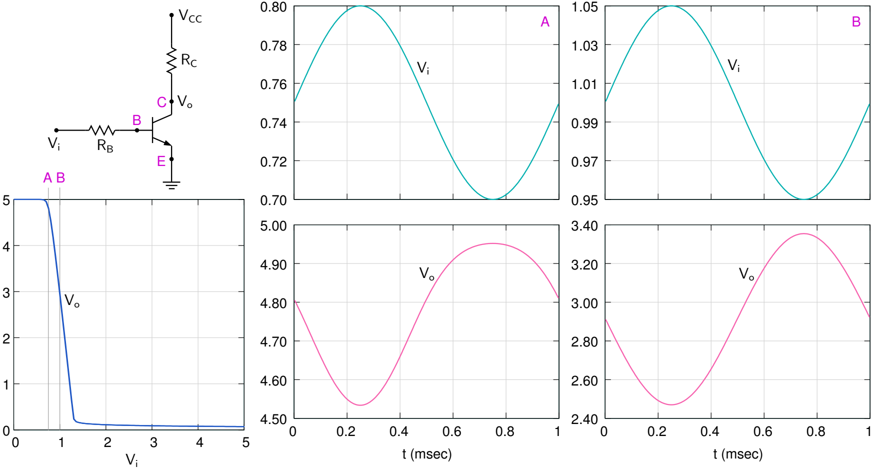

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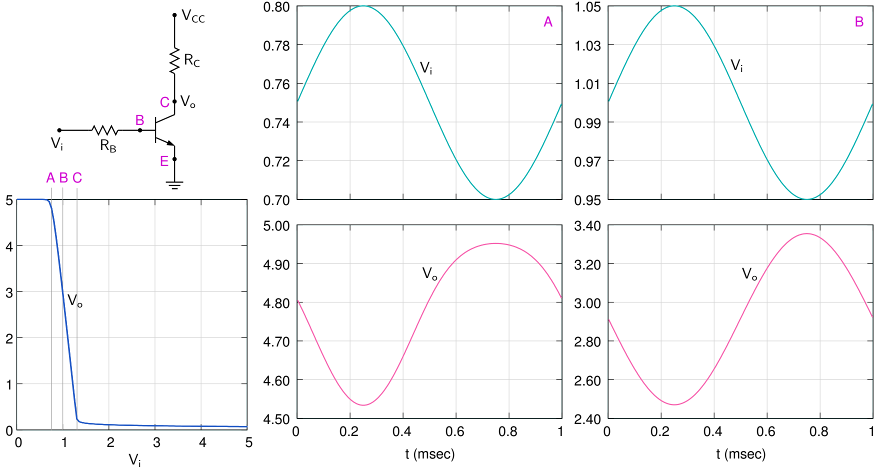
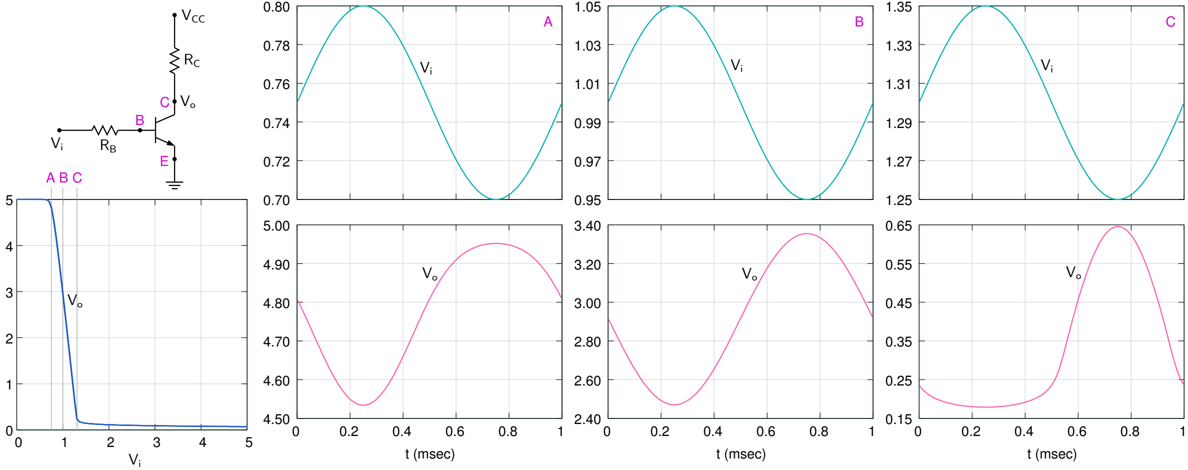
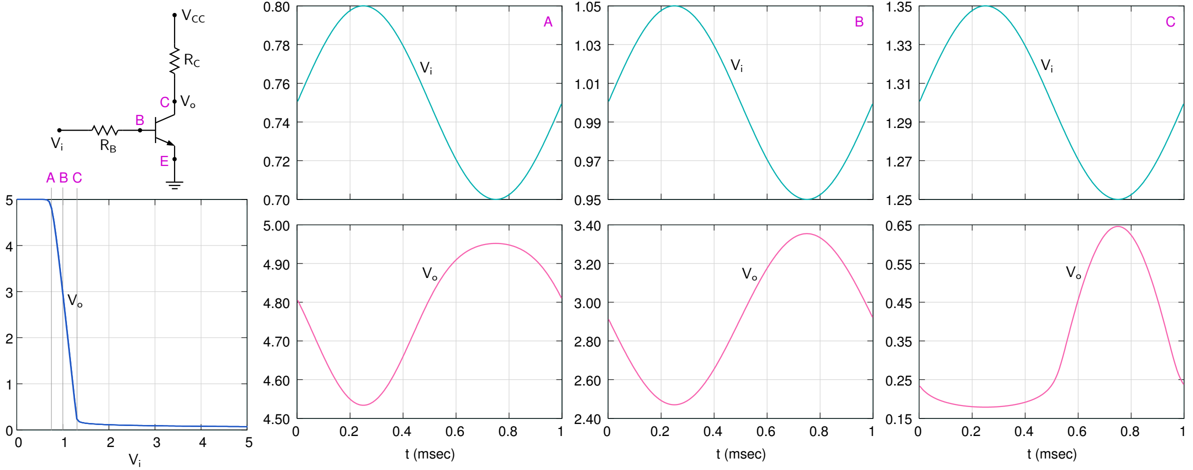


BJT amplifier biasing



BJT amplifier biasing



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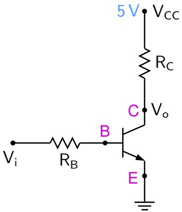
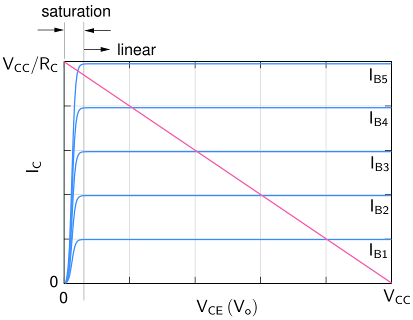
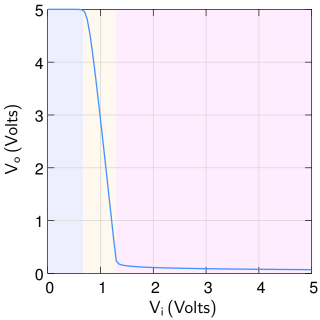
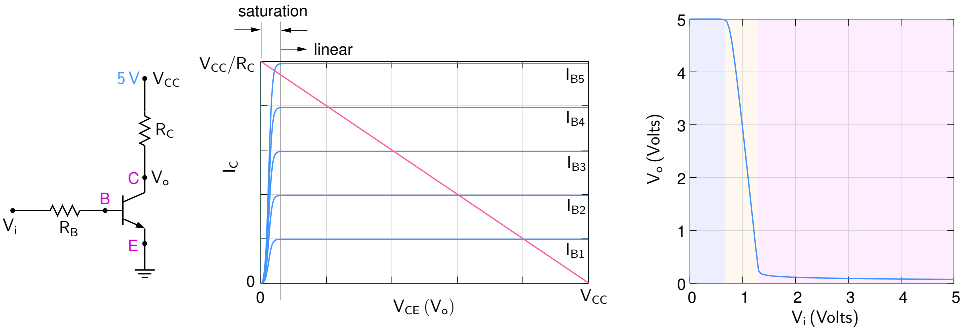


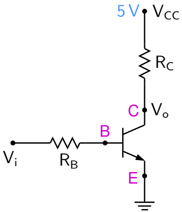


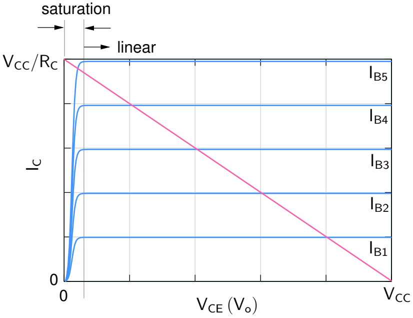


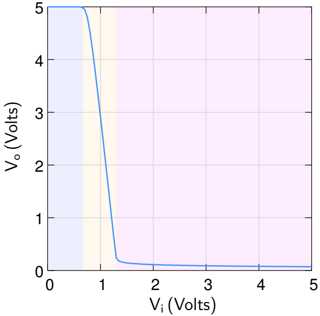
* The key challenges in realizing this amplifier in practice are



* The key challenges in realizing this amplifier in practice are
  - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of V_{BE} (or I_C).

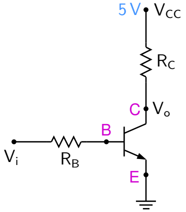


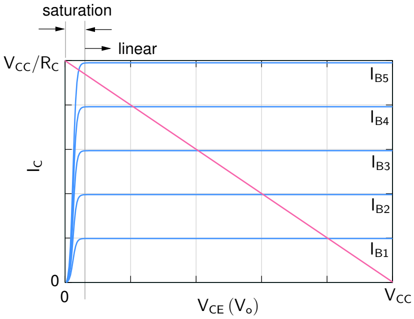


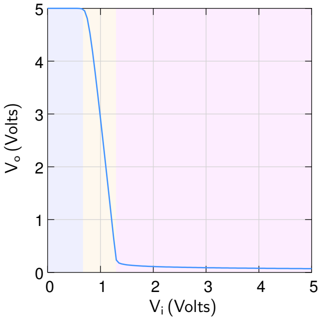


* The key challenges in realizing this amplifier in practice are
  - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of V_{BE} (or I_C).
  - mixing the input DC bias with the signal voltage.

BJT amplifier







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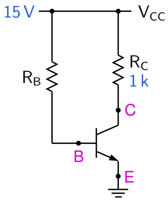
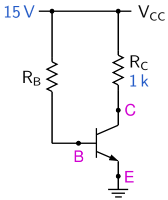
  - adjusting the input DC bias to ensure that the BJT remains in the linear (active) region with a certain bias value of V_{BE} (or I_C).
  - mixing the input DC bias with the signal voltage.
* The first issue is addressed by using a suitable biasing scheme, and the second by using "coupling" capacitors.

BJT amplifier: a simple biasing scheme



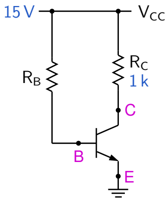
"Biasing" an amplifier ⇒ selection of component values for a certain DC value of I_C (or V_{BE}) (i.e., when no signal is applied).



"Biasing" an amplifier $\Rightarrow$ selection of component values for a certain DC value of I_C (or V_{BE}) (i.e., when no signal is applied).

Equivalently, we may bias an amplifier for a certain DC value of V_{CE}, since I_C and V_{CE} are related: $V_{CE} + I_C R_C = V_{CC}$.

BJT amplifier: a simple biasing scheme



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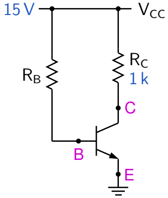
As an example, for $R_C = 1\,\mathrm{k}$, $\beta = 100$, let us calculate R_B for $I_C = 3.3\,\mathrm{m}A$, assuming the BJT to be operating in the active mode.

BJT amplifier: a simple biasing scheme



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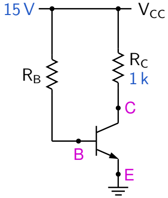
As an example, for $R_C = 1\,\mathrm{k}$, $\beta = 100$, let us calculate R_B for $I_C = 3.3\,\mathrm{m}A$, assuming the BJT to be operating in the active mode.

$$I_B = \frac{I_C}{\beta} = \frac{3.3\,\mathrm{m}A}{100} = 33\,\mu A = \frac{V_{CC} - V_{BE}}{R_B} = \frac{15 - 0.7}{R_B}$$

BJT amplifier: a simple biasing scheme



"Biasing" an amplifier $\Rightarrow$ selection of component values for a certain DC value of I_C (or V_{BE}) (i.e., when no signal is applied).

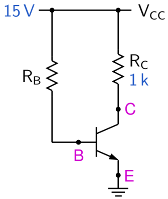
Equivalently, we may bias an amplifier for a certain DC value of V_{CE}, since I_C and V_{CE} are related: $V_{CE} + I_C R_C = V_{CC}$.

As an example, for $R_C = 1\,\text{k}$, $\beta = 100$, let us calculate R_B for $I_C = 3.3\,\text{m}A$, assuming the BJT to be operating in the active mode.

$$I_B = \frac{I_C}{\beta} = \frac{3.3\,\text{m}A}{100} = 33\,\mu A = \frac{V_{CC} - V_{BE}}{R_B} = \frac{15 - 0.7}{R_B}$$

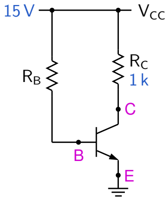
$$\rightarrow R_B = \frac{14.3\,V}{33\,\mu A} = 430\,\text{k}\Omega\,.$$

BJT amplifier: a simple biasing scheme (continued)



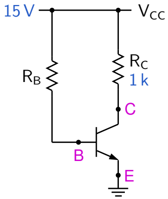
With $R_B = 430\,\text{k}$, we expect $I_C = 3.3\,\text{m}A$, assuming $\beta = 100$.

BJT amplifier: a simple biasing scheme (continued)



With $R_B = 430\,\text{k}$, we expect $I_C = 3.3\,\text{m}A$, assuming $\beta = 100$.

However, in practice, there is a substantial variation in the β value (even for the same transistor type). The manufacturer may specify the nominal value of β as 100, but the actual value may be 150, for example.



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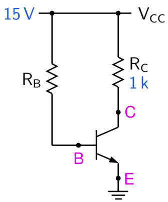
However, in practice, there is a substantial variation in the β value (even for the same transistor type). The manufacturer may specify the nominal value of β as 100, but the actual value may be 150, for example.

With $\beta = 150$, the actual I_C is,

$$I_C = \beta \times \frac{V_{CC} - V_{BE}}{R_B} = 150 \times \frac{(15 - 0.7)\,V}{430\,\text{k}} = 5\,\text{m}A\,,$$

which is significantly different than the intended value, viz., 3.3 mA.



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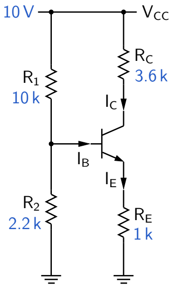
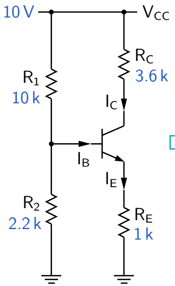
which is significantly different than the intended value, viz., 3.3 mA.

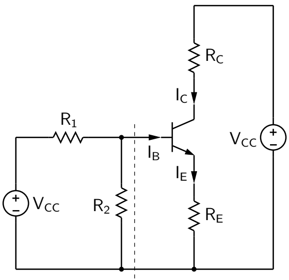
$\rightarrow$ need a biasing scheme which is not so sensitive to β.

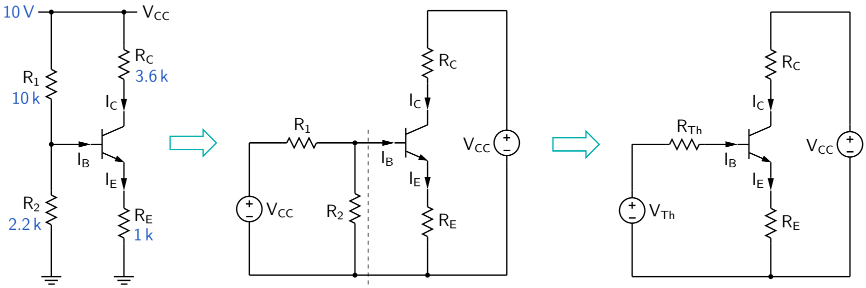
BJT amplifier: improved biasing scheme



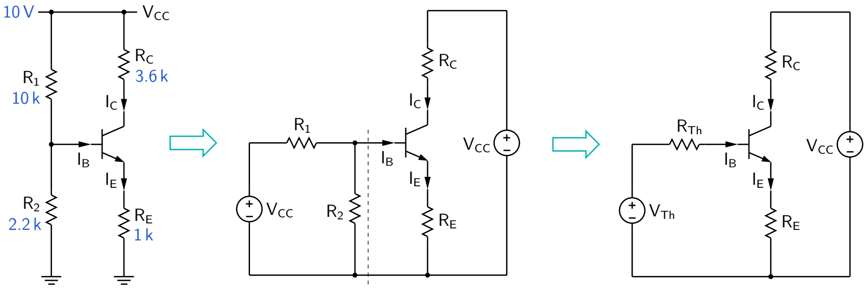
BJT amplifier: improved biasing scheme





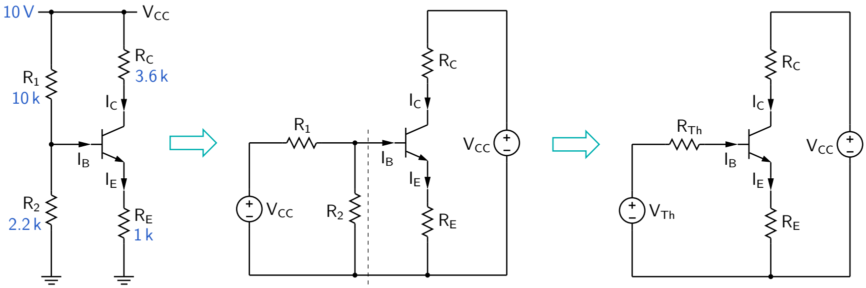

10 V
V_CC
R_C
3.6 k
R_1
10 k
I_C
I_B
I_E
R_2
2.2 k
R_E
1 k
R_1
R_C
I_C
I_B
V_CC
I_E
V_CC
R_2
R_E
R_Th
R_C
I_C
V_CC
I_B
I_E
V_Th
R_E

BJT amplifier: improved biasing scheme



$$V_{Th} = \frac{R_2}{R_1 + R_2}\, V_{CC} = \frac{2.2\,\text{k}}{10\,\text{k} + 2.2\,\text{k}} \times 10\ V = 1.8\ V, \quad R_{Th} = R_1 \parallel R_2 = 1.8\,\text{k}$$

BJT amplifier: improved biasing scheme



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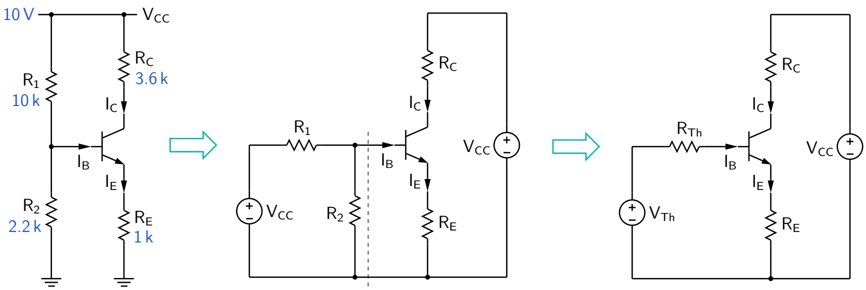
Assuming the BJT to be in the active mode,

KVL: $V_{Th} = R_{Th}\, I_B + V_{BE} + R_E\, I_E = R_{Th}\, I_B + V_{BE} + (\beta + 1)\, I_B\, R_E$

BJT amplifier: improved biasing scheme



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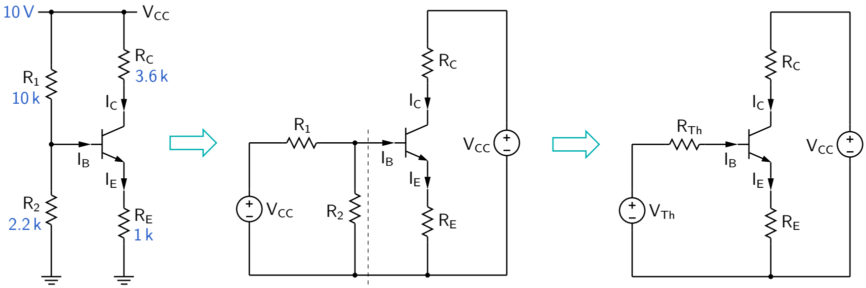
BJT amplifier: improved biasing scheme



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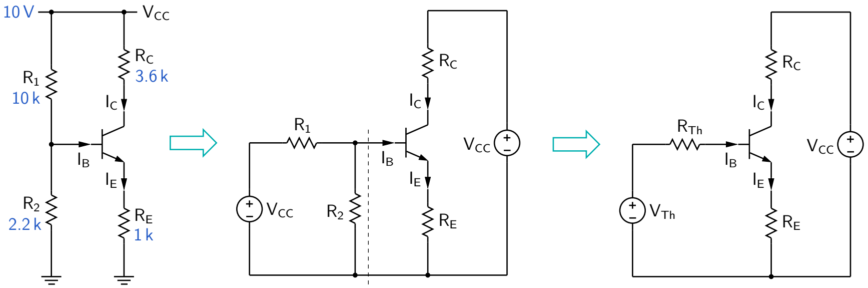
For $\beta = 100$, I_C=1.07 mA.



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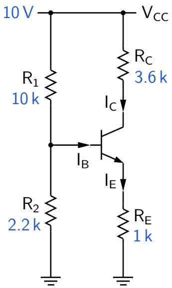
For $\beta = 100$, I_C=1.07 mA.

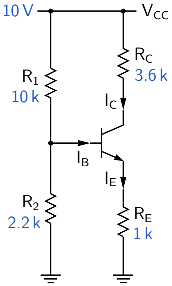
For $\beta = 200$, I_C=1.085 mA.

BJT amplifier: improved biasing scheme (continued)



With $I_C = 1.1\,\mathrm{m}A$, the various DC ("bias") voltages are

BJT amplifier: improved biasing scheme (continued)



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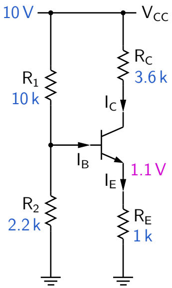

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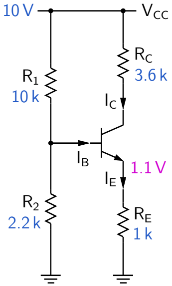

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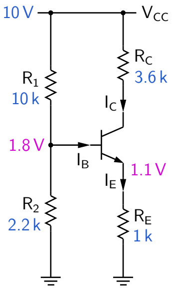
BJT amplifier: improved biasing scheme (continued)



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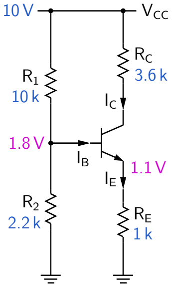
BJT amplifier: improved biasing scheme (continued)



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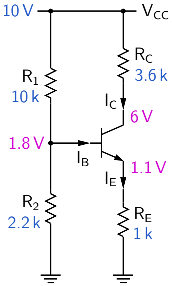
$$V_C = V_{CC} - I_C\,R_C = 10\,V - 1.1\,\mathrm{m}A \times 3.6\,\mathrm{k} \approx 6\,V,$$

BJT amplifier: improved biasing scheme (continued)



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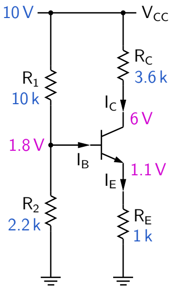

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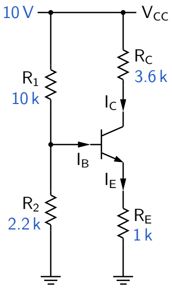
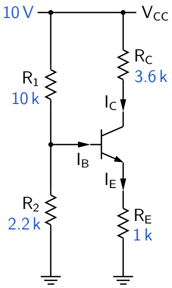
$V_{CE} = V_C - V_E = 6 - 1.1 = 4.9\,V.$



A quick estimate of the bias values can be obtained by ignoring I_B (which is fair if β is large). In that case,



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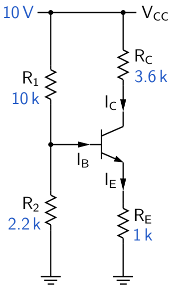

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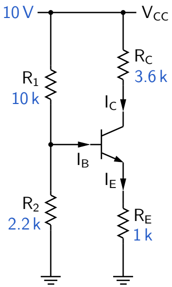
BJT amplifier: improved biasing scheme (continued)



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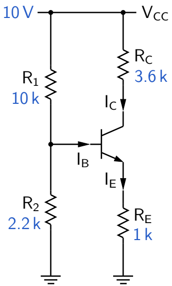
BJT amplifier: improved biasing scheme (continued)



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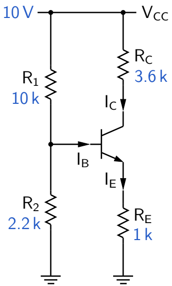

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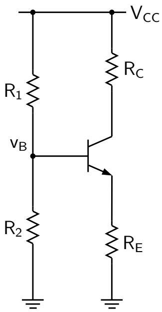
$$V_{CE} = V_{CC} - I_C\, R_C - I_E\, R_E = 10\ V - (3.6\,\text{k} \times 1.1\,\text{m}A) - (1\,\text{k} \times 1.1\,\text{m}A) \approx 5\ V.$$

Adding signal to bias



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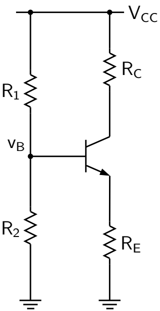
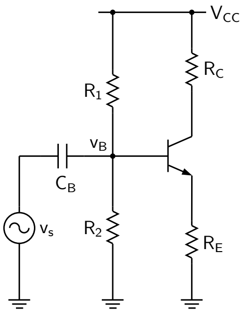


* As we have seen earlier, the input signal $v_s(t) = \widehat{V}\ \sin\,\omega t$ (for example) needs to be mixed with the desired bias value V_B so that the net voltage at the base is $v_B(t) = V_B + \widehat{V}\ \sin\,\omega t$.

Adding signal to bias



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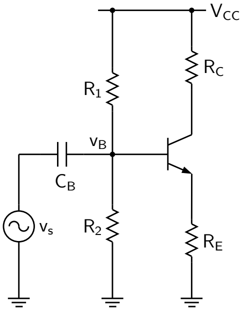
* This can be achieved by using a coupling capacitor C_B.

Adding signal to bias



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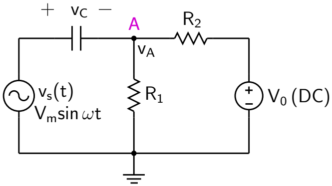
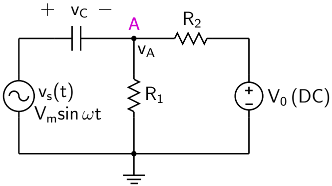
* This can be achieved by using a coupling capacitor C_B.
* Let us consider a simple circuit to illustrate how a coupling capacitor works.



We are interested in the solution (currents and voltages) in the "sinusoidal steady state" when the exponential transients have vanished and each quantity $x(t)$ is of the form X_0 (constant) $+ X_m \sin(\omega t + \alpha)$.



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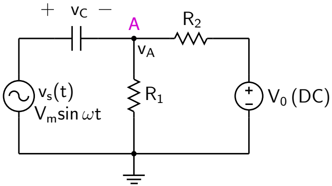
There are two ways to obtain the solution:



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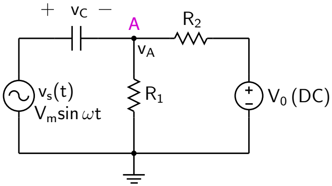
There are two ways to obtain the solution:

(1) Solve the circuit equations directly:

$$\frac{v_A(t)}{R_1} + \frac{v_A(t) - V_0}{R_2} = C\,\frac{d}{dt}\,(v_s(t) - v_A(t))\,.$$



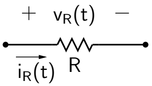
We are interested in the solution (currents and voltages) in the "sinusoidal steady state" when the exponential transients have vanished and each quantity $x(t)$ is of the form X_0 (constant) $+ X_m \sin(\omega t + \alpha)$.

There are two ways to obtain the solution:

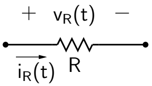
(1) Solve the circuit equations directly:

$$\frac{v_A(t)}{R_1} + \frac{v_A(t) - V_0}{R_2} = C \frac{d}{dt} \left(v_s(t) - v_A(t) \right).$$

(2) Use the DC circuit + AC circuit approach.

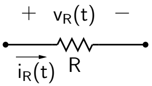


+ $v_R(t)$ −
$i_R(t)$
R



Let $v_R(t) = V_R + v_r(t)$ where V_R = constant, $v_r(t) = \widehat{V}_R \sin(\omega t + \alpha)$,

$i_R(t) = I_R + i_r(t)$ where I_R = constant, $i_r(t) = \widehat{I}_R \sin(\omega t + \alpha)$.

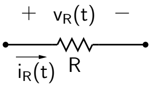


Let $v_R(t) = V_R + v_r(t)$ where V_R = constant, $v_r(t) = \widehat{V}_R \sin(\omega t + \alpha)$,

$i_R(t) = I_R + i_r(t)$ where I_R = constant, $i_r(t) = \widehat{I}_R \sin(\omega t + \alpha)$.

Since $v_R(t) = R \times i_R(t)$, we get $[V_R + v_r(t)] = R \times [I_R + i_r(t)]$.

Resistor in sinusoidal steady state



Let $v_R(t) = V_R + v_r(t)$ where V_R = constant, $v_r(t) = \widehat{V}_R \sin(\omega t + \alpha)$,

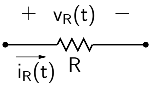
$i_R(t) = I_R + i_r(t)$ where I_R = constant, $i_r(t) = \widehat{I}_R \sin(\omega t + \alpha)$.

Since $v_R(t) = R \times i_R(t)$, we get $[V_R + v_r(t)] = R \times [I_R + i_r(t)]$.

This relationship can be split into two:

$V_R = R \times I_R$, and $v_r(t) = R \times i_r(t)$.

Resistor in sinusoidal steady state



Let $v_R(t) = V_R + v_r(t)$ where V_R = constant, $v_r(t) = \widehat{V}_R \sin(\omega t + \alpha)$,

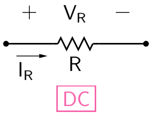
$i_R(t) = I_R + i_r(t)$ where I_R = constant, $i_r(t) = \widehat{I}_R \sin(\omega t + \alpha)$.

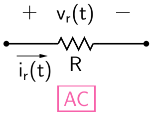
Since $v_R(t) = R \times i_R(t)$, we get $[V_R + v_r(t)] = R \times [I_R + i_r(t)]$.

This relationship can be split into two:

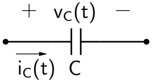
$V_R = R \times I_R$, and $v_r(t) = R \times i_r(t)$.

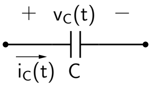
In other words, a resistor can be described by





Capacitor in sinusoidal steady state

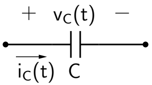




Let $v_C(t) = V_C + v_c(t)$ where V_C = constant, $v_c(t) = \widehat{V}_C \sin(\omega t + \alpha)$,

$i_C(t) = I_C + i_c(t)$ where I_C = constant, $i_c(t) = \widehat{I}_C \sin(\omega t + \beta)$.

Capacitor in sinusoidal steady state

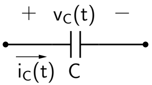


Let $v_C(t) = V_C + v_c(t)$ where V_C = constant, $v_c(t) = \widehat{V}_C \sin(\omega t + \alpha)$,

$i_C(t) = I_C + i_c(t)$ where I_C = constant, $i_c(t) = \widehat{I}_C \sin(\omega t + \beta)$.

Since $i_C(t) = C\,\dfrac{dv_C}{dt}$, we get $[I_C + i_c(t)] = C\,\dfrac{d}{dt}\,(V_C + v_c(t))$.

Capacitor in sinusoidal steady state



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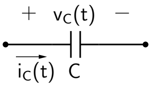
Since $i_C(t) = C\,\dfrac{dv_C}{dt}$, we get $[I_C + i_c(t)] = C\,\dfrac{d}{dt}\,(V_C + v_c(t))$.

This relationship can be split into two:

$I_C = C\,\dfrac{dV_C}{dt} = 0$, and $i_c(t) = C\,\dfrac{dv_c}{dt}$.

Capacitor in sinusoidal steady state



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Since $i_C(t) = C\,\dfrac{dv_C}{dt}$, we get $[I_C + i_c(t)] = C\,\dfrac{d}{dt}\,(V_C + v_c(t))$.

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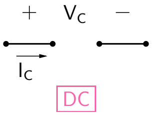
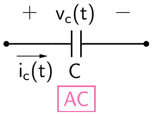
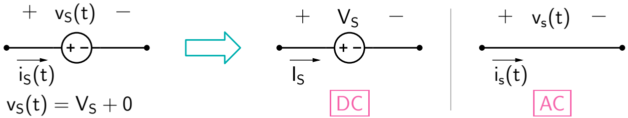
In other words, a capacitor can be described by



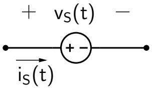


DC voltage source:

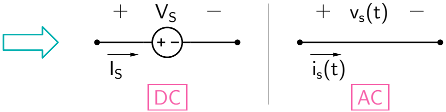


Voltage sources in sinusoidal steady state

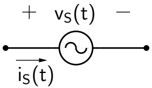
DC voltage source:



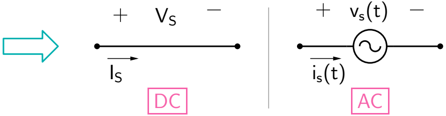
$v_S(t) = V_S + 0$



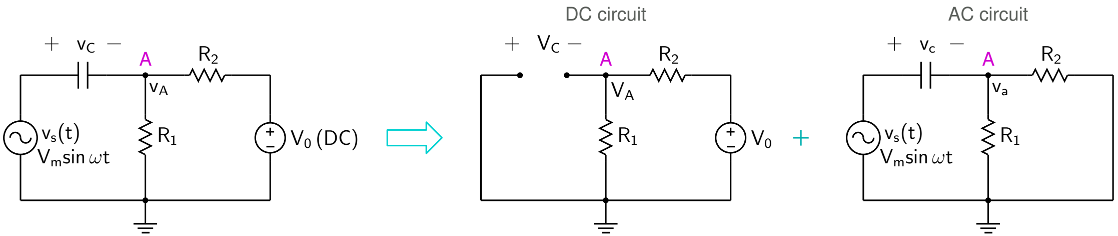
AC voltage source:



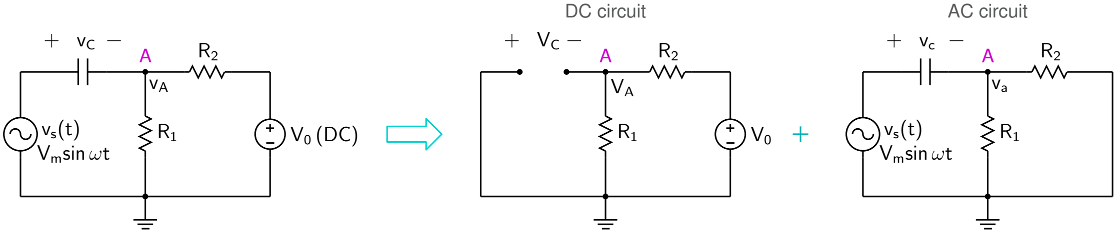
$v_S(t) = 0 + v_s(t)$



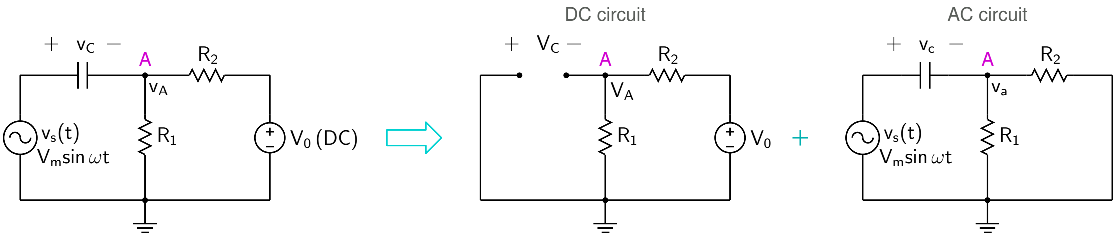
RC circuit with DC + AC sources



RC circuit with DC + AC sources



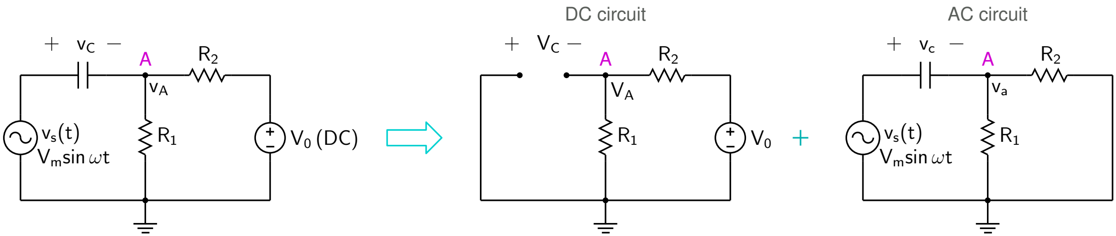
DC circuit: $$\frac{V_A}{R_1} + \frac{V_A - V_0}{R_2} = 0. \quad (1)$$



DC circuit: $$\frac{V_A}{R_1} + \frac{V_A - V_0}{R_2} = 0. \quad (1)$$

AC circuit: $$\frac{v_a}{R_1} + \frac{v_a}{R_2} = C\,\frac{d}{dt}\,(v_s - v_a). \quad (2)$$

RC circuit with DC + AC sources



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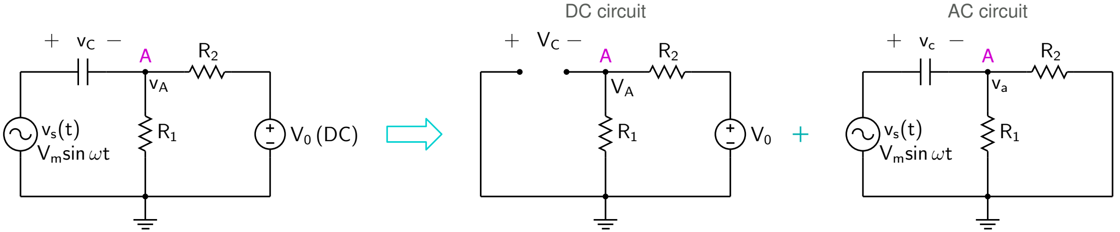
Adding (1) and (2), we get $$\frac{V_A + v_a}{R_1} + \frac{V_A + v_a - V_0}{R_2} = C\,\frac{d}{dt}\,(v_s - v_a). \quad (3)$$



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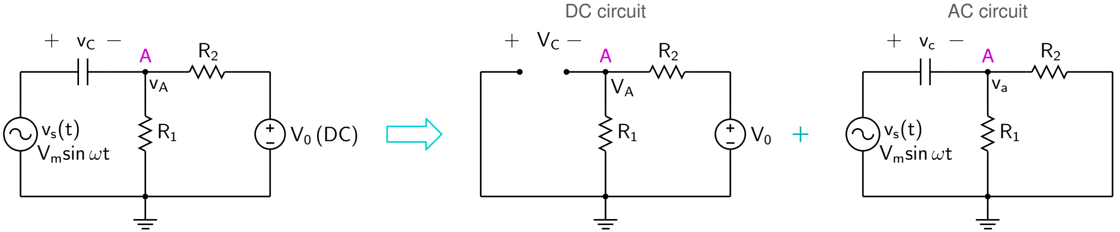
Compare with the equation obtained directly from the original circuit:

$$\frac{v_A}{R_1} + \frac{v_A - V_0}{R_2} = C\,\frac{d}{dt}\,(v_s - v_A). \tag{4}$$



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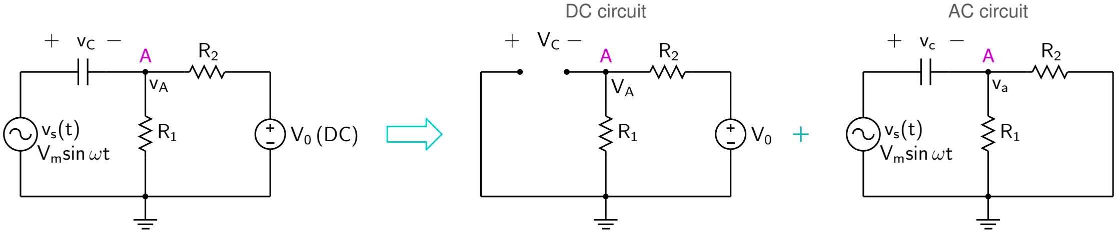
Eqs. (3) and (4) are identical since $v_A = V_A + v_a$.



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Compare with the equation obtained directly from the original circuit:

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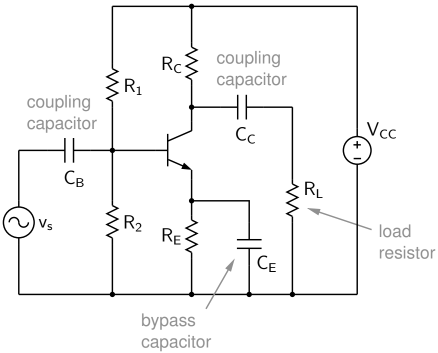
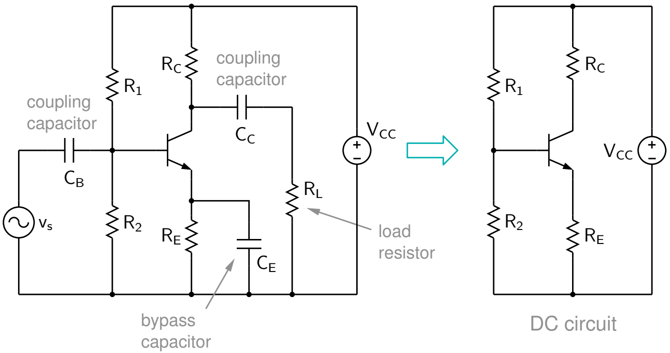
Eqs. (3) and (4) are identical since $v_A = V_A + v_a$.

→ Instead of computing $v_A(t)$ directly, we can compute V_A and $v_a(t)$ separately, and then use $v_A(t) = V_A + v_a(t)$.

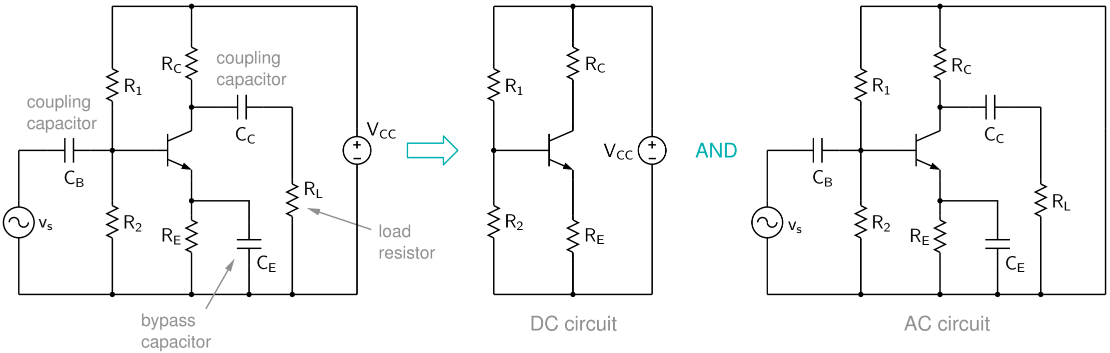
Common-emitter amplifier



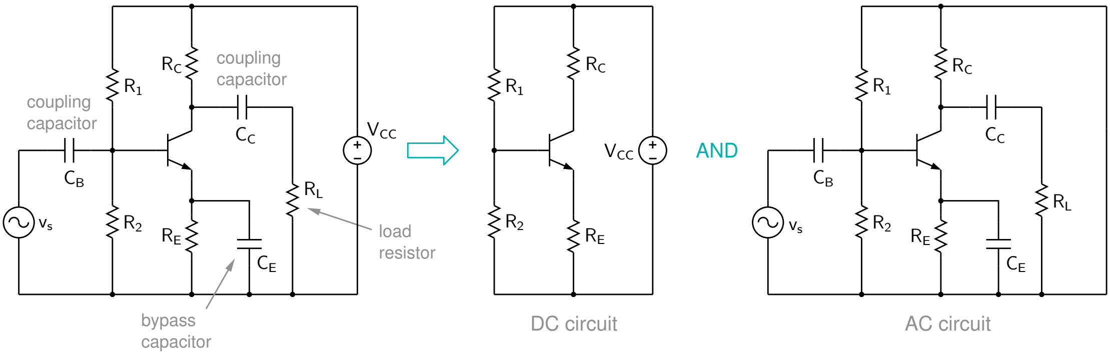
Common-emitter amplifier



Common-emitter amplifier

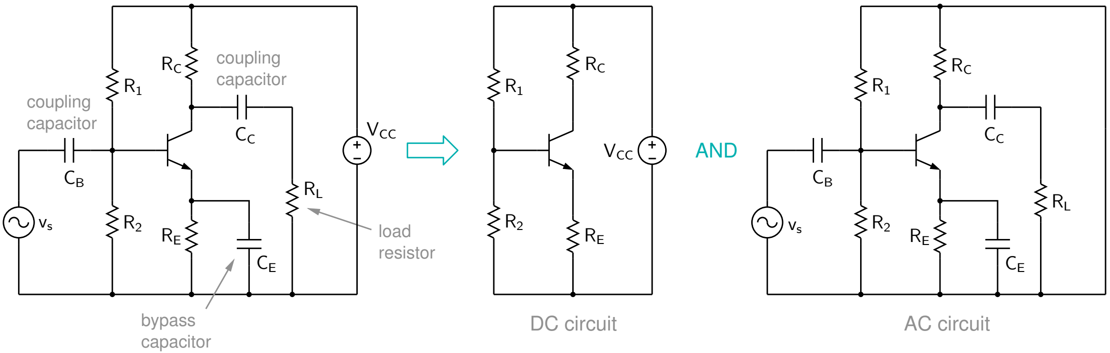


Common-emitter amplifier



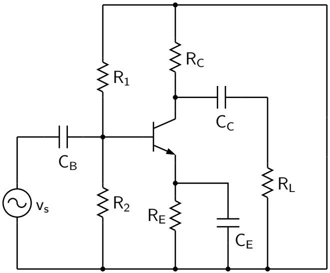
* The coupling capacitors ensure that the signal source and the load resistor do not affect the DC bias of the amplifier. (We will see the purpose of C_E a little later.)

Common-emitter amplifier

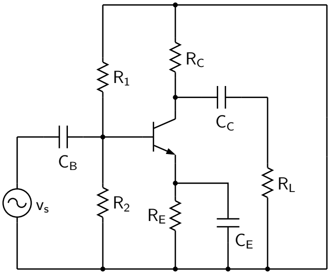


* The coupling capacitors ensure that the signal source and the load resistor do not affect the DC bias of the amplifier. (We will see the purpose of C_E a little later.)
* This enables us to bias the amplifier without worrying about what load it is going to drive.

Common-emitter amplifier: AC circuit



Common-emitter amplifier: AC circuit

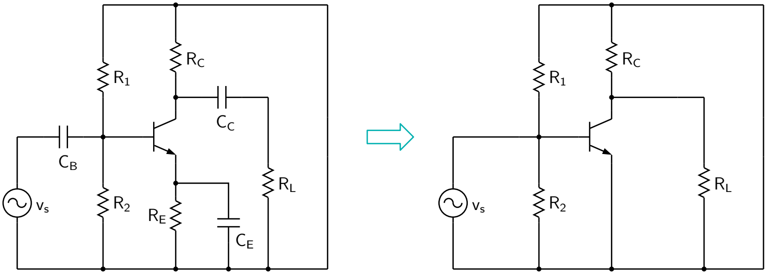


* The coupling and bypass capacitors are "large" (typically, a few μF), and at frequencies of interest, their impedance is small.
  For example, for $C = 10\,\mu F$, $f = 1\,\text{kHz}$,

  $$Z_C = \frac{1}{2\pi \times 10^3 \times 10 \times 10^{-6}} = 16\,\Omega,$$

  which is much smaller than typical values of R_1, R_2, R_C, R_E (a few kΩ).
  $\Rightarrow$ C_B, C_C, C_E can be replaced by short circuits at the frequencies of interest.

Common-emitter amplifier: AC circuit



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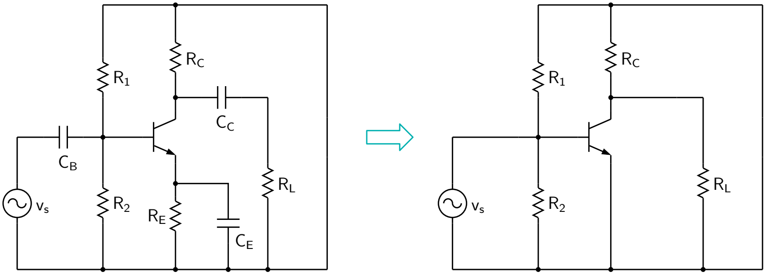

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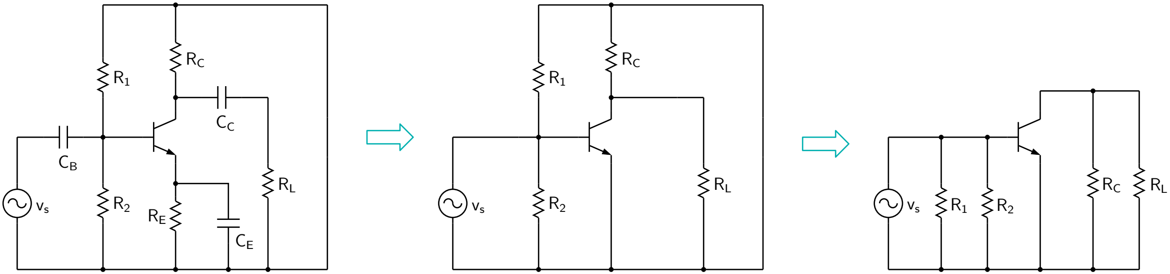
  $\Rightarrow$ C_B, C_C, C_E can be replaced by short circuits at the frequencies of interest.
* The circuit can be re-drawn in a more friendly format.

Common-emitter amplifier: AC circuit



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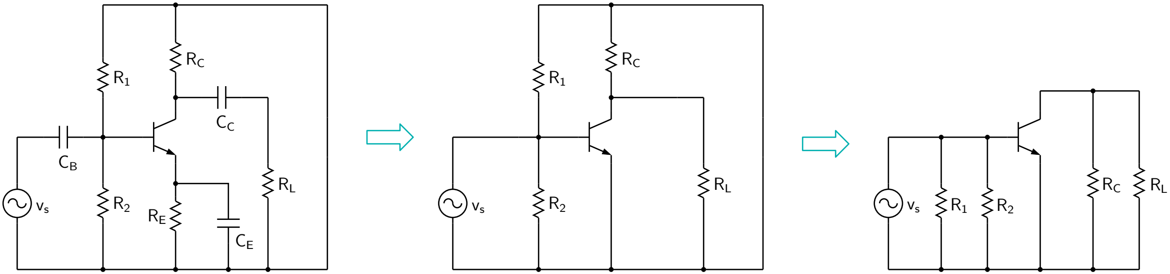
Common-emitter amplifier: AC circuit



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  For example, for $C = 10\,\mu F$, $f = 1\,\text{kHz}$,
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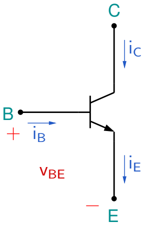
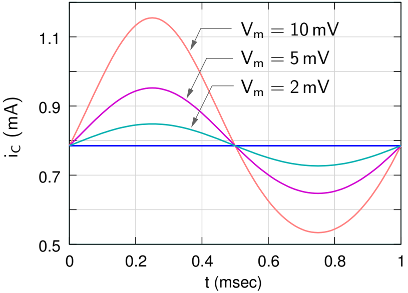
  which is much smaller than typical values of R_1, R_2, R_C, R_E (a few kΩ).
  $\Rightarrow$ C_B, C_C, C_E can be replaced by short circuits at the frequencies of interest.
* The circuit can be re-drawn in a more friendly format.
* We now need to figure out the AC description of a BJT.

BJT: AC model

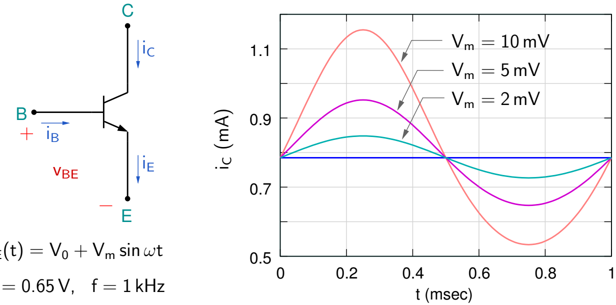


$v_{BE}(t) = V_0 + V_m \sin \omega t$

$V_0 = 0.65\,\text{V}, \quad f = 1\,\text{kHz}$



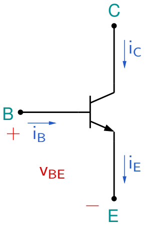
BJT: AC model



$v_{BE}(t) = V_0 + V_m \sin \omega t$

$V_0 = 0.65\,V, \quad f = 1\,kHz$

* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid $\rightarrow$ distortion.



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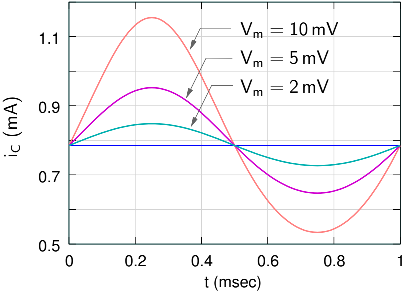
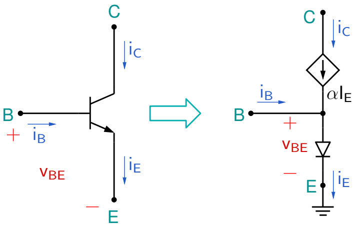
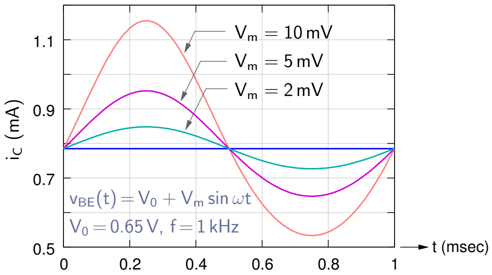


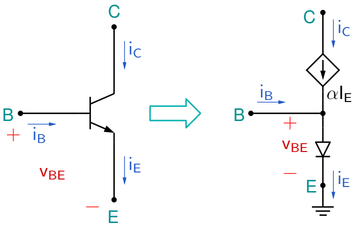
* As the v_{BE} amplitude increases, the shape of $i_C(t)$ deviates from a sinusoid $\rightarrow$ distortion.
* If $v_{be}(t)$, i.e., the time-varying part of v_{BE}, is kept small, i_C varies linearly with v_{BE}. How small? Let us look at this in more detail.

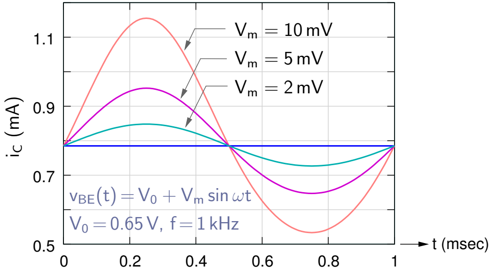




Let $v_{BE}(t) = V_{BE} + v_{be}(t)$ (bias+signal), and $i_C(t) = I_C + i_c(t)$.

BJT: small-signal model





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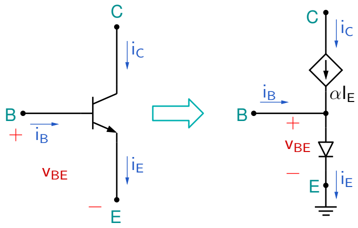
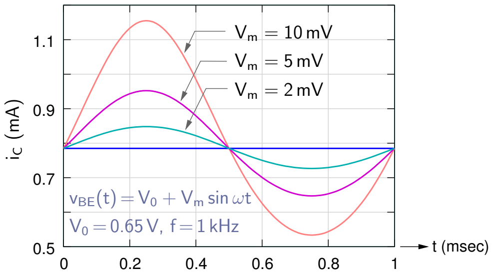
Assuming active mode, $i_C(t) = \alpha\, i_E(t) = \alpha\, I_{ES} \left[\exp\left(\dfrac{v_{BE}(t)}{V_T} \right) - 1 \right]$.





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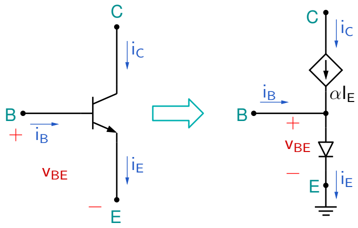
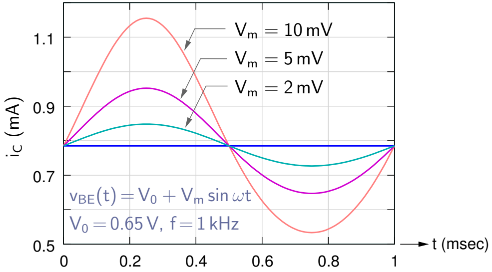
Since the B-E junction is forward-biased, $\exp\left(\dfrac{v_{BE}(t)}{V_T}\right) \gg 1$, and we get

$$i_C(t) = \alpha\, I_{ES}\, \exp\left(\frac{v_{BE}(t)}{V_T}\right) = \alpha\, I_{ES}\, \exp\left(\frac{V_{BE} + v_{be}(t)}{V_T}\right) = \alpha\, I_{ES}\, \exp\left(\frac{V_{BE}}{V_T}\right) \times \exp\left(\frac{v_{be}(t)}{V_T}\right).$$

BJT: small-signal model





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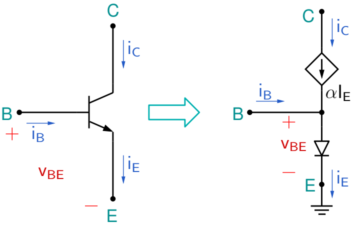
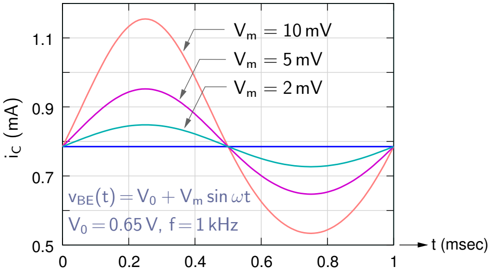
If $v_{be}(t) = 0$, $i_C(t) = I_C$ (the bias value of i_C), i.e., $I_C = \alpha\, I_{ES}\, \exp\left(\dfrac{V_{BE}}{V_T}\right)$

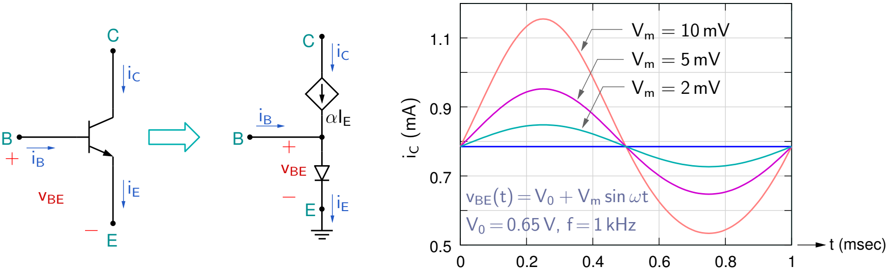
$$\Rightarrow i_C(t) = I_C\, \exp\left(\frac{v_{be}(t)}{V_T}\right).$$

BJT: small-signal model





$$i_C(t) = I_C \, \exp\left(\frac{v_{be}(t)}{V_T}\right) = I_C \left[1 + x + \frac{x^2}{2} + \cdots\right], \quad x = v_{be}(t)/V_T\,.$$



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If x is small, i.e., if the amplitude of $v_{be}(t)$ is small compared to the thermal voltage V_T, we get

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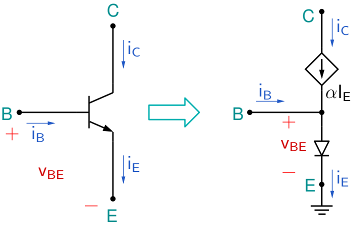
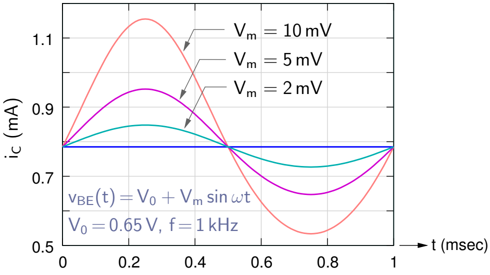
BJT: small-signal model





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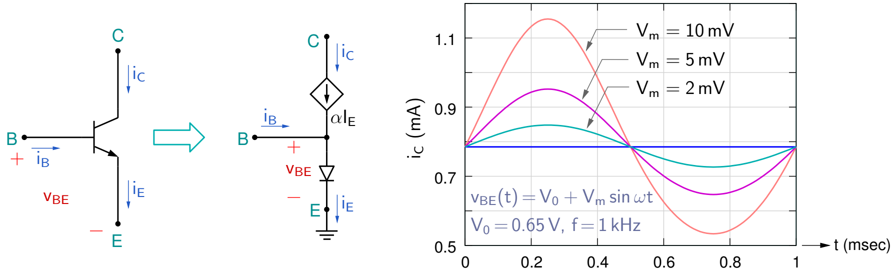

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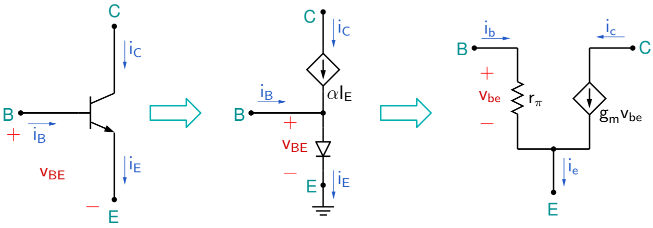
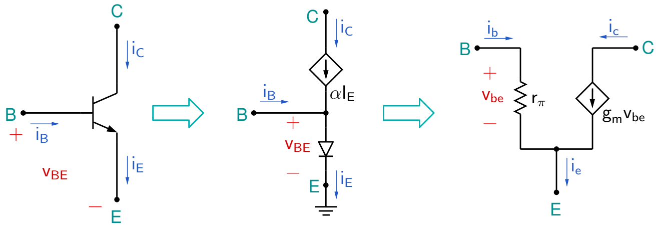
$$i_C(t) = I_C + i_c(t) = I_C\left[1 + \frac{v_{be}(t)}{V_T}\right] \Rightarrow \boxed{i_c(t) = \frac{I_C}{V_T}\,v_{be}(t)}$$



The relationship, $i_c(t) = \dfrac{I_C}{V_T}\, v_{be}(t)$ can be represented by a VCCS, $i_c(t) = g_m\, v_{be}(t)$,

where $g_m = I_C/V_T$ is the "transconductance."



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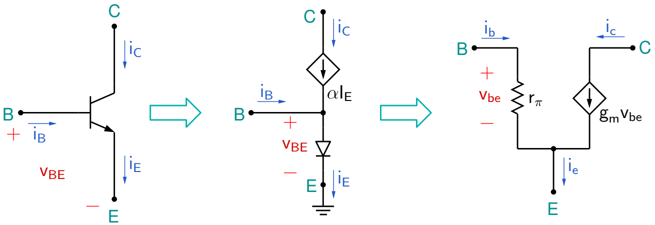
For the base current, we have,

$$i_B(t) = I_B + i_b(t) = \frac{1}{\beta}\,[I_C + i_c(t)]$$

$$\rightarrow i_b(t) = \frac{1}{\beta}\, i_c(t) = \frac{1}{\beta}\, g_m\, v_{be}(t) \rightarrow v_{be}(t) = (\beta/g_m)\, i_b(t).$$



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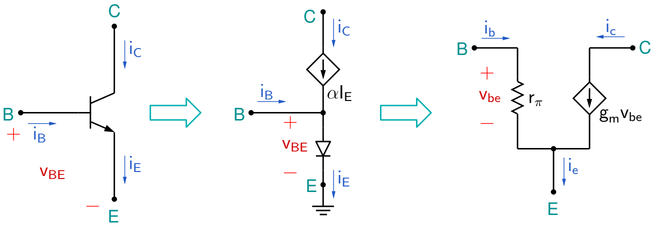
The above relationship is represented by a resistance, $r_\pi = \beta/g_m$, connected between B and E.



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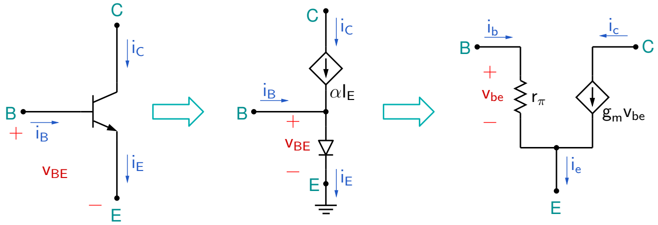
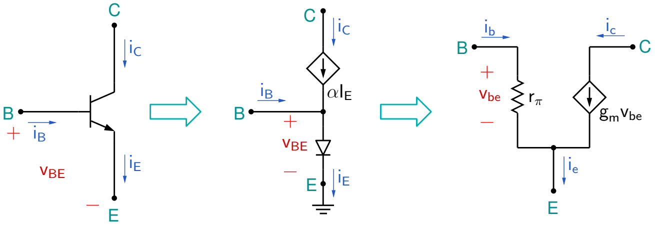
The resulting model is called the π-model for small-signal description of a BJT.

BJT: small-signal model



* The transconductance g_m depends on the biasing of the BJT, since $g_m = I_C/V_T$. For $I_C = 1\,mA$, $V_T \approx 25\,mV$ (room temperature), $g_m = 1\,mA/25\,mV = 40\,m\mho$ (milli-mho or milli-siemens).

BJT: small-signal model



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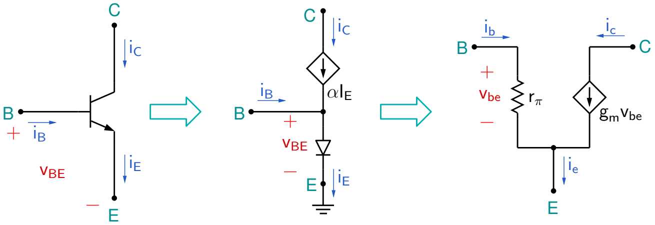
* r_π also depends on I_C, since $r_\pi = \beta/g_m = \beta\,V_T/I_C$. For $I_C = 1\,\text{m}A$, $V_T \approx 25\,\text{m}V$, $\beta = 100$, $r_\pi = 2.5\,\text{k}\Omega$.



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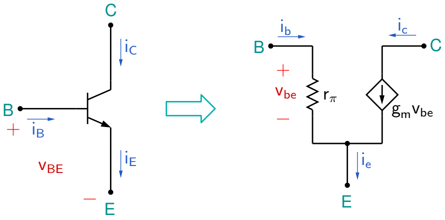
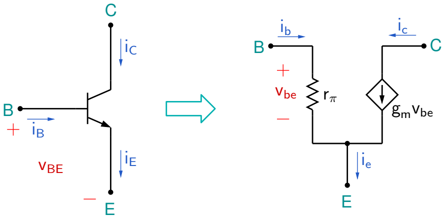
* Note that the small-signal model is valid only for small v_{be} (small compared to V_T).

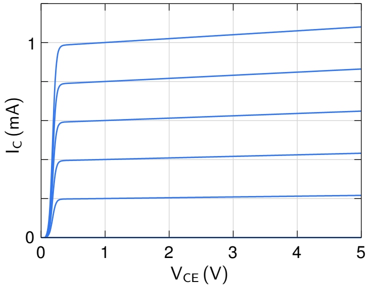
BJT: small-signal model



* In the above model, note that i_c is independent of v_{ce}.

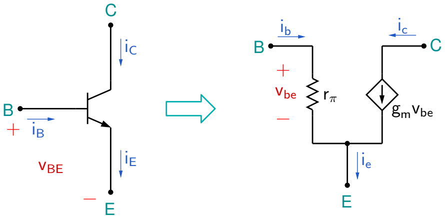
BJT: small-signal model

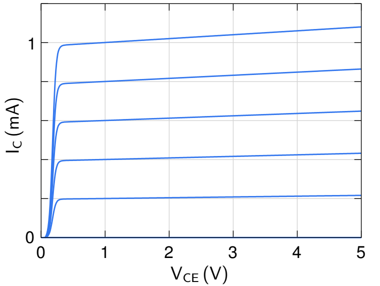




* In the above model, note that i_c is independent of v_{ce}.
* In practice, i_c does depend on v_{ce} because of the Early effect, and $\dfrac{dI_C}{dV_{CE}} \approx$ constant $= 1/r_o$, where r_o is called the output resistance.

BJT: small-signal model





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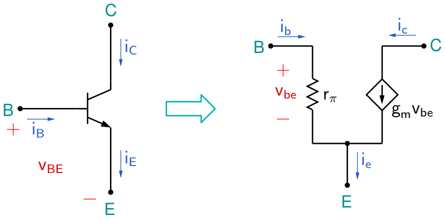
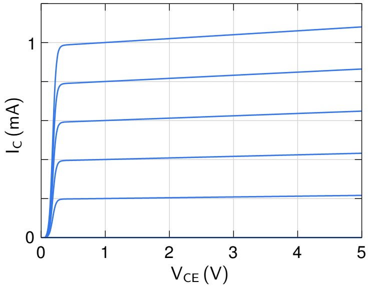
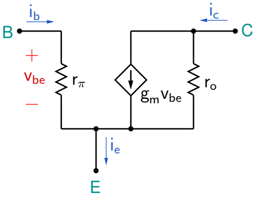
* A more accurate model includes r_o as well.

BJT: small-signal model







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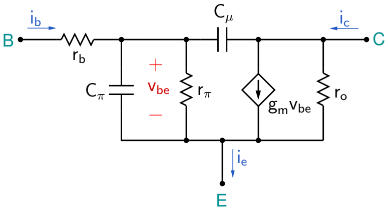
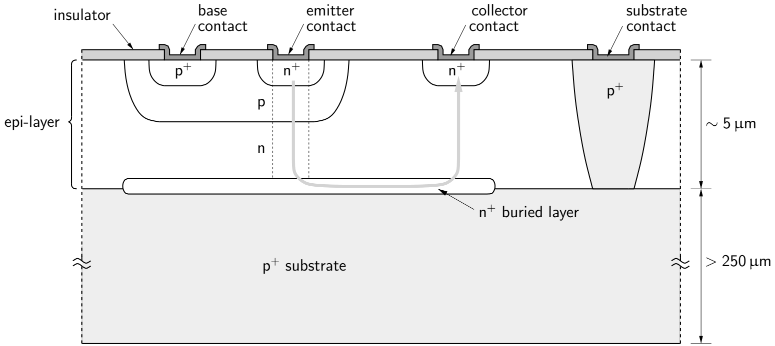
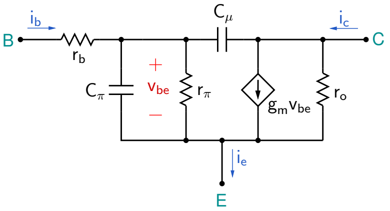
BJT: small-signal model

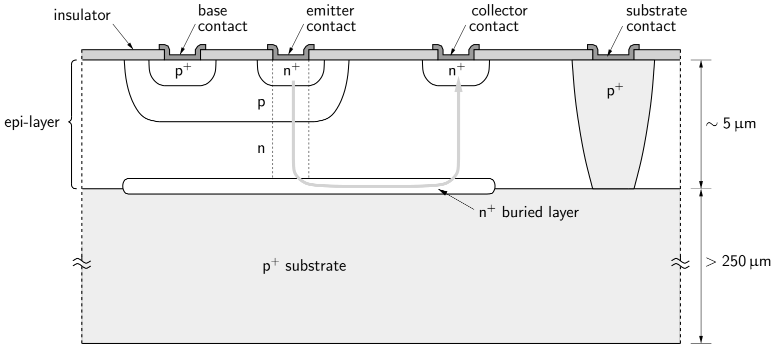




* A few other components are required to make the small-signal model complete:
  r_b: base spreading resistance
  C_π: base charging capacitance + B-E junction capacitance
  C_μ: B-C junction capacitance

BJT: small-signal model





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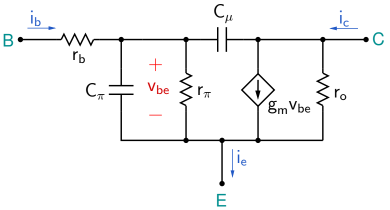
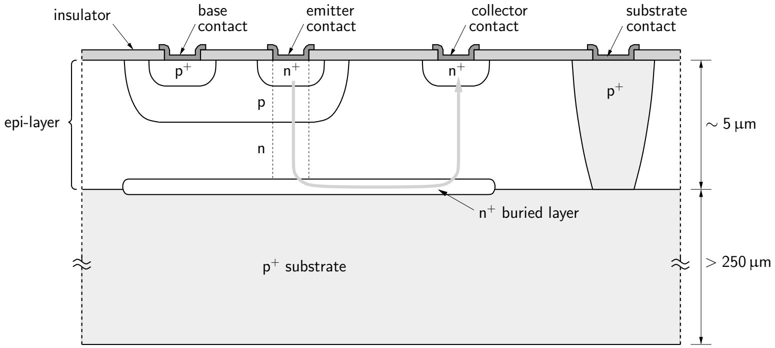
BJT: small-signal model





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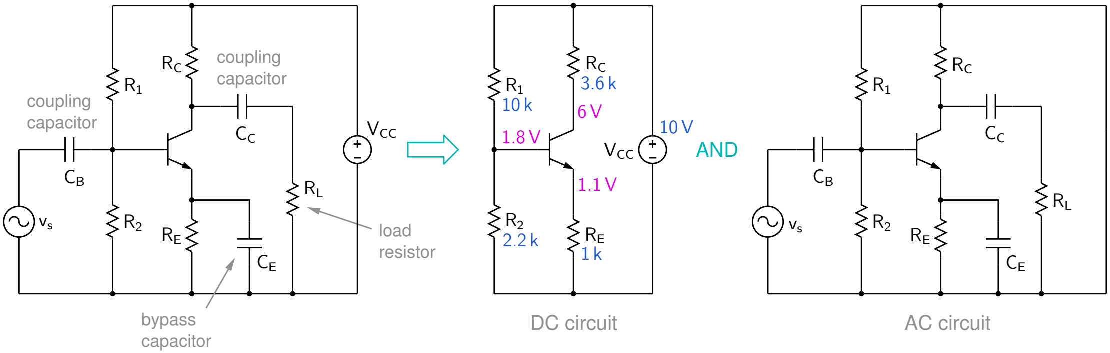
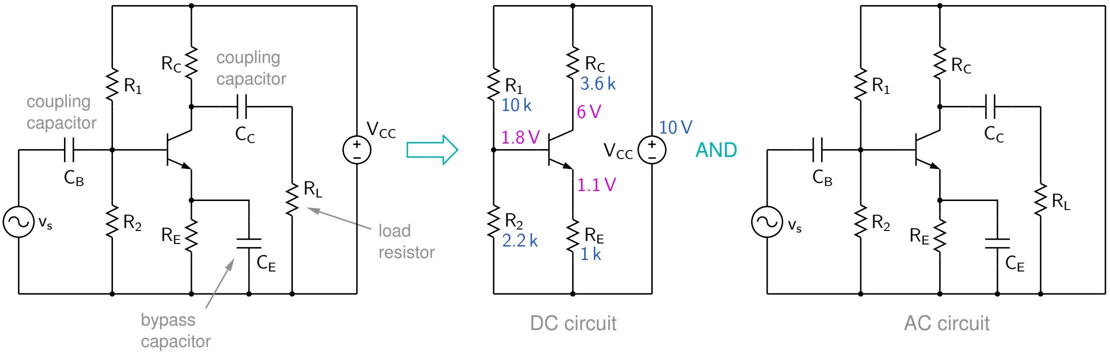
* Note that the small-signal models we have described are valid in the active region only.

Common-emitter amplifier



* We have already analysed the DC (bias) circuit of this amplifier and found that $V_B = 1.8\ V$, $V_E = 1.1\ V$, $V_C = 6\ V$, and $I_C = 1.1\ mA$.

Common-emitter amplifier



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* We now analyse the AC (small-signal) circuit to obtain v_b, v_e, v_c, i_c.

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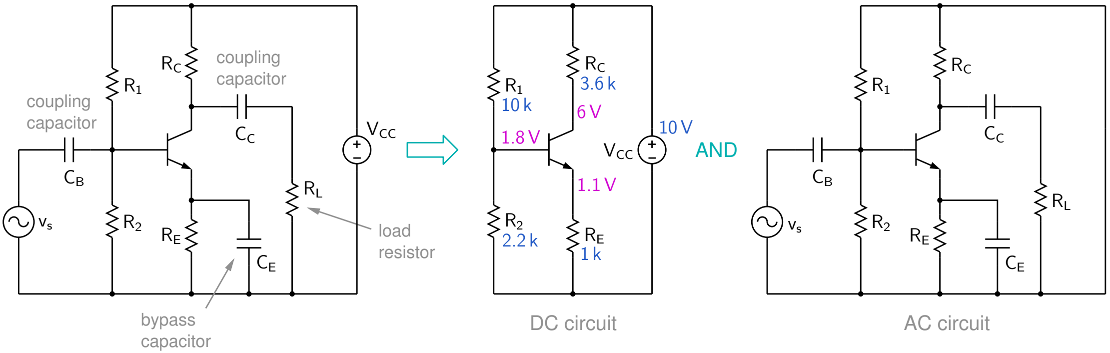

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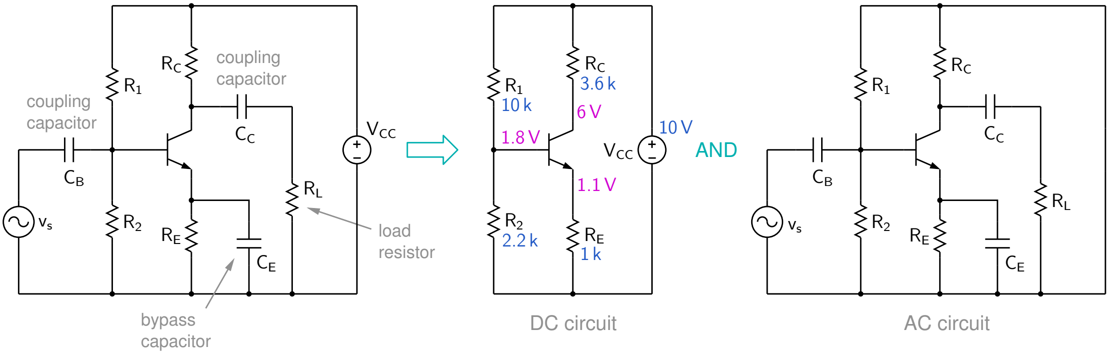
* We now analyse the AC (small-signal) circuit to obtain v_b, v_e, v_c, i_c.
* We will then get the complete solution by simply adding the DC and AC results, e.g., $i_C(t) = I_C + i_c(t)$.

Common-emitter amplifier



* We have already analysed the DC (bias) circuit of this amplifier and found that $V_B = 1.8\ V$, $V_E = 1.1\ V$, $V_C = 6\ V$, and $I_C = 1.1\ mA$.
* We now analyse the AC (small-signal) circuit to obtain v_b, v_e, v_c, i_c.
* We will then get the complete solution by simply adding the DC and AC results, e.g., $i_C(t) = I_C + i_c(t)$.
* We will assume that C_B, C_C, C_E are large enough so that, at the signal frequency (say, 1 kHz), they can be replaced by short circuits.

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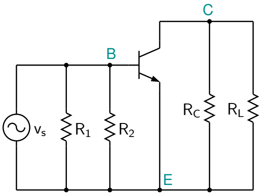

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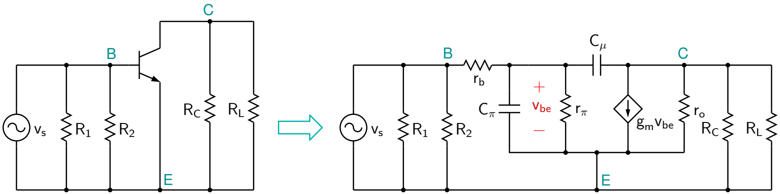

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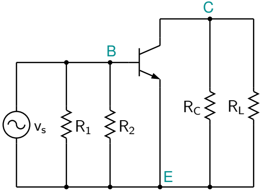

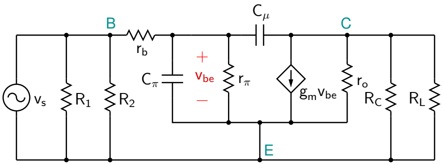

* The parasitic capacitances C_π and C_μ are in the pF range. At a signal frequency of 1 kHz, the impedance corresponding to these capacitances is

$$\mathbf{Z} \sim \frac{-j}{\omega C} = \frac{-j}{2\pi \times 10^3 \times 10^{-12}} \sim -j\,100\,\text{M}\Omega$$

$\rightarrow$ C_π and C_μ can be replaced by open circuits.

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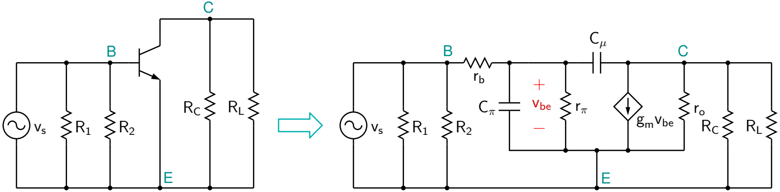

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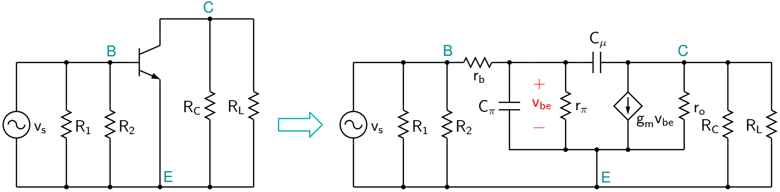

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* The above considerations significantly simplify the AC circuit.

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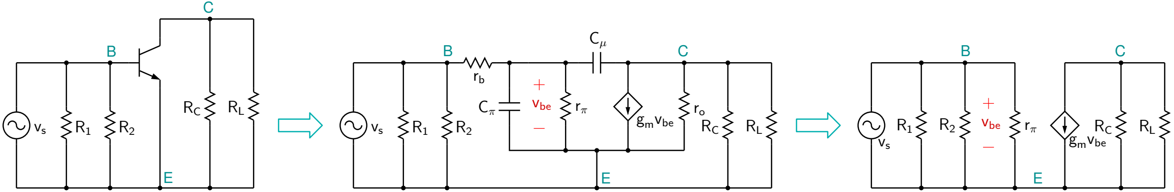

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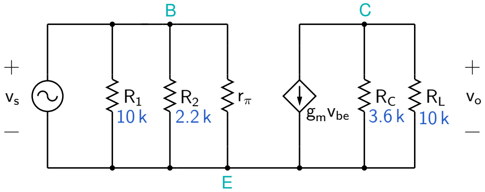

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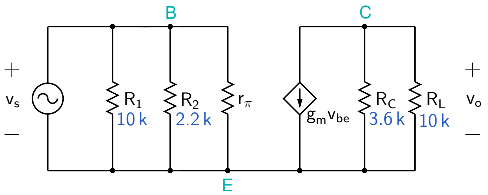

$v_o = -(g_m\, v_{be}) \times (R_C \parallel R_L) = -(g_m\, v_s) \times (R_C \parallel R_L)$

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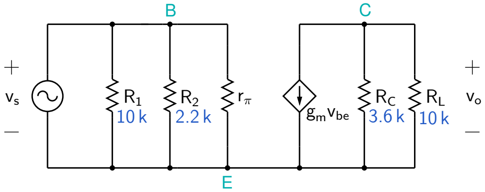
(superscript L is used because the gain includes the effect of R_L.)



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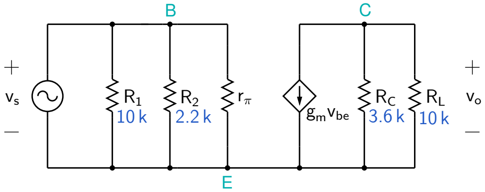

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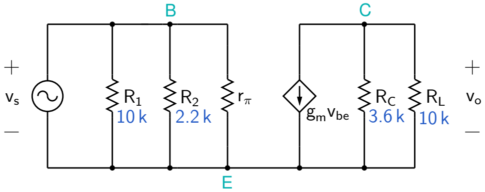

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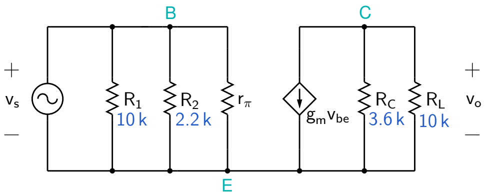

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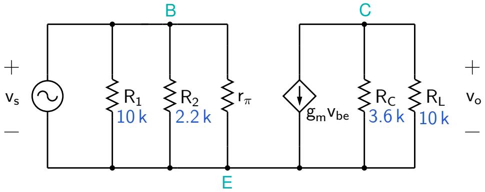

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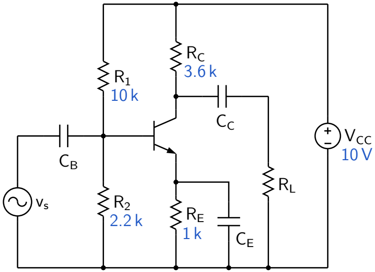
The AC collector current is,

$i_c = g_m\, v_{be} = g_m\, v_s = 42.5\,\text{m}\mho \times (2\,\text{m}V)\,\sin\omega t = 85\,\sin\omega t\ \mu A$.



For $v_s(t) = (2\,\mathrm{m}V)\,\sin\omega t$, we can now obtain expressions for the instantaneous currents and voltages:

$$v_C(t) = V_C + v_c(t) = V_C + v_o(t) = 6\;V - (225\,\mathrm{m}V)\,\sin\omega t\,.$$
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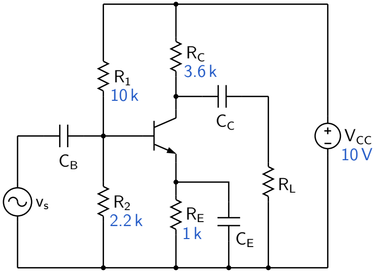

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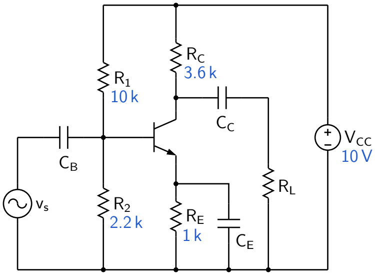
Note that the above procedure (DC + AC analysis) can be used *only if* the small-signal approximation (i.e., $|v_{be}| \ll V_T$) is valid. In the above example, the amplitude of v_{be} is 2 mV, which is much smaller than V_T (about 25 mV).



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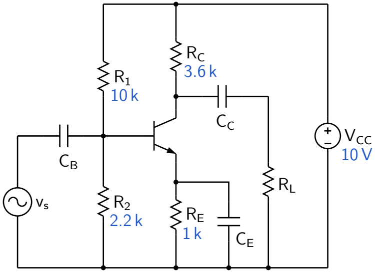
For $v_s(t) = (20\,\mathrm{m}V)\ \sin\omega t$, for example, the small-signal approximation will not hold, and a numerical simulation will be required to obtain the currents and voltages of interest.



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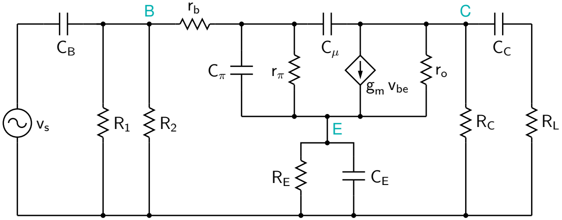
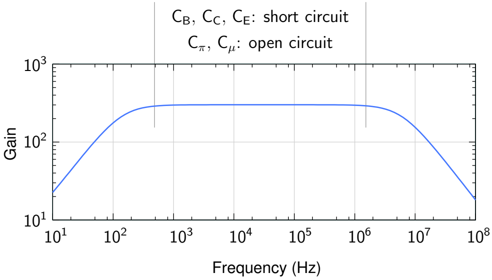
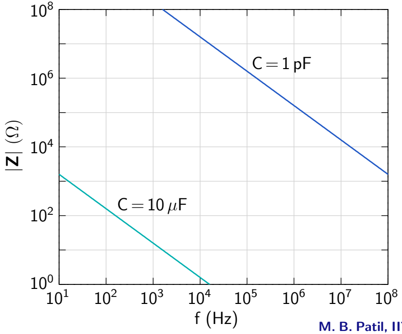
In practice, such a situation is anyway not prevalent (because it gives rise to distortion in the output voltage) except in special types of amplifiers.

Frequency response of common-emitter amplifier







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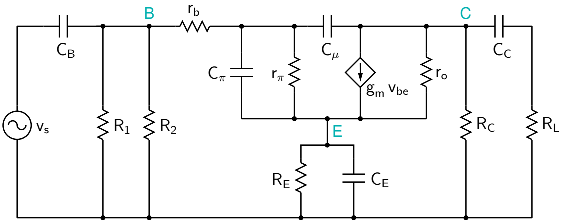

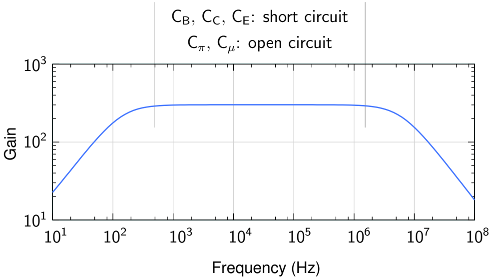

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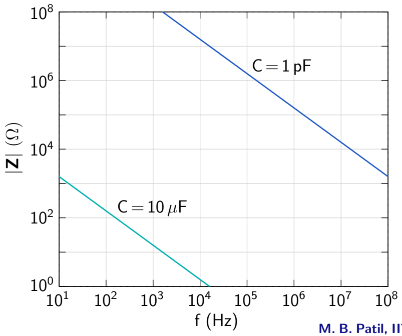

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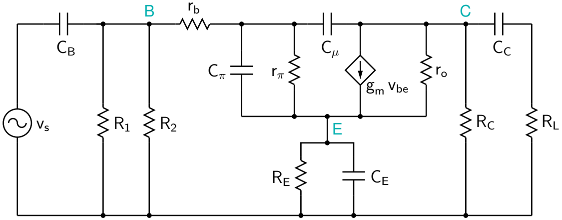

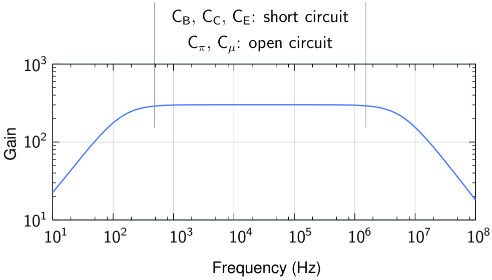

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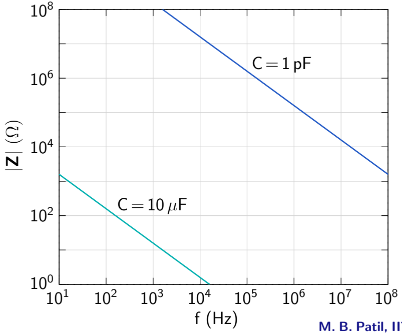

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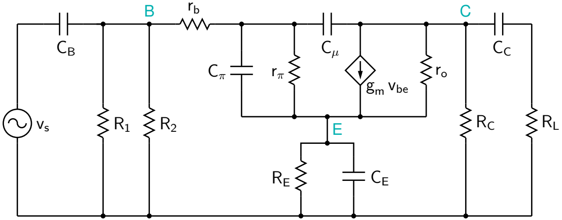

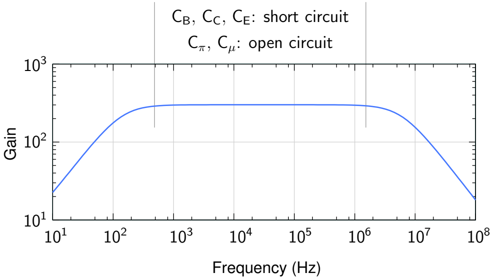

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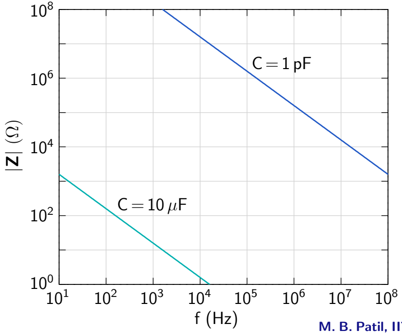
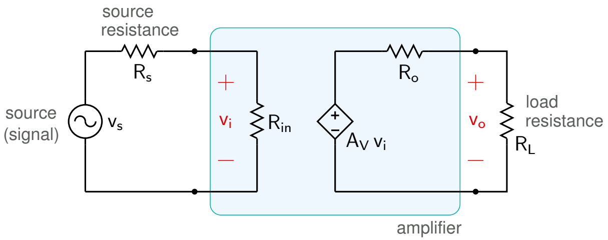
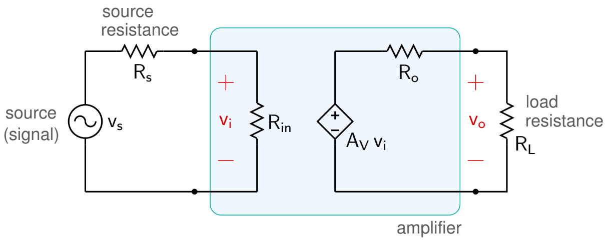
* In the "mid-band" range (which we have considered so far), the large capacitances behave like short circuits, and the small capacitances like open circuits. In this range, the gain is independent of frequency.





* An amplifier is represented by a voltage gain, an input resistance R_{in}, and an output resistance R_o. For a voltage-to-voltage amplifier, a large R_{in} and a small R_o are desirable.



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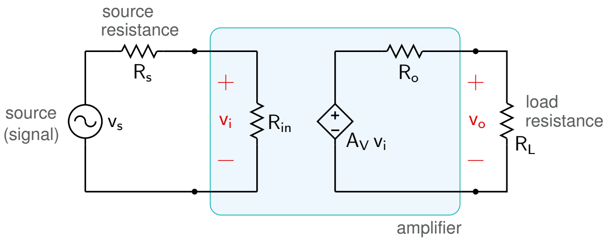

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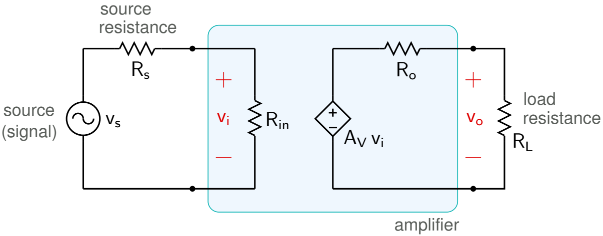

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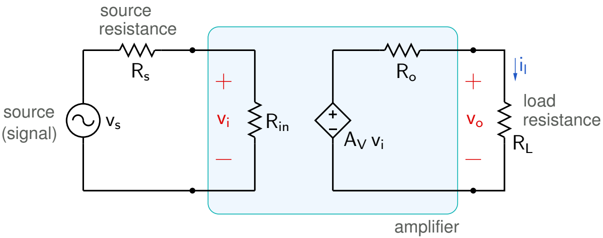
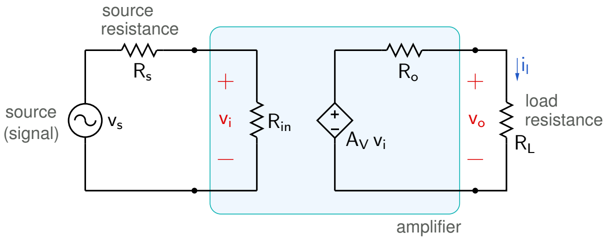
* Suppose we are given an amplifier as a "black box" and asked to find A_V, R_{in}, and R_o. What experiments would give us this information?

Voltage gain A_V

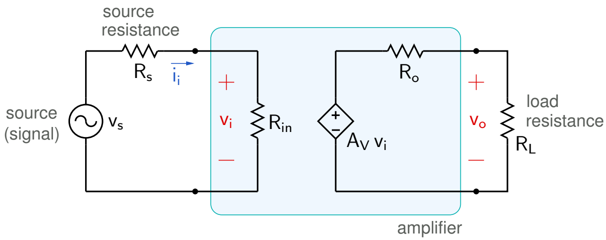


If $R_L \rightarrow \infty$, $i_l \rightarrow 0$, and $v_o \rightarrow A_V\ v_i$.

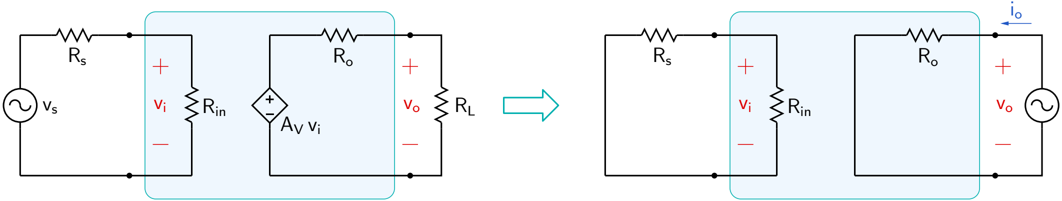


If $R_L \rightarrow \infty$, $i_l \rightarrow 0$, and $v_o \rightarrow A_V\, v_i$.

We can remove R_L (i.e., replace it with an open circuit), measure v_i and v_o, then use $A_V = v_o/v_i$.



Measurement of v_i and i_i yields $R_{in} = v_i/i_i$.



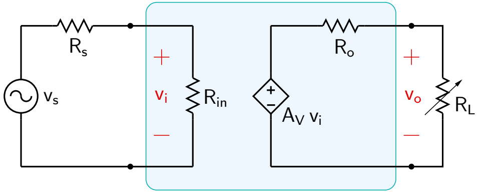
Method 1:

If $v_s \rightarrow 0$, $A_V \, v_i \rightarrow 0$.

Now, connect a test source v_o, and measure $i_o \rightarrow R_o = v_o/i_o$.

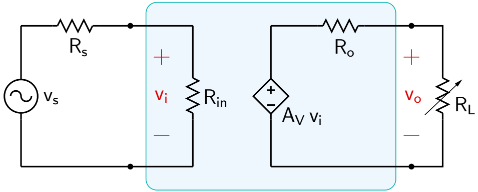
(This method works fine on paper, but it is difficult to use experimentally.)

Output resistance R_o



Method 2:

$$v_o = \frac{R_L}{R_L + R_o} A_V \, v_i \, .$$



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If $R_L \rightarrow \infty$, $v_{o1} = A_V \, v_i$.

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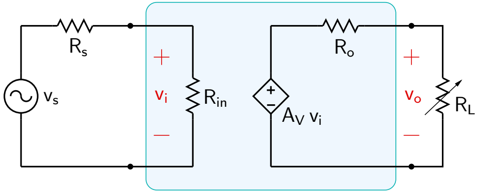
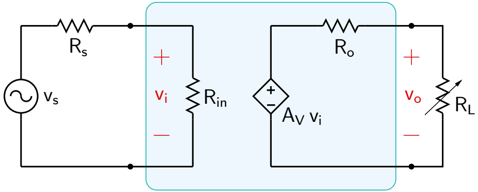


Method 2:

$$v_o = \frac{R_L}{R_L + R_o} A_V \, v_i .$$

If $R_L \rightarrow \infty, \; v_{o1} = A_V \, v_i .$

If $R_L = R_o, \; v_{o2} = \frac{1}{2} A_V \, v_i = \frac{1}{2} \, v_{o1} .$



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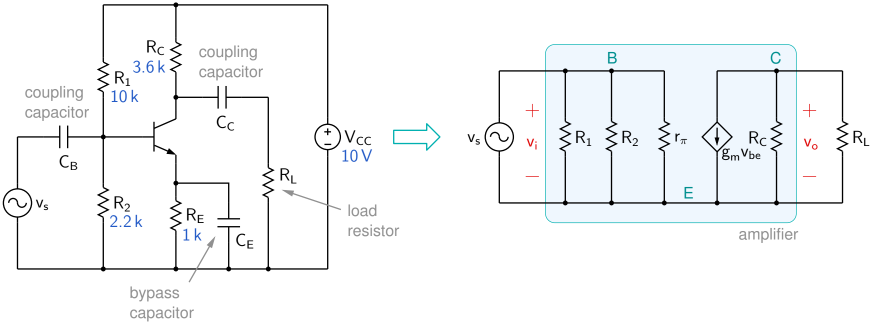
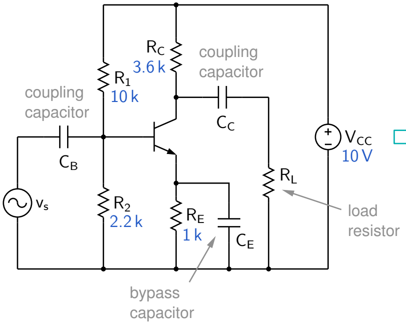
Procedure:

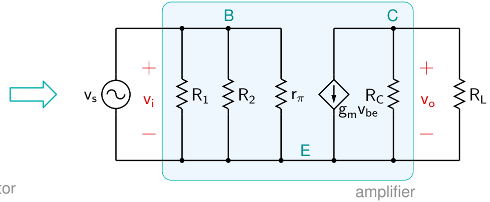
- Measure v_{o1} with $R_L \rightarrow \infty$ (i.e., R_L removed).
- Vary R_L and observe v_o.
- When v_o is equal to $v_{o1}/2$, measure R_L (after removing it).
- R_o is the same as the measured resistance.

Common-emitter amplifier



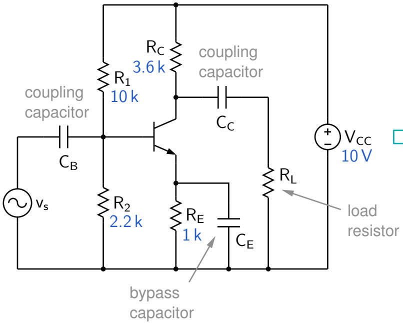
Common-emitter amplifier

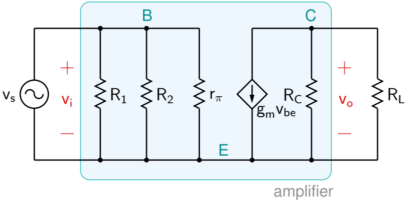




$$A_V = \frac{v_o}{v_i}, \text{ with } R_L \rightarrow \infty.$$

$$A_V = \frac{-g_m v_{be} R_C}{v_i} = -g_m R_C = -42.5\,\text{m}\mho \times 3.6\,\text{k} = 153.$$





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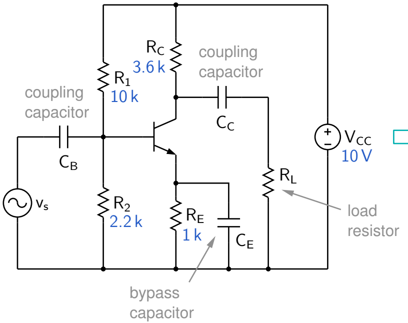
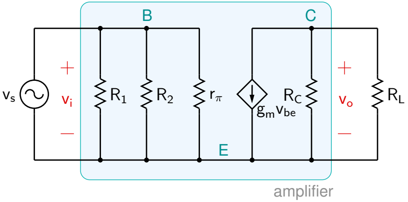
The input resistance of the amplifier is, by inspection, $R_{\text{in}} = (R_1 \parallel R_2) \parallel r_\pi$.

$$r_\pi = \beta/g_m = 100/42.5\,\text{m℧} = 2.35\,\text{k} \rightarrow R_{\text{in}} = 1\,\text{k}.$$





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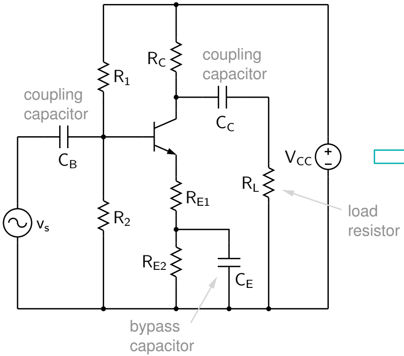
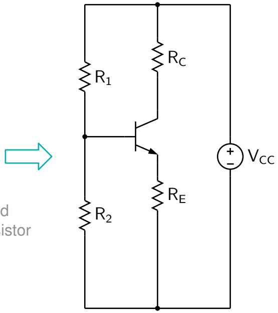
The output resistance is R_C (by "Method 1" seen previously).

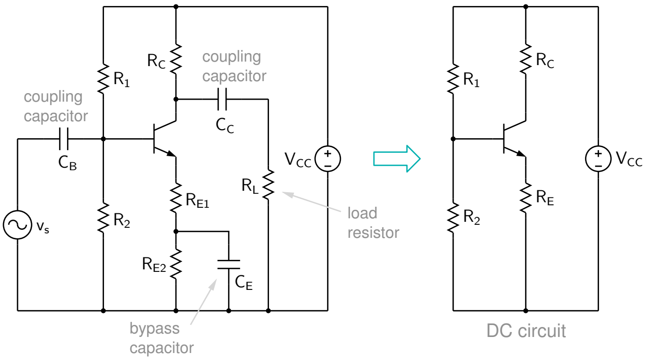
Common-emitter amplifier with partial bypass





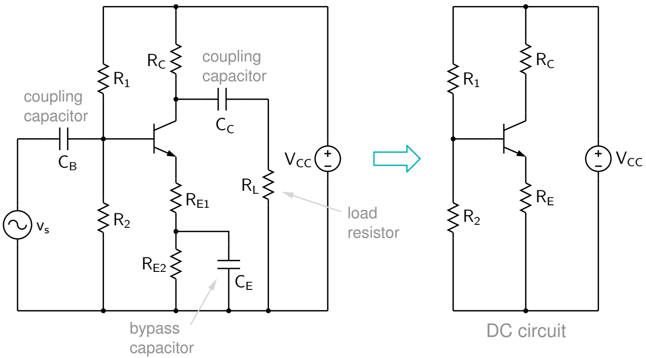
DC circuit

Common-emitter amplifier with partial bypass



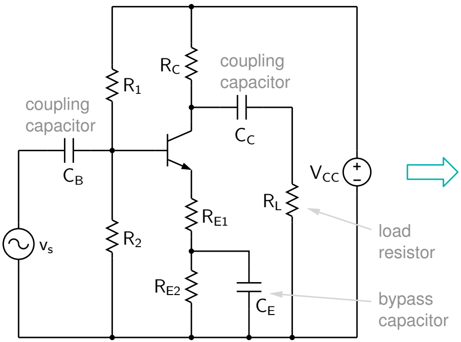
* For DC computation, C_E is open, and the DC analysis is therefore identical to our earlier amplifier, with $R_E \leftarrow R_{E1} + R_{E2}$.

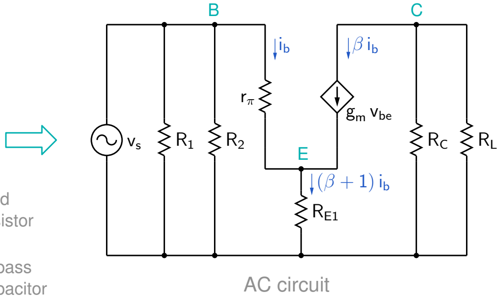
Common-emitter amplifier with partial bypass



* For DC computation, C_E is open, and the DC analysis is therefore identical to our earlier amplifier, with $R_E \leftarrow R_{E1} + R_{E2}$.
* Bypassing a part of R_E (as opposed to all of it) does have an impact on the voltage gain (see next slide).

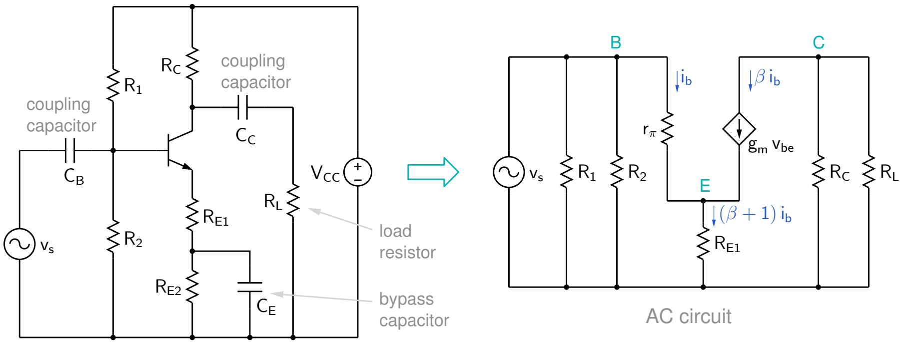
Common-emitter amplifier with partial bypass





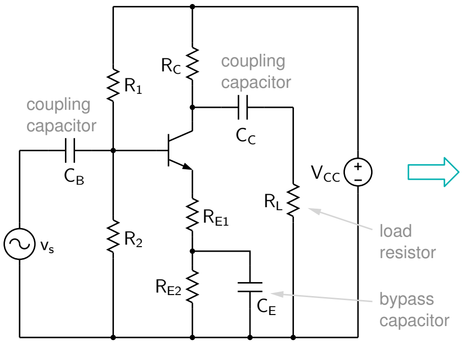
Again, assume that, at the frequency of operation, C_B, C_C, C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

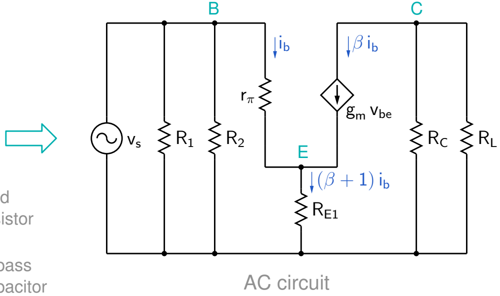
Common-emitter amplifier with partial bypass



Again, assume that, at the frequency of operation, C_B, C_C, C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

$$v_s = i_b\, r_\pi + (\beta + 1)\, i_b\, R_{E1} \rightarrow i_b = \frac{v_s}{r_\pi + (\beta + 1)\, R_{E1}}.$$

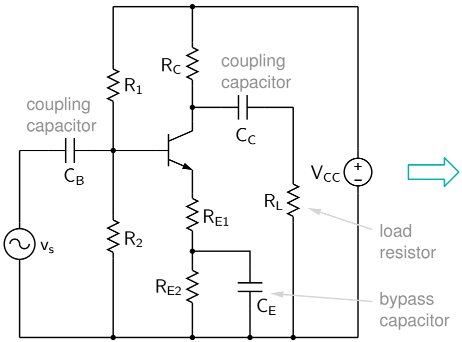


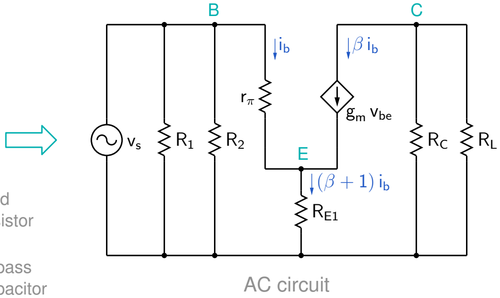


Again, assume that, at the frequency of operation, C_B, C_C, C_E can be replaced by short circuits, and the BJT parasitic capacitances by open circuits.

$$v_s = i_b\, r_\pi + (\beta+1)\, i_b\, R_{E1} \rightarrow i_b = \frac{v_s}{r_\pi + (\beta+1)\, R_{E1}}\,.$$

$$v_o = -\beta\, i_b \times (R_C \parallel R_L) \rightarrow \frac{v_o}{v_s} = -\frac{\beta\,(R_C \parallel R_L)}{r_\pi + (\beta+1)\, R_{E1}} \approx -\frac{(R_C \parallel R_L)}{R_{E1}} \text{ if } r_\pi \ll (\beta+1)\, R_{E1}.$$





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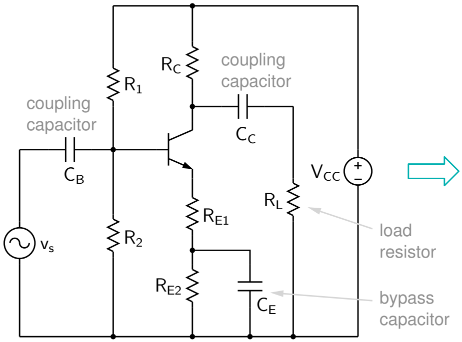
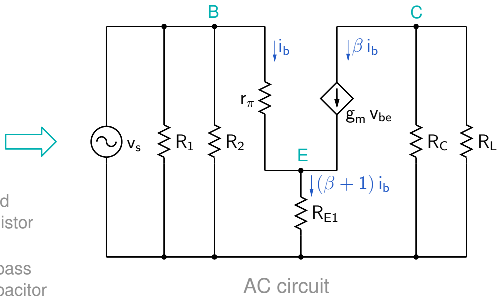
Note: R_{E1} gets multiplied by $(\beta + 1)$.

Common-emitter amplifier with partial bypass

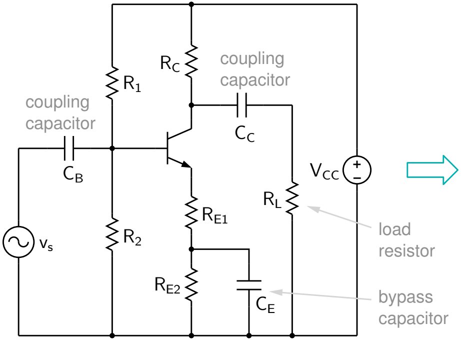


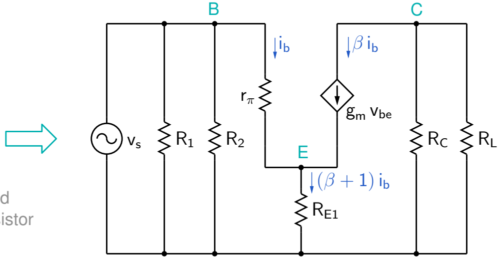


AC circuit

$$\frac{v_{be}}{v_s} = \frac{r_\pi\, i_b}{r_\pi\, i_b + R_E\,(\beta+1)\, i_b} = \frac{r_\pi}{r_\pi + R_E\,(\beta+1)}$$

Common-emitter amplifier with partial bypass





AC circuit

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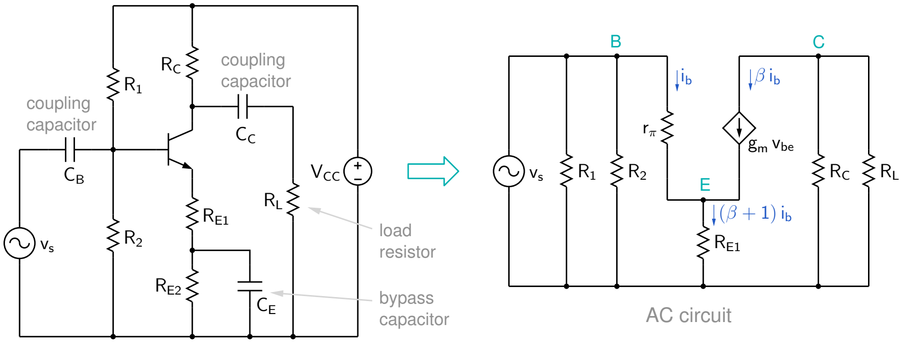
The small-signal condition, viz., $|v_{be}(t)| \ll V_T$ now implies

$$|v_s|\,\frac{r_\pi}{r_\pi + R_E\,(\beta+1)} \ll V_T \quad \text{or} \quad |v_s| \ll V_T \times \frac{r_\pi + R_E\,(\beta+1)}{r_\pi},$$ which is much larger than V_T.

Common-emitter amplifier with partial bypass



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$\rightarrow$ Although the gain is reduced, partial emitter bypass allows larger input voltages to be applied without causing distortion in $v_o(t)$. (For comparison, we required $|v_s| \ll V_T$ for the CE amplifier.)