

# Combinatorial Testing



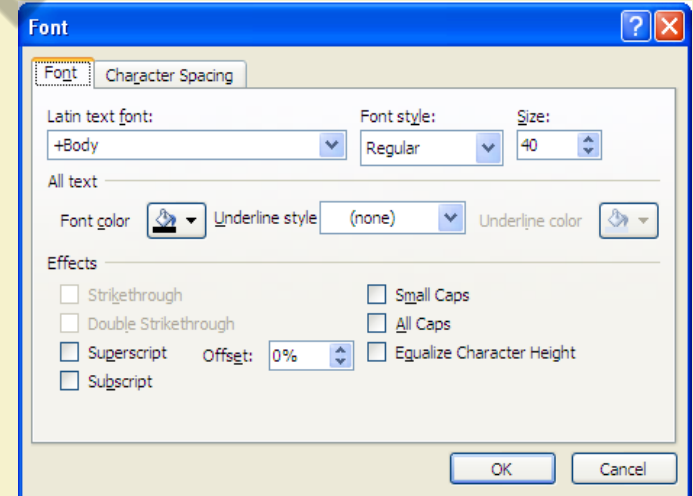
# Combinatorial Testing: Motivation

- The behavior of a program may be affected by many factors:
  - **Input parameters,**
  - **Environment configurations (global variables),**
  - **State variables. ..**
- Equivalence partitioning of an input variable:
  - Identify the possible types of input values requiring different processing.
- If the factors are many:
  - It is impractical to test all possible combinations of values of all factors.

**Combinatorial:** Relating to, or involving combinations

# Combinatorial Testing: Motivation

- Many times, the specific action to be performed depends on the value of a set of Boolean variable:
  - **Controller applications**
  - **User interfaces**



# Combinatorial Testing

- Several combinatorial testing strategies exist:
  - **Decision table-based testing**
  - **Cause-effect graphing**
  - **Pair-wise testing (reduced number of test cases)**

- Applicable to requirements involving conditional actions.
- This is represented as a decision table:
  - Conditions = inputs
  - Actions = outputs
  - Rules = test cases
- Assume independence of inputs
- Example
  - If c1 AND c2 OR c3 then A1

## Decision table-based Testing (DTT)

	Rule1	Rule2	Rule3	Rule4
Condition1	Yes	Yes	No	No
Condition2	Yes	X	No	X
Condition3	No	Yes	No	X
Condition4	No	Yes	No	Yes
Action1	Yes	Yes	No	No
Action2	No	No	Yes	No
Action3	No	No	No	Yes

## Combinations

		Combinations			
		Rule1	Rule2	Rule3	Rule4
Conditions	Condition1	Yes	Yes	No	No
	Condition2	Yes	X	No	X
	Condition3	No	Yes	No	X
	Condition4	No	Yes	No	Yes
Actions	Action1	Yes	Yes	No	No
	Action2	No	No	Yes	No
	Action3	No	No	No	Yes

- A decision table consists of a number of columns (rules) that comprise all test situations
- Example: the triangle problem
  - C1: a, b,c form a triangle
  - C2: a=b
  - C3: a= c
  - C4: b= c
  - A1: Not a triangle
  - A2:scalene
  - A3: Isosceles
  - A4:equilateral
  - A5: Right angled

Sample  
Decision  
table

	r1	r2	...				rn
C1	0	1					0
c2	-	1					0
C3	-	1					1
C4	-	1					0
a1	1	0					0
a2	0	0					1
a3	0	0					0
a4	0	1					0
a5	0	0					

# Test cases from Decision Tables

Test Case ID	a	b	c	Expected output
TC1	4	1	2	Not a Triangle
TC2	2888	2888	2888	Equilateral
TC3	?		)	Impossible
TC4				
...				
TC11				

C1: a, b,c form a triangle

C2: a=b

C3: a= c

C4: b= c



## More Complete Decision Table for the Triangle Problem

Conditions											
C1: $a < b+c?$	F	T	T	T	T	T	T	T	T	T	T
C2: $b < a+c?$	-	F	T	T	T	T	T	T	T	T	T
C3: $c < a+b?$	-	-	F	T	T	T	T	T	T	T	T
C4: $a=b?$	-	-	-	T	T	T	T	F	F	F	F
C5: $a=c?$	-	-	-	T	T	F	F	T	T	F	F
C6: $b=c?$	-	-	-	T	F	T	F	T	F	T	F
Actions											
A1: Not a Triangle	X	X	X								
A2: Scalene											X
A3: Isosceles							X		X	X	
A4: Equilateral				X							
A5: Impossible					X	X		X			

## Test Cases for the Triangle Problem

Case ID	a	b	c	Expected Output
DT1	4	1	2	Not a Triangle
DT2	1	4	2	Not a Triangle
DT3	1	2	4	Not a Triangle
DT4	5	5	5	Equilateral
DT5	?	?	?	Impossible
DT6	?	?	?	Impossible
DT7	2	2	3	Isosceles
DT8	?	?	?	Impossible
DT9	2	3	2	Isosceles
DT10	3	2	2	Isosceles
DT11	3	4	5	Scalene



## Decision Table – Example 2

### Printer Troubleshooting

Conditions	Printer does not print	Y	Y	Y	Y	N	N	N	N
	A red light is flashing	Y	Y	N	N	Y	Y	N	N
	Printer is unrecognized	Y	N	Y	N	Y	N	Y	N
Actions	Check the power cable			X					
	Check the printer-computer cable	X		X					
	Ensure printer software is installed	X		X		X		X	
	Check/replace ink	X	X			X	X		
	Check for paper jam		X		X				

# Quiz: Develop BB Test Cases

- Policy for charging customers for certain in-flight services:

**If the flight is more than half-full and ticket cost is more than Rs. 3000, free meals are served unless it is a domestic flight. Otherwise, no meals are served. Meals are charged on all domestic flights.**

Fill all combinations in the table.

		POSSIBLE COMBINATIONS							
CONDITONS	<i>more than half-full</i>	N	N	N	N	Y	Y	Y	Y
	<i>more than Rs.3000 per seat</i>	N	N	Y	Y	N	N	Y	Y
	<i>domestic flight</i>	N	Y	N	Y	N	Y	N	Y
ACTIONS									

Analyze column by column to determine which actions are appropriate for each combination

		POSSIBLE COMBINATIONS							
CONDITONS	<i>more than half-full</i>	N	N	N	N	Y	Y	Y	Y
	<i>more than Rs. 3000 per seat</i>	N	N	Y	Y	N	N	Y	Y
	<i>domestic flight</i>	N	Y	N	Y	N	Y	N	Y
ACTIONS	<i>serve meals</i>					Y	Y	Y	Y
	<i>free</i>							Y	

Reduce the table by eliminating redundant columns.

		POSSIBLE COMBINATIONS							
CONDITONS	<i>more than half-full</i>	N	N	N	N	Y	Y	Y	Y
	<i>more than Rs. 3000 per seat</i>	N	N	Y	Y	N	N	Y	Y
	<i>domestic flight</i>	N	Y	N	Y	N	Y	N	Y
ACTIONS	<i>serve meals</i>					X	X	X	X
	<i>free</i>							X	

# Final solution

		Combinations			
CONDITONS	<i>more than half-full</i>	N	Y	Y	Y
	<i>more than 3000 per seat</i>	-	N	Y	Y
	<i>domestic flight</i>	-	-	N	Y
ACTIONS	<i>serve meals</i>		X	X	X
	<i>free</i>			X	



–Rules need to be complete:

## Assumptions regarding rules

- That is, every combination of decision table values including default combinations are present.

–Rules need to be consistent:

- That is, there is no two different actions for the same combinations of conditions

## Guidelines and Observations

- Decision table testing is appropriate for programs:
  - There is a lot of decision making
  - Output is a logical relationship among input variables
  - Results depend on calculations involving subsets of inputs
  - There are cause and effect relationships between input and output
- Decision tables do not scale up very well

## Quiz: Design test Cases

- Customers on a e-commerce site get following discount:
  - A member gets 10% discount for purchases lower than Rs. 2000, else 15% discount
  - Purchase using SBI card fetches 5% discount
  - If the purchase amount after all discounts exceeds Rs. 2000/- then shipping is free.

# Cause-effect Graphs

- Overview:
  - Explores combinations of possible inputs
  - Specific combination of inputs (causes) results in outputs (effects)
  - Represented as nodes of a cause effect graph
  - The graph also includes constraints and a number of intermediate nodes linking causes and effects

- If depositing less than Rs. 1 Lakh, rate of interest:
  - 6% for deposit upto 1 year
  - 7% for deposit over 1 year but less than 3 yrs
  - 8% for deposit 3 years and above
- If depositing more than Rs. 1 Lakh, rate of interest:
  - 7% for deposit upto 1 year
  - 8% for deposit over 1 year but less than 3 yrs
  - 9% for deposit 3 years and above

**Cause-Effect  
Graph  
Example**

# Cause-Effect Graph Example

## Causes

C1: Deposit < 1yr

C2: 1yr < Deposit < 3yrs

C3: Deposit > 3yrs

C4: Deposit < 1 Lakh

C5: Deposit ≥ 1Lakh

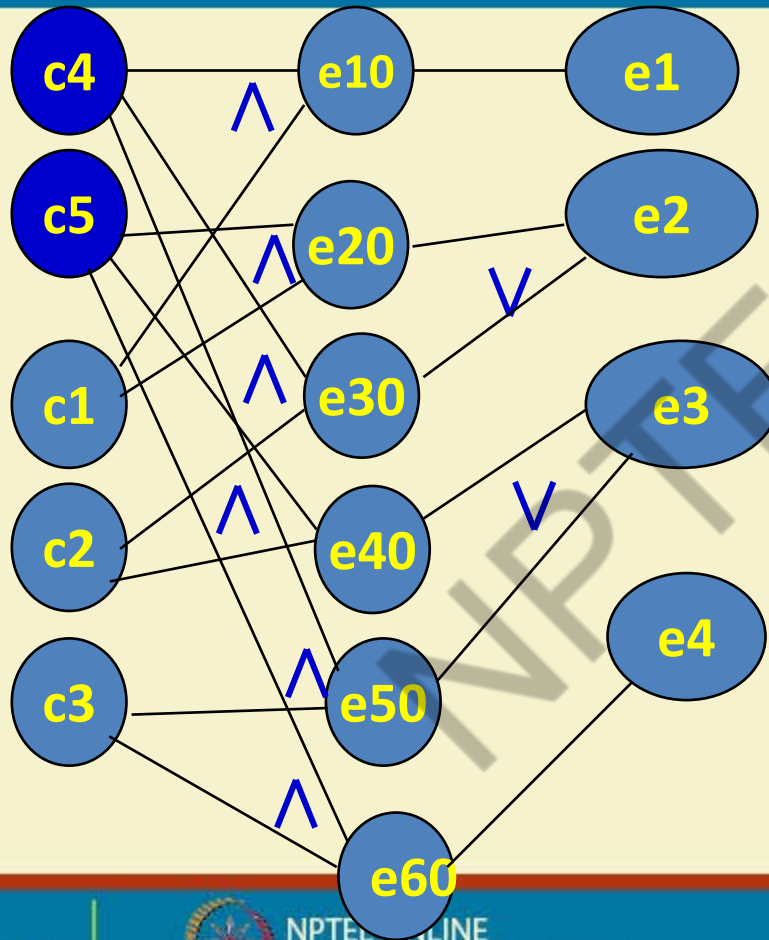
## Effects

e1: Rate 6%

e2: Rate 7%

e3: Rate 8%

e4: Rate 9%



## Cause-Effect Graphing

## Develop a Decision Table

C1	C2	C3	C4	C5	e1	e2	e3	e4
1	0	0	1	0	1	0	0	0
1	0	0	0	1	0	1	0	0
0	1	0	1	0	0	1	0	0
0	1	0	0	1	1	0	1	0

- Convert each row to a test case



# Pair-wise Testing



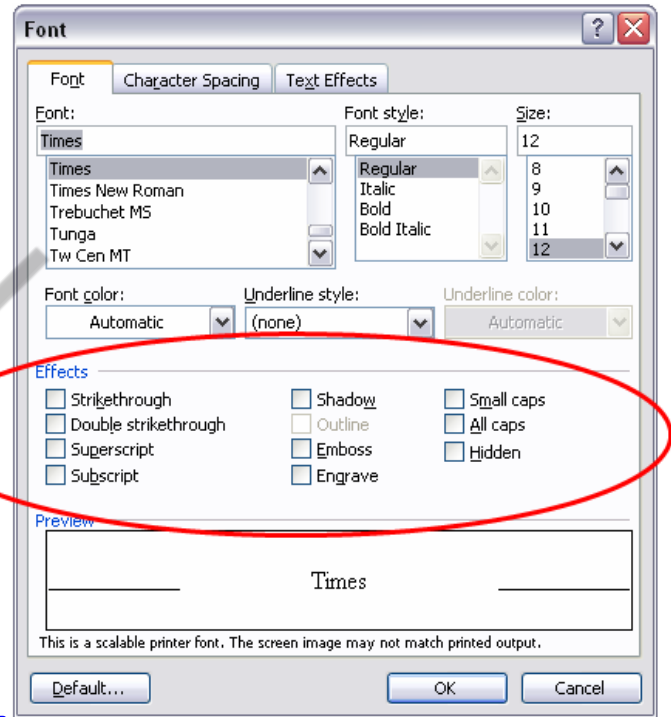
# Combinatorial Testing of User Interface

0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1
1	1	1	0	1	0	0	0	0	1
1	0	1	1	0	1	0	1	0	0
1	0	0	0	1	1	1	0	0	0
0	1	1	0	0	1	0	0	1	0
0	0	1	0	1	0	1	1	1	0
1	1	0	1	0	0	1	0	1	0
0	0	0	1	1	1	0	0	1	1
0	0	1	1	0	0	1	0	0	1
0	1	0	1	1	0	0	1	0	0
1	0	0	0	0	0	0	1	1	1
0	1	0	0	0	1	1	1	0	1

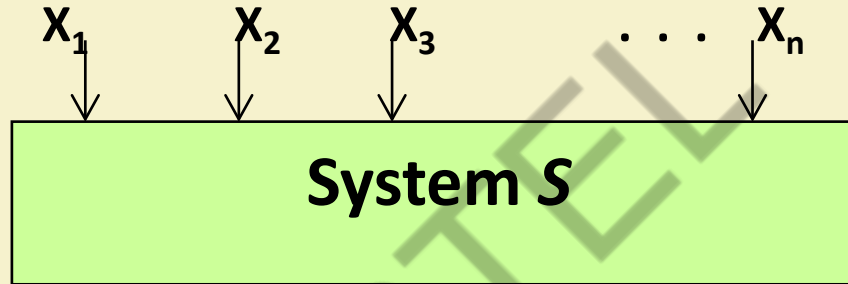
0 = effect off  
1 = effect on

$2^{10} = 1,024$  tests for all combinations

$* 10^3 = 1024 * 1000$  .... Just too many to tests



# Combinatorial Testing Problem



- Combinatorial testing problems generally follow a simple input-process-output model;
- The “state” of the system is not the focus of combinatorial testing.

## t-way Testing

- Instead of testing all possible combinations:
  - A subset of combinations is generated.
- Key observation:
  - **It is often the case that a fault is caused by interactions among a few factors.**
- t-way testing can dramatically reduce the number of test cases:
  - but remains effective in terms of fault detection.

# t-way Interaction Testing

Interest Rate | Amount | Months | Down Pmt | Pmt Frequency

All combinations:  
every value of  
every parameters

All pairs: every  
value of each  
pair of  
parameters

etc. . .

t-way interactions:  
every value of every t-  
way combination of  
parameters

# Pairwise Testing

Pressure | Temperature | Velocity | Acceleration | Air Density

A  
B

T1  
T2  
T3

1  
2  
3  
4  
5  
6

10  
0  
20  
0

1.1  
2.1  
3.1

Pressure	Temperature
A	T1
A	T2
A	T3
B	T1
B	T2
B	T3

# Pairwise Reductions

Number of inputs	Number of selected test data values	Number of combinations	Size of pairwise test set
7	2	128	8
13	3	$1.6 \times 10^6$	15
40	3	$1.2 \times 10^{19}$	21

- **A t-way interaction fault:**
  - Triggered by a certain combination of  $t$  input values.
  - A simple fault is a 1-way fault
  - Pairwise fault is a  $t$ -way fault where  $t = 2$ .
- **In practice, a majority of software faults consist of simple and pairwise faults.**



# Single-mode Bugs

- The simplest bugs are single-mode faults:
  - **Occur when one option causes a problem regardless of the other settings**
  - **Example:** A printout is always gets smeared when you choose the duplex option in the print dialog box
    - Regardless of the printer or the other selected options

# Double-mode Faults

- **Double-mode faults**

- Occurs when two options are combined
- Example:** The printout is smeared only when duplex is selected and the printer selected is model 394

# Multi-mode Faults

- **Multi-mode faults**

- Occur when three or more settings produce the bug
- This is the type of problems that make complete coverage necessary

# Example of Pairwise Fault

- begin
  - int x, y, z;
  - input (x, y, z);
  - if (x == x1 and y == y2)
    - output (f(x, y, z));
  - else if (x == x2 and y == y1)
    - output (g(x, y));
  - Else      **// Missing (x == x2 and y == y1) f(x, y, z) – g(x, y);**
    - output (f(x, y, z) + g(x, y))
- end
- **Expected:**  $x = x1 \text{ and } y = y1 \Rightarrow f(x, y, z) - g(x, y);$   
 $x = x2, y = y2 \Rightarrow f(x, y, z) + g(x, y)$

## Example: Android smart phone testing

- Apps should work on all combinations of platform options, but there are  $3 \times 3 \times 4 \times 3 \times 5 \times 4 \times 4 \times 5 \times 4 = 172,800$  configurations

HARDKEYBOARDHIDDEN\_NO  
HARDKEYBOARDHIDDEN\_UNDEFINED  
HARDKEYBOARDHIDDEN\_YES

KEYBOARDHIDDEN\_NO  
KEYBOARDHIDDEN\_UNDEFINED  
KEYBOARDHIDDEN\_YES

KEYBOARD\_12KEY  
KEYBOARD\_NOKEYS  
KEYBOARD\_QWERTY  
KEYBOARD\_UNDEFINED

NAVIGATIONHIDDEN\_NO  
NAVIGATIONHIDDEN\_UNDEFINED

NAVIGATIONHIDDEN\_YES

NAVIGATION\_DPAD  
NAVIGATION\_NONAV  
NAVIGATION\_TRACKBALL  
NAVIGATION\_UNDEFINED  
NAVIGATION\_WHEEL

ORIENTATION\_LANDSCAPE  
ORIENTATION\_PORTRAIT  
ORIENTATION\_SQUARE  
ORIENTATION\_UNDEFINED

SCREENLAYOUT\_LONG\_MASK  
SCREENLAYOUT\_LONG\_NO  
SCREENLAYOUT\_LONG\_UNDEFINED

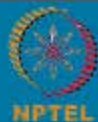
SCREENLAYOUT\_LONG\_YES

SCREENLAYOUT\_SIZE\_LARGE  
SCREENLAYOUT\_SIZE\_MASK  
SCREENLAYOUT\_SIZE\_NORMAL

SCREENLAYOUT\_SIZE\_SMALL  
SCREENLAYOUT\_SIZE\_UNDEFINED

TOUCHSCREEN\_FINGER  
TOUCHSCREEN\_NOTOUCH  
TOUCHSCREEN\_STYLUS  
TOUCHSCREEN\_UNDEFINED

# White-Box Testing



# What is White-box Testing?

- White-box test cases designed based on:
  - Code structure of program.
  - White-box testing is also called structural testing.

# White-Box Testing Strategies

- **Coverage-based:**
  - Design test cases to cover certain program elements.
- **Fault-based:**
  - Design test cases to expose some category of faults



- Several white-box testing strategies have become very popular :

- Statement coverage
- Branch coverage
- Path coverage
- Condition coverage
- MC/DC coverage
- Mutation testing
- Data flow-based testing

## White-Box Testing

# Why Both BB and WB Testing?

## Black-box

- Impossible to write a test case for every possible set of inputs and outputs
- Some code parts may not be reachable
- **Does not tell if extra functionality has been implemented.**

## White-box

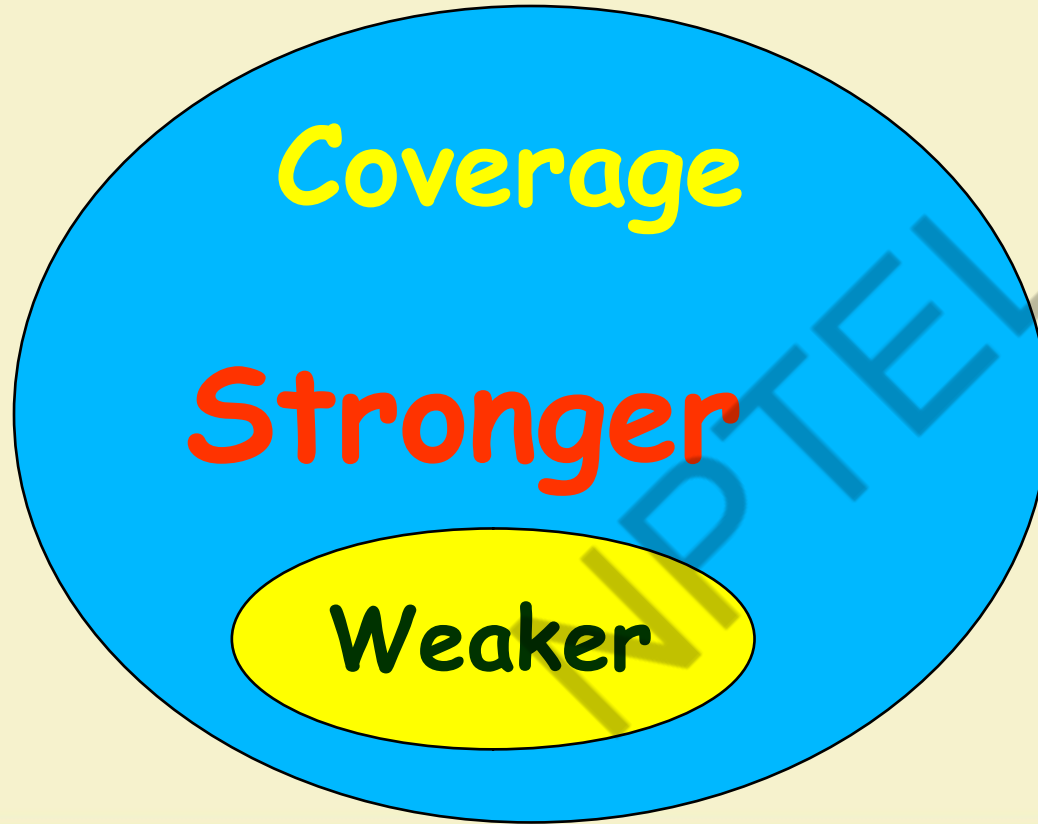
- Does not address the question of whether a program matches the specification
- Does not tell if all functionalities have been implemented
- **Does not uncover any missing program logic**

# Coverage-Based Testing Versus Fault-Based Testing

- Idea behind coverage-based testing:
  - Design test cases so that certain program elements are executed (or covered).
  - Example: statement coverage, path coverage, etc.
- Idea behind fault-based testing:
  - Design test cases that focus on discovering certain types of faults.
  - Example: Mutation testing.

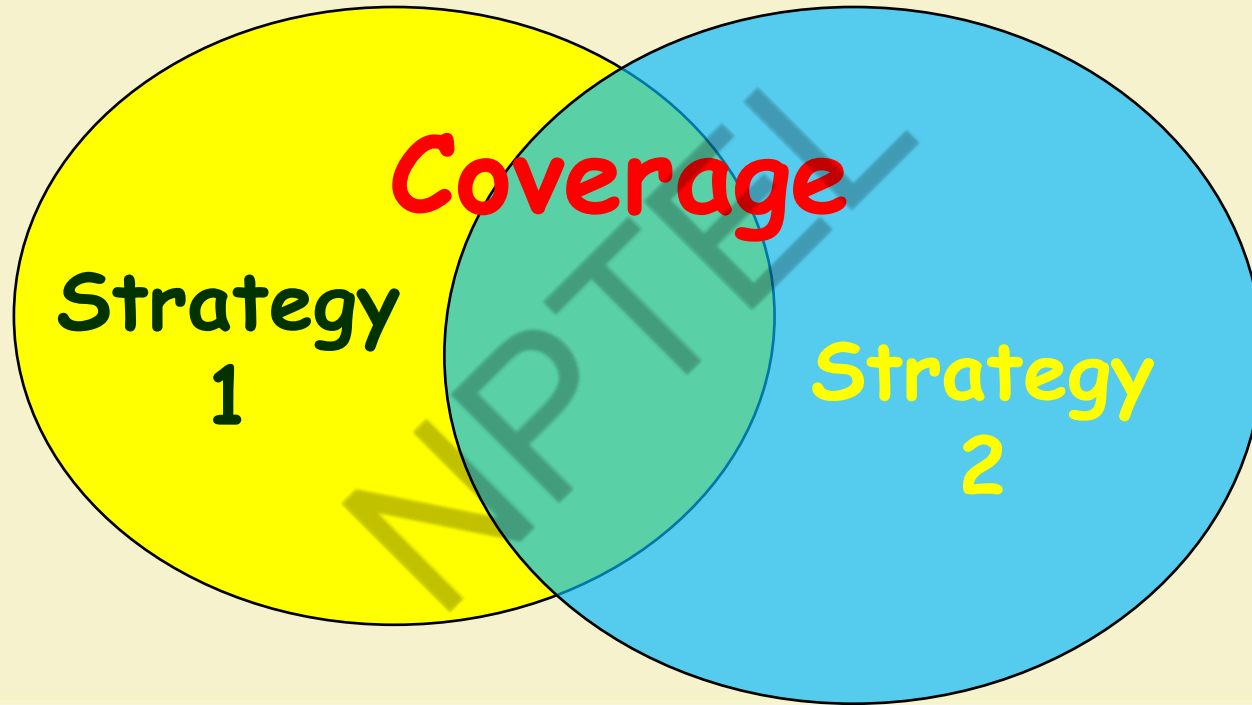
- **Statement:** each statement executed at least once
- **Branch:** each branch traversed (and every entry point taken) at least once
- **Condition:** each condition True at least once and False at least once
- **Multiple Condition:** All combination of Condition covered
- **Path:**
- **Dependency:**

**Types of program element Coverage**

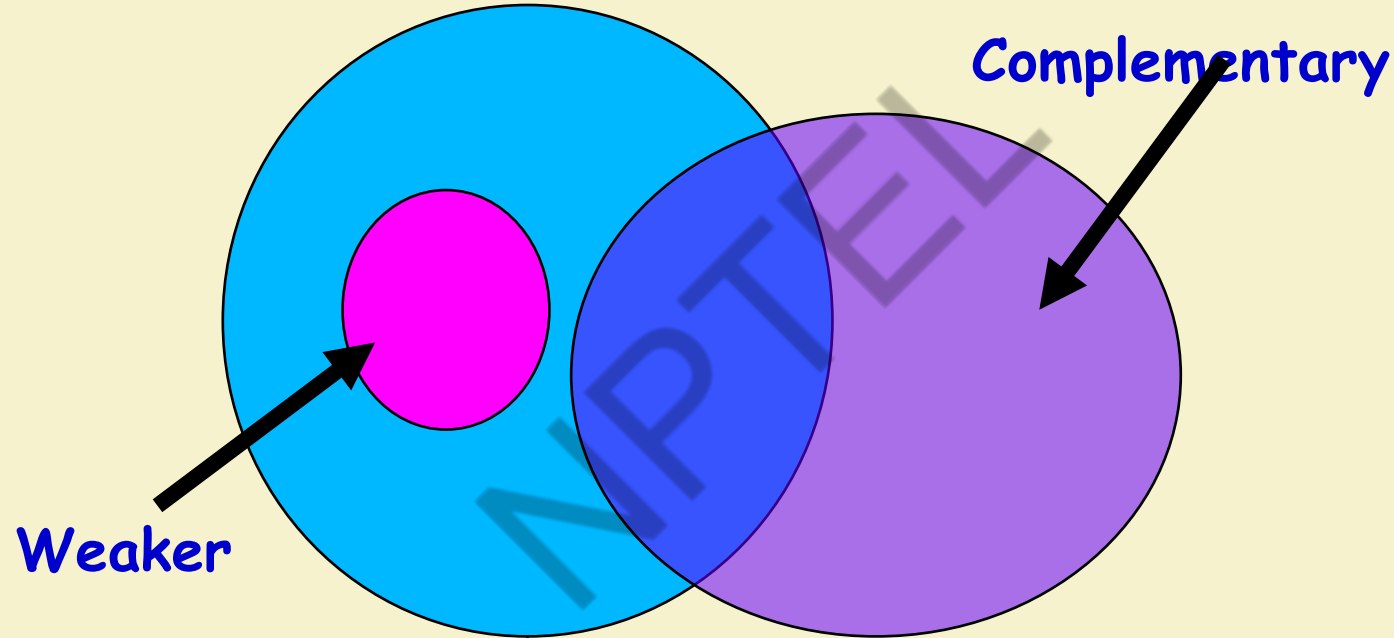


**Stronger and  
Weaker Testing**

# Complementary Testing



# Stronger, Weaker, and Complementary Testing



# Statement Coverage

- Statement coverage strategy:
  - Design test cases so that every statement in the program is executed at least once.



# Statement Coverage

- The principal idea:
  - Unless a statement is executed,
  - We have no way of knowing if an error exists in that statement.

# Statement Coverage Criterion

- However, observing that a statement behaves properly for one input value:
  - **No guarantee that it will behave correctly for all input values!**

# Statement Coverage

- Coverage measurement:

# executed statements

# statements

- **Rationale:** a fault in a statement can only be revealed by executing the faulty statement

# Example

- `int f1(int x, int y){`
- `1 while (x != y){`
- `2   if (x>y) then`
- `3       x=x-y;`
- `4   else y=y-x;`
- `5 }`
- `6 return x;       }`

Euclid's GCD Algorithm

# Example

```
int f1(int x,int y){  
1 while (x != y){  
2   if (x>y) then  
3     x=x-y;  
4   else y=y-x;  
5 }  
6 return x; }
```

Euclid's GCD Algorithm

# Euclid's GCD Algorithm

- By choosing the test set  $\{(x=3, y=3), (x=4, y=3), (x=3, y=4)\}$ 
  - All statements are executed at least once.

# Branch Coverage

- Also called decision coverage.
- Test cases are designed such that:
  - Each branch condition
    - Assumes true as well as false value.

# Example

```
int f1(int x,int y){  
1 while (x != y){  
2   if (x>y) then  
3     x=x-y;  
4   else y=y-x;  
5 }  
6 return x;    }
```



# Example

- Test cases for branch coverage can be:
- $\{(x=3, y=3), (x=3, y=2), (x=4, y=3), (x=3, y=4)\}$

# Branch Testing

- **Adequacy criterion:** Each branch (edge in the CFG) must be executed at least once
- Coverage:

# executed branches

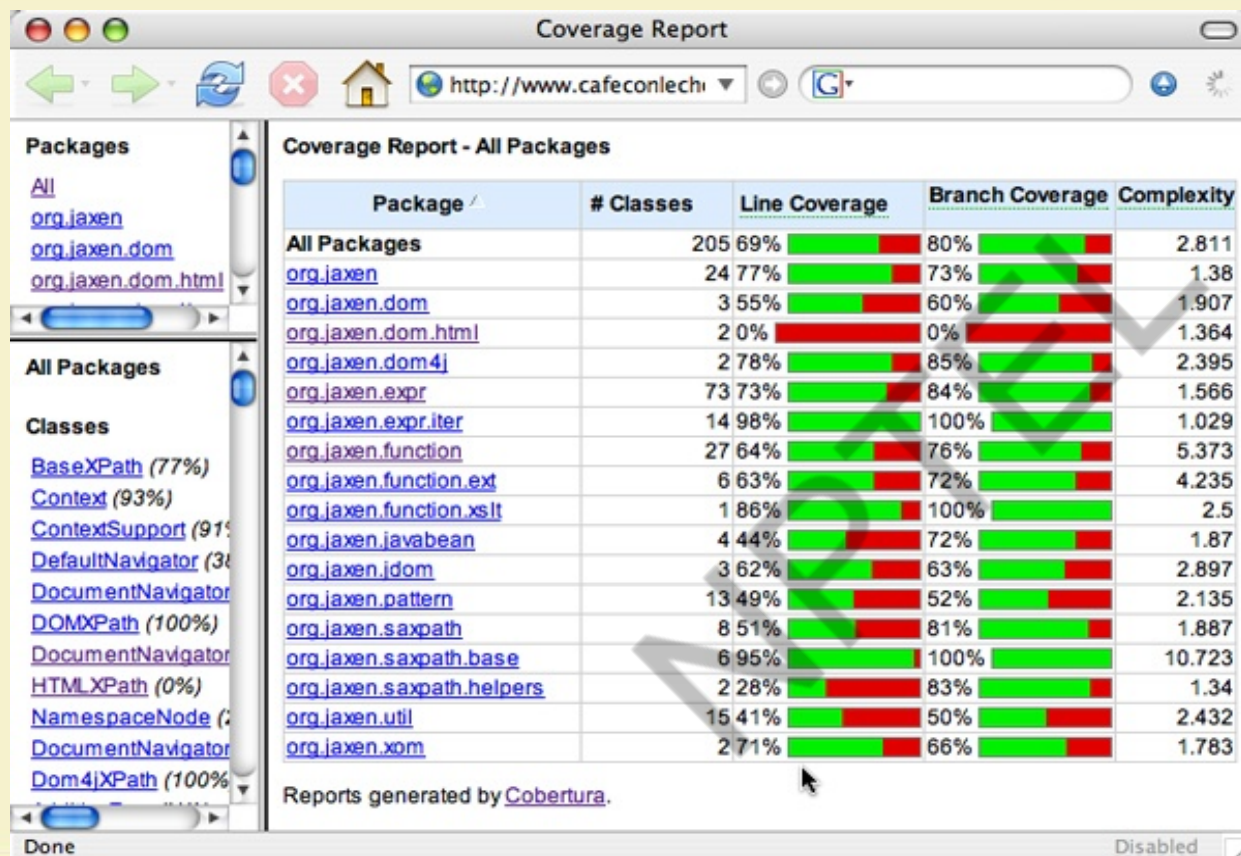
# branches

# Quiz 1: Branch and Statement Coverage: Which is Stronger?

- Branch testing guarantees statement coverage:
  - A stronger testing compared to the statement coverage-based testing.

# Stronger Testing

- Stronger testing:
  - Superset of weaker testing
  - A stronger testing covers all the elements covered by a weaker testing.
  - Covers some additional elements not covered by weaker testing



# Sample Coverage Report

Coverage Report

<http://www.cafeconlechi>

```
110 128      else if ( nav.isElement( first ) )
111          {
112 100          return nav.getElementQName( first );
113          }
114 28      else if ( nav.isAttribute( first ) )
115          {
116 0          return nav.getAttributeQName( first );
117          }
118 28      else if ( nav.isProcessingInstruction( first ) )
119          {
120 0          return nav.getProcessingInstructionTarget( first );
121          }
122 28      else if ( nav.isNamespace( first ) )
123          {
124 0          return nav.getNamespacePrefix( first );
125          }
126 28      else if ( nav.isDocument( first ) )
127          {
128 28          return "";
129          }
130 0      else if ( nav.isComment( first ) )
131          {
132 0          return "";
133          }
134 0      else if ( nav.isText( first ) )
135          {
136 0          return "";
137          }
138      else {
139 0          throw new FunctionCallException("The argument to the name
140      )
141  }
142  }
143 8      return "";
144
```

Done Disabled

# Statements vs Branch Testing

- Traversing all edges of a graph causes all nodes to be visited
  - So a test suite that satisfies branch adequacy criterion also satisfies statement adequacy criterion for the same program.
- The converse is not true:
  - A statement-adequate (or node-adequate) test suite may not be branch-adequate (edge-adequate).

# White-box Testing

- Statement coverage
- Branch coverage (aka decision coverage)
- Basic condition coverage
- Condition/Decision coverage
- Multiple condition coverage
- MC/DC coverage
- Path coverage
- Data flow-based testing
- Mutation testing



# All Branches can still miss testing specific conditions

- Assume failure occurs when `c==DIGIT`

**if((`c == ALPHABET`) || (`c == DIGIT`))**

- Branch adequacy criterion can be satisfied by `c==alphabet` and `c==splchar`
  - **The faulty sub-expression might not be tested!**
  - Even though we test both outcomes of the branch

# Basic Condition Coverage

- Also called **condition coverage** or **simple condition coverage** .
- Test case design: **((c == ALPHABET) || (c == DIGIT))**
  - Each component of a composite conditional expression
    - Made to assume both true and false values.

# Basic Condition Testing

- **Simple or (basic) Condition Testing:**
  - Test cases make each atomic condition assume T and F values
  - Example: **if (a>10 && b<50)**
- **Following test inputs would achieve basic condition coverage**
  - **a=15, b=30**
  - **a=5, b=60**
- Does basic condition coverage subsume decision coverage?

# Example: BCC

- Consider the conditional expression  
– **$((c1.and.c2).or.c3)$** :
  - Each of  $c1$ ,  $c2$ , and  $c3$  is exercised with all possible values,
    - That is, given true and false values.

# Basic condition testing

- **Adequacy criterion:** each basic condition must be executed at least once
- Coverage:

# truth values taken by all basic conditions

$2 * \# \text{ basic conditions}$

# Is BCC Stronger than Decision Coverage?

- Consider the conditional statement:  
  
– If(((a>5).and.(b<3)).or.(c==0)) a=10;
- Two test cases can achieve basic condition coverage: (a=10, b=2, c=2) and (a=1, b=10, c=0)
- **BCC does not imply Decision coverage and vice versa**

# Condition/Decision Coverage Testing

- **Condition/decision coverage:**
  - Each atomic condition made to assume both T and F values
  - Decisions are also made to get T and F values
- **Multiple condition coverage (MCC):**
  - Atomic conditions made to assume all possible combinations of truth values

# MCC

- Test cases make Conditions to assume all possible combinations of truth values.
- Consider: **if (a || b && c) then ...**

Test	a	b	c
(1)	T	T	T
(2)	T	T	F
(3)	T	F	T
(4)	T	F	F
(5)	F	T	T
(6)	T	T	F
(7)	F	F	T
(8)	F	F	F

**Exponential in the  
number of basic  
conditions**



## Multiple Condition Coverage (MCC)

- Consider a Boolean expression having  $n$  components:
  - For condition coverage we require  $2^n$  test cases.
- MCC testing technique:
  - Practical only if  $n$  (the number of component conditions) is small.

# MCC for Compound conditions: Exponential complexity

$((a \parallel b) \&\& c) \parallel d \&\& e$

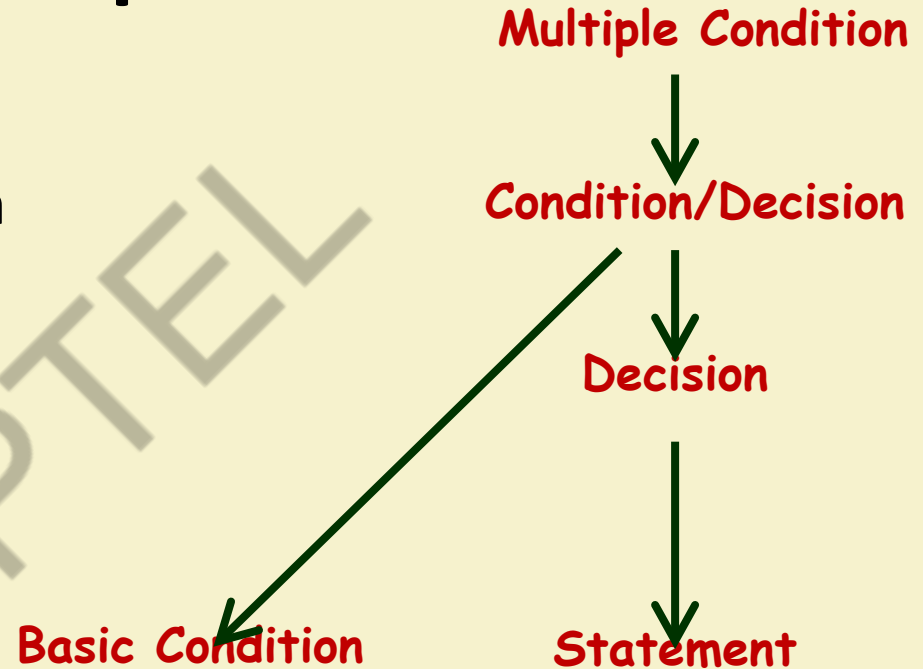
$$2^5=32$$

Test Case	a	b	c	d	e
(1)	T	—	T	—	T
(2)	F	T	T	—	T
(3)	T	—	F	T	T
(4)	F	T	F	T	T
(5)	F	F	—	T	T
(6)	T	—	T	—	F
(7)	F	T	T	—	F
(8)	T	—	F	T	F
(9)	F	T	F	T	F
(10)	F	F	—	T	F
(11)	T	—	F	F	—
(12)	F	T	F	F	—
(13)	F	F	—	F	—

•Short-circuit evaluation often reduces number of test cases to a more manageable number, but not always...

# Subsumption

- Condition testing:
  - Stronger testing than branch testing.
- Branch testing:
  - Stronger than statement coverage testing.



## Shortcomings of Condition Testing

- **Redundancy of test cases:** Condition evaluation could be compiler-dependent:
  - **Reason: Short circuit evaluation of conditions**
- **Coverage may be Unachievable:** Possible dependencies among variables:
  - Example: `((chr=='A') || (chr=='E'))` can not both be true at the same time

# Short-circuit Evaluation

- **if(a>30 && b<50)...**
  - If a>30 is FALSE compiler need not evaluate (b<50)
- Similarly, **if(a>30 || b<50)...**
  - If a>30 is TRUE compiler need not evaluate (b<50)